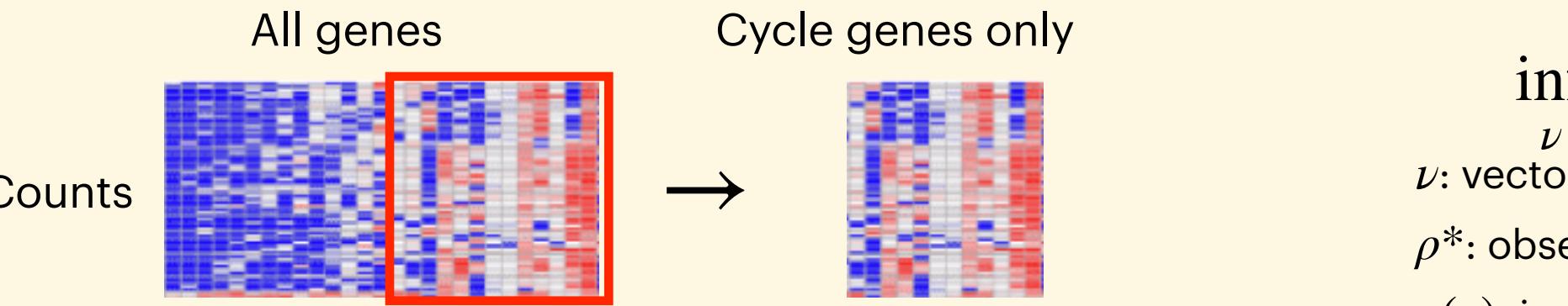


## A Input: scRNA-seq data with no time labels



## B Learning cyclic flow and gene velocity via Lie derivative

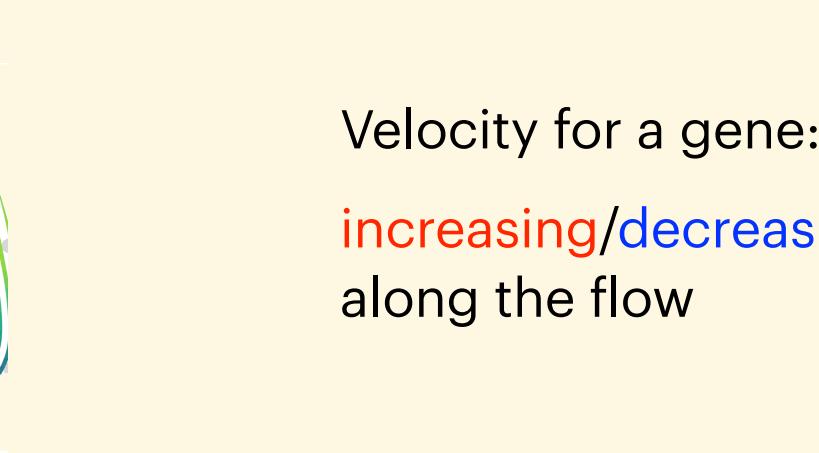
$\inf_{\nu} \mathcal{J}(\rho(\nu), \rho^*)$

$\nu$ : vector field on cycle manifold

$\rho^*$ : observed cell density

$\rho(\nu)$ : invariant measure under  $\nu$

$\mathcal{J}$ : loss between densities

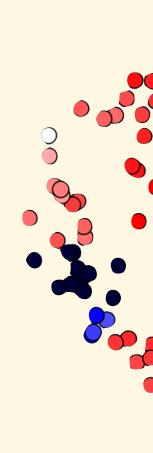


Construct a cell-cell transition matrix  $L$ .

For any gene  $g$  with expression level  $x_g$ ,

directional change:  $\mathcal{L}_\nu x_g \approx V(x_g) = (L - I)x_g$

Velocity for a gene:  
**increasing/decreasing**  
along the flow



## C Gene network inference

For any pair of genes  $g_1, g_2$ ,

$$C(g_1, g_2) = \sum_{c, \tilde{c}} V(x_{g_1}^c) \cdot V(x_{g_2}^{\tilde{c}}) \cdot \mathbb{P}(\text{cell } c \text{ transitions to cell } \tilde{c})$$

