

P8130_hw3_wq2160

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Problem 1

Draw a random sample without replacement of 200 observations (100 men and 100 women) from the entire CE data set named `ce8130entire.csv`. Call this first sample “A” and save the sample. In “sex” variable, **men** are identified by “1”, and **women** by “2”. Note: To obtain the sample data set of approximately 200 observations, you can use the following code. Replace the “set.seed” number with an integer of your choice (3 points).

```
population = read.csv("./ce8130entire.csv")
```

```
set.seed(33)
```

```
A =
```

```
  population %>%  
  group_by(sex) %>%  
  sample_n(100)
```

```
A
```

```
## # A tibble: 200 x 7  
## # Groups:   sex [2]  
##   provnum  sex  race smoker totchg  age  year  
##   <int> <int> <int> <int> <int> <int> <int>  
## 1     28     1     0     0  13434    78  1995  
## 2     21     1     0     0   7701    60  1995  
## 3      7     1     0     0   2225    67  1991  
## 4     20     1     0     0   8369    68  1994  
## 5     47     1     0     0   3095    77  1992  
## 6      7     1     0     0   4134    68  1992  
## 7     14     1     0     0   3226    63  1992  
## 8     27     1     0     0   4592    71  1992  
## 9     48     1     0     0   4988    60  1993  
## 10    14     1     0     0   5180    68  1994  
## # ... with 190 more rows
```

Problem 2

Now use the same seed as before but this time draw a random sample without replacement of 60 observations (30 men and 30 women) and call it sample “B” (Note that Sample “B” is more than 3 times smaller than sample “A”). Save it as a separate sample. Replace the seed number with the same seed number as you used above (3 points).

```
set.seed(33)
B =
  population %>%
  group_by(sex) %>%
  sample_n(30)
```

```
B
```

```
## # A tibble: 60 x 7
## # Groups:   sex [2]
##   provnum sex race smoker totchg age year
##   <int> <int> <int> <int> <int> <int> <int>
## 1     28     1     0     0  13434    78  1995
## 2     21     1     0     0   7701    60  1995
## 3      7     1     0     0   2225    67  1991
## 4     20     1     0     0   8369    68  1994
## 5     47     1     0     0   3095    77  1992
## 6      7     1     0     0   4134    68  1992
## 7     14     1     0     0   3226    63  1992
## 8     27     1     0     0   4592    71  1992
## 9     48     1     0     0   4988    60  1993
## 10    14     1     0     0   5180    68  1994
## # ... with 50 more rows
```

Problem 3

Using sample “A”, display the distribution of CE cost in \$USD (variable name: “totchg”) separately for men and women using side-by-side boxplots and histograms. Label your figures appropriately.

Histogram

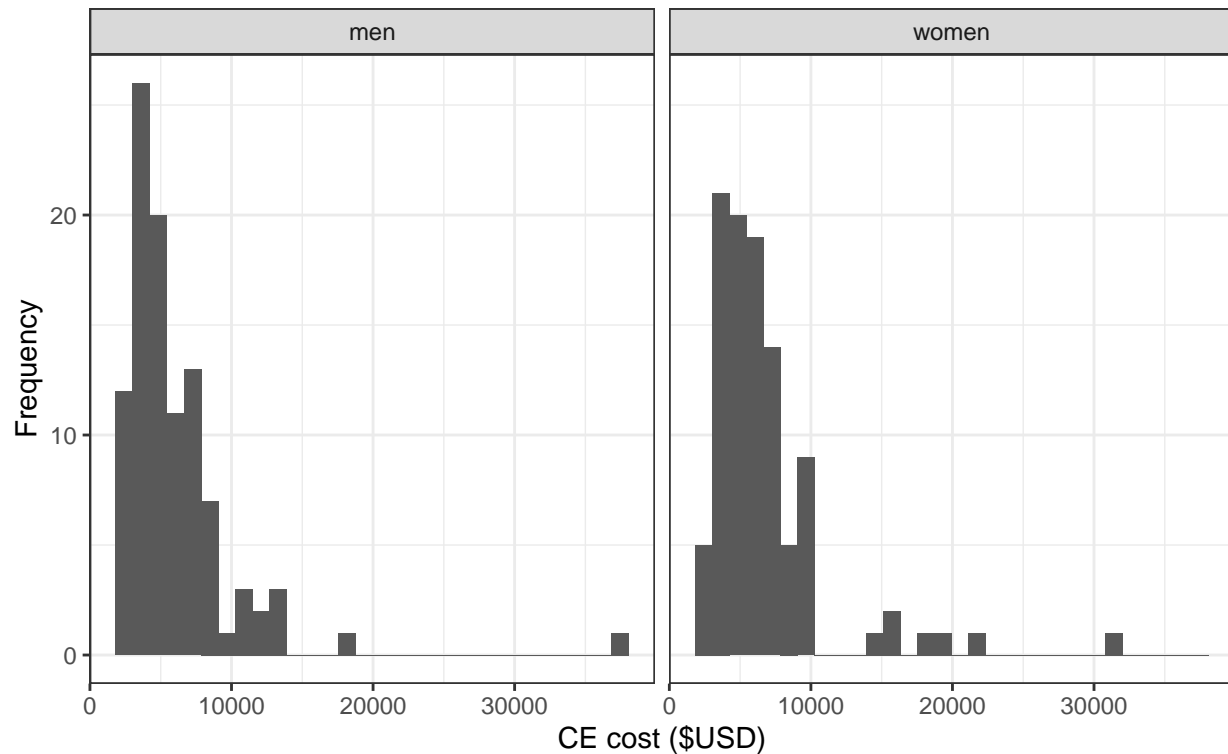
```
hist_plot =
A %>%
  mutate(sex = recode(sex, "1" = "men", "2" = "women")) %>%
  ggplot(aes(x = totchg)) +
  geom_histogram() +
  facet_grid(. ~ sex) +
  labs(
    title = "The Distribution of CE Cost (Histogram)",
    subtitle = "Separately for Men and Women",
    x = "CE cost ($USD)",
    y = "Frequency"
  ) +
  theme_bw()

hist_plot
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

The Distribution of CE Cost (Histogram)

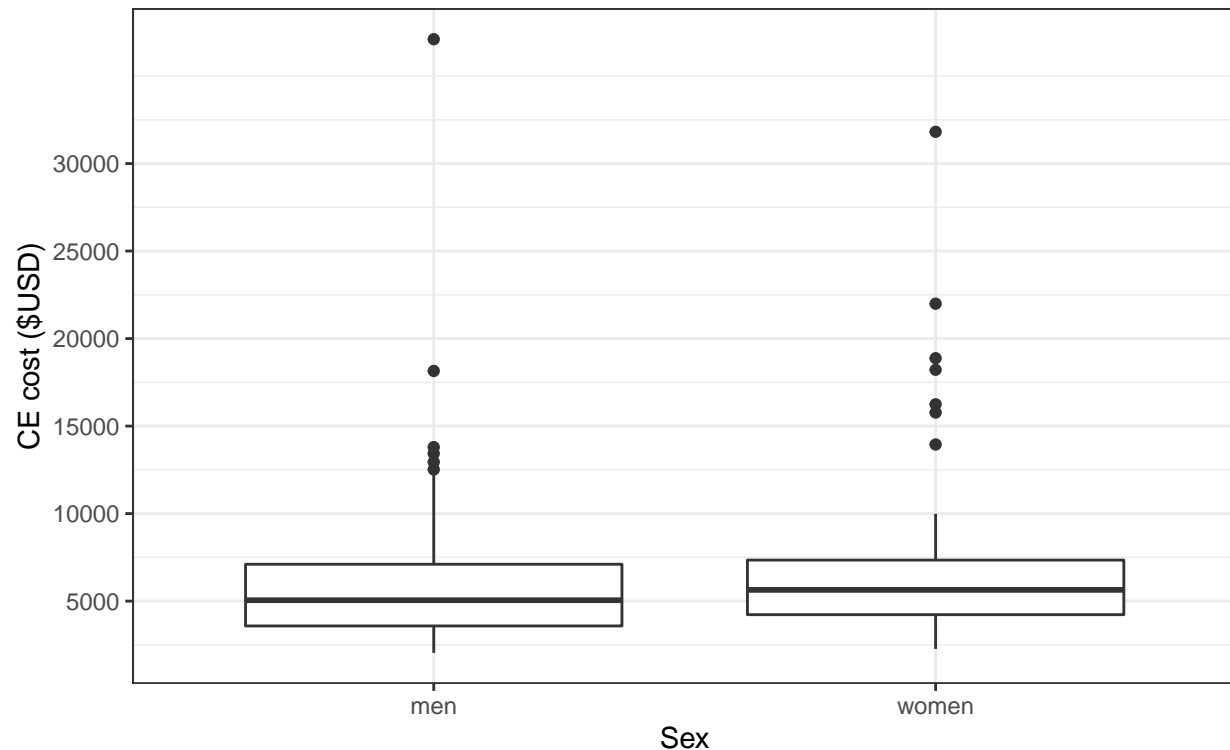
Separately for Men and Women



Boxplot

```
boxp_plot =  
A %>%  
  mutate(sex = recode(sex, "1" = "men", "2" = "women")) %>%  
  ggplot(aes(x = sex, y = totchg)) +  
  geom_boxplot() +  
  labs(  
    title = "The Distribution of CE Cost (Boxplot)",  
    subtitle = "Separately for Men and Women",  
    x = "Sex",  
    y = "CE cost ($USD)"  
  ) +  
  scale_y_continuous(  
    breaks = c(5000, 10000, 15000, 20000, 25000, 30000),  
    labels = c("5000", "10000", "15000", "20000", "25000", "30000")  
  ) +  
  theme_bw()  
  
boxp_plot
```

The Distribution of CE Cost (Boxplot) Separately for Men and Women



Problem 4

Calculate the mean CE cost and 95% confidence interval separately for men and women in sample “A” as well as sample “B”. Assume we don’t know the population variance. Plot the sample “A” and sample “B” confidence intervals next to each other (by sex). How do they differ, which confidence intervals are wider? Explain why.

Plot CI of sample A and B

```
A_s =
  A %>%
  mutate(sample = c("Sample A"))

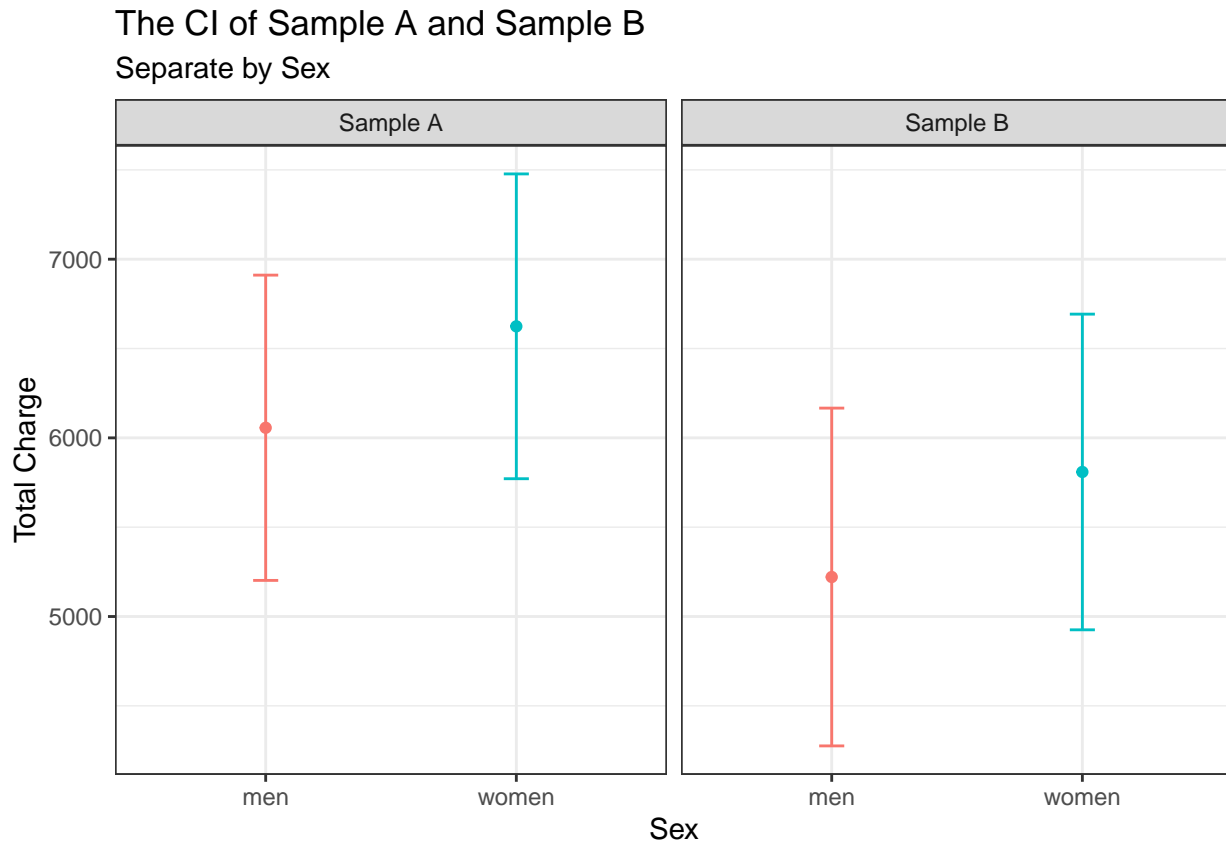
B_s =
  B %>%
  mutate(sample = c("Sample B"))

AB_df = rbind(A_s, B_s)

p_dodge = position_dodge(0.1)

AB_df %>%
  mutate(sex = recode(sex, "1" = "men", "2" = "women")) %>%
  summarise(measurevar = "totchg", groupvars = c("sex", "sample")) %>%
  ggplot(aes(x = sex, y = totchg, color = sex)) +
  geom_point(position = p_dodge) +
```

```
geom_errorbar(aes(ymin = totchg - ci, ymax = totchg + ci), width = .1, position = p_dodge) +
facet_grid(. ~ sample) +
labs(
  title = "The CI of Sample A and Sample B",
  subtitle = "Separate by Sex",
  x = "Sex",
  y = "Total Charge"
) +
theme_bw() +
theme(legend.position = "none")
```



mean CE cost and 95% confidence interval for MEN in Sample A

```
A_men =
A %>%
  filter(sex == "1")

## Mean CE cost for MEN
mean1 =
A_men %>%
  summarize(mean_cost_men = mean(totchg))

mean1
##   mean_cost_men
## 1           6056.47
## 95% CI for MEN
x1 = mean(pull(A_men, totchg))
```

```

s1 = sd(pull(A_men, totchg))
n1 = 100
t1 = qt(0.975, df = 100 - 1)

upper1 = x1 + t1 * s1 / sqrt(n1)
lower1 = x1 - t1 * s1 / sqrt(n1)

upper1
## [1] 6910.899
lower1
## [1] 5202.041

```

mean CE cost and 95% confidence interval for WOMEN in Sample A

```

A_women =
A %>%
  filter(sex == "2")

## Mean CE cost for WOMEN
mean2 =
A_women %>%
  summarize(mean_cost_women = mean(totchg))

mean2
##   mean_cost_women
## 1           6624.28
## 95% CI for WOMEN
x2 = mean(pull(A_women, totchg))
s2 = sd(pull(A_women, totchg))
n2 = 100
t2 = qt(0.975, df = 100 - 1)

upper2 = x2 + t2 * s2 / sqrt(n2)
lower2 = x2 - t2 * s2 / sqrt(n2)

upper2
## [1] 7477.367
lower2
## [1] 5771.193

```

mean CE cost and 95% confidence interval for MEN in Sample B

```

B_men =
B %>%
  filter(sex == "1")

## Mean CE cost for MEN
mean3 =
B_men %>%
  summarize(mean_cost_men = mean(totchg))

mean3
##   mean_cost_men

```

```
## 1      5221.133
## 95% CI for MEN
x3 = mean(pull(B_men, totchg))
s3 = sd(pull(B_men, totchg))
n3 = 30
t3 = qt(0.975, df = 30 - 1)

upper3 = x3 + t3 * s3 / sqrt(n3)
lower3 = x3 - t3 * s3 / sqrt(n3)

upper3
## [1] 6166.488
lower3
## [1] 4275.779
```

mean CE cost and 95% confidence interval for WOMEN in Sample B

```
B_women =
B %>%
  filter(sex == "2")

## Mean CE cost for WOMEN
mean4 =
B_women %>%
  summarize(mean_cost_women = mean(totchg))

mean2
## mean_cost_women
## 1      6624.28
## 95% CI for WOMEN
x4 = mean(pull(B_women, totchg))
s4 = sd(pull(B_women, totchg))
n4 = 30
t4 = qt(0.975, df = 30 - 1)

upper4 = x4 + t4 * s4 / sqrt(n4)
lower4 = x4 - t4 * s4 / sqrt(n4)

upper4
## [1] 6692.608
lower4
## [1] 4925.459
```

Q1: How do they differ?

A1: There is a obvious trend that the mean CE cost of women is higher than men in both samples, and this cost difference is about 570 USD. (Note: According to Sample A and B.)

Q2: Which CI are wider?

A2: I think the CI width of two sample groups are nearly the same, and the CI width of men and women in each sample group are also nearly the same. According to calculation, in Sample A, the CI width of Men is 1708.86, of Women is 2273.98; in Sample B the CI width of Men is 1890.71, of Women is 1767.15. Consequently, there is no significant difference exists.

Q3: Explain why?

A3: Sample A and B come from the same source population, then the total “trend” in these two samples should be nearly the same, which means they are shows that 1) the CE cost of women is higher than men; and 2) there is no significantly CI difference between women and men. However, Sample A have a size of 200, Sample B only have 60 sample size, so the latter one may more far away from the truth.

```
## We can simply calculate the CI interval for each group
upper1-lower1 ## Sample A Men
## [1] 1708.858
upper2-lower2 ## Sample A Women
## [1] 1706.173
upper3-lower3 ## Sample B Men
## [1] 1890.709
upper4-lower4 ## Sample B Women
## [1] 1767.149
```

Problem 5

Conduct test of equality of variance of CE cost among men vs women in sample A and interpret your results.

Method 1

```
var.test(totchg ~ sex, data = A)
```

```
##
## F test to compare two variances
##
## data: totchg by sex
## F = 1.0031, num df = 99, denom df = 99, p-value = 0.9875
## alternative hypothesis: true ratio of variances is not equal to 1
## 95 percent confidence interval:
## 0.6749611 1.4909153
## sample estimates:
## ratio of variances
## 1.00315
```

Method 2

```
## Test statistic for F test
F_test = s1^2 / s2^2
## F critical value
F_crit = qf(0.975, 99, 99)
## Decision
ifelse(F_test > F_crit, "reject", "fail to reject")
```

```
## [1] "fail to reject"
```

Conclusion: In Method 1, the p-value is $0.9875 > 0.05$, we can not reject null hypothesis; In Method 2, the test statistic is $1.00315 < F$ critical value = 1.486, which means it falls into the fail to reject region. Combining results above, we can say that **The variance of CE cost among men and women in sample A statistically have no significant difference. In other words, in sample A, the variances of CE cost among men and women are equal.**

Problem 6

Using sample “A”, calculate the difference between the mean CE costs for men and women (cost in men - cost in women). Calculate a 95% CI for this difference. Assume we don’t know the population variance. Your decision of equal vs unequal variance should be based on your answer in Problem 5.

```
## Difference between the mean CE costs for men and women
```

```
diff_A = x1 - x2
diff_A
```

```
## [1] -567.81
```

```
# calculate pooled sd
```

```
sd_pooled = sqrt((s1^2 * 99 + s2^2 * 99) / 198)
```

```
## 95% CI
```

```
diffci_lower = diff_A - qt(0.975, 198) * sd_pooled * sqrt((1/99) + (1/99))
```

```
diffci_upper = diff_A + qt(0.975, 198) * sd_pooled * sqrt((1/99) + (1/99))
```

```
diffci_lower
```

```
## [1] -1773.828
```

```
diffci_upper
```

```
## [1] 638.2081
```

Conclusion: difference between the mean CE costs for men and women is -567.81, and the 95% CI for this difference is (-1773.83, 638.21).

Problem 7

Now use sample “A” to test the hypothesis whether men and women have a different CE cost. State the null and alternative hypotheses and interpret your results.

Note: in Problem 5, my data shows that the variances are equal. Therefore, we conduct a two-sample t-test for equal variances by pooling the standard deviations.

The **null hypothesis** is: The mean CE costs for men and women are equal. The **alternative hypothesis** is: The mean CE costs for men and women are different (not equal).

```
# calculate test statistic
```

```
t_stat = (x1 - x2) / (sd_pooled * sqrt((1/99) + (1/99)))
```

```
# calculate critical value
```

```
t_crit = qt(0.975, df = 99 + 99 - 2)
```

```
# decision
```

```
ifelse(abs(t_stat) > t_crit, "reject", "fail to reject")
```

```
## [1] "fail to reject"
```

Also, we can use this method:

```
t.test(totchg ~ sex, data = A, var.equal = TRUE)
```

```
##
## Welch Two Sample t-test
##
## data: totchg by sex
## t = -0.93313, df = 198, p-value = 0.3519
## alternative hypothesis: true difference in means between group 1 and group 2 is not equal to 0
## 95 percent confidence interval:
## -1767.7829 632.1629
## sample estimates:
## mean in group 1 mean in group 2
## 6056.47 6624.28
```

Interpretation: Combining two results, we fail to reject the null hypotheses, and we can conclude that there is no significant difference of CE cost between men and women.

Problem 8

Use your results from Sample A: graphs, estimates, confidence intervals, and/or test results, to write a one paragraph summary of your findings regarding the average costs of CE for men and women. Write as if for an audience of health services researchers. Be quantitative and use health-services language, rather than statistical jargon in your write-up.

Answer

We randomly sampled 200 individuals from HSCRC database with 100 men and 100 women, for the purpose of finding the difference of mean CE costs between men and women in Maryland for the period 1990 through 1995. After plotting our sample with histogram and boxplot, we can see that there is a slight difference between the mean CE costs of men and women, specifically, the average cost for women is about 570 USD higher than men in this sample. Using this sample, we try to assume the true value for underlying population. After calculation, we are 95% confident that there is no significant difference of average CE cost between men and women.

Problem 9

Now for the truth, which we have the luxury of knowing in this problem set. Compute the actual mean CE cost for men and for women for the whole population (CE8130entire.csv). Also calculate the difference. Do your 95% CIs include the true means?

```
pop_men =
population %>%
  filter(sex == "1")

## Actual mean CE cost for men
pop_men %>%
  summarize(pop_men_mean = mean(totchg))

## pop_men_mean
## 1 6890.872
```

```
pop_women =
  population %>%
  filter(sex == "2")

## Actual mean CE cost for women
pop_women %>%
  summarize(pop_women_mean = mean(totchg))
```

```
##   pop_women_mean
## 1          7014.377
```

```
xa = mean(pull(pop_men, totchg))
xb = mean(pull(pop_women, totchg))
```

```
## Mean difference
xa - xb
```

```
## [1] -123.5047
```

My 95% CI for women is (5771.193, 7477.367), and it includes the true value 7014,377; My 95% CI for men is (5202.041, 6910.899), and it includes the true value 6890.872; My 95% CI for mean difference is (-1773.828, 638.2081), and it includes the true difference -123.5047.

Problem 10

If each student in a class of 140 calculates a 95% confidence interval for $(\mu_M - \mu_W)$, how many of these intervals do you expect to contain the true population mean difference? Calculate the probability that all 140 will contain the true population mean difference.

Answer

For the definition of CI, it's "Over the collection of all 95% CIs that could be constructed from repeated samples of size n , 95% of them will contain the true population mean". So for each student, they have the probability of 0.95 to contain true mean difference. Then the probability of all 140 contain the true population mean difference:

```
## Expectation
0.95*140
```

```
## [1] 133
```

```
## Probability
0.95^140
```

```
## [1] 0.00076086
```

It is 0.00076, which is surprisingly low.