Week02 Project
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Problem 1 (3pts)

1. Using the normalized formulas in the Week 1 notes, I got the following results:

   Mean: 1.0489703904839585

   Varience: 5.427220681881727

   Skewness: 0.06975396906312004

   Kurtosis: 23.218000186050283

The Mean

$$\hat{\mu}_1 = E[X] = \frac{1}{n}\sum_{i}^{n} x_i$$

Variance – in small samples, dividing by n leads to a biased estimator.

$$\hat{\mu}_2 = E[(X - \hat{\mu}_1)^2] = \frac{1}{n-1}\sum_{i}^{n}(x_i - \hat{\mu}_1)^2$$

Unnormalized skew

$$\hat{\mu}_3 = E[(X - \hat{\mu}_1)^3] = \frac{n}{(n-1)(n-2)}\sum_{i}^{n}(x_i - \hat{\mu}_1)^3$$

Normalize to the formula above by dividing by σ^3

Unnormalized kurtosis

$$\hat{\mu}_4 = E[(X - \hat{\mu}_1)^4] = \frac{n^2}{(n-1)^3(n^2-3n+3)}[[(n(n-1)^2 + (6n-9))]K_{\bar{x}}(x) - n(6n-9)\sigma_{\bar{x}}^4(x)$$

Where

$K_{\bar{x}}(x)$ is the biased estimator for kurtosis (see link for formula)

$\sigma_{\bar{x}}^4(x)$ is the square of the biased estimator for variance.

2. Using my chosen statistical package, I got the following results:

   Mean: 1.0489703904839585

   Variance: 5.427220681881727

   Skewness: 0.8806086425277365

   Kurtosis: 26.12220078998973

3. The values of Skewness and Kurtosis are different in (1) and (2), and the functions I used in (1) are unbiased, so my statistical package functions are biased.

Problem 2 (5pts)

1. The table below shows the result of the OLS and MLE regression I performed in Python：

	OLS	MLE
B0	-0.087384	-0.08738448
B1	0.775274099	0.775274099
Standard deviation	1.0062751598133177	1.0037563156250344

The slight differences in the β and σ values between the OLS and MLE methods indicate that the assumptions of linearity and normally distributed errors are likely valid for this dataset.

The slight difference also due to finite sample size or slight violations of the assumptions.

2. The table below shows the result of MLE regressions and the AIC value of models:

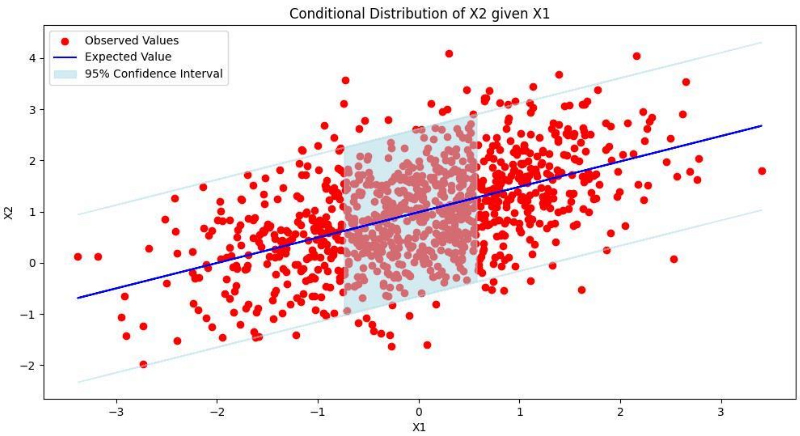
	MLE-normality assumption	MLE-T distribution assumption
Beta value	-0.08738448	0.667256883609476
Standard deviation	1.0037563156250344	0.8594749199340328
AIC	577.0238897254426	570.6422788011275

The MLE given the assumption of a T distribution of errors is better because of the lower AIC value.

3. The conditional distributions:

COV	X1	X2
X1	1.0697746428027168	0.9614732933624849
X2	0.9614732933624849	0.530684554713422

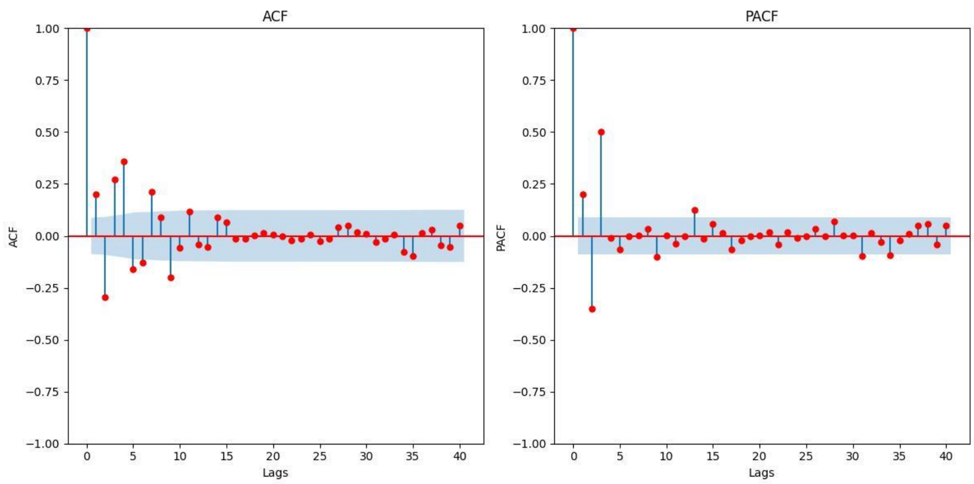
Ploted:



4. The maximum likelihood estimators:
Estimated Beta: 0.76903146
Estimated Sigma: 1.0075356745714914

Problem3(2pts)

Firstly: Look at the ACF and PACF graphs. I noticed that the ACF value shows a decreasing trend, so I predict that the AR model performs better.



The table below shows the result of AR(1) - AR(3) and MA(1) - MA(3) models:

	Coefficient		Coefficient
AR(1)	1.700242	MA(1)	2.123608
AR(2)	2.291383	MA(2)	2.125498
AR(3)	1.125224	MA(3)	2.125867

I used the Akaike Information Criterion (AIC) method to choose the model. The table below shows the AIC values of AR(1) - AR(3) and MA(1) - MA(3) models:

	AIC		AIC
AR(1)	1644.6555047688475	MA(1)	1567.4036263707876
AR(2)	1581.079265904978	MA(2)	1537.9412063807395
AR(3)	1436.6598066945812	MA(3)	1536.8677087350306

Since the AIC value for the AR(3) model is the smallest, it means this model fits the data best.