Week07 project

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Problem1

1.1 The result of comparing the values between the two methods for both a call and a put:

```
delta: (call, put)
GBSM:0.08301107089626869, -0.9169889291037313
FDD:0.08297130374668171, -0.9165496329472944
gamma: (call, put)
GBSM:0.016830979206204362, 0.016830979206204362
FDD:0.016822911064195978, 0.016822951920403284
vega: (call, put)
GBSM:6.942036604441163, 6.942036604441163
FDD:6.938653056250743, 6.93865305626673
theta: (call, put)
GBSM:-8.126522359668838, -1.9409914783019566
FDD:-8.126308803761084, -1.9407779203106656
rho: (call, put)
GBSM:1.1025939156368187, -13.758003122735788
FDD:-0.030359909416688424, -1.2427313238703164
carry rho: (call, put)
GBSM:1.132953825011723, -12.515271800549371
FDD:1.1329550097096686, -12.515270634423814
```

The reasons for discrepancy between the two methods for calculating Rho (GBSM vs. FDD) may be: 1) The FDD method is highly sensitive to numerical errors (step size Δr); 2) The underlying pricing model in FDD may differ in assumptions compared to the GBSM closed-form solution.

Carry Rho (sensitivity of option value to cost of carry, b) is typically not used for American options with dividends. This is because the cost of carry (b) does not directly appear as a parameter in pricing models for such options.

1.2 The binomial tree valuation for American options with and without discrete dividends:

```
Binomial tree value without dividend for call: 0.341603945513609
Binomial tree value without dividend for put: 14.020014650639181

Binomial tree value with dividend for call: 0.30041508863955924
Binomial tree value with dividend for put: 14.560397204371986
```

1.3 The Greeks of the call and the put are:

```
delta: (call, put)
0.07564529288392463 -0.9304253539674789
gamma: (call, put)
0.01624307464913788 0.014779533669457834
vega: (call, put)
6.42956107974818 6.0562654358991
theta: (call, put)
-7.5594852571106985 -0.8555281205477883
rho: (call, put)
0.9528888093222332 -12.430363528616262
```

1.4 Sensitivity to dividend amount:

Call: -0.034; Put: 0.934

Problem2

2.1 Calculate Mean, VaR and ES:

	Mean	VaR($)	ES($)
Call	3.097361	6.799017	6.799953
CallSpread	2.659462	4.589017	4.589953
CoveredCall	149.117511	7.260838	10.602005
ProtectedPut	156.756094	3.003057	4.272518
Put	9.631896	-0.268636	0.799042
PutSpread	5.011379	-0.219114	0.373946
Stock	150.074736	11.310838	14.652005
Straddle	12.729257	0.973351	0.984207
SynLong	-6.534535	17.095658	19.424308

2.2 Calculate VaR and ES using Delta-Normal:

Portfolio	Mean	VaR	ES
Call	0	9.922584	12.443321
CallSpread	0	3.36575	4.220786
CoveredCall	0	3.717425	4.661801
ProtectedPut	0	9.887767	12.399659
Put	0	2.265449	2.840964
PutSpread	0	1.102982	1.383185
Stock	0	12.108229	15.184207
Straddle	0	7.657136	9.602357
SynLong	0	12.188033	15.284285

2.3 Compare these results to last week's results (rerun the code from last week using these inputs):

	Portfolio	Current Value ($)	Mean Simulated Value ($)
0	Call	68000.0	NaN
1	CallSpread	45900.0	NaN
2	CoveredCall	1469800.0	NaN
3	ProtectedPut	1540400.0	83.861954
4	Put	48500.0	4.371051
5	PutSpread	30100.0	1.394586
6	Stock	1510300.0	165.063209
7	Straddle	116500.0	NaN
8	SynLong	19500.0	NaN

	VaR 95% ($ Loss)	ES 95% ($ Loss)
0	NaN	NaN
1	NaN	NaN
2	NaN	NaN
3	1.540225e+06	1.540321e+06
4	4.849190e+04	4.849590e+04
5	3.009294e+04	3.009898e+04
6	1.510122e+06	1.510136e+06
7	NaN	NaN
8	NaN	NaN

Compared to European options, the values of American options, as calculated in this week's problem, are generally higher. This is consistent with observations from last week's assignment, though there are exceptions—such as puts that are unlikely to be exercised, which may have lower values. Typically, higher values or returns are associated with greater risks, which explains why the VaR and ES for American options are often higher.

Problem3

3.1 The result of expected returns for each stock (annualized) in Python:

```
Expected Returns for Each Stock (Annualized):
AAPL: 0.000705
META: 0.001032
UNH: 0.000235
MA: 0.000591
MSFT: 0.000779
NVDA: 0.001424
HD: 0.000597
PFE: 0.000334
AMZN: 0.000963
BRK-B: 0.000483
PG: 0.000342
XOM: 0.000311
TSLA: 0.001256
JPM: 0.000525
V: 0.000568
DIS: 0.000513
GOOGL: 0.000861
JNJ: 0.000295
BAC: 0.000620
CSCO: 0.000535
```

3.2 The result of the annual covariance matrix for the given stocks is:

Annualized Covariance Matrix:

	AAPL	META	UNH	MA	MSFT	NVDA	HD
AAPL	0.050501	0.020870	-0.001577	0.010380	0.021561	0.033939	0.010867
META	0.020870	0.136272	-0.011660	0.015539	0.041722	0.077580	0.010079
UNH	-0.001577	-0.011660	0.049432	0.005066	-0.001927	-0.016474	0.006551
MA	0.010380	0.015539	0.005066	0.027007	0.012275	0.021657	0.011621
MSFT	0.021561	0.041722	-0.001927	0.012275	0.039604	0.045418	0.011498
NVDA	0.033939	0.077580	-0.016474	0.021657	0.045418	0.253770	0.023843
HD	0.010867	0.010079	0.006551	0.011621	0.011498	0.023843	0.042762
PFE	0.002864	0.000547	0.010636	0.003087	0.005221	-0.016984	0.005594
AMZN	0.024202	0.064354	-0.006390	0.015380	0.035189	0.067779	0.016908
BRK-B	0.006499	0.008382	0.006006	0.010321	0.006653	0.003171	0.010077
PG	-0.000199	-0.000426	0.007212	0.004023	0.002121	-0.009521	0.003260
XOM	-0.002407	-0.005812	0.001574	0.004131	-0.005784	-0.017253	0.005945
TSLA	0.046972	0.033953	0.001457	0.021594	0.029237	0.071756	0.028368
JPM	0.001670	0.006064	0.004938	0.008724	0.004695	0.005717	0.013261
V	0.010306	0.014285	0.003748	0.019691	0.012743	0.018001	0.010632
DIS	0.003784	0.010023	0.000456	0.007901	0.008303	0.015492	0.012559
GOOGL	0.025260	0.043208	-0.004703	0.011537	0.029757	0.051591	0.007758
JNJ	0.000410	-0.005947	0.008443	0.004104	-0.000898	-0.020411	0.006870
BAC	0.005001	0.005183	0.005151	0.006302	0.004584	-0.000161	0.022778
CSCO	0.008293	0.007240	0.005028	0.009910	0.008667	0.010232	0.013129

	PFE	AMZN	BRK-B	PG	XOM	TSLA	JPM
AAPL	0.002864	0.024202	0.006499	-0.000199	-0.002407	0.046972	0.001670
META	0.000547	0.064354	0.008382	-0.000426	-0.005812	0.033953	0.006064
UNH	0.010636	-0.006390	0.006006	0.007212	0.001574	0.001457	0.004938
MA	0.003087	0.015380	0.010321	0.004023	0.004131	0.021594	0.008724
MSFT	0.005221	0.035189	0.006653	0.002121	-0.005784	0.029237	0.004695
NVDA	-0.016984	0.067779	0.003171	-0.009521	-0.017253	0.071756	0.005717
HD	0.005594	0.016908	0.010077	0.003260	0.005945	0.028368	0.013261
PFE	0.062464	0.002466	0.010833	0.002163	0.003940	0.002940	0.010246
AMZN	0.002466	0.081874	0.008978	-0.001785	-0.005728	0.046419	0.008206
BRK-B	0.010833	0.008978	0.016801	0.004501	0.008345	0.009049	0.013116
PG	0.002163	-0.001785	0.004501	0.022672	0.002867	-0.003999	0.000597
XOM	0.003940	-0.005728	0.008345	0.002867	0.041071	-0.001924	0.011345
TSLA	0.002940	0.046419	0.009049	-0.003999	-0.001924	0.293876	0.020562
JPM	0.010246	0.008206	0.013116	0.000597	0.011345	0.020562	0.033249
V	0.004419	0.013531	0.009401	0.004643	0.002715	0.021799	0.008395
DIS	0.008145	0.013054	0.008012	-0.000892	0.009776	0.026769	0.011302
GOOGL	0.003924	0.045175	0.008250	-0.000272	-0.004480	0.031925	0.006162
JNJ	0.016889	-0.002814	0.007518	0.005866	0.005893	-0.001402	0.007898
BAC	0.010263	0.008630	0.014265	0.002463	0.016620	0.028866	0.028629
CSCO	0.010953	0.018381	0.007840	0.002357	0.008121	0.017523	0.009708

	V	DIS	GOOGL	JNJ	BAC	CSCO
AAPL	0.010306	0.003784	0.025260	0.000410	0.005001	0.008293
META	0.014285	0.010023	0.043208	-0.005947	0.005183	0.007240
UNH	0.003748	0.000456	-0.004703	0.008443	0.005151	0.005028
MA	0.019691	0.007901	0.011537	0.004104	0.006302	0.009910
MSFT	0.012743	0.008303	0.029757	-0.000898	0.004584	0.008667
NVDA	0.018001	0.015492	0.051591	-0.020411	-0.000161	0.010232
HD	0.010632	0.012559	0.007758	0.006870	0.022778	0.013129
PFE	0.004419	0.008145	0.003924	0.016889	0.010263	0.010953
AMZN	0.013531	0.013054	0.045175	-0.002814	0.008630	0.018381
BRK-B	0.009401	0.008012	0.008250	0.007518	0.014265	0.007840
PG	0.004643	-0.000892	-0.000272	0.005866	0.002463	0.002357
XOM	0.002715	0.009776	-0.004480	0.005893	0.016620	0.008121
TSLA	0.021799	0.026769	0.031925	-0.001402	0.028866	0.017523
JPM	0.008395	0.011302	0.006162	0.007898	0.028629	0.009708
V	0.022884	0.005352	0.011023	0.004906	0.008003	0.009791
DIS	0.005352	0.071514	0.009315	0.001750	0.015920	0.010569
GOOGL	0.011023	0.009315	0.077409	-0.002217	0.006355	0.013982
JNJ	0.004906	0.001750	-0.002217	0.023668	0.010143	0.006289
BAC	0.008003	0.015920	0.006355	0.010143	0.056120	0.011909
CSCO	0.009791	0.010569	0.013982	0.006289	0.011909	0.040735

3.3 The super-efficient portfolio is:

```
Optimal Portfolio Weights:
Stock  Weight (%)
 AAPL        0.0
 META       30.0
  UNH        0.0
   MA        0.0
 MSFT        0.0
 NVDA       30.0
   HD        0.0
  PFE        0.0
 AMZN       10.0
BRK-B        0.0
   PG        0.0
  XOM        0.0
 TSLA       30.0
  JPM        0.0
    V        0.0
  DIS        0.0
GOOGL        0.0
  JNJ        0.0
  BAC        0.0
 CSCO        0.0
```

I set up a diversification constraint: restrict the maximum allocation for each stock to avoid overly concentrated portfolios (Limit each stock to a maximum of 30% of the portfolio).

From the debug outputs, the portfolio weights are highly concentrated in a few stocks (META, NVDA, TSLA, and AMZN).