Team Control Number

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Problem Chosen

A

2015 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to your solution paper.)

The 2014 outbreak of Ebola Virus Disease has become a worldwide dilemma because of its fatality. In order to accelerate the eradication of Ebola, or prevent the deterioration of the situation at least, we proposed four models to solve different side of the problem-disease spreading model, locations of stations and the delivery system model, model of medicine choice and model of allocation.

In the disease spreading model, we present a susceptibles-infected-bacteria-recovered (SIBR) model, and a more accurate model modified with graph theory. The SIBR model is based on ODEs, and with reasonable assumptions, the system is solvable. We compare our theory with actual epidemic curve (i.e. how the number of infected individuals varies with time), and predict the trend of the outbreak in null-medical-help situation as well as how the predicted epidemic curve respond to medical treatment and vaccination. Also, a few factors appeared in the model will be analyzed in detail.

In trying to locate the medical stations in a most appropriate way and to design a delivery system to transport the supplies at the lowest expense, we create two models: The medical station locating model and the Delivery System Model. In these two models we serve the faith of human equity, allowing every person to have an approach to medical care. The models also serve the principle of economy. The supplies can be sent to the destination in a shortest distance.

In the model of medicine choice, we investigate the manufacturing speed and cost of vaccine and drugs. In order to figure out how to minimize the cost while supplying abundant medicine, we use basic skills of operational research to solve the dilemma.

In the model of medicine allocation, we use primitive ways of thinking, which means that patients infected later should be more probable to get medicine than those infected earlier. Then we set up recursion formula according to the situation of patients and get the number of waiting patients.

Our models provide an integrated scheme of macro scheduling and are in touch of facts to a large extent. Furthermore, they are combination of sense and sensibility in that we tried to minimize the discrimination to vulnerable groups, especially regional discrimination.

Macro Scheduling in Face of Ebola

Contents

1. Introduction

Few people know about Ebola virus disease (EVD) before its outbreak in 2014. According to official data from CDC by 2015 February 6, the number of total cases in countries with widespread transmission (Guinea, Liberia and Sierra Leon) was 22525 and total deaths was 9004. There are also several cases found in other countries such as the USA, Spain, Senegal, Nigeria and Mali, but fortunately, because over 42 days (double the 21-day incubation period of the Ebola virus) has elapsed since the last patient in isolation became laboratory negative for EVD, these countries are now considered free of Ebola. Figure 1-1 shows how the total cases are distributed in Africa.

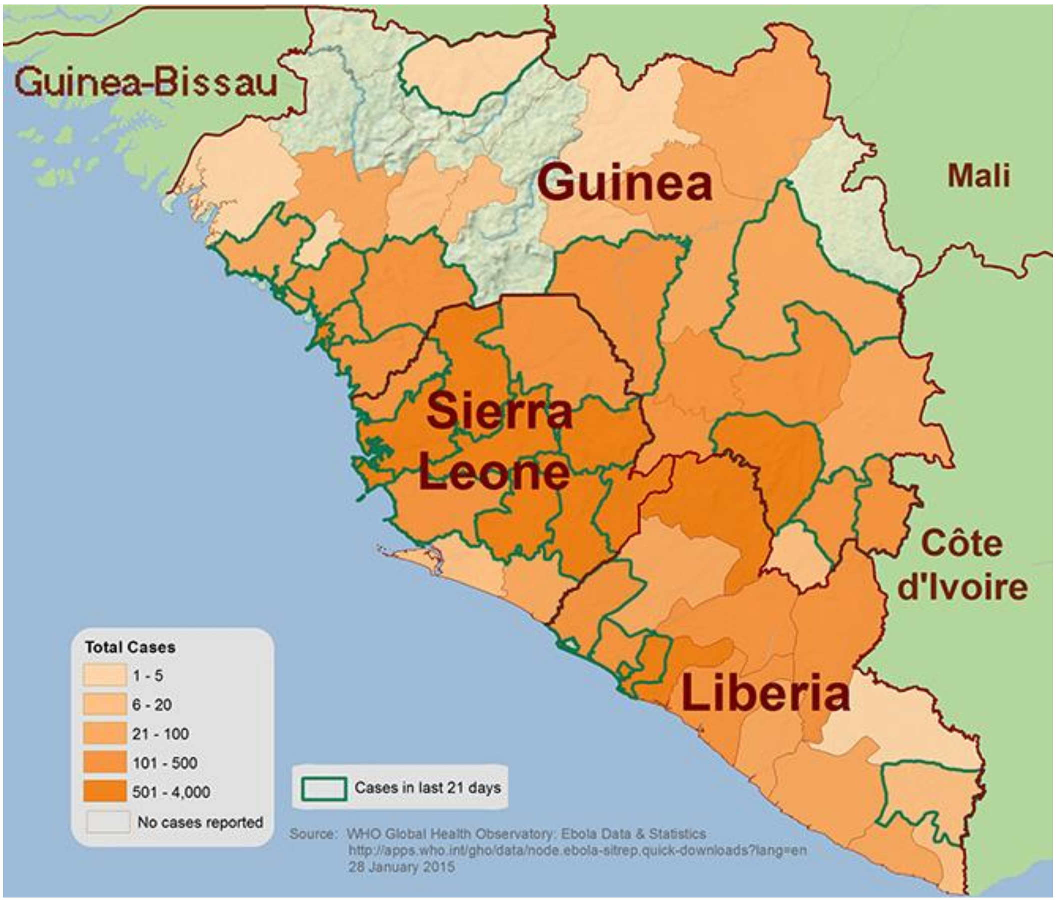

Figure 1-1 West Africa Cases Distribution Map

EVD is a severe and often fatal hemorrhagic fever in humans and other mammals caused by Ebola virus. This virus is a kind of most dangerous virus in history which was first discovered in 1976 near a river called 'Ebola'. Since then, it breaks sporadically in Africa and destroys several villages before it disappear suddenly with no signal every time. The outbreak in 2014 is caused by the kind of Ebola virus which has the highest case-fatality rate. What this cruel killer looks like is shown in Figure 1-2.

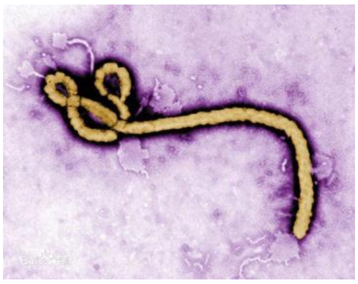

Figure 1-2 Ebola Virus

Why is Ebola virus spreading so rapidly in Africa? Ebola is spread through direct contact with blood or body fluids (including but not limited to urine, saliva, sweat, feces, vomit, breast milk, and semen) from a person sick with Ebola. It is a custom to touch the dead body in African culture, which greatly increases the risk of being infected at a funeral. The second reason is about other creatures. Human-like animals such as monkeys, gorillas and chimpanzees also can be infected with this kind of virus. In Africa, many people eat various wild meat from the forest, including these human-like animals. Thus, they may be infected when they deal with those meat.

It has been one year since the first patient in this outbreak was confirmed to be infected by Ebola virus. However, the virus did not disappear as it did before and is still spreading rapidly in Africa. Recently, the world medical association announced that their new medicine could stop the Ebola and cure patients whose disease is not advanced. Therefore, we are tasked to build a model to optimize the eradication of Ebola, or at least its current strain.

2. General assumptions

- **We do not consider the variation of Ebola virus.** That is, the methods of treating patients do not need to change a lot during the plan. Under usual circumstances, the time virus needs to get a new variation is much longer than that of our plan. Thus, we do not have to spend extra time trying to develop many different kinds of medicine aiming at different variations of the virus and the medicine is effective all the time during this period.
- **Individual differences are ignored in all of our models.** The population of Africa is really enormous and we cannot take everyone into consideration. For instance, some people are immune to Ebola virus without vaccination and some people can heal themselves without medicine even if they were infected. However, this phenomenon is so rare that it do not have the ability to influence our conclusion.
- **The total human community size remains unchanged.** Guinea has a population of 10.6 million, but the infected is one the scale of thousand. The huge difference in scale makes the loss of population ignorable.

3. Notations

SYMBOLS	DEFINITIONS
S	The abundance of susceptibles
I	The abundance of infected individuals
B	The concentration of Ebola pathogen in the environment
R	The number of people who recovered and keep immunity against Ebola
H	Population of that area
μ	The natality and mortality rates in that area
β	Transmission parameter in a Michaelis-Menten equation
K	Another parameter in a Michaelis-Menten equation
γ	The rate of recovery
α	The rate people die of Ebola
p	The rate infected people contribute to the concentration of pathogen
μ_B	The rate the pathogen concentration lowers because of anti-infection measurement
l	Transport rate that describes the speed of pathogen carried by infected individuals travel through counties
P_{ij}	The normalized rate that pathogen travel from node i to node j
N	The total number of the counties
V	Numbers of people get vaccination per day
Q	Numbers of people be cured and recover per day
φ	Time for a patient from getting infected to death without treatment
A/B/P	A medium or big medical station
a/b	A small medical station
XX/XXX/XXXX	The circumference of the drawing(the letters represent its zeniths)
μ	Number of people infected by each infected person per week
t	Time
a_t	Increasement of infected cases at time t, a(0)=0
γ_t	Ratio of isolated patients among those who were not isolated at time t-1
b_t	Number of patients (not isolated at t-1) alive at time t
c_t	Number of patients (not isolated at t-1) dead at time t
r_t	Supposed to be isolated at time t
e_t^k	Number of cured patients at time t who was infected at time k
f_t	Number of patients waiting for medicine
g_t	Number of isolated patients dead at time t
b_t^k	Number of patients in b(t) who was infected at time k

4. Disease Spreading Model

In this chapter, we will first discuss the spread of the disease if there is no medical help at all and all the people are well mixed. Then we will modify our model with graph theory, assuming each county as a node. Next, we will compare our theory with actual epidemic curve (i.e. how the number of infected individuals varies with time), to determine a few parameters, and predict the trend of the outbreak in null-medical-help situation as well as how the predicted epidemic curve respond to medical cure and vaccination. Finally, a few factors appeared in the model will be analyzed in detail.

4.1 SIBR Epidemic Model

In order to understand the general tendency of disease spreading, we first build a model combining the standard susceptible-infected-recovered epidemic model and the susceptible-infected-bacteria (SIB) epidemic model introduced by Codeco (2001). This Susceptible- Infected –Bacteria-Recovered (SIBR) model has four variables, namely the amount of susceptibles (S), the abundance of infected individuals (I), the concentration of Ebola pathogen in the environment (B), and the number of people who recovered and keep immunity against Ebola (R).

4.1.1 Assumptions

- **All the people, including the susceptibles, the infected, and the immured ones, as well as the concentration of pathogen in the environment, are uniform distributed and well mixed.** This might not be an accurate description, but is one convenient way for calculations, and we will prove that it could provide a reasonably accurate estimate later.
- **The concentration of Ebola pathogen in the environment (B) has the same natality and mortality, so it can only be added by infected people, and be removed by anti-infection measurement.** As we all know, Ebola pathogen is a kind of virus, which can only be reproduced in a host. So it cannot propagate itself without a chimpanzee or human being. Also, Ebola pathogen is originated from chimpanzees. Thus it would be natural to assume that once the outbreak begins, people would stop to get too close to chimpanzee. So how the concentration of pathogen varies in chimpanzee group would have little significance for human being.
- **The newborns are susceptibles.** Newborns are generally considered with lower immunity.
- **People recovered from Ebola are considered immune.** As immunity usually lasts longer than the time scale of one outbreak, we do not take loss of immunity into account.

4.1.2 ODES in the model

The model can be described by following differential equations:

$$\frac{dS}{dt}=\mu(H-S)-\beta\frac{BS}{K+B}$$
$$\frac{dI}{dt}=\beta\frac{BS}{K+B}-(\gamma+\alpha+\mu)I$$
$$\frac{dB}{dt}=pI-\mu_B B$$
$$\frac{dR}{dt}=\gamma I-\mu R$$

SYMBOLS	DEFINITIONS
S	The abundance of susceptibles
I	The abundance of infected individuals
B	The concentration of Ebola pathogen in the environment
R	The number of people who recovered and keep immunity against Ebola
H	Population of that area
μ	The natality and mortality rates in that area
β	Transmission parameter in a Michaelis-Menten equation
K	Another parameter in a Michaelis-Menten equation
γ	The rate of recovery
α	The rate people die of Ebola
p	The rate infected people contribute to the concentration of pathogen
μ_B	The rate the pathogen concentration lowers because of anti-infection measurement

Table 1 Quick Review of Notations

The first equation describes how the abundance of susceptibles varies with time. It would increase because of the newborns, and decrease because people get infected. The item $\beta BS/(K+B)$ borrowed its form from Michaelis-Menten equation, consider the susceptibles as enzyme and Ebola pathogen as substrate.

The second equation describes the infected individuals are added because of the reaction of Ebola pathogen and reduced because people recover or die.

As assumed above, the concentration of Ebola pathogen can only be added by infected people, and be removed by anti-infection measurement, such as burn the body and clothes of the dead, as is described in the third equation.

The fourth equation describes the people immune to Ebola increase because the infected recover, and decreased because of natural death.

With an initial condition of $S(0)=H$, $I(0)>0$, $B(0)=0$, an outbreak occurs only if (E. Bertuzzo,2009)

$$\frac{H\beta p}{(\gamma+\alpha+\mu)K\mu_B}>1$$

4.1.3 Parameters estimate

In this model, there are 7 parameters. As the absolute value of concentration of Ebola pathogen does not interest us, we can define as $B^*=B/K$, to minimize the amount of parameters. Substitute B^* into equations above, it is clear that we can use $p^*=p/K$ instead of p and K without any more changes. Thus the amount of parameters is lowered by one.

The natality and mortality μ of the area of Ebola epidemic, i.e. Guinea, Liberia and Sierra Leone, which is listed in the internet, is about 4.2e-4 per day. It is said the mortality of Ebola is about 70%, and the disease course is about 7 days. Thus the rate of recovery should be 0.04 per day, and rate of mortality should be 0.1 per day. Because the research on Ebola is not impeccable, these data might not be of high reliability, but only for estimate. If the research on this data can be completed in the future, our calculation can be more accurate as well.

And here left β, μ_B, p^*. Because the research on this data is almost devoid, there is no way we can use the result from former research. In this case, we have to use the epidemic curve in this outbreak to reckon this three parameters.

We use least square method to estimate the parameter and use Guinea as an example. Figure 5-1 is the epidemic curve of Guinea.

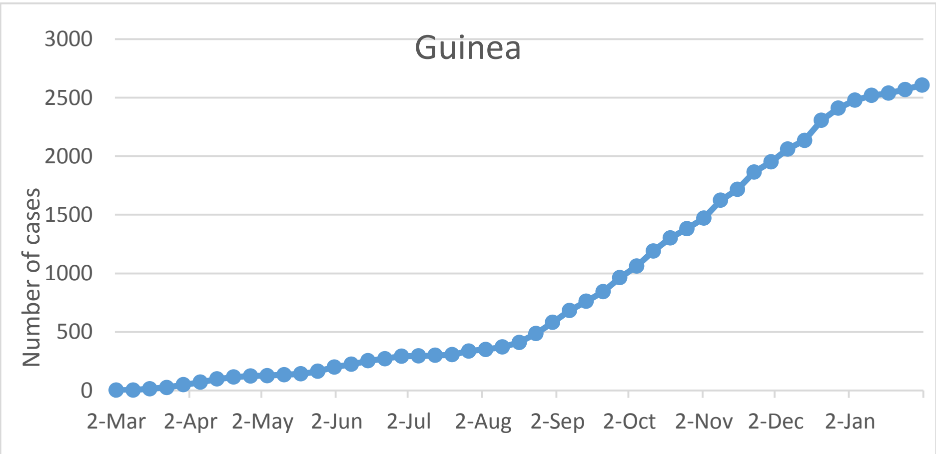

Figure 4-1 Actual Epidemic Curve in Guinea

As it is easy to be affected by random factor when the number of cases is small, we only use the data that above 500 cases. Let those parameters traverse the discretized numbers in a reasonable interval, and record the group of parameters that minimize the variance between numerical solution and actual epidemic data. The parameters are listed below. And the theoretical solution is shown in Figure 5-2.

$$\beta = 4.8 \times 10^{-5} / d$$
$$u_B = 0.001 / d$$
$$p^* = 1 \times 10^{-5} / d$$

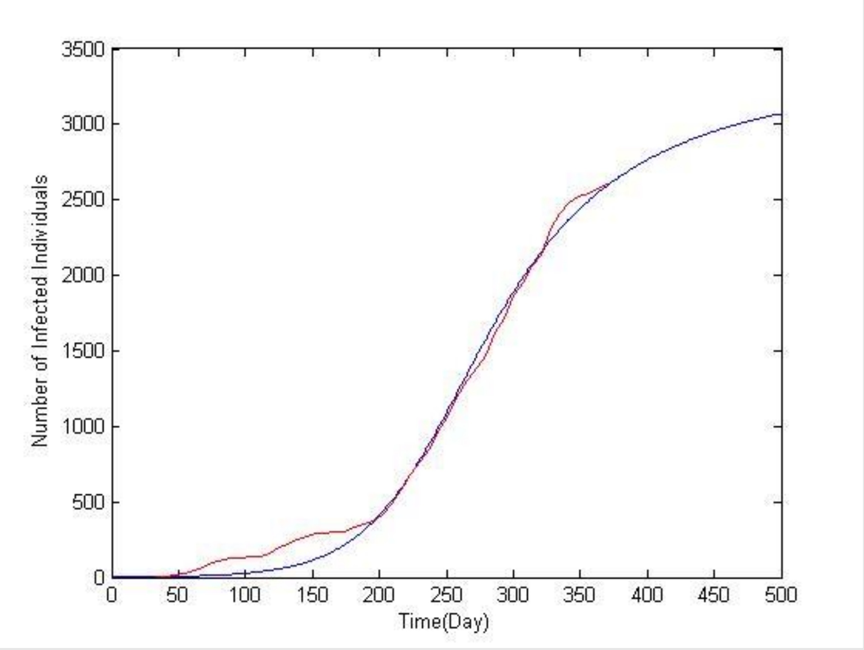

Figure 4-2 Theoretical Solution and Actual Outbreak

4.2 A modified model with graph theory

As we know, the all-people-are-well-mixed assumption is not solid. If we treat each county as a node in a graph, and modify our model with graph theory, the simulation could be of more dependability.

4.2.1 ODEs in the model

We will continue to use the assumptions listed in 5.1.1 except assumption one, and the equations set with 4N variables is given by:

$$\frac{dS_i}{dt} = \mu(H_i - S_i) - \beta\frac{B_i S_i}{K + B_i}$$
$$\frac{dI_i}{dt} = \beta\frac{B_i S_i}{K + B_i} - (\gamma + \alpha + \mu)I_i - lI_i + \sum_{\substack{j=1\\ i\neq j}}^{N} lP_{ji}I_j$$
$$\frac{dB_i}{dt} = pI_i - \mu_B B_i$$
$$\frac{dR_i}{dt} = \gamma I_i - \mu R_i$$

where subscript i indicates the i^{th} county, and P_{ij} needs to be normalized to follow the equation:

$$\sum_{i=1}^{N}\frac{dI_i}{dt} = \sum_{i=1}^{N}\beta\frac{B_i S_i}{K + B_i} - (\gamma + \alpha + \mu)\sum_{i=1}^{N} I_i$$

which means the graph system cannot have extra influence to itself because of pathogen transport inside. Thus P_{ij} should follow the equation:

$$\sum_{i\neq j} P_{ij} = N$$

SYMBOLS	DEFINITIONS
l	Transport rate that describes the speed of pathogen carried by infected individuals travel through counties
P_{ij}	The normalized rate that pathogen travel from node i to node j
N	The total number of the counties

Table 2 Quick Review of Notations

4.2.2 Parameter Estimate

The population of each county, i.e. H_i, is listed in Wikipedia. For example, the population of Guinea's 8 counties is listed in Table 3.

Nzerekore	*Mamou*	*Labe*	*kindia*	*Kankan*	*Faranah*	*Conakry*	*Boke*
1,663,582	732,117	995,717	1,559,185	1,986,329	942,733	1,667,864	1,081,445

Table 3 Populations of Counties in Guinea

As for P_{ij}, the longer distance from node i's capital to node j's capital, the smaller P_{ij} should be. So, it is convenient to let P_{ij} be proportional to the inverse of distance from node i's capital to node j's capital. For example, Guinea has 8 counties, and the distances from each other is listed in Table 4.

(km)	*Nzerekore*	*Mamou*	*Labe*	*kindia*	*Kankan*	*Faranah*	*Conakry*	*Boke*
Nzerekore	0	574	721	709	369	386	838	1040
Mamou	574	0	147	138	396	190	267	469
Labe	721	147	0	247	497	336	376	347
kindia	709	138	247	0	533	325	137	339
Kankan	369	396	497	533	0	204	662	864
Faranah	386	190	336	325	204	0	454	656
Conakry	838	267	376	137	662	454	0	269
Boke	1040	469	347	339	864	656	269	0

Table 4 Distances Between Counties in Guinea

If we inverse every of the elements in Table 4 and get them normalized, we can get matrix formed by P_{ij}.

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
		Nzerekore	*Mamou*	*Labe*	*kindia*	*Kankan*	*Faranah*	*Conakry*	*Boke*
1	*Nzerekore*	∞	0.0824	0.0656	0.0668	0.1282	0.1226	0.0564	0.0455
2	*Mamou*	0.0824	∞	0.3220	0.3430	0.1195	0.2491	0.1772	0.1009
3	*Labe*	0.0656	0.3221	∞	0.1916	0.0952	0.1408	0.1258	0.1364
4	*kindia*	0.0667	0.3430	0.1916	∞	0.0888	0.1456	0.3455	0.1396
5	*Kankan*	0.1282	0.1195	0.0952	0.0888	∞	0.2320	0.0715	0.0547
6	*Faranah*	0.1226	0.2491	0.1409	0.1456	0.2320	∞	0.1042	0.0721
7	*Conakry*	0.0564	0.1772	0.1259	0.3455	0.0715	0.1042	∞	0.1759
8	*Boke*	0.0455	0.1009	0.1364	0.1396	0.0547	0.0721	0.1759	∞

Table 5 Node Matrix in Guinea

As for the transport rate l, as there is not enough data available, we are going to use the method that used in 5.1.3 again. The numerical result is listed below:

$$\beta = 5.5 \times 10^{-5} / d$$
$$u_B = 0.001 / d$$
$$p^* = 8.3 \times 10^{-5} / d$$
$$l = 6.5 \times 10^{-4} / d$$

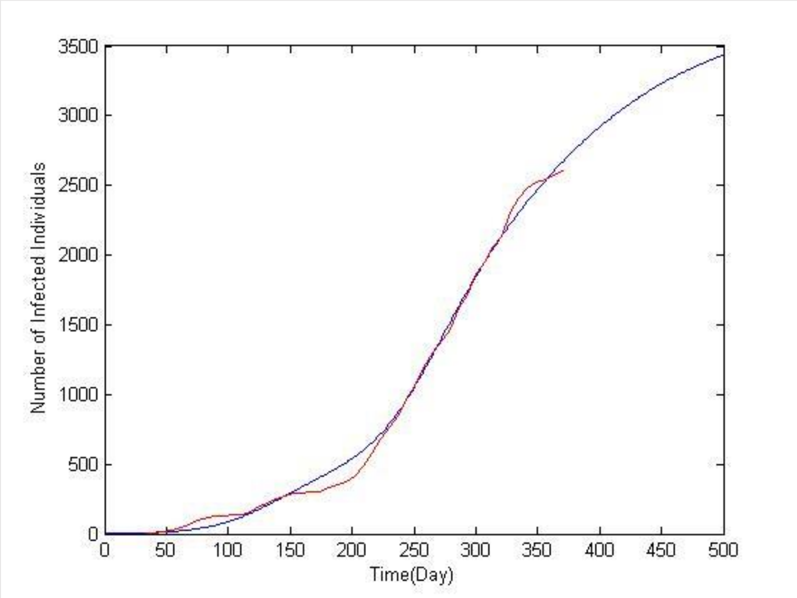

Figure 4-3 Theoretical Solution and Actual Outbreak

4.3 Predictions

Now that we have got all the parameters needed, we could go on to predict the epidemic curve.

4.3.1 Epidemic curve without medical help

4.4.1.1 Predicted Epidemic Curve with SIBR Model

With the parameters we have got in 4.1.3, the predicted epidemic curve is shown in Figure 4-4.

As we can see, if there is no medical interfere at all, the epidemic would last for decades. The increasing trend of infected individuals would slow down in a few months, and then it will reach a climax of 3330 and start to decrease slowly. Although in this case, assumption three (i.e. the total human community size remains unchanged) might not be valid because of too many deaths, the prediction still alert us what a disaster this would be-more than half of the country would die of Ebola, noticing the population of Guinea is 10,628,972.

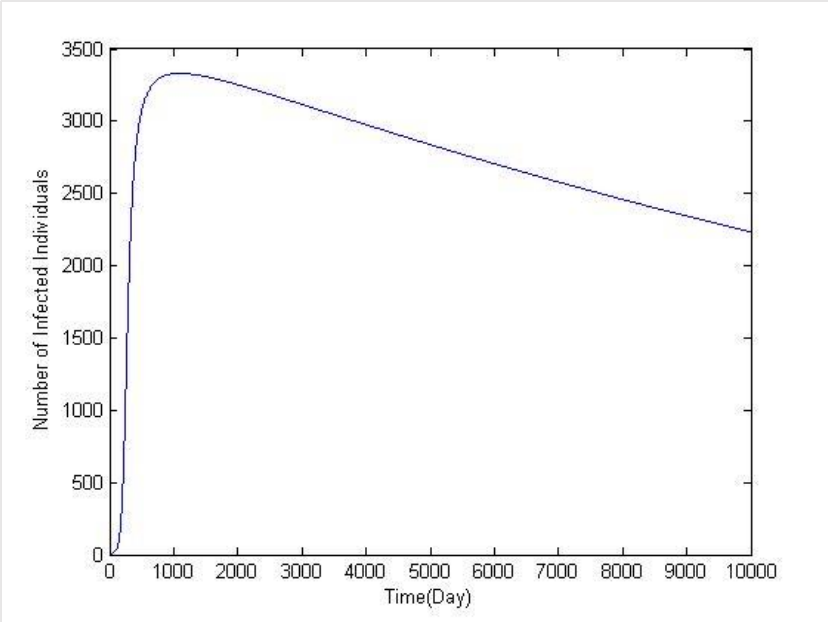

Figure 4-4 Predicted Epidemic Curve with SIBR Model

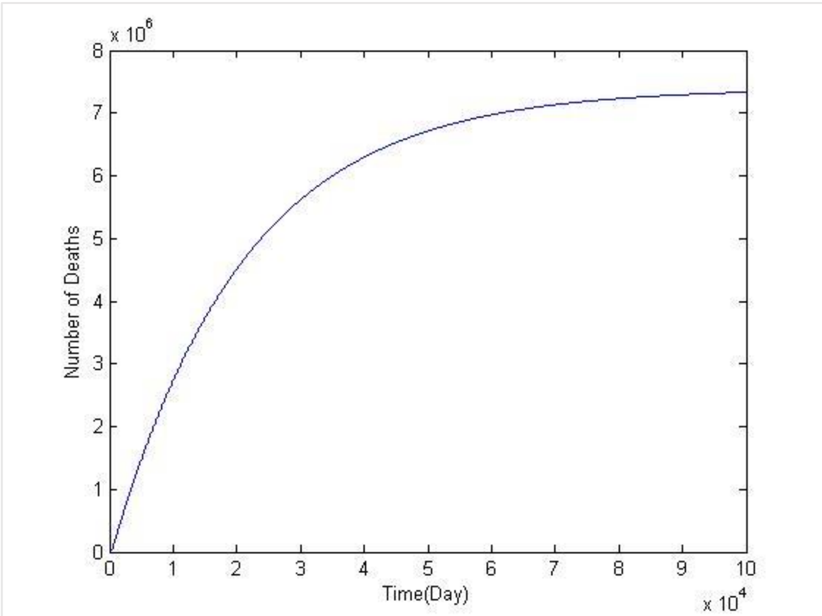

Figure 4-5 Predicted Deaths with SIBR Model

4.3.1.2 Predicted Epidemic Curve with Modified Model

The result from modified model shown in Figure 4-4 is almost identical to the SIBR model shown in Figure 4-6, except they have different climax. As we all know, model is never a 100% accurate method to stimulate the real world, as this world is so complicated and constantly changing. As both prediction have almost the same trend, we will use SIBR model to lower the amount of calculation in the next section, although the modified model is slightly more accurate indeed.

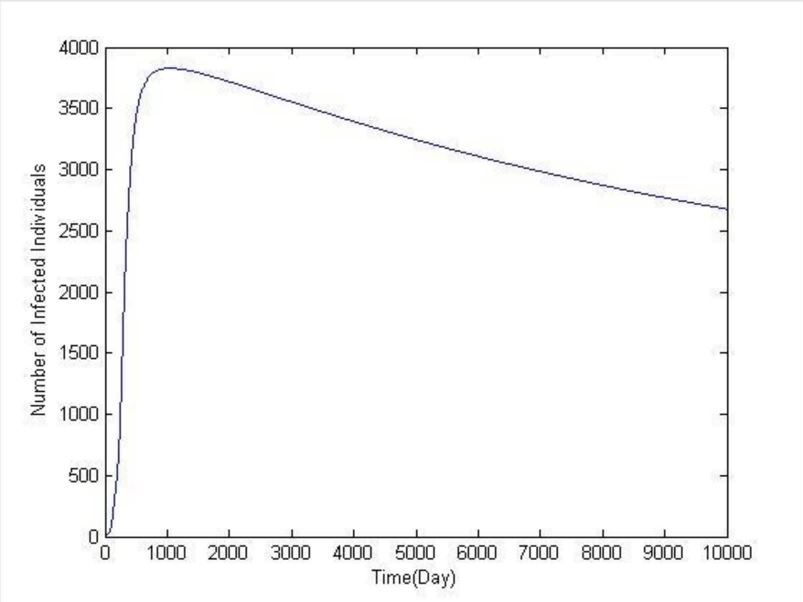

Figure 4-6 Predicted Deaths with Modified Model

4.3.3 Predictions with Medical Help

4.3.3.1 Adding medical help to ODEs

$$\frac{dS}{dt} = \mu(H - S) - \beta\frac{BS}{K + B} - V$$

$$\frac{dI}{dt} = \beta\frac{BS}{K + B} - (\gamma + \alpha + \mu)I - Q$$

$$\frac{dB}{dt} = pI - \mu_B B$$

$$\frac{dR}{dt} = \gamma I - \mu R + V$$

SYMBOLS	DEFINITIONS
V	Numbers of people get vaccination per day
Q	Numbers of people be cured and recover per day

Table 6 Quick Review of Notations

With medical help, V of the susceptibles per day get vaccinated and become immune to Ebola, and Q of the infected individuals will be cured and recover. Obviously, when programming to solve the equations, a judgment statement is necessary to prevent the numbers of susceptibles and infected individuals to become negative.

4.3.3.2 Numerical solutions of how medical help matters

So, if we start medical help to West African countries, how would the epidemic curve change?

4.3.3.2.1 If Treatment only

If we provide treatment to people only without any vaccination from now on, the time needed to eliminate Ebola is shown in aaaaa. Noticing horizontal coordinate is people that are treated and recover per day, the actual number of people being treating should multiply by the course of a treatment, which is generally 7 days.

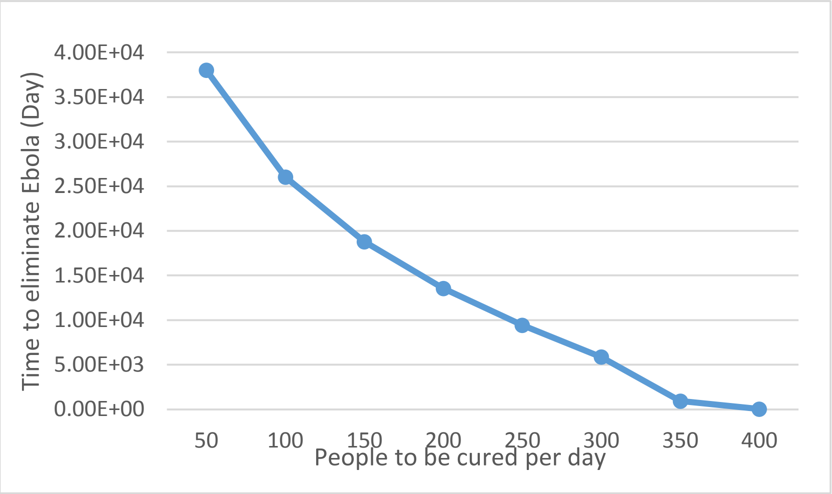

Figure 4-7 Predicted Time to Eliminate Ebola

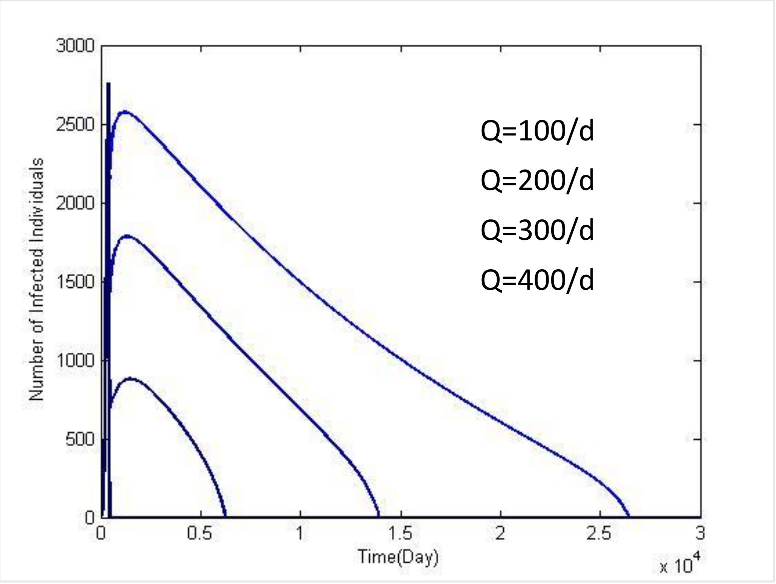

Figure 4-8 Predicted Epidemic Curve With Different Q

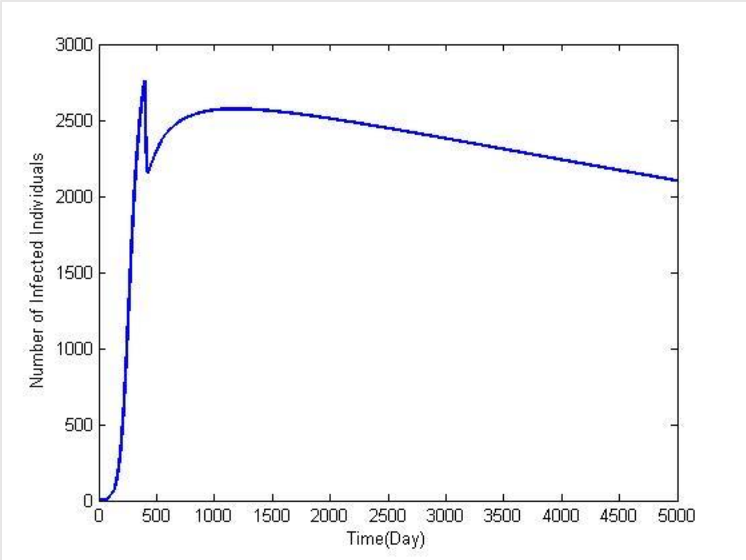

Figure 4-9 Predicted Epidemic Curve With Q=200/d

As we can see from Figure 4-8 and Figure 4-9, when we start the treatment, there will be a sharp drop. But the drop would not last to the end. The number of infected individuals would bounce after the sharp drop. Then it slowly declines to the end.

4.3.3.2.2 If Vaccination only

If we provide treatment to people only without any vaccination from now on, the time needed to eliminate Ebola is shown in Figure 4-10.

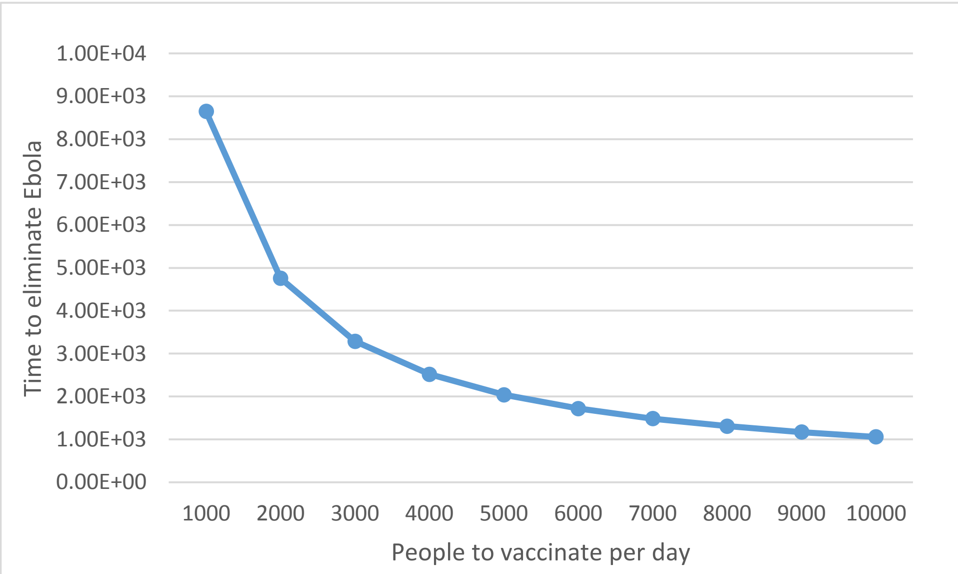

Figure 4-10 Predicted Time to Eliminate Ebola

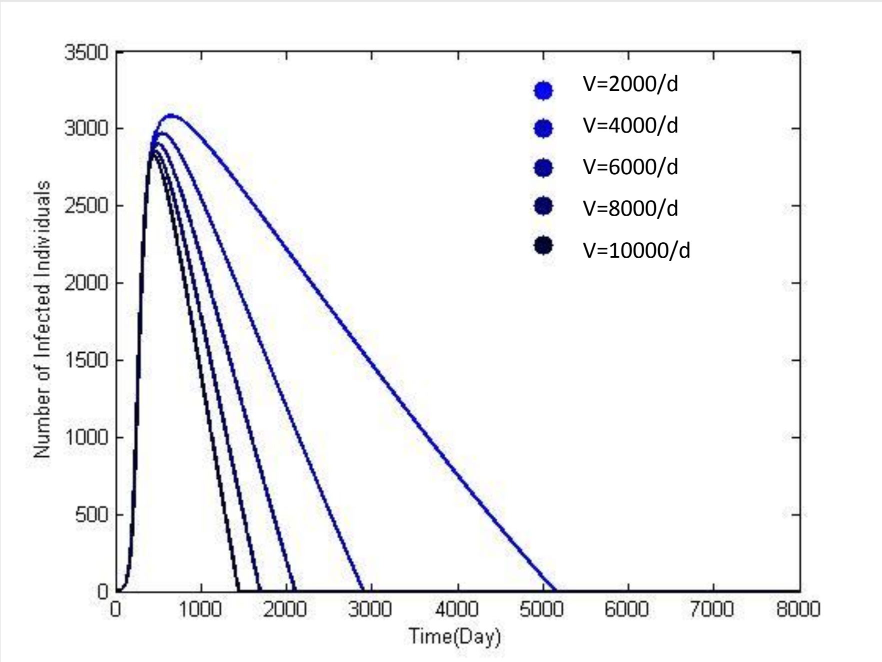

Figure 4-11 Predicted Epidemic Curve With Different V

There would not be any sharp drop or bounce like Figure 4-9 if we use vaccination only. Comparing Figure 4-8 to Figure 4-11, we can easily draw the conclusion that treatment would take short term effect, and vaccination would take long term effect.

4.3.3.2.3 The combination of Treatment and Vaccination

	Q(/d)	**0**	**100**	**200**	**300**	**400**
V(/d)	Days					
0		9.9602E+04	2.60E+04	1.36E+04	5.85E+03	28.6
2000		4.75E+03	3.59E+03	2.45E+03	1.10E+03	27.29
4000		2.52E+03	1.93E+03	1.33E+03	594.1	26.27
6000		1.72E+03	1.32E+03	904.64	408.51	25.44
8000		1.31E+03	1.00E+03	683.57	312.21	24.74
10000		1.06E+03	806.5	548.59	253.23	24.13

Table 7 Predicted Time to Eliminate

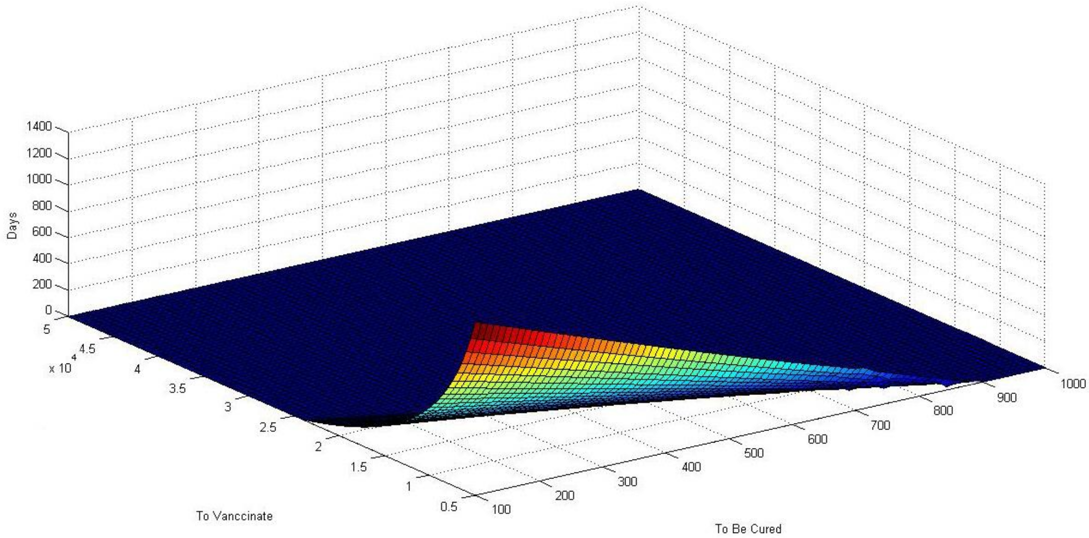

Figure 4-12 Predicted Time to Eliminate Ebola

Obviously, the more people we cure or vaccinate in a day, the more quickly Ebola to be eliminated.
Also, although treatment seems more helpful in Table 7, we should provide vaccination as well to prevent the next outbreak.

4.4 Analysis on a few parameters

Although all the parameters are estimated already, there are a few parameters we would like to analyze further. The rate infected people contribute to the concentration of pathogen p and transport rate that describes the speed of pathogen carried by infected individuals travel through counties l are two parameters that should have been controlled when the epidemic started. If there are adequate human factor against Ebola, the outbreak might not have happened.

4.4.1 The Effect of Parameter p

The estimate of p is 1×10^{-5} per day and the epidemic curve is shown again in Figure 4-13. If the government could quarantine 9/10 of the infected in ten days after the first case, the epidemic curve should be like Figure 4-14. The climax is reduced by more than 1000 and there would be more time for the world to react. If the government could quarantine almost all the infected, the epidemic curve should be like Figure 4-15. There will be no more than 35 infected at the same time. There will not be any large-scale outbreak.

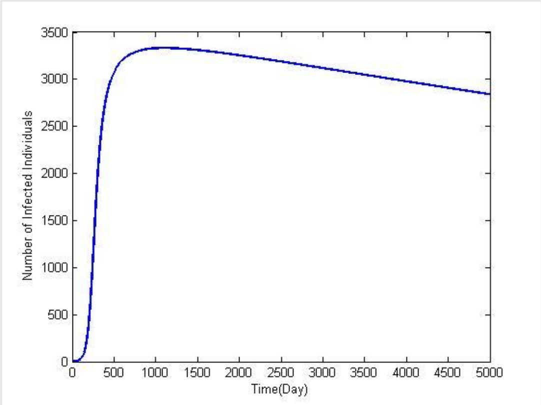

Figure 4-13

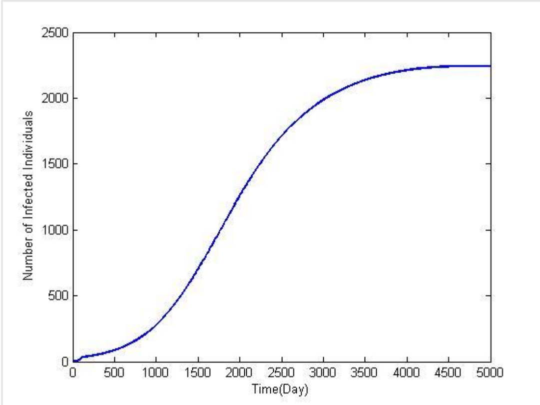

Figure 4-14

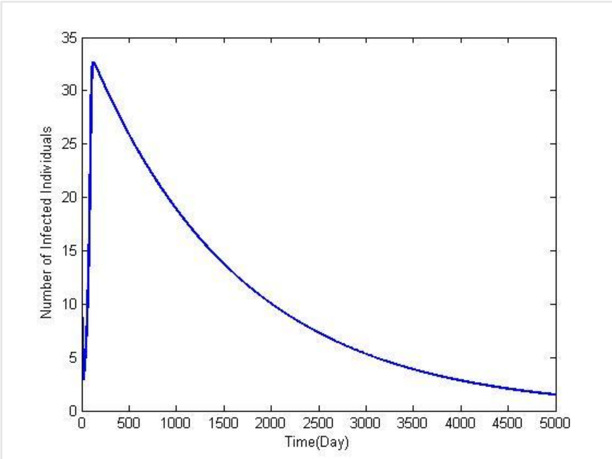

Figure 4-15

4.4.2 The Effect of Parameter l

The estimate of l is 6.5×10^{-4} per day. As we can see in Figure 4-16, neither the peak value nor the time that the number of the infected reaches the peak value respond sensitively to l. But, if $l = 0$, i.e. the area that the first case is found is isolated, the difference is huge.

Indeed, facing infectious disease, quarantine and isolation are the best ways to minimize the number of victims, as what we are doing to those West African countries.

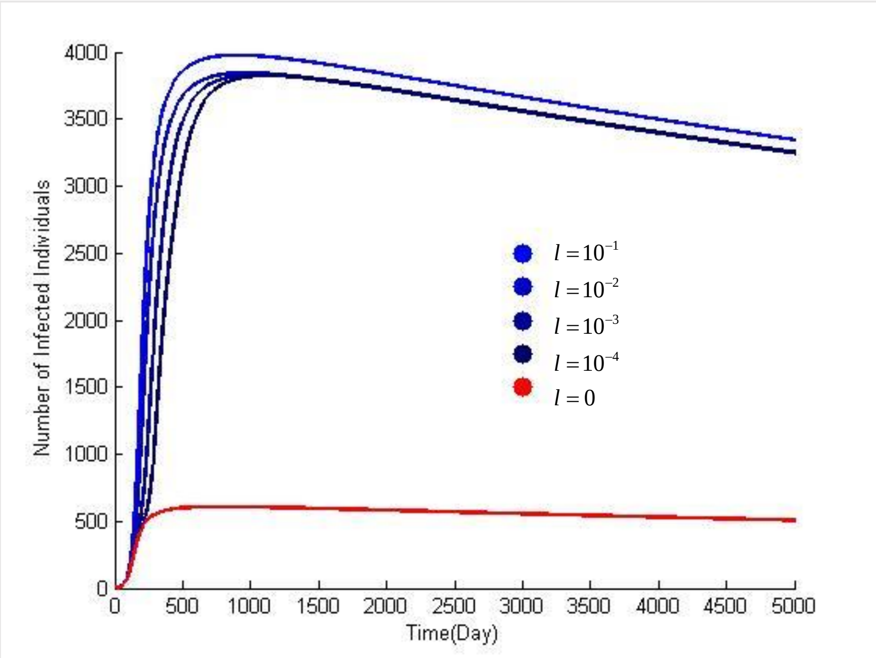

Figure 4-16 Predicted Epidemic Curve With Different l

5. Locations of Stations and the Delivery System Model

In this chapter we use Liberia, one of the countries badly attacked by Ebola, as an example.

The medical stations need to be located in some reasonable sites so as to be made full use of. The design of the delivery system is also very essential. These two models will be discussed in this chapter.

5.1 The medical station locating model

The model is about how to locate medical stations properly to guarantee all people has an access to medical care, with minimized number of stations and distance between them and roads. It considers 4 factors: transportation methods, roads, cities, and population density.

5.1.1 Assumptions

- **Residents of the country has to go to every medical station on foot.** As most Africa families live in poverty and owns no vehicles, they have to travel on foot. Even if there are some who own vehicle, it does not affect the results because the medical station serves for the general public.

- **A person can walk 10 hours a day, at a speed of 1a person can walk 10 hours a day, at a speed of 1m/s.**10 hours a day is a general walking tolerance of a person, and 1m/s is also a general walking speed of human.
- **People need to go home within a day.** In Liberia, there are often no shelters along the road, and people have to go home to sleep.
- **In each county, the number of residents in the capital city is a lot more than people in other areas.** It is very natural because there are only a few cities in Liberia, which support so many people. Capital of each county are often the largest city in the county and holds many people.
- **The towns (including cities) has a lot more people than other areas.** It is also true because towns are places that people gather. Other areas are mostly rural, with low productivity, and can not support many people.
- **In other areas of each county, people are in a uniform distribution according to the population density of the county.** In counties with higher population density, it is natural that there are more people in the rural areas because people there have better access to larger cities, and can get what they want more easily.

5.1.2 Honeycomb model

This model only considers the assumption 1,2,3 of the above. Our model supports the idea that all men are equal and that everyone should have an opportunity to get ample medical care. So we designed a model in which the medical stations are accessible to everyone. This is a basic model and further adjustment will be presented in the following sections.

According to the City of Central Place Theory, we know that a medical center's serving area is a hexagon, so the longest distance equals its side. According to the assumptions above, a person can take 5 hours to walk to the medical station, which is 18km. This means the side of the hexagon is 18km, so the distances between two stations should be 18 $\sqrt{3}$ km, and its structure should be triangle.

As a consequence, we draw a distribution map below(Figure 5-2). The distance between two stations is 18 $\sqrt{3}$ km.

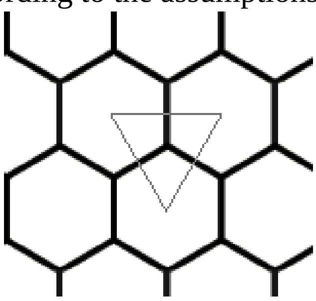

Figure 5-1 Honeycomb Model

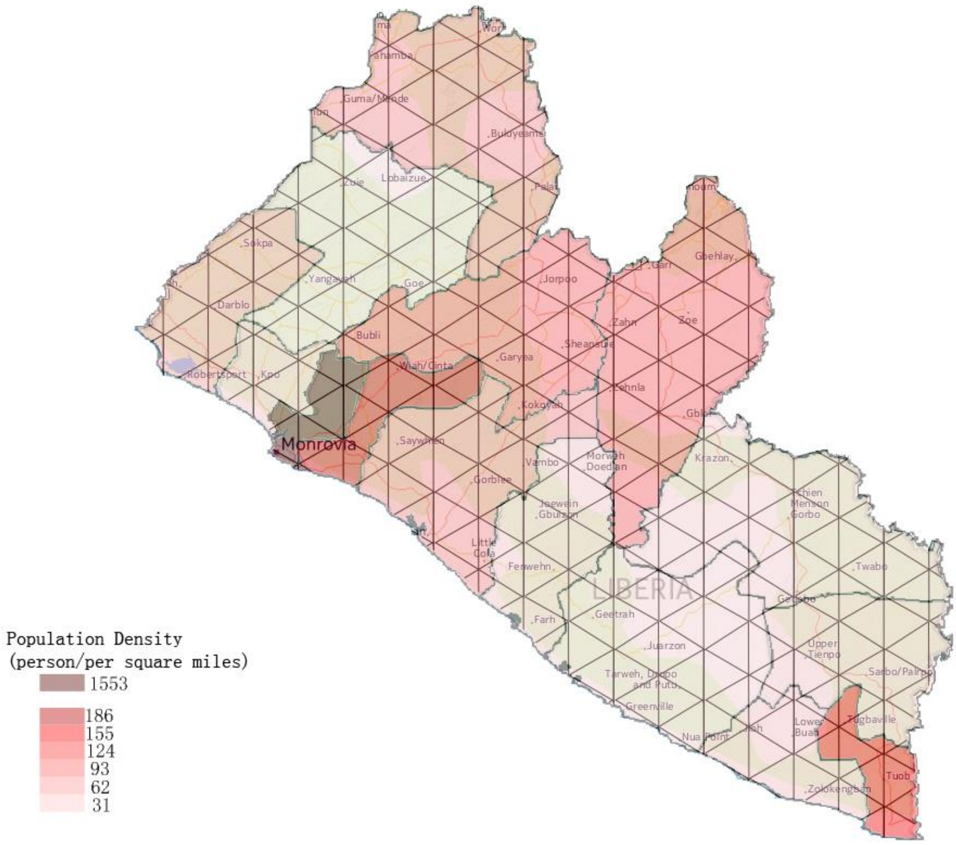

Figure 5-2 First Mesh in Liberia

5.1.3 The extension of Honeycomb model

In this section we are going to consider the affection of road, population density, and the location of cities and towns. We will adjust the locations of each stations and divide them into three levels: Big, Medium, and Small. Such adjustments can make the transportation of medicines more easily, as well as allocating the resources more reasonable so that the medical station can never be too crowded or too sparse.

5.1.3.1 The roads

In order to meet the demand of transportation, we need to lean the stations towards the road, so we develop the idea if there is a road near the station, we can put it along the road.

Therefore, the algorithm is that we draw a circle with the station, 10.4 ($6\sqrt{3}$) is the radius of the area. If roads are included in the circle, choose the nearest one and locate the stations there. After that, for the roads with no roads around, find six nearest points of it and locate the station at a point which minimize the sum of the distances from it to its six nearest stations.

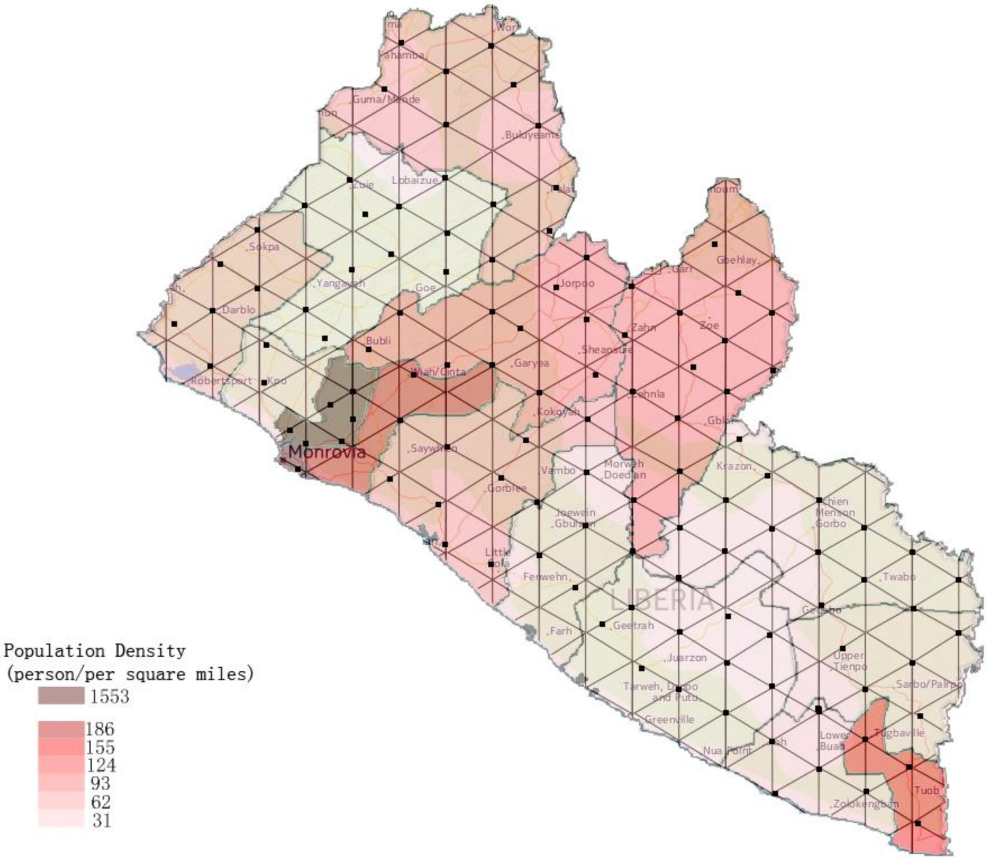

Figure 5-3 First Adjustment to the Mesh

5.1.3.2 Large cities and population density

In this section we consider the location of large cities and the population density, and divide the medical stations into 2 types: Big and Small. There are many people in the large cities. As the population density in large cities are all very high, there should be a big medical station there, which is able to hold large amounts of people, and give the infected medical treatment.

Here is the algorithm:
Step1: Find a station which is the nearest to the large city.
Step2: Move the station to the city's location and mark them yellow(which means Big). The result is shown in Figure 5.4.

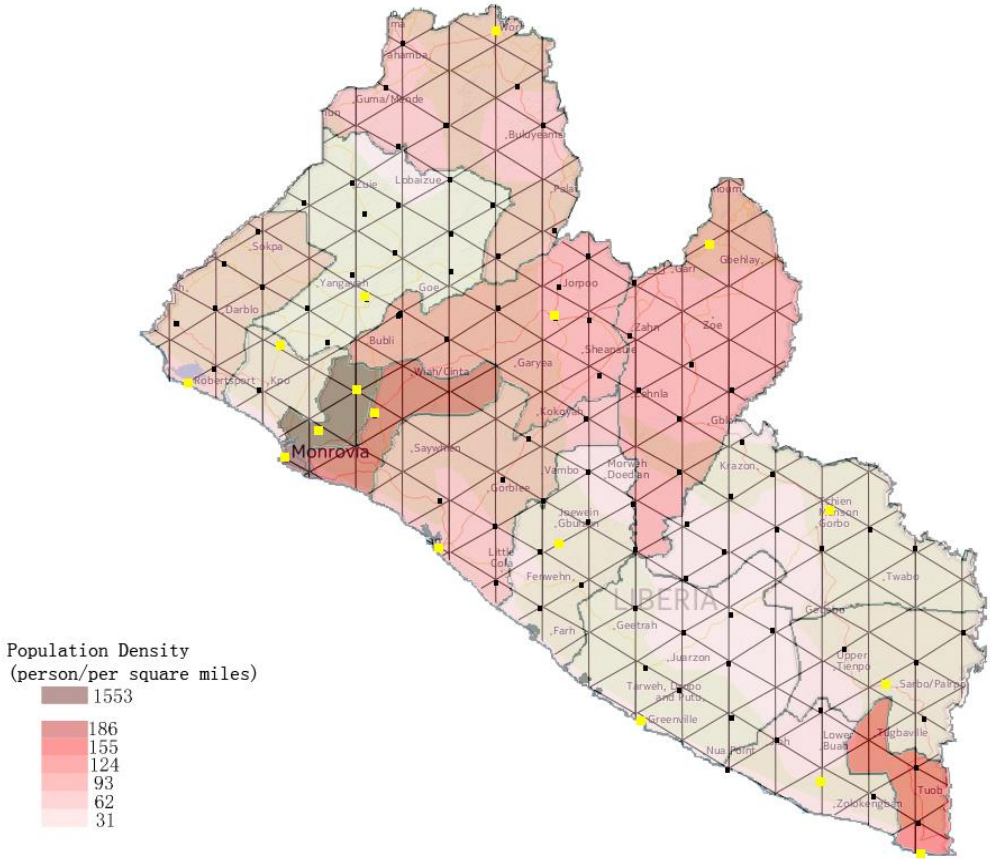

Figure 5-4 Second Adjustment to the Mesh

5.1.3.3 Towns

This section considers the factor of towns, and the station is divided into 3 levels: Big, Medium, and Small. Many people resident in town, but not that many as big cities. What's more, there are not many cities in Liberia, generally only one in a county (which is the capital), so there are still many infected people unable to get treatment. As a result, we need to set up a medium medical station in a town, which can do medical treatments to some extent, and to send the badly infected people to large cities.

Here is the algorithm:

Step1: Find a station which is the nearest to the town.

Step2: Move the station to the town's location and mark them green (which means Medium).

The result is shown in Figure 5.5.

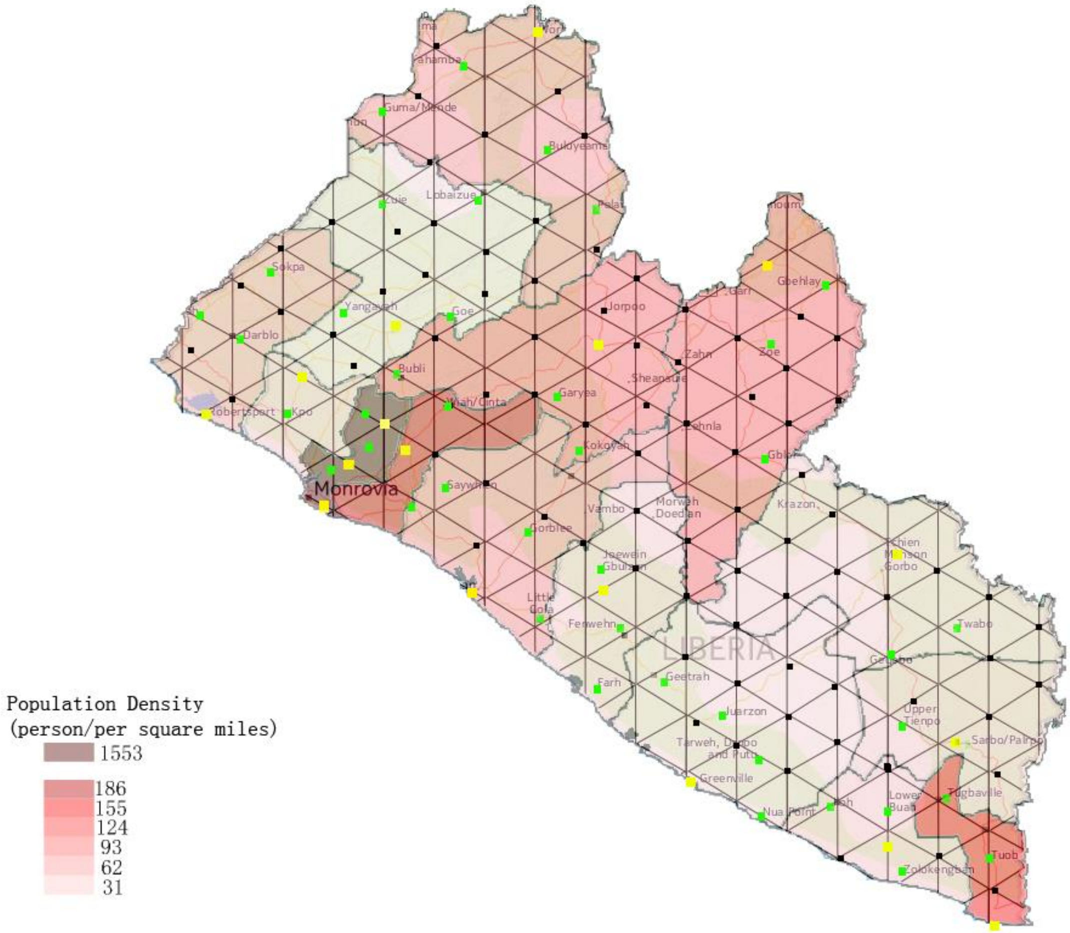

Figure 5-5 Third Adjustment to the Mesh

5.1.4 Final result

As some stations are too close to each other, they can be mixed as one station to save the limited resources. Therefore, we mix these points as one, and the size of the station is the same as the biggest station mixed before.
The result is shown in figure 5.6.

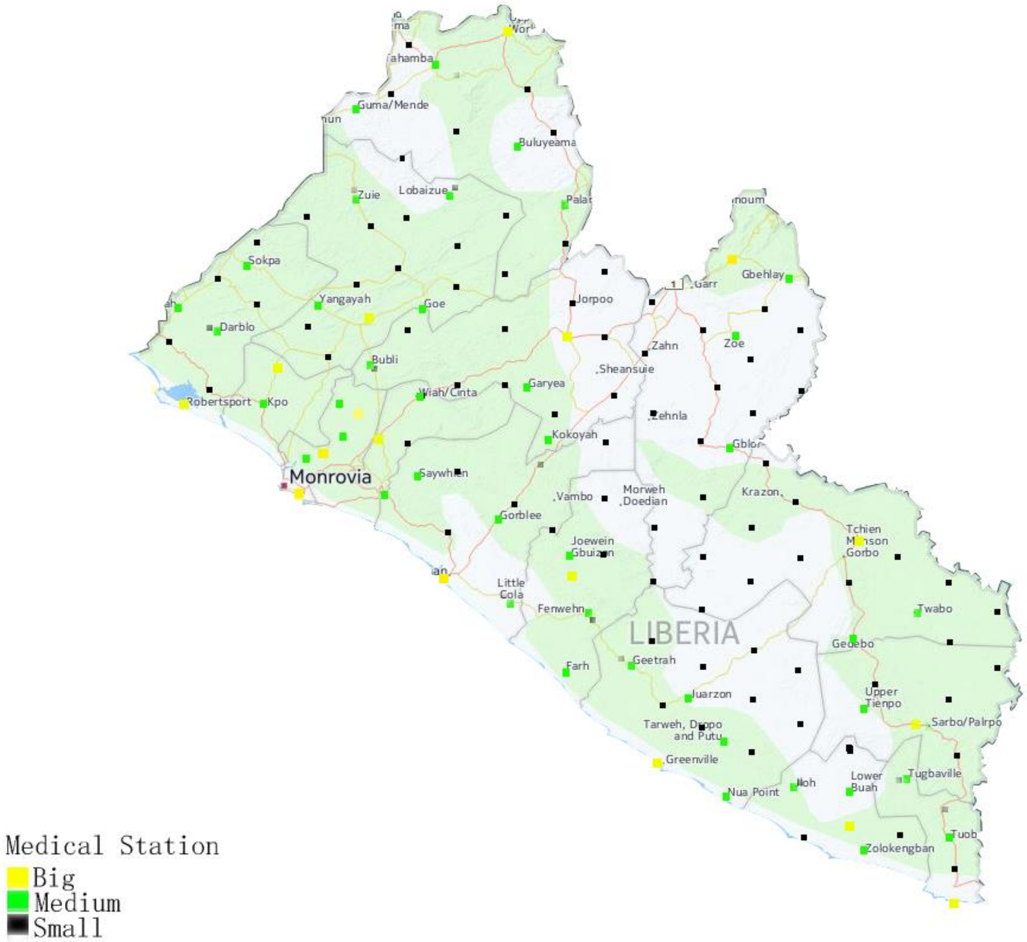

Figure 5-6 Final Result of the Location

By using software, we can also get the relative coordinates of every point
Take the big stations as an example. Its coordinates (x, y) are shown in the sheet below：

x	16. 19	22. 88	17. 92	12. 05	9. 35	6. 56	11. 73	12. 35
y	1. 34	8. 22	10. 5	9. 95	11. 47	12. 56	12. 84	13. 58
x	9. 97	14. 29	18. 08	26. 63	20. 67	28. 33	26. 35	29. 44
y	15. 26	17. 82	17. 74	16. 69	23. 35	22. 21	25. 31	27. 62

Table 8 Coordinates of Big Stations

5.2 Delivery System Model

After we locate all the stations, we have to find an appropriate way to deliver the medicine to each station. The system needs to minimize the overall distances of all routes (multiplying the number of cars needed).

5.2.1 Assumptions

- **All the medical supplies are transported form the capital of Liberia.** This is because Liberia relies a lot on foreign aids. These aids are usually transported by air, and have to arrive at the airport.
- **Large cities transport the supplies from the capital of Liberia.** Large cities need to handle many people in its county and requires a lot of supplies, so transporting them directly from the capital of the country can be a good idea.
- **Towns get the supplies from the nearest large city.** Towns are also in a great demand of supplies. However, the capital of Liberia is too far away and it would not be economical to get the supplies from it. So it is wise to get them from the nearest large city.
- **A truck of medical supplies from large cities can meet the demand of 4 small stations, and a truck of it from towns can meet the demand of 3 small stations.** The small stations are only used for immunization, so they do not need lots of supplies. The truck that large cities can provide can be bigger than towns. Also, large cities can use smaller trucks for transportation, but larger trucks are can hold more goods and are more economical.

5.2.2 Delivery System Model--from big stations to medium stations

According to assumption 3, towns get supplies from the nearest big stations. Therefore, for every green point, we choose the nearest yellow point and link them.
The result is shown in figure 5.7.

SYMBOLS	DEFINITIONS
A/B/P	A medium or big medical station
a/b	A small medical station
XX/XXX/XXXX	The circumference of the drawing(the letters represent its zeniths)

Table 9 Quick Review to Notations

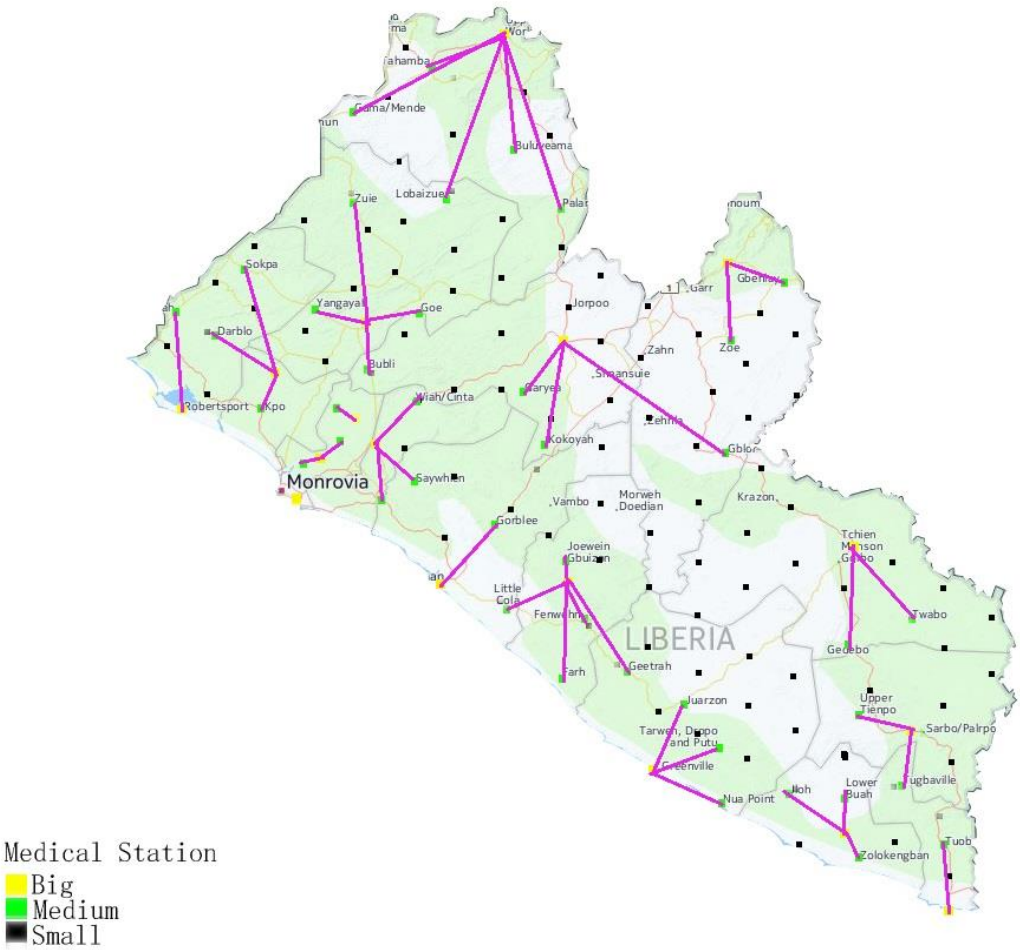

Figure 5-7 First Lever of Delivery

5.2.3 Delivery system model--from big&medium stations to small ones

According to assumption 4, we need to find a proper strategy to transport supplies from larger stations to small stations. The car needs to pass though several small stations and then finally go back to the original city it started before. The aim of the model is to make the number of the total distances as small as possible. The question is similar to the Traveling Salesman Problem (TSP), which is very hard to find the best solution. Therefore we developed an algorithm to draw our solution to the best solution as near as possible.

Here is the algorithm:

Step1:

Every small medical station finds its nearest larger station (medium or big), and links with them.

Step2:

Medium station:
If N(linked dots)mod3==0, link these dots(including the larger station) and leave them apart (which means the 3 dots will not be included in the following consideration).
Big station:
Do the same as the above except that the number 3 is needed to be changed into 4.

Step3:
Traversal all the medium and large stations.
If there exists a station (marked A) which has only one dot (marked a) linked to it:
Choose a next nearest station (marked B);
Switch (the number of dots already linked to the station):
0: choose a next nearest station and switch again;
1: (marked b): Aab>Bab? Link a to B: Link b to A;
2: (marked b, c): Babc>2Aa+Bbc? (The radio>3? Link them apart: choose the next nearest station and switch again) : link a to B, if B is medium, link them and leave the apart.
3: (marked b,c,d): Babcd>2Aa+Bbcd?(the radio>3? Link the apart: choose the next nearest station and switch again): link a to B, leave them apart.
Return to Step3.

Step 4:
(Now the remaining station has 2 or 3 dots.)
Traversal all the medium and large stations (with dots linked).
If there are 2 dots linked to the station
Each of the dots linked to it chooses its second nearest station.
If all (both) stations are the same, all (both) of them choose a nearest station.
If the station is big and with 3 dots linked, and if only 2 neighbored stations has been chosen, find their third nearest station (s).
Link every (each) dot to the station it has chosen
If 2 dots select one station (P)
If there are 2 dots (a,b,and and there second nearest stations are A&B):
Find the smaller one between aP+Bb and bP+aA, and choose that strategy.
If this strategy has a smaller distance than the initial one, then adopt the strategy (leave the dots apart).
Return to step 4

Step 5:
Link the remaining dots

The result is shown in the figure 5.8.

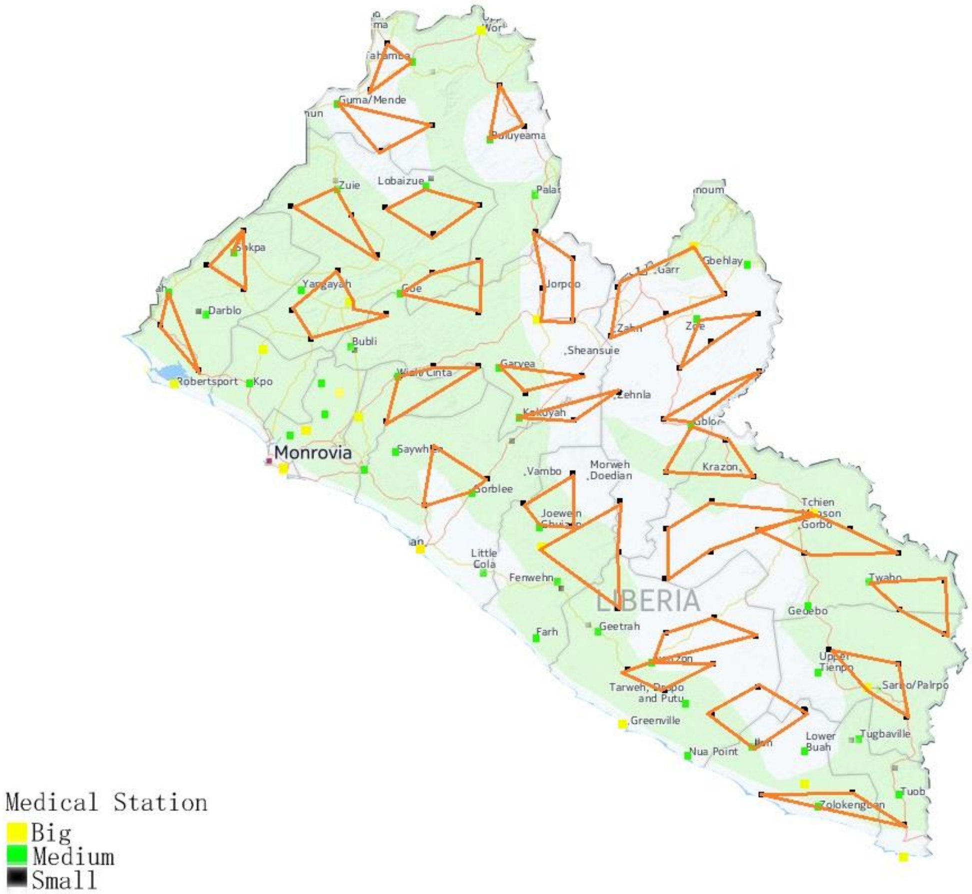

Figure 5-8 Second Lever of Delivery

6. Model of medicine choice

6.1 Introduction

Now scientists over the world are all concerned with the developing process of Ebola *vaccine* and cure. However, the speed of producing medicine of each single medicine company cannot catch up with the increasing number of cases who got EVD. Thus, in order to supply enough medicine to stop the spread of Ebola, medicine companies need to cooperate together. Considering the finite budget at present, the problem we are faced is about how to supply enough medicine to African people in a short time and minimize the cost we spend as well.

As there do not exist a proved effective medicine now, we supposed that these kinds of medicine have large possibility to work well. They are: GSK, ZMapp, Vsv, SinCon (SC) and Advanc. Then we set a standard to investigate them and decide how many doses of each kind of them are needed.

6.2 Assumptions

- **As the quantity is really big to calculate that in Africa, we take Guinea for example.** Due to model one, we assume that 400 people have to be cured every day and 0.01% uninfected people get vaccinated each day. Then 33.6 thousand vaccines are needed.
- **One uninfected person only has one chance to be vaccinated and he is assumed to be immune to Ebola virus.**
- **One infected person only use one kind of drug.** A mixture of medicine is not permitted.
- **The medicine effect are all the same.** If patient A got enough doses of ZMapp while patient B got enough doses of TKM, they will both be cured.
- **The assumed data about these medicine:**

Unit of cost: dollar

No.1 vaccines:

name	S	C	D
GSK	50 thousand doses	50	1
Vsv	25 thousand doses	48	1
Sc	30 thousand doses	52	1

Table 10 Data of Vaccines

No.2 drugs

Name	S	C	D
ZMapp	9000	300	3
TKM	24000	500	2

Table 11 Data of Drugs

SYMBOLS	DEFINITIONS
S	Production per month
C	Cost per dose
D	Number of doses a patient needs
Y_1	Number of thousand people get vaccinated each month with GSK
Y_2	Number of thousand people get vaccinated each month with Vsv
Y_3	Number of thousand people get vaccinated each month with Sc
TC1	Total cost of vaccine
X_1	The number of people cured by TKM per month
X_2	The number of people cured by ZMapp per month
TC2	Total cost of drugs

Table 12 Quick Review to the Notations

6.3 Model

6.3.1 Vaccine

This is a simple question in operational research.
As the quantity is so big, the unit of Y is million.

According to the population of Africa, we have: $Y_1 + Y_2 + Y_3 = 33.6$

$Y_1 \le 50$
$Y_2 \le 25$
$Y_3 \le 30$

Total cost of vaccine： $TC1 = 50Y_1 + 48Y_2 + 52Y_3$

Total cost minimize when

$Y_1 = 8.6$
$Y_2 = 25$
$Y_3 = 0$

Then $TC1 = 1630$ thousand dollars

6.3.2 Drugs

It is very similar to the analysis above.

Then $X_1 + X_2 = 12000$.

$2X_1 \le 24000$

$3X_2 \le 9000$

The total cost $TC2 = 500 \cdot 2X_1 + 300 \cdot 3X_2$

It minimizes when

$X_1 = 9000$
$X_2 = 3000$
$TC2 = 11700000$

7. Model of allocation

7.1 Introduction

Because of lack of abundant medicine for everyone infected and the finite efficiency of medicine, how to allocate these medicine has become a significant problem. In this model, we tried to think in a primitive way and guarantee the survival rate of patients. As mentioned in the announcement of world medical association, their new medicine can only cure those whose disease is not so advanced. In this way, we should try to allocate the medicine to those who just get infected for a short time.

7.2 Assumptions:

- Considering that infected people has a survival rate of no more than 15% according to official data and the data has not taken the large quantity of new cases every day into consideration, the survival rate can be very low, thus we assume that each of the infected patients need equal doses of medicine and ignore those who were healed without medicine.
- The ratio of isolated patients largely grows with public concern about EVD improving at the same time. Otherwise, this disease will certainly lose control.

φ	**Time for a patient from getting infected to death without treatment**
μ	**Number of people infected by each infected person per week**
t	**Time**
$\boldsymbol{a_t}$	**Increasement of infected cases at time t, a(0)=0**
$\boldsymbol{\gamma_t}$	**Ratio of isolated patients among those who were not isolated at time t-1**
$\boldsymbol{b_t}$	**Number of patients (not isolated at t-1) alive at time t**
$\boldsymbol{c_t}$	**Number of patients (not isolated at t-1) dead at time t**
$\boldsymbol{r_t}$	**Supposed to be isolated at time t**
$\boldsymbol{e_t^k}$	**Number of cured patients at time t who was infected at time k**
$\boldsymbol{f_t}$	**Number of patients waiting for medicine**
$\boldsymbol{g_t}$	**Number of isolated patients dead at time t**
$\boldsymbol{b_t^k}$	**Number of patients in b(t) who was infected at time k**

Table 13 Quick Review to the Notations

7.3 Model

EVD is a fatal disease which can take a healthy man's life in less than 10 days. Because of our lack of abundant medicine, how to allocate them has become a significant problem.

As for the allocation of vaccinate, it's no doubt that we should vaccinate those who is more easily infected such as the young and the old. Many people in Africa have the tradition to touch the dead body in their family and it greatly increased the risk of being infected. In this way, those who have some family members infected and dead should be considered easily infected and get vaccinated first.
For the cure medicine, we have to think about what kind of patient is the most worthy to be treated first. Patients can be divided into several groups. This picture below shows how situations of the patients change.

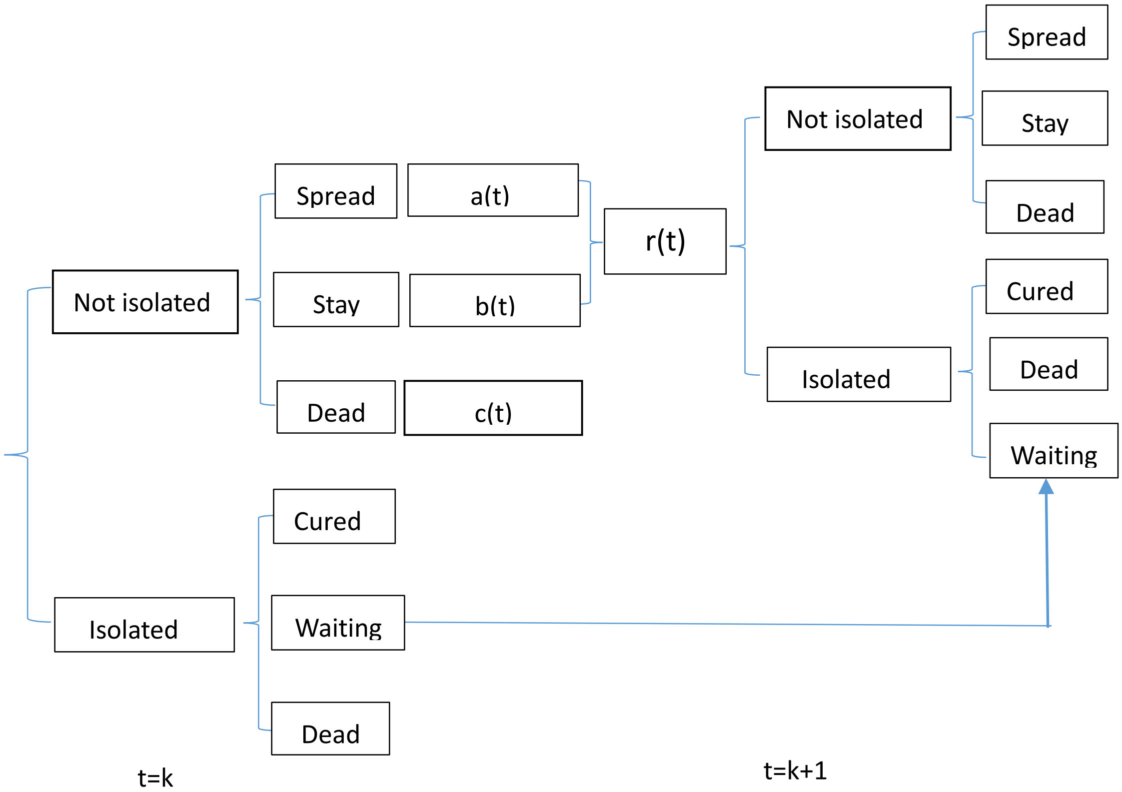

Figure 7-1 situation changes of patients

From the picture, we can see that：

$$a_t = \mu \cdot (1-\gamma_t) \cdot r_t$$

Because patients infected die after ψ days, then we have:

$$c_k = b_{k-1}^{k-\varphi}$$

Thus, we can calculate the number of patients need to be isolated at time k:

$$r_k = a_{k-1} + b_{k-1}^{k-1} + b_{k-1}^{k-2} + \cdots + b_{k-1}^{k-\varphi+1}$$

Similarly, we can calculate:

$e_k^k = \min[a_{k-1}, r_k \cdot \gamma_k]$ and

$b_k^k = \max[0, a_{k-1} - e_k^k]$

When $r_k \cdot \gamma_k \le a_{k-1}$, all patients are able to be isolated.

$e_k^{k-1} = e_{k-1}^{k-1}, \cdots, e_k^{k-\varphi+1} = e_{k-1}^{k-\varphi+1}$ and

$b_k^{k-1} = b_{k-1}^{k-1}, \cdots, b_k^{k-\varphi+1} = b_{k-1}^{k-\varphi+1}$

It is obvious that if a patient is given medicine right after infected, then he is more probable to survive. Thus we should think primitively about the situation and give the medicine to those just confirmed to be infected rather than patients who is suffering the last period.

So, if $r_k \cdot \gamma_k > a_{k-1}$,

$e_k^{k-2} = e_{k-1}^{k-2} + \min[\max(r_k \cdot \gamma_k - a_{k-1} - b_{k-1}^{k-1}, 0), b_{k-1}^{k-2}]$
$\cdots$

$e_k^{k-\varphi+1} = e_{k-1}^{k-\varphi+1} + \min[\max(r_k \cdot \gamma_k - a_{k-1} - b_{k-1}^{k-1} - \cdots - \mathrm{b}_{k-1}^{k-\varphi+2}, 0), b_{k-1}^{k-\varphi+1}]$

$b_k^{k-1} = b_{k-1}^{k-1} - (e_k^{k-1} - e_{k-1}^{k-1})$
$b_k^{k-2} = b_{k-1}^{k-2} - (e_k^{k-2} - e_{k-1}^{k-2})$
$\cdots$
$b_k^{k-\varphi+1} = b_{k-1}^{k-\varphi+1} - (e_k^{k-\varphi+1} - e_{k-1}^{k-\varphi+1})$

and we also have this for isolated patients at time k:

$d_k = 0,\ f_k = e_{k-1}^{k-\varphi}$

In this way, we can calculate the new cases of patients at time k:

$a_k = (b_k^k + b_k^{k-1} + \cdots + b_k^{k-\varphi+1}) \cdot \mu$

Then we are able to use this model to finger out how many patients are waiting for medicine. This may help with the allocation of medicine.

7.4 Test

Take this for an example:

Assume that the initial number of patients is 2, μ=10 and φ=7, according to official data. The unit of time is week. Then we get the result using the algorithm above:

T	γ_t	r_t	NEED
0	0	2	0
1	0.5	22	0
2	0.8	121	11
3	0.9	275	107
4	0.95	308	366
5	0.96	176	654
6	0.97	88	819
7	0.98	31	899
8	0.99	10	910
9	0.99	11	809
10	1	11	570
11	1	0	300
12	1	0	140
13	1	0	60
14	1	0	30
15	1	0	20
16	1	0	10
17	1	0	0
18	1	0	0
19	1	0	0

Table 14 Test Data for Model 4

From the form above, we can see that the number of the least supply of medication lies in the last row.

8. Strengths and weaknesses

8.1 Strengths

- **In spreading model in chapter 4**, we considered that different community has different sizes and those who recovered from EVD. It makes our results closer to the reality.
- **In the medical station locating model in chapter 5**, we achieved that:
  1).Everyone can have an access to medical care, regardless of their economic condition, because they can always go there on foot.
  2).Most of the medical stations are settled beside the road, which makes the transposition of supplies from other areas more easily.
  3).The size of the medical stations are adjusted according to the population density of that area, as well as the city's location. This can arrange the limited resources more

reasonably.

- **In the delivery system model**, the transportation routes are designed to make the total distances need for delivering supplies as short as possible. And it is a great solution which is fairly close to the best answer.
- **The model of allocating in chapter 7** is based on the isolation and the death time of an infected person. Because of the primitive thinking style, it makes saving more people while we are short of enough medicine possible. As you can see, this model is also a simple one and easy to apply.

8.2 Weaknesses

- **In spreading model in chapter 4**, while considering the distance between provinces, we just take the distance between their capitals. And we estimated the transport rate with method in 5.3.1 because of lack of enough data. Thus further investigation is needed
- **In the medical station locating model in chapter 5,**
  1).There are too many medical stations required. It may be beyond the county's financial capacity.
  2).Quite a few stations are still located far from the road, and it can be very hard for them to get medical supplies from the outside.
  3)The slight adjustment of the stations' location may result in a longer distance for people in some areas. They may find it very hard to get to the place and return home in one day.
  4)Perhaps people of Liberia are able to go outside home for more than one day. In this case the distance between two stations can be longer.
- **In the delivery system model**,

  1).The routes design is not the best one due to the limitation of the computer's calculating ability.
  2).The assumptions that the model based are not definitely true. For instance, supplies from foreign countries may not arrive only in the capital city. Also, a truck of supplies may not be able to fill 3 or 4 small station, or maybe it can meet the demand of more than those stations. Moreover, there may be not people in some areas at all, and there is no need to build a medical station there.
- **In the model for medicine choice**, because of lack of new EVD medicine (no such one kind of medicine is proved safe in human at present), we can only analysis this in much simpler ways. And we do not know how many days a medicine can cure a patient, thus there is a delay time at the beginning of medical supply. If we can get more information about the medicines and the companies they were produced by, the model can be more rational and certainly much closer to reality.

- **In the model of allocating in chapter 7,** the time from getting infected to death can not easily be determined due to an incubation period which is of great individual difference. And how many people a patient can spread the virus to is hard to figure out. Different places may have various environment and social habits and we have to fix our model in various situation. And we do not consider those people isolated at home, whose amount can not be found in official data. Further investigations have to be made with more information. And, in addition, giving those deadly patients up seems against humanity.

9. Letter for the world medical association

To whom it may concern,

Health is the basis of career and source of happiness. However, the outbreak of Ebola broke the peaceful life of African people and turned the Africa continent into a hell on earth. Over twenty thousand people have been confirmed to be infected with Ebola virus, which leads to the death of over 90% of its human hosts. Without medical control, it may deteriorate into a worldwide disaster. To help with this dilemma, we proposed two suggestions in different aspects:

1. **Medical assistance**
   Vaccination of healthy people and treatment of infected patients are both of significant importance. Each one can accelerate the eradication of Ebola to a large extent.
   Not only infected patients should be isolated but the transportation from epidemic area to the outside should be cut off. Thus we can prevent the spread of EVD.
   Make sure that every citizen stay away from wild animals, especially human-like ones.

2. **How to ensure medical assistance**
   In order to ensure that everyone has an access to medical assistance, we should set up mini medical station in the range that anyone can reach a station in one day on foot. Medicines should be delivered according to the hierarchy of administrative districts.
   To ensure the survival rate of infected patients, we should guarantee that those who got infected a short time ago take the medicine first. Considering the speed of manufacturing of vaccinate and drugs, we should choose different kinds of them to cost the least.
   In order to ensure that enough medicine is produced in time, cooperation between different companies should be encouraged.

Thank you for your time and consideration on this letter. For the sake of all human beings, please consider our suggestions!

Sincerely,

Team #38432
2015/2/9

References

[1]Codeco,C. 2001 Endemic and epidemic dynamics ofcholera: the role of the aquatic reservoir. *BMC Infect. Dis.* **1**,1.(doi:10.1186/1471-2334-1-1)

[2]E.Bertuzzo et al. 2009 On spatially explicit models of cholera epidemics. *J.R.SocInterface (2010)* **7**, 321-333.(doi:10.1098/rsif.2009.0204)

[3]Parham, P.E. & Ferguson,N.M 2006 Space and contact networks:capturing the locality of disease transmission. *J.R. Soc. Interface* **3**, 483-493. (doi:10.1098/rsif.2005.0105)

[4]Triska, F., Duff, J. & Avanzino, R. 1003 Patterns of hydrological exchange and nutrient transformation in the hyporheic zone of a gravel-bottom stream-examining terrestrial aquatic linkages. *Freshwater boil.* **29**, 259-274. (doi:10.1111/j.1365-2427.1993.tb00762.x)

[5]Gu Yong.2010 Demand analysis of emergency medical supplies based on the prevention and control mechanism of infectious diseases. *Journal of WuHan University of Technology (Transportation Science & Engineering).* 2010,34(4)

[6]Liberia. Wikipedia, 9 February 2015.

[7]Guinea. Wikipedia, 4 February 2015.

[8]Ebola virus disease. Wikipedia, 8 February 2015.

[9]Yahoo maps:https://maps.yahoo.com/

[10]About Ebola Virus Disease *cdc.gov* 8 December 2014.

[11]2014 Ebola Outbreak in West Africa cdc.gov 6 February 2015.