HW 6
Due: Fri, May 13

These equations should be read in concert with the posted review notes for the course. We will go through some (or maybe all) of them in class on May 9 and May 11.

1. In control theory, one often wants to plot a *transfer function*

$$h(s) = c^T (A - sI)^{-1} b.$$

   The transfer function can be computed in $O(n^2)$ time using a Hessenberg reduction on A. Write a code to do this (you may take advantage of the fact that MATLAB automatically detects and solves Hessenberg systems in $O(n^2)$ time).

2. If A is strictly *column* diagonally dominant, Jacobi iteration converges. Why?

3. Consider the fixed point iteration $x_{k+1} = x_k + \cos(x_k)$. Show that for x_0 near enough to $x_* = \pi/2$, the iteration converges, and describe the convergence behavior.

4. Describe a fast variable projection algorithm to solve

$$Ax = b(x_n)$$

   where $A \in \mathbb{R}^{n \times n}$ is a fixed matrix and $b : \mathbb{R} \to \mathbb{R}^n$ is twice differentiable.

5. Write a system of equations to characterize the minimum of a linear function $\phi(x) = v^T x$ on the ellipsoid $x^T M x = 1$ where M is symmetric and positive definite.