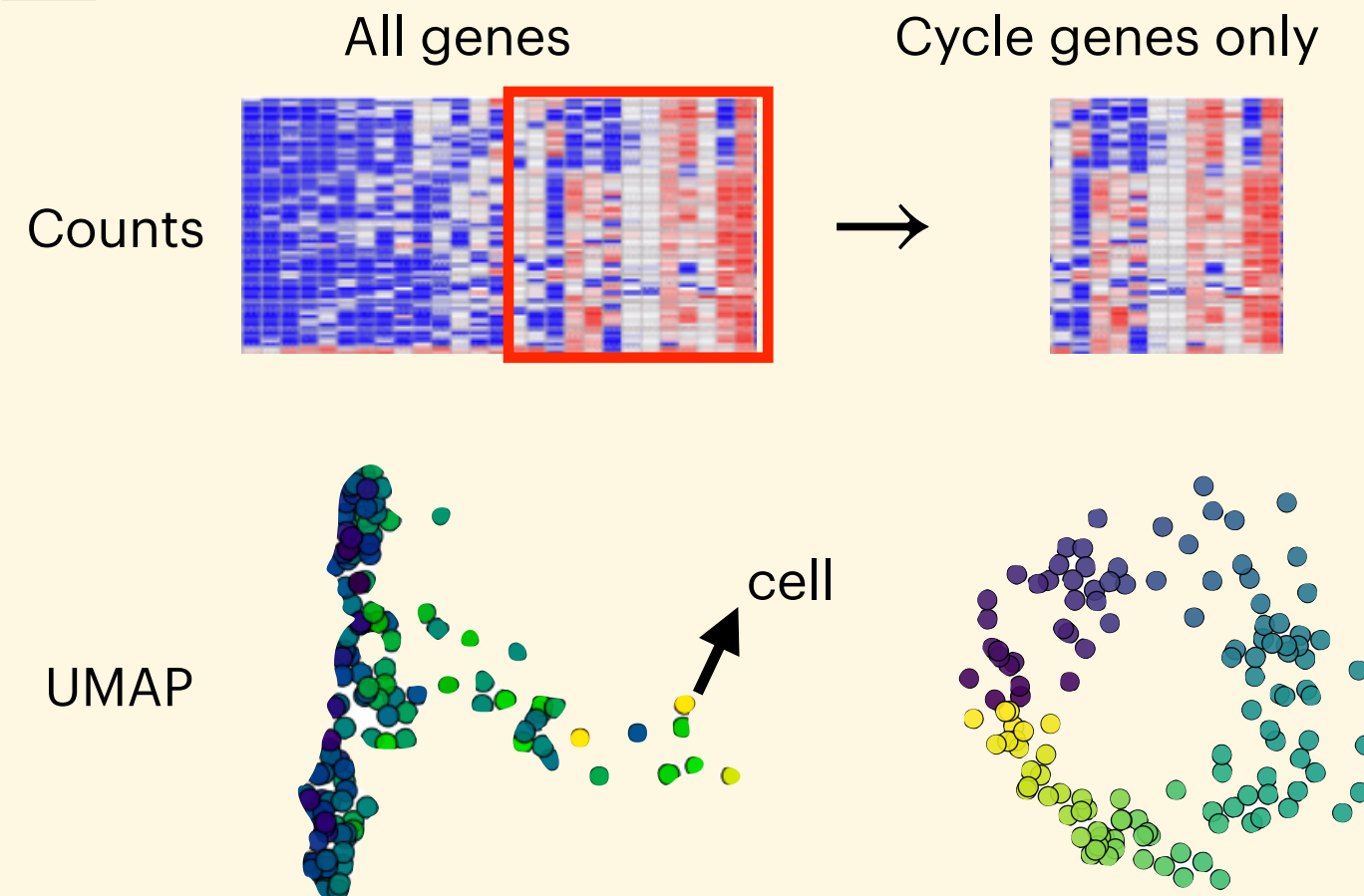


A Input: scRNA-seq data with no time labels



B Learning cyclic flow and gene velocity via Lie derivative

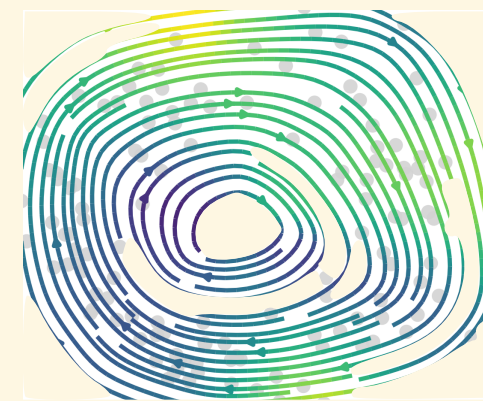
$$\inf_{\nu} \mathcal{J}(\rho(\nu), \rho^*)$$

ν : vector field on cycle manifold

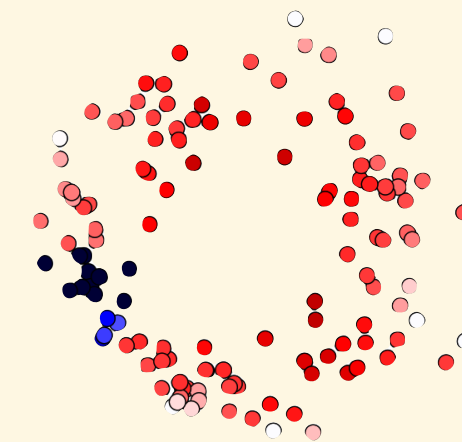
ρ^* : observed cell density

$\rho(\nu)$: invariant measure under ν

\mathcal{J} : loss between densities



Velocity for a gene:
increasing/decreasing
along the flow



C Gene network inference

For any pair of genes g_1, g_2 ,

$$C(g_1, g_2) = \sum_{c, \tilde{c}} V(x_{g_1}^c) \cdot V(x_{g_2}^{\tilde{c}}) \cdot \mathbb{P}(\text{cell } c \text{ transitions to cell } \tilde{c})$$

$\mathbb{P}(\text{cell } c \text{ transitions to cell } \tilde{c})$

m genes



m genes

