HW 1

Due: Fri, Feb 5

The first two problems should be plausible given what you know as of Jan 29. Ideally, the material presented on Monday, Feb 1 will allow you to do the other two problems. Don't be shy about asking for help in office hours or on Piazza!

1: Placing parens Suppose $A, B \in \mathbb{R}^{n\times n}$ are square matrices, $D = \operatorname{diag}(d) \in \mathbb{R}^{n\times n}$ is a diagonal matrix, and $u, v \in \mathbb{R}^n$ are vectors. Write short fragments of MATLAB to evaluate them as efficiently as possible, and give the complexity in terms of n:

1. $v^T(I + DAD)v$
2. $u^T A^2 v$
3. $\operatorname{tr}(uv^T A)$

2: Recognizing rank Consider the MATLAB fragment

```
function [y] = hw1mult(x)
  n = length(x);
  A = reshape(1:n^2, n, n);
  y = A*x;
```

1. What is A for $n = 3$?
2. Show that A has rank two (independent of n).
3. Rewrite `hw1mult` so that it runs in $O(n)$ time.

3: Norms!

1. Show that $x \mapsto \|x\|_1 + \|x\|_\infty$ is a norm.
2. The space $\mathcal{P}_3$ of polynomials with degree less than or equal to three has a norm $\|p\|$ given by
$$\|p\|^2 = \int_{-1}^{1} p(x)^2\, dx$$
For a general cubic $p(x) = ax^3 + bx^2 + cx + d$, write $\|p\|$ in terms of a, b, c, d.

4: Pushing products Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric and positive definite. The A-norm of a vector $v \in \mathbb{R}^n$ is $\|v\|_A = \sqrt{v^T A v}$. Describe how to reconstruct A given a function that computes $\|v\|_A$ for any given vector. Code it up in a function with the following interface:

```
function [A] = hw1normA(normfun, n)
% Given a function to evaluate the Euclidean norm of a length n vector
% v with respect to the A inner product, reconstruct A.
```