Midterm

Due: 2016-03-11

For the midterm, you are allowed to use texts, papers, or other references (with citation). You should not ask help from any other person, whether inside or outside the class. You should not worry if you do not get all the answers; this is a sign that the test is doing a proper job! We reserve the right to ask follow-up questions in person (e.g. to determine whether it makes sense to assign partial credit). You may ask us to clarify ambiguities or perceived errors in the prompt, but please do not ask for hints.

In addition to a PDF document detailing any derivation work, you should submit a MATLAB file with your solutions, following the format of the `mt_codes.m` file provided on the web page.

1. Given $A = QR$, solve $Ax = b$ in $O(n^2)$ time.

2. Suppose $A = QR$ is an economy QR with $A \in \mathbb{R}^{m \times n}$ and $m \gg n$. Minimize $\|Ax - b\|^2 + \lambda^2 \|x\|^2$ with $O(mn) + O(n^3)$ additional work.

3. Suppose A is symmetric and positive definite and $A = LU$ is an ordinary LU factorization. Show how to get the Cholesky factorization $A = R^T R$ in $O(n^2)$ additional time.

4. Write a function to compute $\sqrt{x+1} - \sqrt{x-1}$ for $x > 1$. You should retain accuracy in floating point for $x \gg 1$.

5. Write an $O(n)$ time code equivalent to `x = triu(u*v')\b;` where $u_i, v_i \neq 0$ for $i = 1, 2, \ldots, n$.

6. The matrix $A \in \mathbb{R}^{n \times n}$ is upper triangular except for a nonzero α in position (i, j) (with $i > j$). Give an efficient ($O(n^2)$ time) algorithm to solve $Ax = b$.

7. Suppose $A \in \mathbb{R}^{m \times n}$ has many repeated rows. In MATLAB notation, $A = W(p, :)$ — that is, row i of A is the same as row p_i of $W \in \mathbb{R}^{k \times n}$. Write a code to minimize $\|Ax - b\|^2$ in $O(m) + O(kn^2)$ time, assuming A full rank.

8. Compute the R factor in the QR decomposition of A in $O(n^2)$ time for A of the form

$$A = \begin{bmatrix} \alpha_1 & & & & \\ & \alpha_2 & & & \\ & & \ddots & & \\ & & & \alpha_{n-1} & \\ \beta_1 & \beta_2 & \dots & \beta_{n-1} & \alpha_n \end{bmatrix}.$$