HW 5

Due: Fri, Apr 15

1: Simple Singularity Write Newton's iteration for finding the zero of the function $f(x) = x^2$. Is the iteration superlinearly convergent? Why or why not?

2: Noodling with Newton Write a Newton iteration to find the intersection of the parametric curves

$$t \mapsto (t\cos(t), t\sin(t))$$
$$s \mapsto (2\cos(s) - \cos(2s), 2\sin(s) - \sin(2s))$$

Use the initial guess $t = s = \pi$. Plot a residual versus the iteration number on a semilog scale to illustrate quadratic convergence.

3: Canny Convergence Suppose $x^{k+1} = G(x^k)$ where G is Lipschitz with constant $\alpha < 1$. Show that

$$\|e^k\| \leq \frac{\|x^{k+1} - x^k\|}{1-\alpha}.$$

4: Beyond Jacobi Suppose $f, g : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, and for some $M > K$, for any $x, y, \hat{x}, \hat{y} \in \mathbb{R}^n$ that

$$\|f(\hat{x}, y) - f(x, y)\| \geq M\|\hat{x} - x\| \qquad \|g(\hat{x}, y) - g(x, y)\| \leq K\|\hat{x} - x\|$$
$$\|f(x, \hat{y}) - f(x, y)\| \leq K\|\hat{y} - y\| \qquad \|g(x, \hat{y}) - g(x, y)\| \geq M\|\hat{y} - y\|.$$

Consider the Jacobi iteration

$$f(x^{k+1}, y^k) = 0$$
$$g(x^k, y^{k+1}) = 0.$$

Argue that if $x^k = x^* + u^k$ and $y^k = y^* + v^k$, then we have the error iteration

$$\max(\|u^{k+1}\|, \|v^{k+1}\|) \leq \frac{K}{M} \max(\|u^k\|, \|v^k\|)$$

Hint: Apply triangle inequality and the problem conditions to

$$f(x^* + u^{k+1}, y^* + v^k) - f(x^* + u^{k+1}, y^*) + f(x^* + u^{k+1}, y^*) - f(x^*, y^*) = 0$$

and similarly for g to get a recurrence in $\|u^k\|$ and $\|v^k\|$.