A Low Complexity Algorithm for Proportional Resource Allocation in OFDMA Systems

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Abstract— **Orthogonal Frequency Division Multiple Access (OFDMA) basestations allow multiple users to transmit simultaneously on different subcarriers during the same symbol period. This paper considers basestation allocation of subcarriers and power to each user to maximize the sum of user data rates, subject to constraints on total power, bit error rate, and proportionality among user data rates. Previous allocation methods have been iterative nonlinear methods suitable for offline optimization. In the special high subchannel SNR case, an iterative root-finding method has linear-time complexity in the number of users and $N \log N$ complexity in the number of subchannels. We propose a non-iterative method that is made possible by our relaxation of strict user rate proportionality constraints. Compared to the root-finding method, the proposed method waives the restriction of high subchannel SNR, has significantly lower complexity, and in simulation, yields higher user data rates.**

I. INTRODUCTION

OFDMA, also referred to as Multiuser-OFDM [1], is being considered as a modulation and multiple access method for 4th generation wireless networks [2]. OFDMA is an extension of Orthogonal Frequency Division Multiplexing (OFDM), which is currently the modulation of choice for high speed data access systems such as IEEE 802.11a/g wireless LAN [3] and IEEE 802.16a fixed wireless broadband access [4] systems.

OFDM systems divide a broadband channel into many narrowband subchannels. Each subchannel carries a quadrature amplitude modulated (QAM) signal. The subcarriers are combined in a computationally efficient manner by means of an inverse fast Fourier transform (IFFT) in the transmitter. Each complex-valued IFFT input is obtained from a QAM constellation mapping (lookup table). The IFFT outputs form the transmitted symbol. Before transmission, a cyclic prefix (copy of the last few symbol samples) is prepended to the symbol. The receiver performs the dual operations of cyclic prefix (CP) removal, FFT, and QAM demapping.

In current OFDM systems, only a single user can transmit on all of the subcarriers at any given time, and time division or frequency division multiple access is employed to support multiple users. The major setback to this static multiple access scheme is the fact that the different users see the wireless channel differently is not being utilized. OFDMA, on the other hand, allows multiple users to transmit simultaneously on the different subcarriers per OFDM symbol. Since the probability that all users experience a deep fade in a particular subcarrier is very low, it can be assured that subcarriers are assigned to the users who see good channel gains on them.

The problem of assigning subcarriers and power to the different users in an OFDMA system has recently been an area of active research. In [5], the margin-adaptive resource allocation problem was tackled, wherein an iterative subcarrier and power allocation algorithm was proposed to minimize the total transmit power given a set of fixed user data rates and bit error rate (BER) requirements. In [6], the rate-adaptive problem was investigated, wherein the objective was to maximize the total data rate over all users subject to power and BER constraints. It was shown in [6] that in order to maximize the total capacity, each subcarrier should be allocated to the user with the best gain on it, and the power should be allocated using the water-filling algorithm across the subcarriers. However, no fairness among the users was considered in [6]. This problem was partially addressed in [7] by ensuring that each user would be able to transmit at a minimum rate, and also in [8] by incorporating a notion of fairness in the resource allocation through maximizing the minimum user's data rate. In [9], the fairness was extended to incorporate varying priorities. Instead of maximizing the minimum user's capacity, the total capacity was maximized subject to user rate proportionality constraints. This is very useful for service level differentiation, which allows for flexible billing mechanisms for different classes of users. However, the algorithm proposed in [9] involves solving non-linear equations, which requires computationally expensive iterative operations and is thus not suitable for a cost-effective real-time implementation.

This paper extends the work in [9] by developing a subcarrier allocation scheme that linearizes the power allocation problem while achieving approximate rate proportionality. The resulting power allocation problem is thus reduced to a solution to simultaneous linear equations. In simulation, the proposed algorithm achieves a total capacity that is consistently higher than the previous work, requires significantly less computation, while achieving acceptable rate proportionality.

II. SYSTEM MODEL

The block diagram for the downlink of a typical OFDMA system is shown in Fig. 1. At the base station transmitter, the bits for each of the different K users are allocated to the N subcarriers, and each subcarrier n ($1 \leq n \leq N$) of user
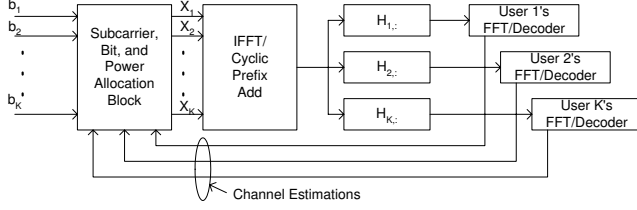
Fig. 1. OFDMA system block diagram for K users. Each user is allocated different set of subcarriers by the basestation.

k $(1 \leq k \leq K)$ is assigned a power $p_{k,n}$. It is assumed that subcarriers are not shared by different users. Each of the user's bits are then modulated into N M-level QAM symbols, which are subsequently combined using the IFFT into an OFDMA symbol. This is then transmitted through a slowly time-varying, frequency-selective Rayleigh channel with a bandwidth B. The subcarrier allocation is made known to all the users through a control channel; hence each user needs only to decode the bits on their assigned subcarriers.

It is assumed that each user experiences independent fading and the channel gain of user k in subcarrier n is denoted as $g_{k,n}$, with additive white gaussian noise (AWGN) $\sigma^2 = N_o \frac{B}{N}$ where N_o is the noise power spectral density. The corresponding subchannel signal-to-noise ratio (SNR) is thus denoted as $h_{k,n} = g_{k,n}^2/\sigma^2$ and the kth user's received SNR on subcarrier n is $\gamma_{k,n} = p_{k,n} h_{k,n}$. The slowly time-varying assumption is crucial since it is also assumed that each user is able to estimate the channel perfectly and these estimates are made known to the transmitter via a dedicated feedback channel. These channel estimates are then used as input to the resource allocation algorithms.

In order that the BER constraints be met, the effective SNR has to be adjusted accordingly. The BER of a square M-level QAM with Gray bit mapping as a function of received SNR $\gamma_{k,n}$ and number of bits $r_{k,n}$ can be approximated to within 1 dB for $r_{k,n} \geq 4$ and BER $\leq 10^{-3}$ as [10]

$$\text{BER}_{\text{MQAM}}(\gamma_{k,n}) \approx 0.2 \exp \left[\frac{-1.6\gamma_{k,n}}{2^{r_{k,n}} - 1} \right]. \qquad (1)$$

Solving for $r_{k,n}$, we have

$$r_{k,n} = \log_2 \left(1 + \frac{\gamma_{k,n}}{\Gamma} \right) = \log_2 \left(1 + p_{k,n} H_{k,n} \right) \qquad (2)$$

where $\Gamma \triangleq -\ln(5\text{BER})/1.6$ is a constant SNR gap, and $H_{k,n} \triangleq h_{k,n}/\Gamma$ is the effective subchannel SNR.

The objective of the resource allocation is formulated as

$$\max_{c_{k,n}, p_{k,n}} \frac{B}{N} \sum_{k=1}^{K} \sum_{n=1}^{N} c_{k,n} \log_2(1 + p_{k,n} H_{k,n}) \qquad (3)$$

subject to:
C1: $c_{k,n} \in \{0,1\} \ \forall k, n$
C2: $p_{k,n} \geq 0 \ \forall k, n$
C3: $\sum_{k=1}^{K} c_{k,n} = 1 \ \forall n$
C4: $\sum_{k=1}^{K} \sum_{n=1}^{N} c_{k,n} p_{k,n} \leq P_{tot}$
C5: $R_i : R_j = \phi_i : \phi_j \ \forall i, j \in \{1, \cdots, K\}, i \neq j$

where $c_{k,n}$ is the subcarrier allocation indicator such that $c_{k,n} = 1$ if and only if subcarrier n is assigned to user k, and P_{tot} is the transmit power constraint. In C5,

$$R_k = \frac{B}{N} \sum_{n=1}^{N} c_{k,n} r_{k,n} \qquad (4)$$

is the total data rate for user k and $\phi_1 : \phi_2 : \cdots : \phi_K$ are the normalized proportionality constants where $\sum_{k=1}^{K} \phi_k = 1$.

Note that constraints C1 and C2 in (3) ensure the correct values for the subcarrier allocation indicator and the power, respectively. C3 imposes the restriction that each subcarrier can only be assigned to one user, and C4 and C5 are the power and proportional rate constraints respectively.

III. RELATED WORK

Note that (3) is an NP-hard combinatorial optimization problem [11] with non-linear constraints. Hence, it is highly improbable that polynomial time algorithms can be used to solve it optimally. Therefore certain simplifications are needed in order to make the problem tractable.

The approach in [7] was to decouple the solution into an initial resource allocation step, wherein the number of subcarriers and amount of power for each user is determined; and subsequently a subcarrier assignment step, wherein each user is assigned their corresponding subcarriers. In [9] on the other hand, the approach was to first determine the subcarrier allocation, followed by the power allocation. We shall discuss the method developed in [9], and improved in [12] next.

The subcarrier allocation was determined by allowing each user to take turns choosing the best subcarrier for him. In each turn, the user with the least proportional capacity gets the priority to choose his best subcarrier. After the subcarrier allocation, the problem in (3) is then simplified into a maximization over continuous variables $p_{k,n}$ given by

$$\max_{p_{k,n}} \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2(1 + p_{k,n} H_{k,n}) \qquad (5)$$

subject to:
C1: $p_{k,n} \geq 0 \ \forall k, n$
C2: $\sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} \leq P_{tot}$
C3: $R_i : R_j = \phi_i : \phi_j \ \forall i, j \in \{1, \cdots, K\}, i \neq j$

where Ω_k refers to the set of subcarriers assigned to user k, $r_{k,n}$ is as defined in (2), and

$$R_k = \frac{B}{N} \sum_{n \in \Omega_k} r_{k,n} \qquad (6)$$

is the total data rate for user k.

The set of total power assigned for each user k, denoted as P_k for $1 \leq k \leq K$, can be solved using Lagrangian multiplier techniques [13], and was derived as

$$\begin{aligned} &\frac{1}{\phi_1} \frac{N_1}{N} \left(\log_2(1 + H_{1,1} \frac{P_1 - V_1}{N_1}) + \log_2 W_1 \right) = \\ &\frac{1}{\phi_k} \frac{N_k}{N} \left(\log_2(1 + H_{k,1} \frac{P_k - V_k}{N_k}) + \log_2 W_k \right) \end{aligned} \qquad (7)$$

for $k = 1, 2, \cdots, K$ where

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$$V_k = \sum_{n=2}^{N_k} \frac{H_{k,n} - H_{k,1}}{H_{k,n} H_{k,1}}, \qquad (8)$$

$$W_k = \left(\prod_{n=2}^{N_k} \frac{H_{k,n}}{H_{k,1}} \right)^{\frac{1}{N_k}}, \qquad (9)$$

and N_k is the number of subcarriers assigned to user k. Note that the effective subchannel SNR's $H_{k,n}$ are assumed to be arranged in ascending order.

Adding the total power constraint

$$\sum_{k=1}^{K} P_k = P_{tot} \qquad (10)$$

we arrive at K non-linear equations with K unknowns $\{P_k\}_{k=1}^{K}$. These equations could be solved numerically using the Newton-Raphson method and its variants, but the high computational complexity of these algorithms make them impractical for real-time systems.

An approximation proposed in [9], which requires the subchannel SNR to be high, reduced the problem into solving a single non-linear equation in one variable. This approximation assumes $V_k = 0$ and $H_{k,1} P_k / N_k \gg 1$, and the resulting non-linear equation is

$$\sum_{k=1}^{K} c_k (P_1)^{d_k} - P_{total} = 0 \qquad (11)$$

where

$$c_k = \begin{cases} 1 & \text{if } k = 1 \\ \frac{N_k}{H_{k,1} W_k} \left(\frac{H_{1,1} W_1}{N_1} \right)^{\frac{N_1 \phi_k}{N_k \phi_1}} & \text{if } k = 2, 3, \ldots, K \end{cases} \qquad (12)$$

and

$$d_k = \begin{cases} 1 & \text{if } k = 1 \\ \frac{N_1 \phi_k}{N_k \phi_1} & \text{if } k = 2, 3, \ldots, K. \end{cases} \qquad (13)$$

We shall refer to this method of subcarrier and power allocation as *ROOT-FINDING*. Although (11) can be solved with less computation than solving (7), iterative methods for root finding are still needed. This motivates a different approach to the resource allocation problem.

IV. PROPOSED SOLUTION

We utilize a combination of the approaches in [7] and [9] to exploit the nature of the problem and reap significant complexity reduction benefits while maintaining reasonable performance. The proposed steps are as follows:

Step 1 Determine the number of subcarriers N_k to be initially assigned to each user;

Step 2 Assign the subcarriers to each user in a way that ensures rough proportionality;

Step 3 Assign the total power P_k for user k to maximize the capacity while enforcing the proportionality;

Step 4 Assign the powers $p_{k,n}$ for each user's subcarriers subject to his total power constraint P_k

The underlying premise behind these steps is that in practical systems, adherence to the proportionality constraints need not be strictly enforced. Note that the proportionality constraints are used to differentiate various services, wherein the service provider may choose to prioritize their customers based on different billing mechanisms. Since the proportion of rates are more of a soft guarantee than a hard one, a rough proportionality is acceptable as long as the capacity is maximized and the algorithm complexity is low. Details of each of these steps are described in the following subsections.

A. Step 1 - Number of subcarriers per user

In this initial step, we determine N_k to satisfy

$$N_1 : N_2 : \cdots : N_K = \phi_1 : \phi_2 : \cdots : \phi_K \qquad (14)$$

This initial step is based on the reasonable assumption also made in [7] that the proportion of subcarriers assigned to each user is approximately the same as their eventual rates after power allocation, and thus would roughly satisfy the proportionality constraints. This is accomplished by

$$N_k = \lfloor \phi_k N \rfloor \qquad (15)$$

This may lead to $N^* = N - \sum_{k=1}^{K} N_k$ unallocated subcarriers. The next subsection discusses how the N_k subcarriers for user k and N^* subcarriers, if any, will be assigned.

B. Step 2 - Subcarrier assignment

This step allocates the per user allotment of subcarriers N_k and then the remaining N^* subcarriers in a way that maximizes the overall capacity while maintaining rough proportionality. This greedy algorithm, which is a modification of the one used in [12], is described below. [1]

(a) Initialize

  $c_{k,n} = 0, \; \forall k \in \{1, \cdots, K\}$ and $\forall n \in \{1, \cdots, N\}$
  $R_k = 0, \; \forall k \in \{1, \cdots, K\}$
  $p = P_{tot}/N$ [2]
  $\mathcal{N} = \{1, 2, \cdots, N\}$

(b) for $k = 1$ to K

  Sort $H_{k,n}$ in ascending order
  $n = \arg\max_{n \in \mathcal{N}} |H_{k,n}|$
  $c_{k,n} = 1$
  $N_k = N_k - 1, \mathcal{N} = \mathcal{N} \setminus \{n\}$
  $R_k = R_k + \frac{B}{N} \log_2(1 + p H_{k,n})$

(c) while $\| \mathcal{N} \| > N^*$

  $\mathcal{K} = \{1, 2, \ldots, K\}$
  $k = \arg\min_{k \in \mathcal{K}} R_k / \phi_k$
  $n = \arg\max_{n \in \mathcal{N}} |H_{k,n}|$
  if $N_k > 0$

    $c_{k,n} = 1$
    $N_k = N_k - 1, \mathcal{N} = \mathcal{N} \setminus \{n\}$
    $R_k = R_k + \frac{B}{N} \log_2(1 + p H_{k,n})$

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[1] We use script letters, e.g. $\mathcal{A}$, to denote sets; $\setminus$ to denote set subtraction; and $\| \mathcal{A} \|$ to denote the cardinality of the set $\mathcal{A}$.

[2] Equal power allocation among the subcarriers is initially assumed.

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else
    K = K \ {k}
(d) K = {1, 2, ..., K}
    for n = 1 to N*
        k = arg max_{k∈K} |H_{k,n}|
        c_{k,n} = 1
        R_k = R_k + B/N log_2(1 + pH_{k,n})
        K = K \ {k}
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The first step of the algorithm initializes all the variables. R_k keeps track of the capacity for each user and $\mathcal{N}$ is the set of yet unallocated subcarriers.

The second step assigns to each user the unallocated subcarrier that has the maximum gain for that user. Note that an inherent advantage is gained by the users that are able to choose their best subcarrier earlier than others, particularly for the case of two or more users having the same subcarrier as their best. However, this bias is negligible when $N \gg K$ since the probability of that happening will be very low.

The third step proceeds to assign subcarriers to each user according to the greedy policy that the user that needs a subcarrier most in each iteration gets to choose the best subcarrier for it. Since we are enforcing proportional rates, the need of a user is determined by the user who has the least capacity divided by its proportionality constant. Once the user gets his allotment of N_k subcarriers, he can no longer be assigned any more subcarriers in this step.

The fourth step assigns the remaining N^* subcarriers to the best users for them, wherein each user can get at most one unassigned subcarrier. This is to prevent the user with the best gains to get the rest of the subcarriers. This policy balances achieving proportional fairness while increasing overall capacity. Notice that as a consequence of our subcarrier allocation scheme,

$$N_1 : N_2 : \cdots : N_K \approx \phi_1 : \phi_2 : \cdots : \phi_K \qquad (16)$$

with the approximation getting tighter as $N \to \infty$ and $N \gg K$. This is a reasonable assumption since current wireless systems that utilize OFDMA [4] satisfy these conditions.

C. Step 3 - Power allocation among users

The output of the first two steps is a subcarrier allocation for each user, which reduces the resource allocation problem to an optimal power allocation as in (5). Notice though, that we could use the approximation in (16) and relax constraint C3 in (5) to

$$R_i : R_j = N_i : N_j; \forall \, i, j \in \{1, \cdots, K\}, i \neq j. \qquad (17)$$

Hence, we could replace ϕ_k with N_k in (7), thus forming simultaneous *linear* equations which can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & a_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & a_{K,K} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_K \end{bmatrix} = \begin{bmatrix} P_{tot} \\ b_2 \\ \vdots \\ b_K \end{bmatrix} \qquad (18)$$

where

$$a_{k,k} = -\frac{N_1}{N_k} \frac{H_{k,1} W_k}{H_{1,1} W_1} \qquad (19)$$

$$b_k = \frac{N_1}{H_{1,1}} \left(W_k - W_1 + \frac{H_{1,1} V_1 W_1}{N_1} - \frac{H_{k,1} V_k W_k}{N_k} \right). \qquad (20)$$

This set of simultaneous linear equations can be easily solved due to its well ordered symmetric and sparse structure. In order to see this, we first reorder the equations in (18) into

$$\begin{bmatrix} a_{K,K} & 0 & \cdots & 1 \\ 0 & a_{K-1,K-1} & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} P_K \\ P_{K-1} \\ \vdots \\ P_1 \end{bmatrix} = \begin{bmatrix} b_K \\ b_{K-1} \\ \vdots \\ P_{tot} \end{bmatrix} \qquad (21)$$

and then perform LU factorization on the coefficient matrix to obtain

$$L = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{K,K}} & \frac{1}{a_{K-1,K-1}} & \cdots & 1 \end{bmatrix} \qquad (22)$$

$$U = \begin{bmatrix} a_{K,K} & 0 & \cdots & 1 \\ 0 & a_{K-1,K-1} & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 - \sum_{k=2}^{K} \frac{1}{a_{k,k}} \end{bmatrix}. \qquad (23)$$

Finally, using forwards-backwards substitution, the individual powers are given by

$$P_1 = \left(P_{tot} - \sum_{k=2}^{K} \frac{b_k}{a_{kk}} \right) \Big/ \left(1 - \sum_{k=2}^{K} \frac{1}{a_{kk}} \right) \qquad (24)$$

and

$$P_k = (b_k - P_1)/a_{kk}, \text{ for } k = 2, \cdots, K \qquad (25)$$

D. Step 4 - Power allocation across subcarriers per user

Step 3 gives the total power P_k for each user k, which are then used in this final step to perform waterfilling across the subcarriers for each user as

$$p_{k,n} = p_{k,1} + \frac{H_{k,n} - H_{k,1}}{H_{k,n} H_{k,1}} \qquad (26)$$

where

$$p_{k,1} = \frac{P_k - V_k}{N_k}. \qquad (27)$$

We shall refer to this proposed 4-step approach as *LINEAR*.

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V. Algorithmic Complexity

In order to analyze the computational complexity of the algorithm, recall that K refers to the total number of users in the system. N on the other hand refers to the number of subcarriers, which is a power of 2 and much larger than K.

Step 1 of the algorithm requires 1 division and K multiplications, and thus has a complexity of $O(K)$.

Step 2(a) requires constant time for initialization. Step 2(b) involves sorting the subcarrier gains $H_{k,n}$ for each user k, therefore requiring $O(KN \log_2 N)$ operations. Step 2(c) searches for the best user k among K users for the remaining $N - K$ unallocated subcarriers, thus requires $O((N - K) * K)$ operations. Step 2(d) allocates the very few remaining N^* subcarriers to the best user, and thus requires $O(K)$ operations. These operations pertain to the subcarrier allocation, and the asymptotic complexity is $O(KN \log_2 N)$.

Step 3 involves solving for the individual powers, which is given by (24) and (25). This requires only 1 division, $2(K-1)$ multiplications and $3(K-1)$ subtractions, thus the complexity is $O(K)$. The power allocation step of the *ROOT-FINDING* method (11) from [9], on the other hand, requires iterative root-finding methods such as Newton-Raphson method, bisection method, secant method, and many others [14]. A popular algorithm for solving these equations combines the bisection and secant methods, and is called the ZEROIN subroutine [15]. This is the method used in the simulations, and the complexity is $O(nK)$, where n is the number of function evaluations. n is typically around 10 for smooth functions [15]. Although also asymptotically linear, each function evaluation of (11) involves taking non-integer powers of real numbers. This is significantly more complex than the simple operations required in the *LINEAR* power allocation method, especially when considering implementation in fixed point arithmetic. Furthermore, the *ROOT-FINDING* power allocation method also needs a high subchannel SNR assumption to function properly, which the *LINEAR* method does not make.

Note that Steps 1 and 2 also correspond to the subcarrier allocation step of *ROOT-FINDING*. Both methods involve similar computations and have the same asymptotic complexity. Thus, the real computational savings of *LINEAR* can be seen in the power allocation step.

VI. Simulation Results

A. Simulation Parameters

The frequency selective multipath channel is modeled as consisting of six independent Rayleigh multipaths, with an exponentially decaying profile. A maximum delay spread of 5μs and maximum doppler of 30 Hz is assumed. The channel information is sampled every 0.5 ms to update the subchannel and power allocation. The total power was assumed to be 1 W, the total bandwidth as 1 MHz, and total subcarriers as 64. The average subchannel SNR is 38 dB, and BER $\leq 10^{-3}$, giving an SNR gap $\Gamma = -\ln(5 \times 10^{-3})/1.6 = 3.3$. This constant is used in the calculation of the rate $r_{k,n}$ of user k in subcarrier n given in (2).
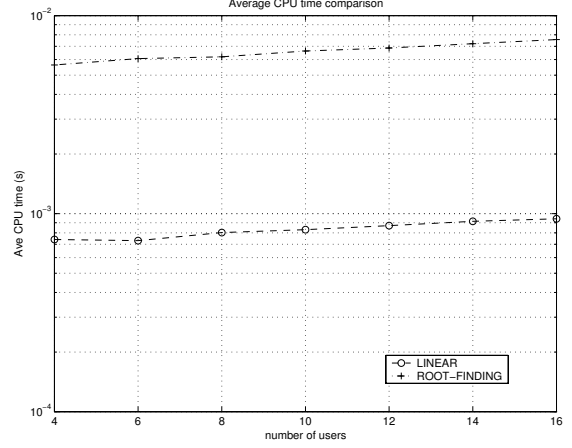

Fig. 2. Average CPU time for compiled Matlab code for the *ROOT-FINDING* method and the proposed *LINEAR* method. Simulation was for $N = 64$ subcarriers with subchannel SNR = 38 dB, SNR gap $\Gamma = 3.3$. *ROOT-FINDING* method has an order of magnitude slower execution time than the proposed *LINEAR* method, yet the *LINEAR* method gives a higher achieved capacity for these simulation parameters.

The number of users for the system is varied from 4-16 in increments of 2. A total of 1000 different channel realizations and 100 time samples for each realization were used for each of the number of users. For each channel realization, a set of proportionality constants (expressed as integers) $\zeta_k = \phi_k / \min \phi_k$ are assigned to each user. It is assumed that these constants follow the probability mass function

$$p_{\zeta_k} = \begin{cases} 1 & \text{with probability } 0.5 \\ 2 & \text{with probability } 0.3 \\ 4 & \text{with probability } 0.2. \end{cases} \quad (28)$$

B. Computational Complexity

Fig. 2 shows the comparison of the computational complexity of the *LINEAR* and *ROOT-FINDING* methods. The algorithms for the two different methods were compiled from MATLAB into C code, and was run on a Pentium-4 3.2 GHz based personal computer running Windows XP professional. Simulation used floating-point arithmetic.

The number of function evaluations needed for the power allocation of *ROOT-FINDING* to converge was found to be around 9. From Fig. 2, *LINEAR* is an order of magnitude faster in execution time than *ROOT-FINDING*. [3]

C. Overall Capacity

Fig. 3 shows the comparison of total capacities between the proposed method *LINEAR* and *ROOT-FINDING*. Notice that the capacities increase as the number of users increases. This is the effect of multiuser diversity gain, which is more prominent in systems with larger number of users. The proposed *LINEAR* method has a consistently higher total capacity than the *ROOT-FINDING* method for all the numbers of users for this set

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[3]This comparison gives an idea on the possible computational savings of using the proposed *LINEAR* method, but is in no way a conclusive indicator of actual speed when implemented in actual systems.
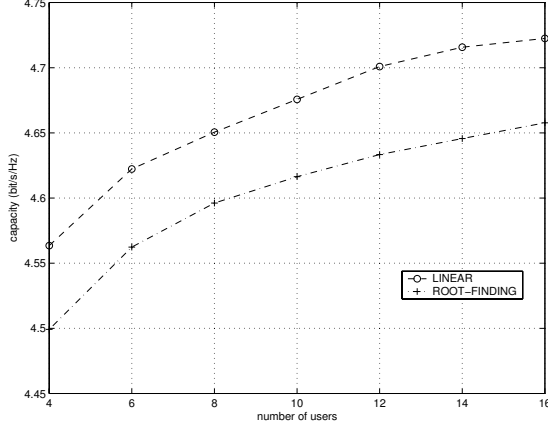
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Fig. 3. Total capacity versus number of users in an OFDMA system with $N = 64$ subcarriers and subchannel SNR $= 38$ dB and SNR gap $\Gamma = 3.3$. The capacity achieved by the proposed *LINEAR* method is consistently higher than for the *ROOT-FINDING* method for a system with 4 up to 16 users.
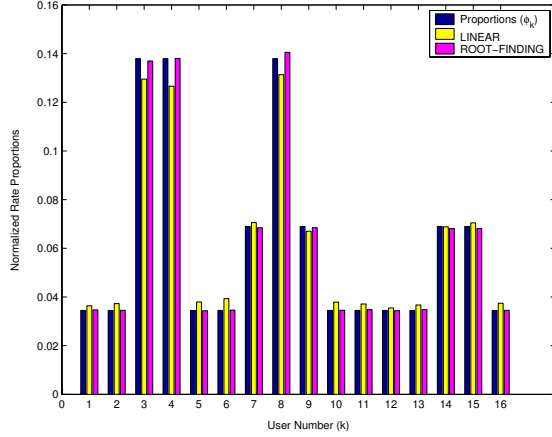

Fig. 4. Normalized capacity ratios per user for 16 users averaged over 100 channels, with the required proportions ϕ_k shown as the leftmost bar for each user. The proposed *LINEAR* method has minimal deviation from the required proportions, but *ROOT-FINDING* adheres to it much better.

of simulation parameters. This advantage can be attributed to the relaxation of the proportionality constraints, and the added freedom of assigning the N^* subcarriers in Step 2 of the proposed algorithm described in Section IV-B.

D. Proportionality

Fig. 4 shows the normalized proportions of the capacities for each user for the case of 16 users averaged over 100 channel samples. The normalized capacities are given by $R_k / \sum_{k=1}^{16} R_k$, and are observed for both *LINEAR* and *ROOT-FINDING*. This is compared to the normalized proportionality constraints $\{\phi_k\}_{k=1}^{16}$. In contrast to the *ROOT-FINDING* method, the proportionality among the users for the *LINEAR* method is no longer being strictly enforced, but this has been argued to be not much of a problem in Section III.

A summary of the comparisons of the two methods is shown in Table I.

Performance Criterion	ROOT-FINDING	LINEAR
Subcarrier Allocation Complexity	$O(KN \log_2 N)$	$O(KN \log_2 N)$
Power Allocation Complexity	$O(nK), n \approx 9$	$O(K)$
Achieved Capacity	High	Higher
Adherence to Proportionality	Tight	Loose
Assumptions on Subchannel SNR	Needs to be high	None

VII. CONCLUSIONS

This paper presents a new method to solve the rate-adaptive resource allocation problem with proportional rate constraints for OFDMA systems. It improves on the previous work in this area [9] by developing a novel subcarrier allocation scheme that achieves approximate rate proportionality while maximizing the total capacity. This scheme was also able to exploit the special linear case in [9], thus allowing the optimal power allocation to be performed using a direct algorithm with a much lower complexity versus an iterative algorithm. It is shown through simulation that the proposed method performs better than the previous work in terms of significantly decreasing the computational complexity, and yet achieving higher total capacities, while being applicable to a more general class of systems.

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