

# Uses of particle filtering in finance

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# Overview

- The potential approach (Tino Kluge)
- The equity premium puzzle (Arnaud Jobert, Alessandro Platania)
- Dynamic Engine for Bayesian Inference (DEBI) (Elie Bassouls, Simon Godsill)

# The potential approach

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- Supermartingale iff

$$(\alpha - \mathcal{G})f \equiv g \geq 0.$$

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- Dimension is large: 5 states, 3 currencies  $\Rightarrow$  dimension = 47.
- Price is a function of state of the chain - so only a few possible prices for any given instrument??

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- PF interpretation resolves conceptual difficulty about finitely many possible prices: *The model price is the population average of the individual particle prices.*

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- Particle-filtering approach gives us posterior for the price of *any* derivative, and could be used provide confidence intervals for the price.

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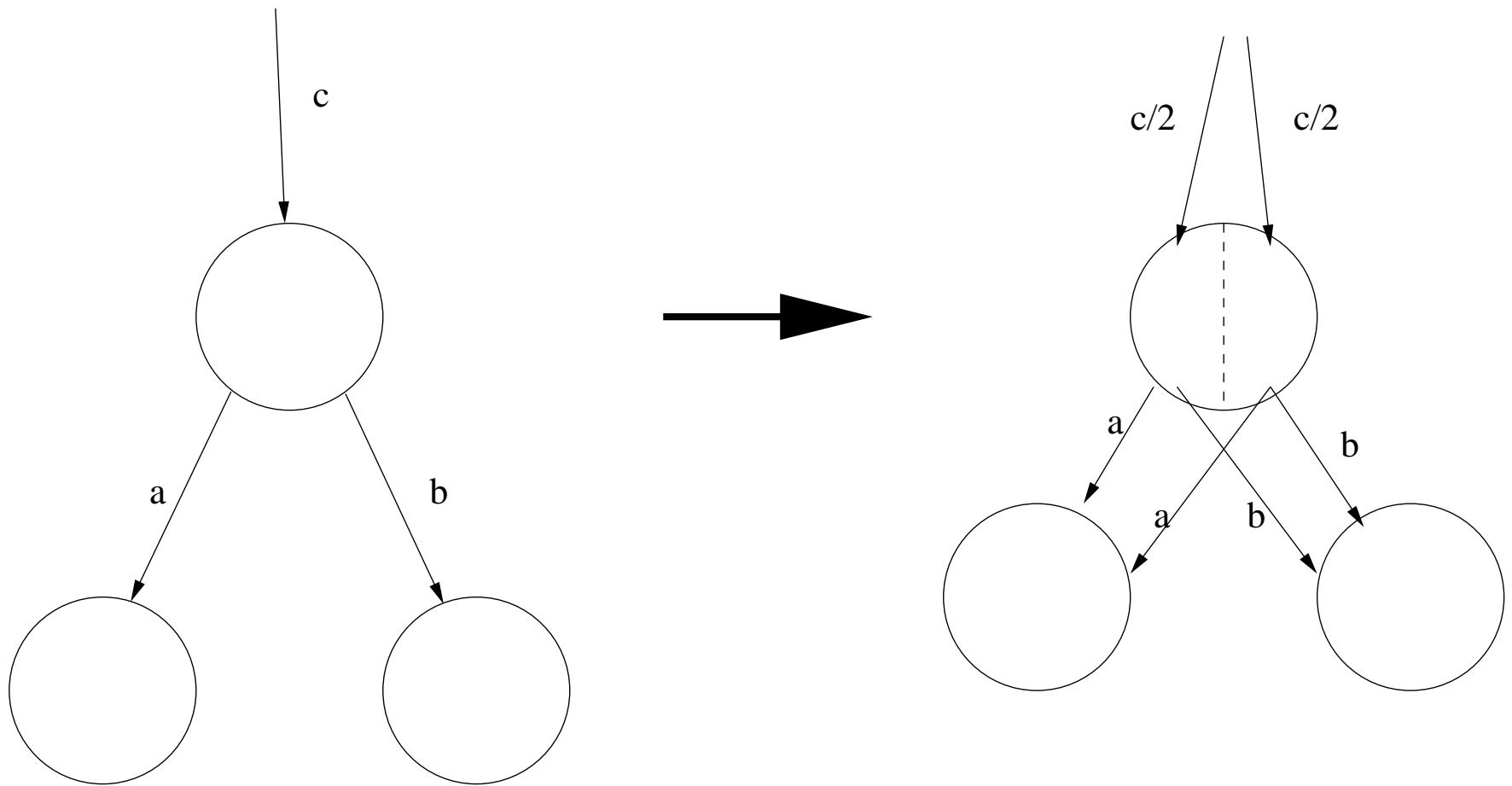


Figure 2: Replacing a state with an equivalent pair of states.

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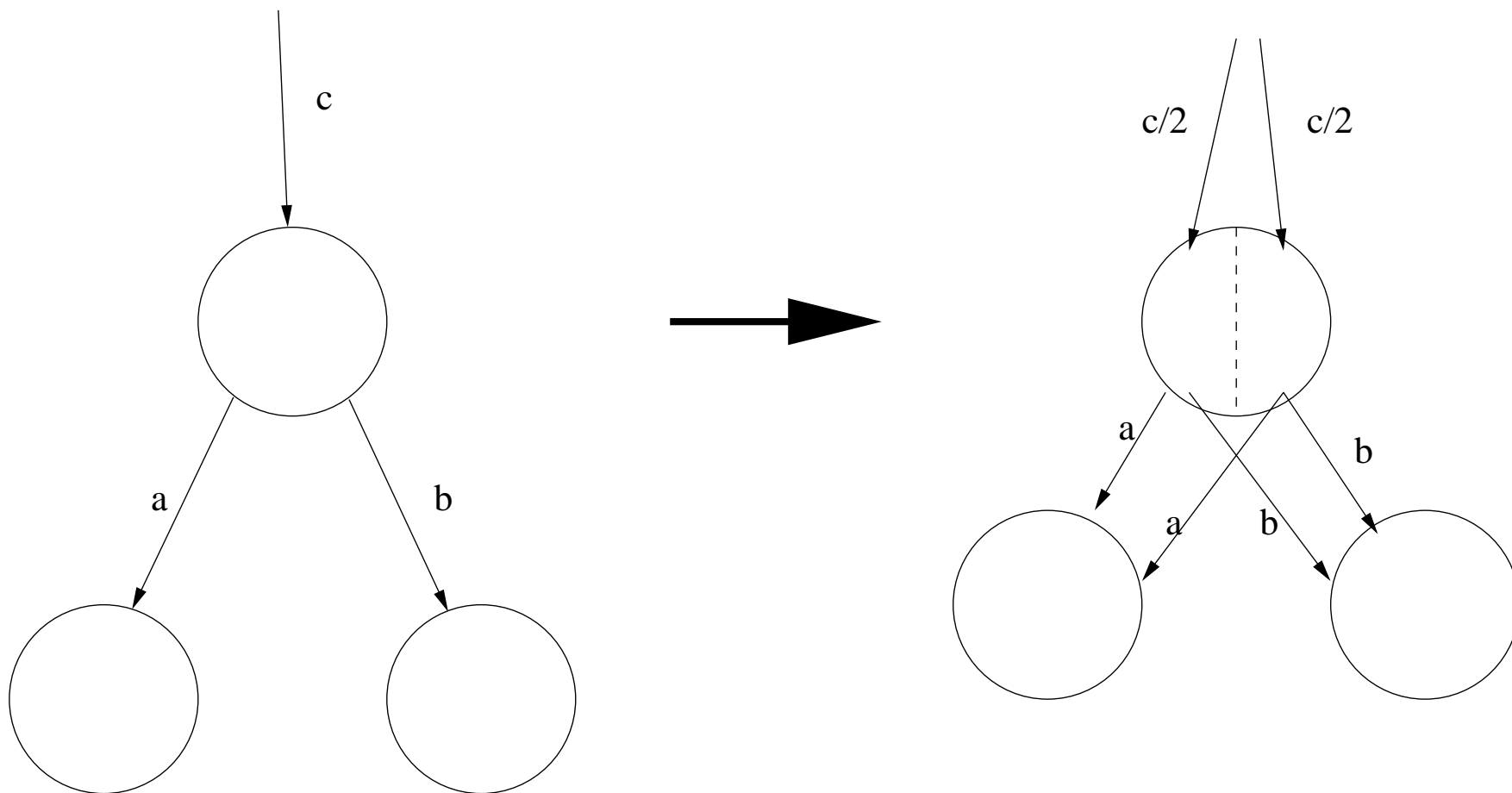


Figure 3: Replacing a state with an equivalent pair of states.

- Allow population to contain models with different numbers of states?

*Some results.*

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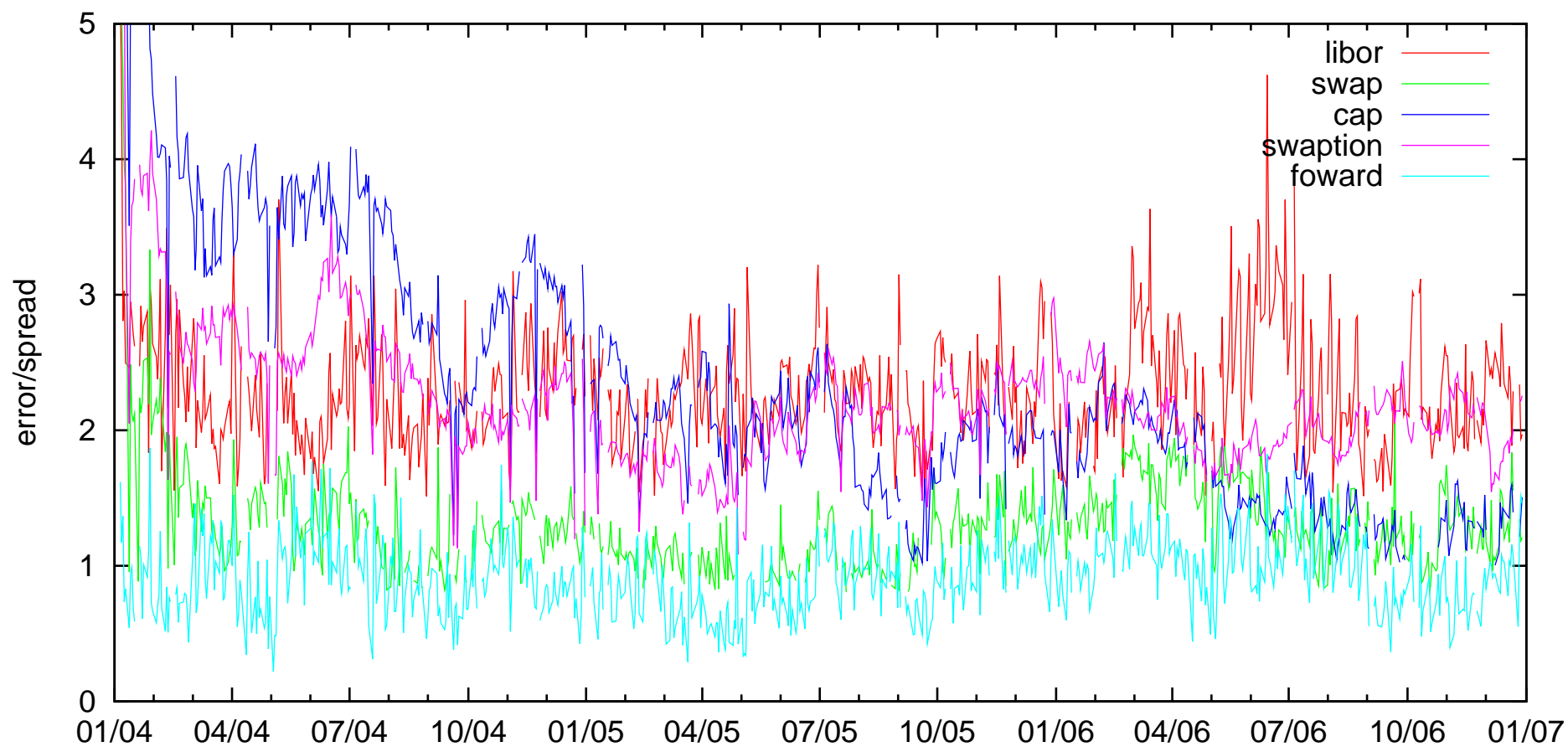


Figure 5: Mean errors (in spreads) for different classes

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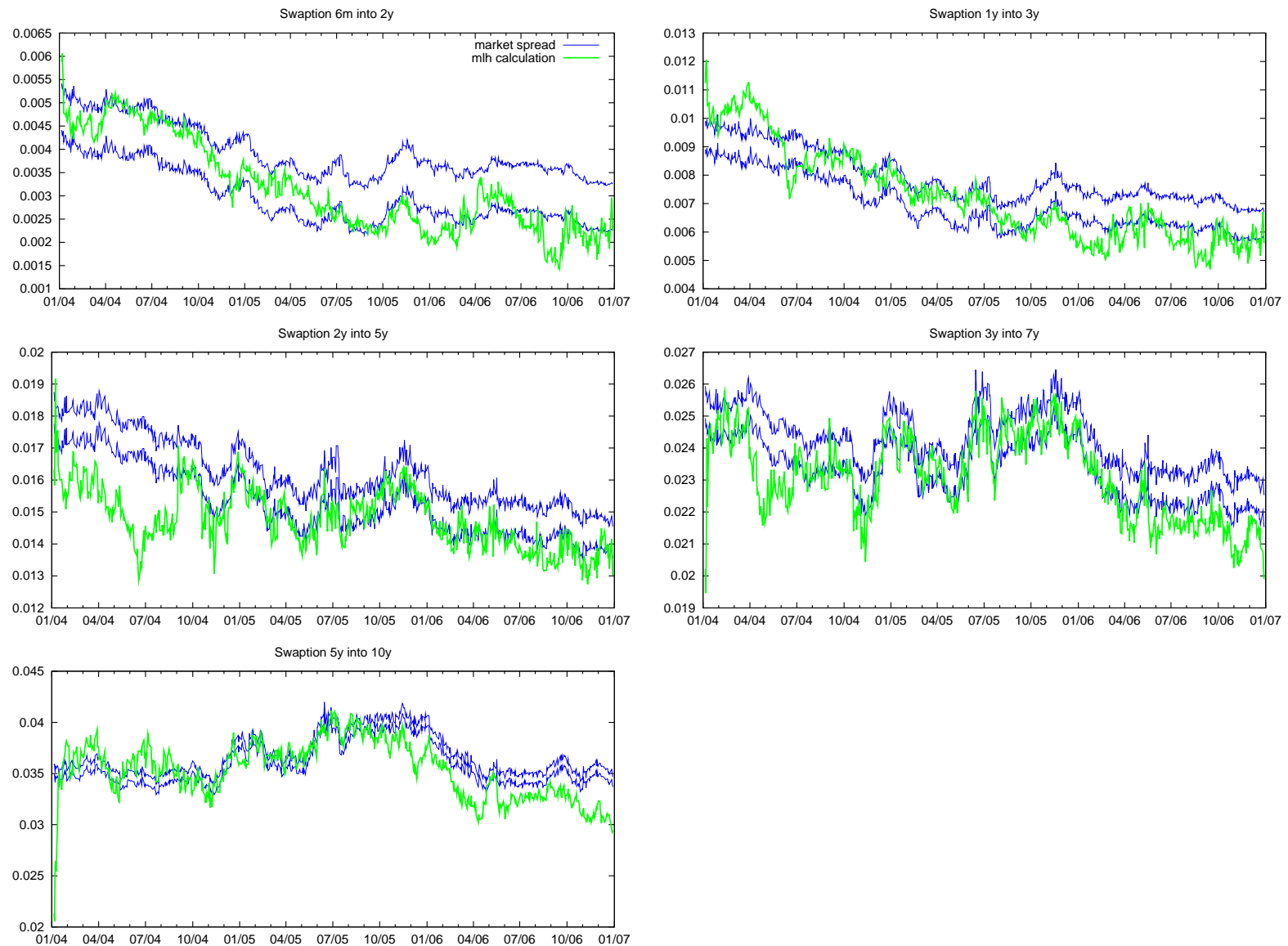


Figure 7: Euro swaption prices

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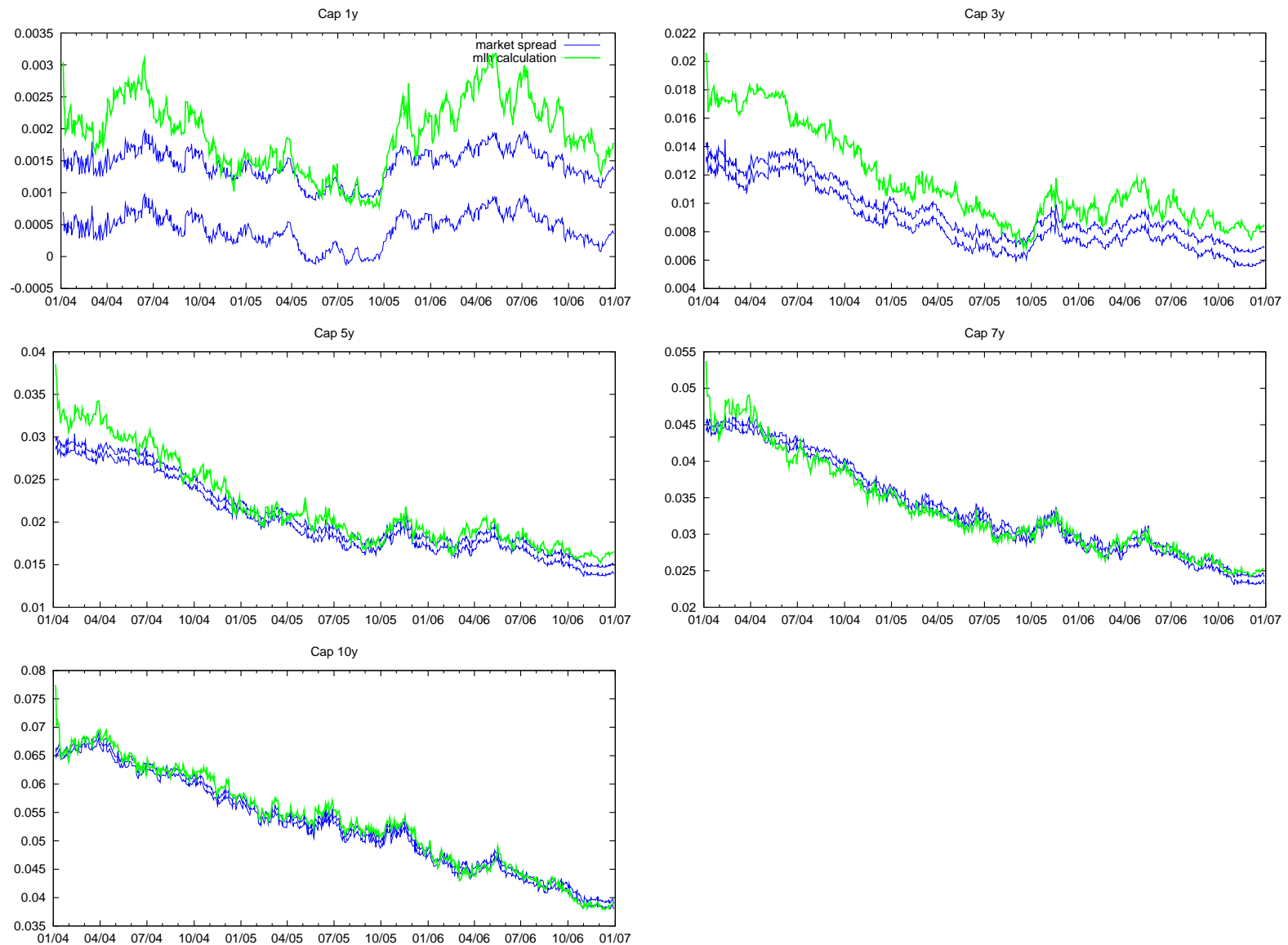


Figure 9: Euro cap prices

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Assume mean (and precision) of the Gaussian steps unknown, assume some observational noise in the reported data.

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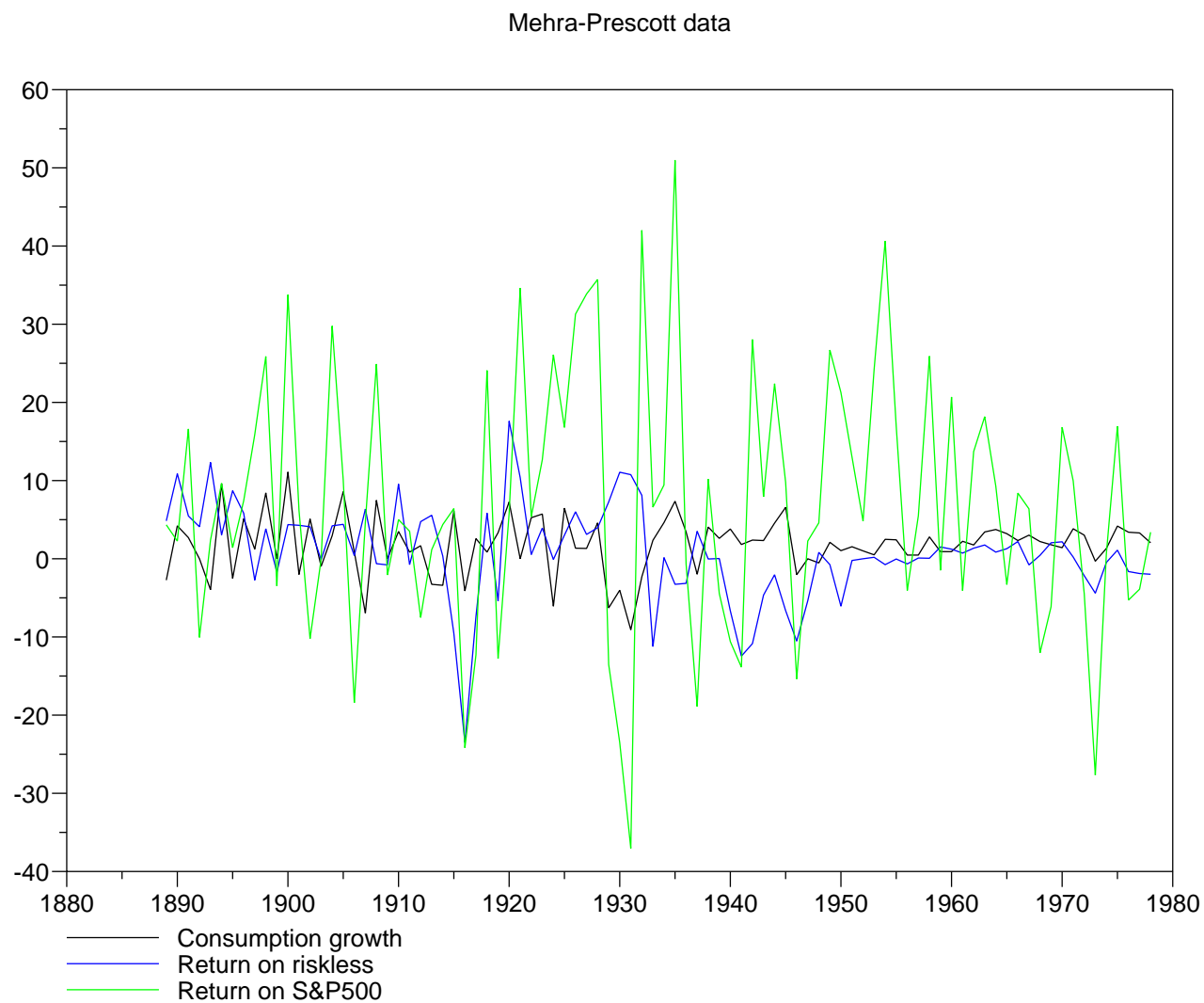


Figure 11: Mehra-Prescott data

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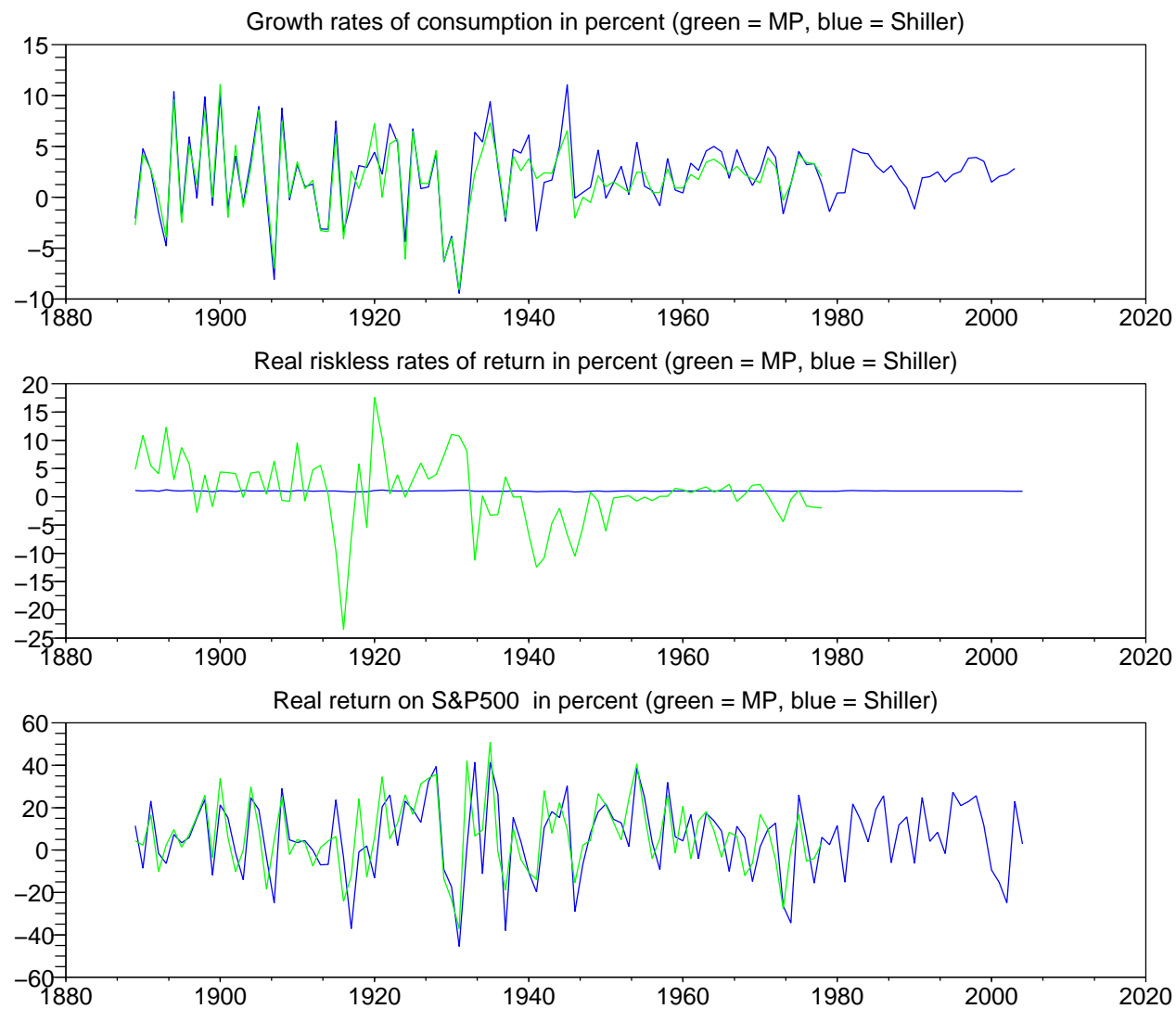


Figure 13: Mehra-Prescott's data, and Shiller's data

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Thus we have a built-in change from 1911 to 1912 of about 5% !

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- How stable do we think the data-generating process is?
- If the data is changing, PF will kill off alternative models which later look better

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- Posteriors for  $\beta$  and  $R$  are numerically sensible ... most of the time.
- Posterior for  $R$  usually quite tight ... but not in the same place next time ... over-resampling?
- Posteriors for observational noise quite high ..
- How stable do we think the data-generating process is?
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- The assumption that the data-generating process is unchanged over the kinds of timescales needed to reliably estimate the parameter is untenable.

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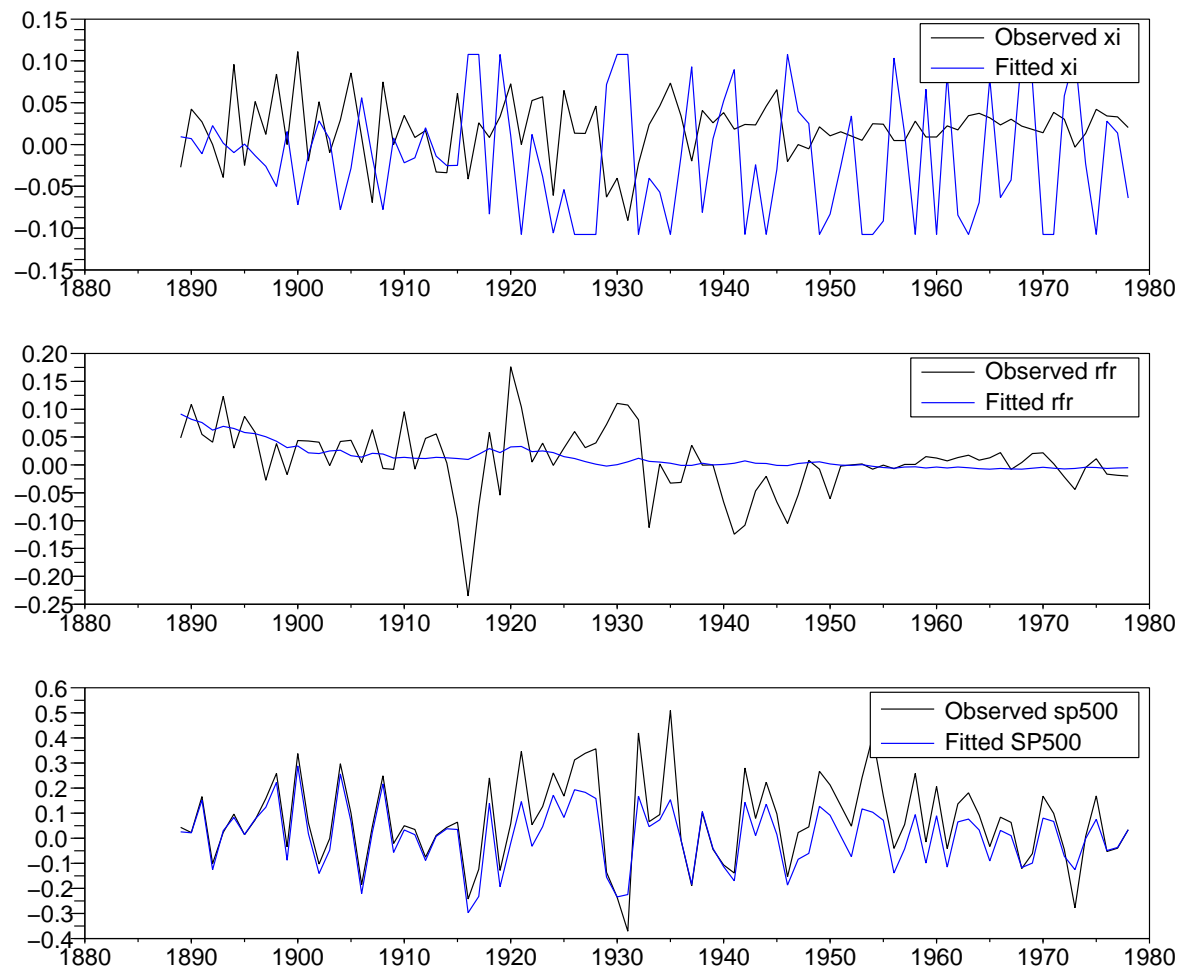


Figure 16: ML values with original MP data



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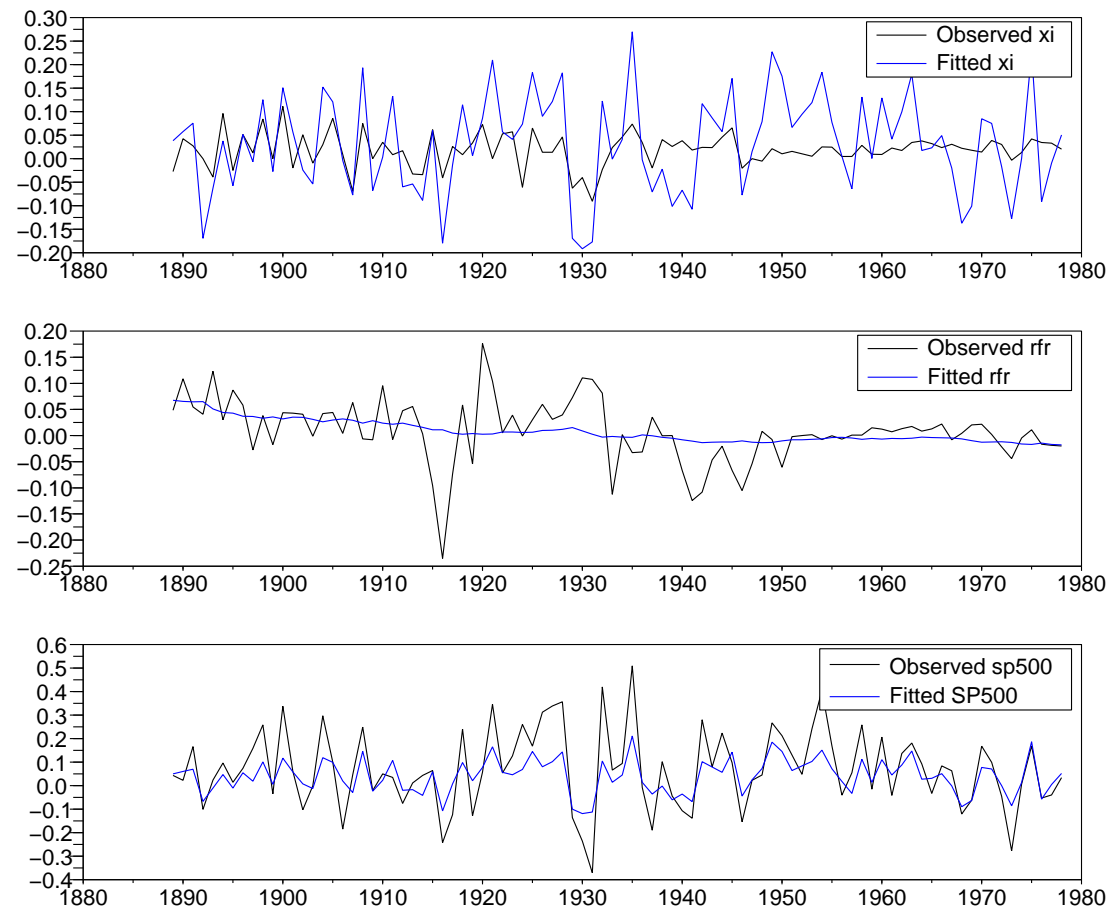


Figure 18: MCMC median values for MP data

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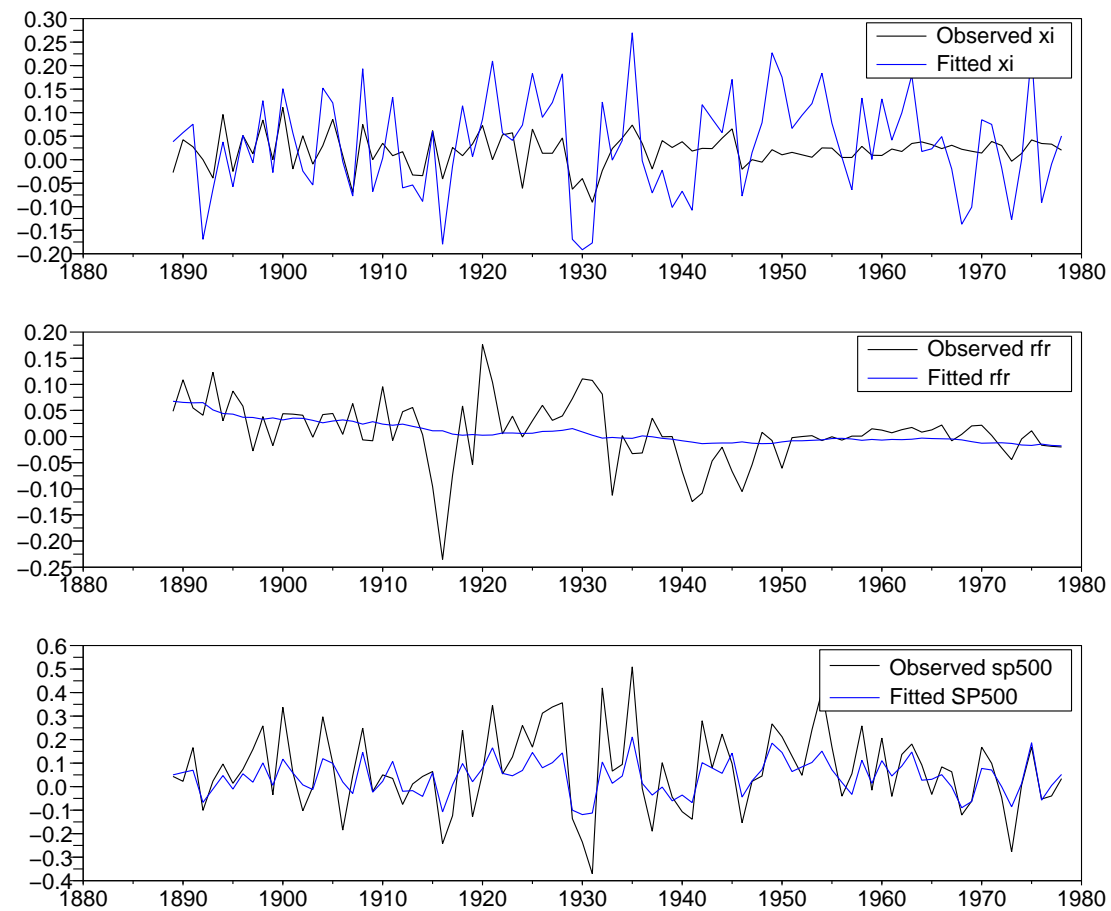


Figure 19: MCMC median values for MP data

Median for posterior of  $R = 10.4$

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- It took 90 years to gather the MP data ... and in 25 years we are no nearer a satisfying explanation of this data.
- Recognise that there are only about 300 numbers in this dataset ... and leave this small bone for the dogs to fight over!

# Dynamic Engine for Bayesian Inference



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- currently have basic lexer/parser, reading in simple models and constructing Java objects from them
- next build Java interface to Scilab, then start with some simple PF examples

# *Summary*

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- Applications in finance will rarely be time-homogeneous, so need to understand how to deal with such examples
- Need a way to take the labour out of building/running models - DEBI?
- Don't be unrealistic in what you expect SMC to be able to do