# HW 3

Due: Fri, Mar 4

**1: Pesky polynomials** We would like the best quartic (degree 4) approximation to $\cos(x)$ on $[-1, 1]$ in a least squares sense; that is,

$$\text{minimize} \int_{-1}^{1} |p(x) - \cos(x)|^2 \, dx$$

Set up and solve in MATLAB, and compare to a solution based on sampling at a uniform mesh of ten points. *Hint:* $\int_{-1}^{1} x^k \cos(x) \, dx$ is $26\sin(1) - 40\cos(1)$ for $k = 4$ and $4\cos(1) - 2\sin(1)$ for $k = 2$.

**2: QR to SVD** Suppose $A = QR$ is an economy QR factorization. Show that the singular values of $A$ are the same as those of $R$.

**3: Vector projector** Suppose $A \in \mathbb{R}^{m \times n}$ where $m > n$ has full column rank. Given $A$ and a vector $b$, write one line of MATLAB to compute the element $c$ in the range space of $A$ that is nearest to $b$ (in the Euclidean norm).

**4: Generally speaking** Often, we use least squares to construct models of the world. We assume that the "truth" is

$$Ax = b,$$

but what we measure is the first few rows of $A$ and $b$ (which we write as $A_1$ and $b_1$), and those measurements are corrupted by noise. Suppose we have $A$ exactly, but only get the noisy partial right hand side $\hat{b}_1 = b_1 + e_1$, from which we form

$$\text{minimize } \|A_1\hat{x} - \hat{b}_1\|^2.$$

Our goal in this problem is to use the error analysis ideas in Section 6.2 to figure out the inherited error in the reconstruction of $\hat{b}_2 = A_2\hat{x}$.

1. Let $e_2 = \hat{b}_2 - b_2$. Argue *briefly* that $e_2 = A_2 A_1^\dagger e_1$.
2. Show that
$$\frac{\|e_2\|}{\|b_2\|} \leq \kappa(A_2 A_1^\dagger) \frac{\|e_1\|}{\|b_1\|}.$$

Things get somewhat more complicated if we also allow the entries of $A$ to be contaminated by error, though the same basic ingredients come into play.