Week04 project

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Problem 1

Given to the 3 types of price returns and $rt \sim N(0, \ \sigma^2)$, we know that the mean and standard deviation as following:

Method	Model	Expected Value	Standard Deviation
Classical Brownian Motion	$P_t = P_{t-1} + r_t$	P_{t-1}	σ
Arithmetic Return System	$P_t = P_{t-1}(1+r)$	P_{t-1}	$P_{t-1} * \sigma$
Log Return or Geometric Brownian Motion	$P_t = P_{t-1} e^{r_t}$	$P_{t-1} * e^{\frac{\sigma^2}{2}}$	$P_{t-1}\sqrt{e^{\sigma^2} - 1}$

Using python to calculate, we can see the tiny difference between estimated one and expected one as following:

Method	Expected Value	Estimated Value	Expected Std	Estimated Std
Classical Brownian Motion	100	99.9988	0.1	0.0996
Arithmetic Return System	100	99.8815	10	9.9642
Log Return or Geometric Brownian Motion	100.5012	100.3785	10.0250	10.0226

Problem 2

2.1

The process and arithmetic returns for all prices can be seen in the python.

```
        SPY       AAPL      MSFT      AMZN      TSLA  ...       SYK        GM       TFC       TJX                Date
0 -0.006723 -0.035793 -0.002009 -0.013914 -0.017817  ...  0.009815 -0.013822 -0.023653 -0.009387  2023-09-06 00:00:00
1 -0.003070 -0.029249 -0.008922  0.018395 -0.001707  ... -0.000277 -0.007617 -0.015141  0.007602  2023-09-07 00:00:00
2  0.001506  0.003492  0.013216  0.002757 -0.011889  ...  0.001799  0.011667  0.017424  0.000000  2023-09-08 00:00:00
3  0.006577  0.006623  0.010979  0.035231  0.100925  ...  0.035159 -0.010015  0.011081  0.001422  2023-09-11 00:00:00
4 -0.005486 -0.017061 -0.018258 -0.013068 -0.022297  ... -0.016515  0.026058 -0.003321  0.006552  2023-09-12 00:00:00
```

2.2

Using the stock prices of META and 5 methods to calculate VaR, the results of VaR (%) are as following:

	VaR
Normal Distribution	-0.0382
EWMA	-0.0319
MLE T Distribution	-0.0324
AR(1)	-0.0380
Historical Simulation	-0.0288

(1) The Normal Distribution method shows a VaR of approximately -0.0382, indicating a potential daily loss of over 3.82% in META.
(2) The EWMA method results in a VaR of about -0.0319, suggesting a slightly lower potential loss than the normal distribution.
(3) The MLE T Distribution provides a VaR of around -0.0324, reflecting similar risks to the EWMA method due to its consideration of fat tails in returns.
(4) The AR(1) Model yields a VaR of roughly -0.0380, close to the normal distribution's value, indicating that it captures similar risk levels.
(5) The Historical Simulation method shows a VaR of about -0.0288, representing the lowest potential loss based on past data.

Problem 3

I chose Delta Normal, Monte Carlo, and Historical Simulation models to compare different approaches for calculating VaR. Delta Normal assumes normally distributed returns, which simplifies the calculation. Monte Carlo uses simulations to account for more complex risks, while Historical Simulation relies on actual past returns, which can better capture extreme events.

Using Delta Normal, Monte Carlo, and Historical methods to calculate the VaR for portfolios:

	A	B	C	ALL
Delta Normal	20037.39	11829.61	27219.96	54418.03
Monte Carlo	17311.31	10740.21	24870.65	49413.94
Historic	16884.39	8976.78	20198.81	46410.02

During the process, I found that some stocks in portfolio C couldn't be matched with corresponding values in DailyPrices.csv. So, when calculating the portfolio VaR, I removed the missing values. The specific stock names are as follows:

```
The following stocks are missing from the price data and will be excluded:
   Portfolio Stock  Holding
71         C   ELV      126
78         C   MMC       84
85         C  VRTX      177
87         C  REGN      173
88         C    CB      190
90         C    CI      159
91         C   ETN       96
92         C   SLB      173
93         C   PGR      174
98         C   BSX      188
```

I found that the VaR values calculated using the Delta Normal model were the highest, followed by Monte Carlo, with the Historical model giving the lowest values. I think this difference is due to the assumptions of each model: Delta Normal assumes normally distributed returns, leading to higher risk estimates, while Monte Carlo simulates a range of outcomes based on historical data, and Historical Simulation uses actual past returns, which may have fewer extreme events and thus lower risk estimates.