COMP30660 - Computer Arch & Org (Conv) -2022/23

Assignment 1: Number System and Logic

[15 Points]

Student Name: Wenqing Zhao
Student Number: 21211886

1. Conversion
   a. Complete this summation by converting the decimal numbers to binary by using Double Dabble Method and then converting the result of the sum back to decimal (Show the steps to get full marks) [1 Point]

      105 ???? ????

      <u>114 ???? ????</u>

      ??? ???? ????

2	105		114	
2	52	1	57	0
2	26	0	28	1
2	13	0	14	0
2	6	1	7	0
2	3	0	3	1
2	1	1	1	1
	0	1	0	1

105   1101001
+ 114   1110010

219   11011011

1 1 0 1 1 0 1 1

$2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$

$= 128 + 64 + 0 + 16 + 8 + 0 + 2 + 1$

$= 219$

   b. Complete this summation by converting the decimal numbers to octal by using Direct Method and then converting the result of the sum back to decimal (Show the steps to get full marks) [1 Point]

      77 ???? ????

      <u>113 ???? ????</u>

      ??? ???? ????

2	77		113	
2	38	1	56	1
2	19	0	28	0
2	9	1	14	0
2	4	1	7	0
2	2	0	3	1
2	1	0	1	1
	0	1	0	1

77   1001101
+ 113   1110001

190   10111110

1 0 1 1 1 1 1 0

$2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$

$= 128 + 0 + 32 + 16 + 8 + 4 + 2 + 0$

$= 190$

2. Represent following decimal numbers using, 8-bit Signed magnitude binary representation, 8 bits Signed 1's complement binary representation and 8 bit Signed 2's complement binary representation [1 Point]

a) 41

41

2 | 41
2 | 20 1
2 | 10 0
2 | 5 0
2 | 2 1
2 | 1 0
0 1

0 0 1 0 1 0 0 1
1's complement: 0 0 1 0 1 0 0 1
2's complement: 0 0 1 0 1 0 0 1

-41

2 | 41
2 | 20 1
2 | 10 0
2 | 5 0
2 | 2 1
2 | 1 0
0 1

1 0 1 0 1 0 0 1
1's complement: 1 1 0 1 0 1 1 0
2's complement: 1 1 0 1 0 1 1 1

b) -41
c) 32
d) -32

32

2 | 32
2 | 16 0
2 | 8 0
2 | 4 0
2 | 2 0
2 | 1 0
0 1

0 0 1 0 0 0 0 0
1's complement: 0 0 1 0 0 0 0 0
2's complement: 0 0 1 0 0 0 0 0

-32

2 | 32
2 | 16 0
2 | 8 0
2 | 4 0
2 | 2 0
2 | 1 0
0 1

1 0 1 0 0 0 0 0
1's complement: 1 1 0 1 1 1 1 1
2's complement: 1 1 1 0 0 0 0 0

e) 101
f) -71

101

2 | 101
2 | 50 1
2 | 25 0
2 | 12 1
2 | 6 0
2 | 3 0
2 | 1 1
0 1

0 1 1 0 0 1 0 1
1's complements 0 1 1 0 0 1 0 1
2's complements 0 1 1 0 0 1 0 1

-71

2 | 71
2 | 35 1
2 | 17 1
2 | 8 1
2 | 4 0
2 | 2 0
2 | 1 0
0 1

1 1 0 0 0 1 1 1
1's complements 1 0 1 1 1 0 0 0
2's complements 1 0 1 1 1 0 0 1

g) 2^7 out of range, so it doesn't has 1's complement and 2's complement.
h) -2^7

-2^7 $\quad -2^7 = -128$

2 | 128
2 | 64 0
2 | 32 0
2 | 16 0
2 | 8 0
2 | 4 0
2 | 2 0
2 | 1 0
0 1

1 0 0 0 0 0 0 0
1's complements 1 1 1 1 1 1 1 1
2's complements 1 0 0 0 0 0 0 0

3. Convert the following numbers with the indicated bases into decimal [1 Point]

a. $(23410)_5$
b. $(645213)_7$

2 3 4 1 0

$2\times5^4 + 3\times5^3 + 4\times5^2 + 1\times5^1 + 0$

$= 1250 + 375 + 100 + 5$

$= 1730$

6 4 5 2 1 3

$= 6\times7^5 + 4\times7^4 + 5\times7^3 + 2\times7^2 + 7^1 + 3\times7^0$

$= 100842 + 9604 + 1715 + 98 + 7 + 3$

$= 112269$

c. $(856125)_9$

d. $(B24AC)_{16}$

8 5 6 1 2 5

$= 8\times 9^5 + 5\times 9^4 + 6\times 9^3 + 9^2 + 2\times 9^1 + 5\times 9^0$

$= 472392 + 32805 + 4374 + 81 + 18 + 5$

$= 509675$

B 2 4 A C

$= 11\times 16^4 + 2\times 16^3 + 4\times 16^2 + 10\times 16^1 + 12\times 16^0$

$= 720896 + 8192 + 1024 + 160 + 12$

$= 730284$

4. Add the following binary numbers [1 Point]
   a. 10101.10101 + 111.1101
   b. 1010.10011 + 0.1101

$$\begin{array}{r} 10101.10101 \\ +\ 111.1101 \\ \hline 11101.01111 \end{array} \qquad \begin{array}{r} 1010.10011 \\ +\ 0.1101 \\ \hline 1011.01101 \end{array}$$

   c. 1011.0101001 + 10011.10101
   d. 10111.11001 + 101.1

$$\begin{array}{r} 1011.0101001 \\ +\ 10011.10101 \\ \hline 11110.1111101 \end{array} \qquad \begin{array}{r} 10111.11001 \\ +\ 101.1 \\ \hline 11101.01001 \end{array}$$

5. Perform the following operations. (Note: Please consider the base) [1 Point]
   a. $(A914)_{16} + (BC12A)_{16}$
   b. $(E189A)_{16} + (BB87A)_{16}$

$$\begin{array}{r} A914 \\ +\ BC12A \\ \hline C6A3E \end{array} \qquad \begin{array}{r} E189A \\ +\ BB87A \\ \hline 19D114 \end{array}$$

   c. $(54623)_7 + (23165)_7$
   d. $(854162)_9 - (264712)_9$

$$\begin{array}{r} 54623 \\ +\ 23165 \\ \hline 111121 \end{array} \qquad \begin{array}{r} 854162 \\ -\ 264712 \\ \hline 578350 \end{array}$$

   e. $(BB78A2)_{16} - (A16B23)_{16}$
   f. $(4213)_5 + (3213)_5$

$$\begin{array}{r} BB78A2 \\ -\ A16B23 \\ \hline 1A0D7F \end{array} \qquad \begin{array}{r} 4213 \\ +\ 3213 \\ \hline 12431 \end{array}$$

   g. $(D4715A)_{16} - (A1527B)_{16}$

$$\begin{array}{r} D4715A \\ -\ A1527B \\ \hline 331EDF \end{array}$$

6. Logic Circuits

A. Write the Boolean equations for each of the logic circuits shown below [0.5 Point]

$$W = \overline{AB} \cdot (B + C)$$

$$X = (A \cdot B + B \cdot C) \cdot B \cdot C$$

$$Y = \overline{AB} \oplus (\overline{C + D}) + \overline{AB} \cdot (\overline{C + D})$$

$$Z = (\overline{AB} \oplus C \oplus \overline{CD}) + D$$

B. Create the truth table for each circuit[1 Point]

(a)

A	B	C	W
0	0	0	0
1	0	0	0
0	1	0	1
0	0	1	1
1	1	0	0
1	0	1	1
0	1	1	1
1	1	1	0

(b)

A	B	C	X
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
1	0	1	0
0	1	1	1
1	1	1	1

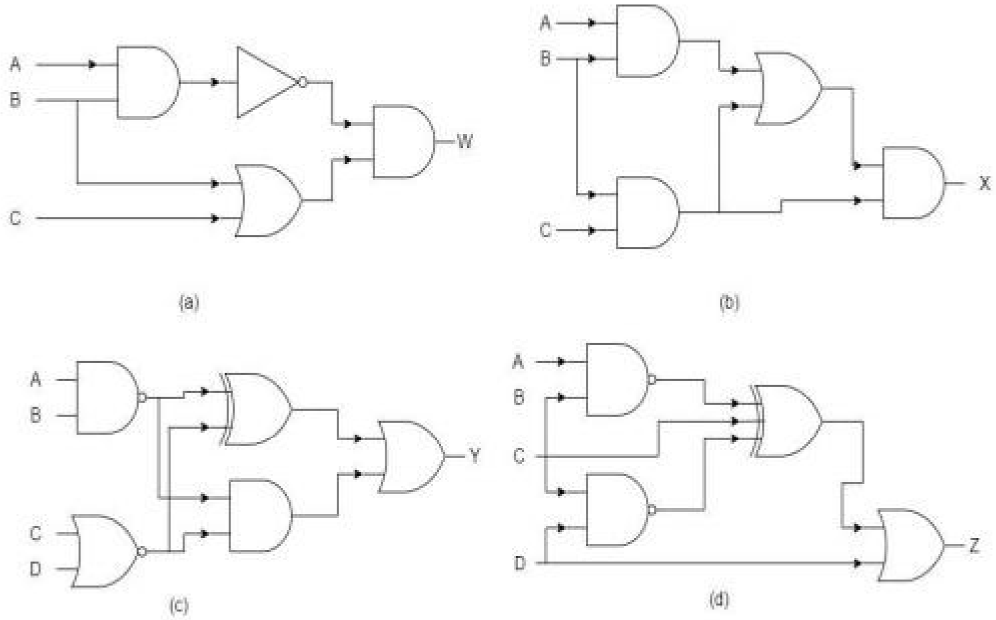
(c)

A	B	C	D	Y
0	0	0	0	1
1	0	0	0	1
0	1	0	0	1
0	0	1	0	1
0	0	0	1	1
1	1	0	0	1
0	1	1	0	1
0	0	1	1	1
1	0	1	0	1

0	1	0	1	1
1	0	0	1	1
1	1	1	0	0
0	1	1	1	1
1	0	1	1	1
1	1	0	1	0
1	1	1	1	0

(d)

A	B	C	D	Z
0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	0	1	0	1
0	0	0	1	1
1	1	0	0	1
0	1	1	0	1
0	0	1	1	1
1	0	1	0	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	0
0	1	1	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1



7.

(a) $X = W\overline{Z}(W+Y) + WY(\overline{Z}+\overline{W})$

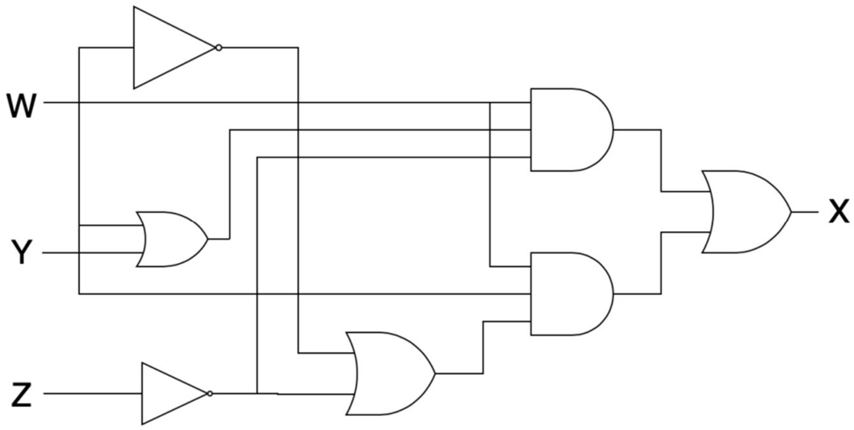
(b) $X = \overline{(A+B)(\overline{C})}$

(c) $X = (\overline{A\overline{B}\,C + B\overline{C}})$

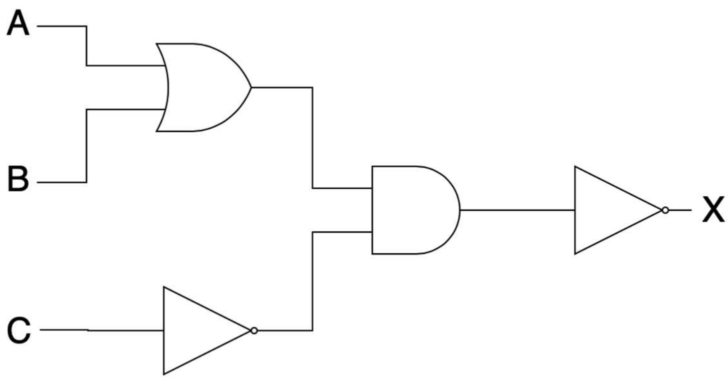
(d) $X = (A\overline{B}\,C + \overline{ABC})\,C$

a. Draw the logic circuit diagram for the Boolean equations above [1 Point]

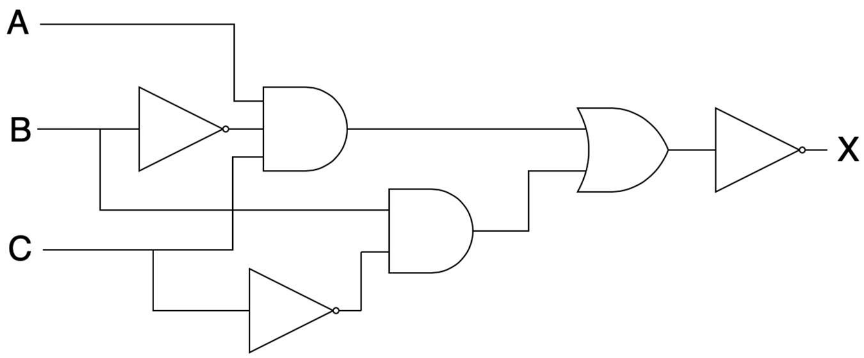
(a)



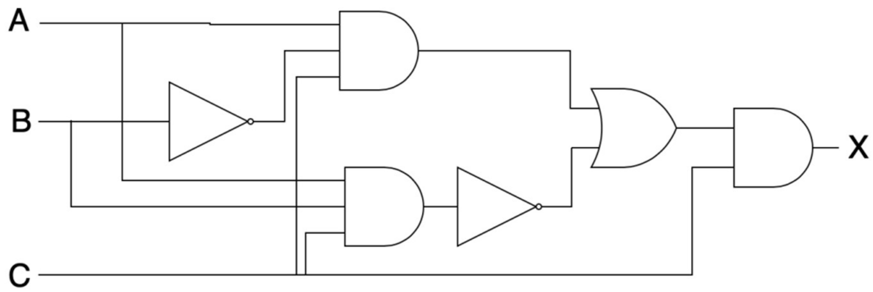
(b)



(c)



(d)



b. Using the DeMorgan's Theorem and Boolean Laws and Rules, simplify and reduce to the stage where the overbars are above a single variable only [1 Point]

(a)

$$
\begin{aligned}
X &= W\bar{Z}(W+Y) + WY(\bar{Z}+\bar{W}) \\
&= WW\bar{Z} + W\bar{Z}Y + WY\bar{Z} + W\bar{W}Y \\
&= W\bar{Z} + W\bar{Z}Y + W\bar{Z}Y \\
&= W\bar{Z}(1+Y) + W\bar{Z}Y \\
&= W\bar{Z}(1+Y) \\
&= W\bar{Z}
\end{aligned}
$$

(b)

$$
\begin{aligned}
X &= \overline{(A+B)(\bar{C})} \\
&= \overline{(A+B)} + \bar{\bar{C}} \\
&= \bar{A}\bar{B} + C
\end{aligned}
$$

(c)

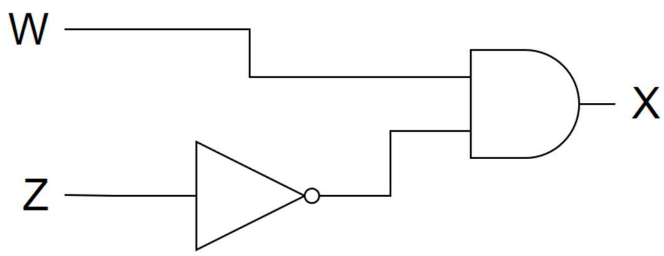
$$
\begin{aligned}
X &= \overline{(A\bar{B}C + B\bar{C})} \\
&= (\overline{A\bar{B}C})(\overline{B\bar{C}}) \\
&= (\bar{A}+B+\bar{C})(\bar{B}+C) \\
&= \bar{A}\bar{B} + B\bar{B} + \bar{B}\bar{C} + \bar{A}C + BC + \bar{C}C \\
&= \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}C + BC
\end{aligned}
$$

(d)

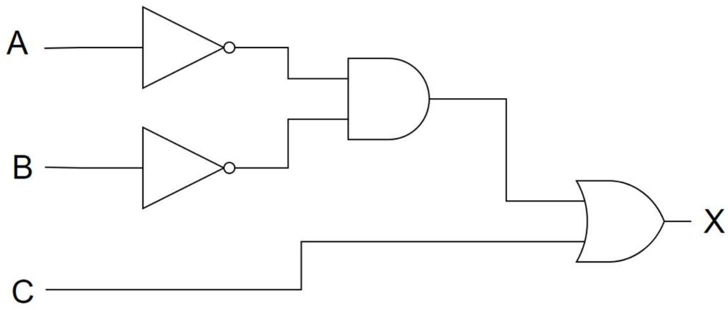
$$
\begin{aligned}
X &= (A\bar{B}C + \overline{ABC})C \\
&= (A\bar{B}C + \bar{A} + \bar{B} + \bar{C})C \\
&= A\bar{B}C + \bar{A}C + \bar{B}C \\
&= (A+1)\bar{B}C + \bar{A}C \\
&= (\bar{A}+\bar{B})C
\end{aligned}
$$

c. Draw the simplified circuit [0.5 Point]

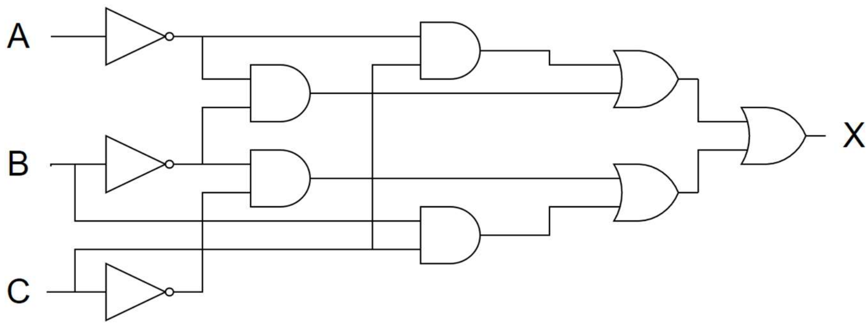
(a)



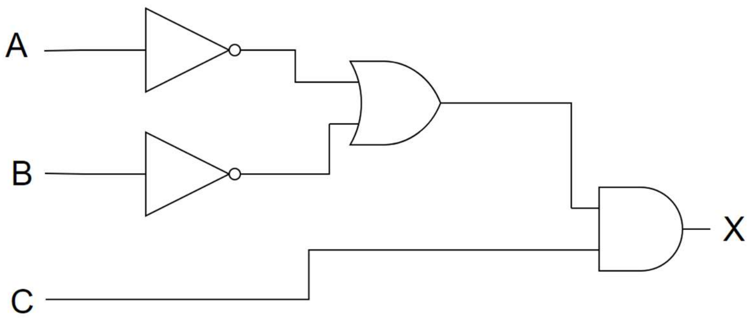
(b)



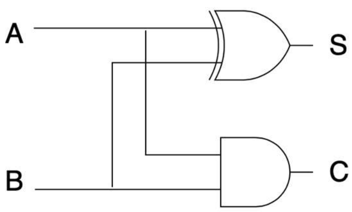
(c)

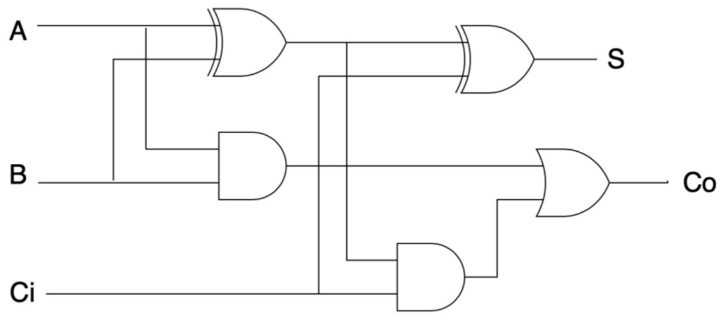


(d)

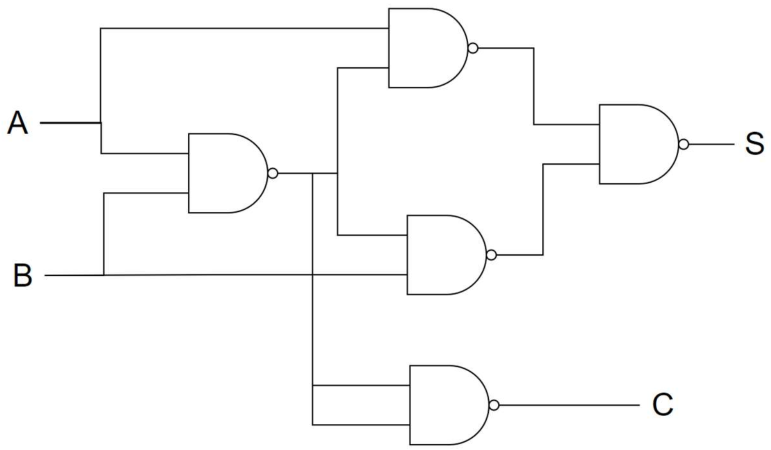


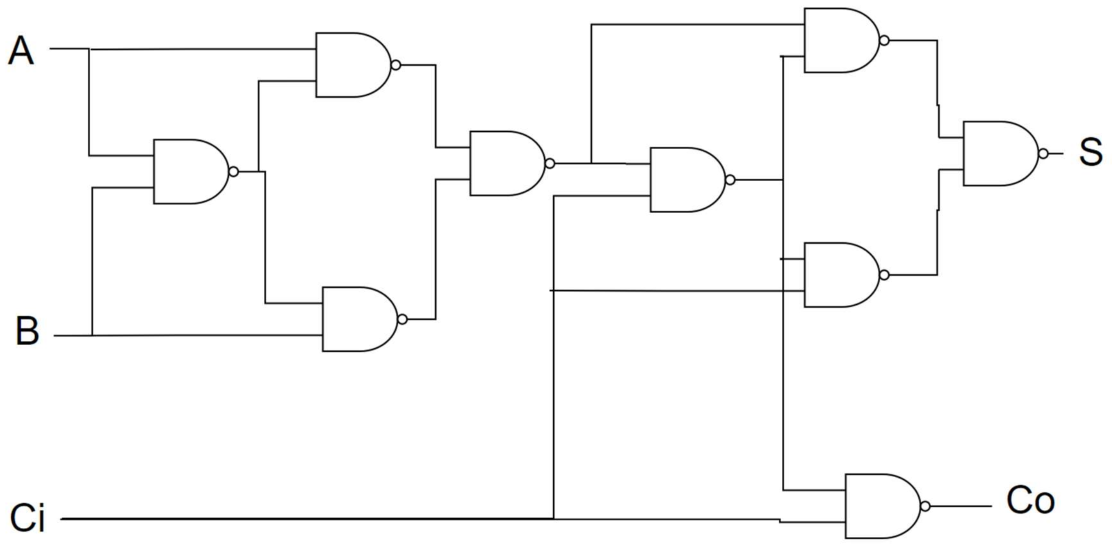
8. a) Draw half adder and full adder circuits [0.5 points]



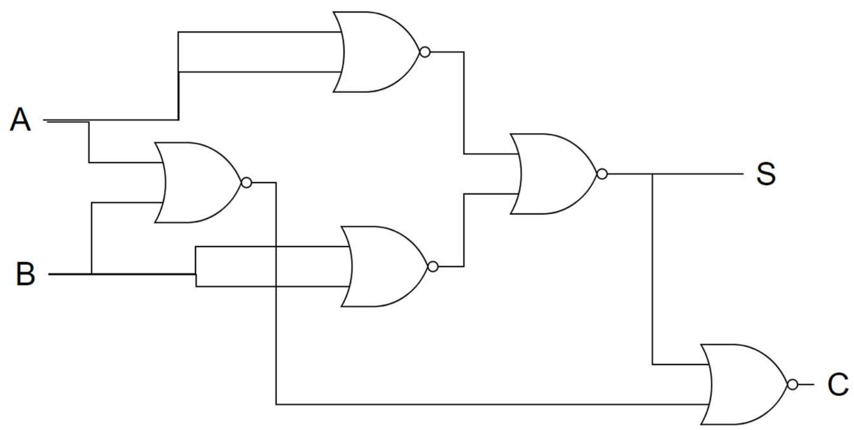


b) Draw them only using NAND gates [0.5 points]





c) Draw them only using NOR gates [1 point]



9. Determine which rule (or rules) are being used in the following Boolean reductions and show the steps of reduction [2 Points]

$$\overline{(\bar{X}.\bar{\bar{Y}}).\overline{(\bar{Y}+Z)}} = \text{x}.\ \bar{Y} + \text{Y}.\ \bar{Z}$$
$$\overline{(\bar{X}+\bar{Z}).(\bar{\bar{X}}\,\bar{Y})} = \text{x}(\overline{Z} + \text{Y})$$
$$\text{AB} + \text{AC} + \text{ABC} = \text{A}(\text{B} + \text{C})$$
$$\text{AB} + \text{A}(\bar{B} + \text{C}) + \text{AB}\,\bar{C} = \text{A}$$
$$(\text{A} + \text{B})(\text{A} + \text{C}) = \text{A} + \text{BC}$$

$$(A+\bar{B}+\bar{C})(A+\bar{B}+C)(A+B+\bar{C}) = A+\bar{B}\bar{C}$$

$$\overline{(X\overline{\overline{Y}})(\overline{Y}+Z)}$$
$$= X\overline{Y} + \overline{(\overline{Y}+Z)}$$
$$= X\overline{Y} + Y\overline{Z}$$

$$\overline{(\overline{X}+Z)(\overline{XY})}$$
$$= \overline{(\overline{X}+Z)} + XY$$
$$= X\overline{Z} + XY$$
$$= X(\overline{Z}+Y)$$

$$AB + AC + ABC$$
$$= A(B+C+BC)$$
$$= A(B(1+C)+C)$$
$$= A(B+C)$$

$$AB + A(\overline{B}+C) + AB\overline{C}$$
$$= A(B+\overline{B}+C+B\overline{C})$$
$$= A(1+C+B\overline{C})$$
$$= A(1+B\overline{C})$$
$$= A$$

$$(A+B)(A+C)$$
$$= AA + AB + AC + BC$$
$$= A + AB + AC + BC$$
$$= A(1+B) + AC + BC$$
$$= A + AC + BC$$
$$= A(1+C) + BC$$
$$= A + BC$$

$$(A+\overline{B}+\overline{C})(A+\overline{B}+C)(A+B+\overline{C})$$
$$= (AA + A\overline{B} + AC + A\overline{B} + \overline{B}\overline{B} + \overline{B}C + A\overline{C} + \overline{B}\overline{C} + \overline{C}C)\cdot(A+B+\overline{C})$$
$$= (A(1+\overline{B}) + AC + A\overline{B} + \overline{B}(1+C) + A\overline{C} + \overline{B}\overline{C})\cdot(A+B+\overline{C})$$
$$= (A+\overline{B})(A+B+\overline{C})$$
$$= AA + AB + A\overline{C} + A\overline{B} + \overline{B}B + \overline{B}\overline{C}$$
$$= A(1+B) + A\overline{C} + A\overline{B} + \overline{B}\overline{C}$$
$$= A + \overline{B}\overline{C}$$

10. Perform following calculations with bitwise operators (consider the base) [1 point]

a) (0b1010 ⊕ 0b1001) ⊕ 0b1100

0b1010 → 1011000100000000100000
⊕ 0b1001 → 10110001000000000000001
000011 ← 00000000000000000100001
⊕ 0b1100 → 101100010001000000000
0b1111 ← 10110001000100010001

b) (0xA10 ^ 0xBA2) V (0x57A)

0xA10 → 101000010000

∧ 0xBA2 → 101110100010

0xA00 ← 101000000000

∨ 0x57A → 010101111010

0xF7A ← 111101111010

c) (0xA1 ^ 0xB6) ⊕ (0xC3 ∨ 0xD2)

0xA1 → 10100001

∧ 0xB6 → 10110110

0xA0 ← 10100000

0xC3 → 11000011

∨ 0xD2 → 11010010

0xD3 ← 11010011

0xA0 → 10100000

⊕ 0xD3 → 11010011

0x73 ← 01110011

d) (¬0xA7) ∨(¬0xBD)

0xA7 → 10100111

0x58 ← 01011000

0xBD → 10111101

0x42 ← 01000010

0x58 → 01011000

∨ 0x42 → 01000010

0x5A ← 01011010