2021年3月13日 7:53

#1

(1)

$P(sunny) = P_1$ 1	$P(free) = P_2$ 1
$P(not\ sunny) = P_3$ 0	$P(not\ free) = P_4$ 0

possiable sample:

$(1,1) = P_1 P_2 \Rightarrow$ sunny + free $\Rightarrow x_1$

$(1,0) = P_1 P_4 \Rightarrow$ sunny + not free $\Rightarrow x_2$

$(0,1) = P_3 P_2 \Rightarrow$ not sunny + free $\Rightarrow x_3$

$(0,0) = P_3 P_4 \Rightarrow$ not sunny + not free $\Rightarrow x_4$

$P_3 = 1 - P_1 \qquad P_4 = 1 - P_2$

$\sum N = 1 = x_1 + x_2 + x_3 + x_4$

x_1	x_2	x_3	x_4
P_1P_2	P_1P_4	P_3P_2	P_3P_4

$P_1^{(x_1+x_2)} \quad P_2^{(x_1+x_3)} \quad P_3^{(x_3+x_4)} \quad P_4^{(x_2+x_4)}$

$\log L = \log(P_1^{(x_1+x_2)}) + \log(P_2^{(x_1+x_3)}) + \log(P_3^{(x_3+x_4)}) + \log(P_4^{(x_2+x_4)})$

$\log L = (x_1+x_2)\log P_1 + (x_1+x_3)\log P_2 + (x_3+x_4)\log P_3 + (x_2+x_4)\log P_4$

$\frac{\partial L}{\partial P_1} = \frac{x_1+x_2}{P_1}$ to $+ \frac{x_3+x_4}{1-P_1}$ to $\qquad P_1 = \frac{x_1+x_2}{N}$

$\frac{\partial L}{\partial P_2} = \frac{x_1+x_3}{P_2} + \frac{x_2+x_4}{1-P_2} \qquad P_2 = \frac{x_1+x_3}{N}$

$\frac{\partial L}{\partial P_3} = \frac{x_3+x_4}{P_3} + \frac{x_1+x_2}{1-P_3} \qquad P_3 = \frac{x_3+x_4}{N}$

$\frac{\partial L}{\partial P_4} = \frac{x_2+x_4}{P_4} + \frac{x_1+x_3}{P_2} \qquad P_4 = \frac{x_2+x_4}{N}$

(b) $N = 10$

P_1: sunny P_2: free

$P_1(MLE) = \frac{\text{number of sunny day}}{10}$

$P_2(MLE) = \frac{\text{number of free day}}{10}$

sunny $\rightarrow P(free)$

$\Rightarrow P_2(MLE)$

assume free(1) not free(0)

$f(x)$= Bernolli: $P_2^{(10-x)}(1-P_2)^x$

(c) morning, afternoon, evening

$\because$ we want to know the table if free (1) or not free (0).

$\therefore$ we need three parameters to show the problities

$P(free \mid morning) \quad P(free \mid afternoon) \quad P(free \mid evening)$

$P(not\ free \mid morning) \quad P(not\ free \mid afternoon) \quad P(not\ free \mid evening)$

#2 $P(x) = 0.5 N(\mu_1, \sigma_1^2) + 0.5 N(\mu_2, \sigma_2^2)$

(a)

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}\,\sigma_1}\cdot \exp\left(\frac{-\frac{1}{2}\sum_{i=1}^{n}(x_i-\mu_1)^2}{\sigma_1^2}\right) + \frac{1}{\sqrt{2\pi}\,\sigma_2}\cdot \exp\left(\frac{-\frac{1}{2}\sum_{i=1}^{n}(x_i-\mu_2)^2}{\sigma_2^2}\right)\right]$$

$$= \frac{1}{2\sqrt{2\pi}}\left[\frac{1}{\sigma_1}\exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu_1)^2}{2\sigma_1^2}\right) + \frac{1}{\sigma_2}\exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu_2)^2}{2\sigma_2^2}\right)\right]$$

$$\Rightarrow -\ln(P(x)) = \ln\left[-\frac{1}{2\sqrt{2\pi}}\left[\frac{1}{\sigma_1}\exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu_1)^2}{2\sigma_1^2}\right) + \frac{1}{\sigma_2}\exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu_2)^2}{2\sigma_2^2}\right)\right]\right]$$

Let C be constant by $\log(a+b) = \log a + \log b + constant$. $\log a + \log b = \log(ab)$

$$\Rightarrow C - \ln\frac{1}{\sigma_1} - \ln\left[\exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu_1)^2}{2\sigma_1^2}\right)\right] - \ln\left(\frac{1}{\sigma_2}\right) - \ln\left[\exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu_2)^2}{2\sigma_2^2}\right)\right]$$

$$\Rightarrow C - \ln\frac{1}{\sigma_1} - \ln\left[\exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu_1)^2}{2\sigma_1^2}\right)\right] - \ln\left(\frac{1}{\sigma_2}\right) - \ln\left[\exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu_2)^2}{2\sigma_2^2}\right)\right]$$

$$\Rightarrow C - \ln\left(\frac{1}{\sigma_1}\right) - \ln\left(\frac{1}{\sigma_2}\right) + \frac{1}{2}\frac{\sum_{i=1}^{n}(x_i-\mu_1)^2}{\sigma_1^2} + \frac{1}{2}\frac{\sum_{i=1}^{n}(x_i-\mu_2)^2}{\sigma_2^2}$$

$$\Rightarrow C + \ln(\sigma_1) + \ln(\sigma_2) + \frac{1}{2}\left[\frac{1}{\sigma_1^2}\sum_{i=1}^{n}(x_i^2 - 2\mu_1 \cdot x_i + \mu_1^2) + \frac{1}{\sigma_2^2}\sum_{i=1}^{n}(x_i^2 - 2\mu_2 \cdot x_i + \mu_2^2)\right]$$

$$\Rightarrow \ln\sigma_1\sigma_2 + \frac{1}{2}\left[\frac{\sum_{i=1}^{n}(x_i^2 - 2\mu_1 x_i + \mu_1^2)}{\sigma_1^2} + \frac{\sum_{i=1}^{n}(x_i^2 - 2\mu_2 - x_i + \mu_2^2)}{\sigma_2^2}\right]$$

(b) $$L = C + \ln(\sigma_1) + \ln(\sigma_2) + \frac{1}{2}\left[\frac{1}{\sigma_1^2}\sum_{i=1}^{n}(x_i^2 - 2\mu_1 \cdot x_i + \mu_1^2) + \frac{1}{\sigma_2^2}\sum_{i=1}^{n}(x_i^2 - 2\mu_2 \cdot x_i + \mu_2^2)\right)$$

$$\frac{\partial L}{\partial \mu_1} = \underline{C} + \underline{\ln(\sigma_1)} + \underline{\ln(\sigma_2)} + \frac{1}{2}\left[\frac{1}{\sigma_1^2}\sum_{i=1}^{n}(x_i^2 - 2\mu_1 \cdot x_i + \mu_1^2) + \underline{\frac{1}{\sigma_2^2}\sum_{i=1}^{n}(x_i^2 - 2\mu_2 \cdot x_i + \mu_2^2)}\right)$$ are constant

$$\Rightarrow \frac{\partial L}{\partial \mu_1} = \frac{1}{2\sigma_1} \cdot -2\sum_{n=1}^{n} x_i + 2n\cdot\mu_1$$

$$= \frac{\sum_{i=1}^{n} x_i + n\mu_1}{\sigma_1^2} = 0$$

$\because \sigma_1^2 > 0$

$\therefore \sum_{i=0}^{n} x_i + n\mu_1 = \sigma_1^2 = 0$

$\therefore n\mu_1 = \sum_{i=0}^{n} x_i$

$\mu_1 = \frac{1}{n}\sum_{i=0}^{n} x_i$

$\mu_1 = \bar{x}$

in the same way

$\mu_2 = \bar{x}$

$$L = C + \ln(\sigma_1) + \ln(\sigma_2) + \frac{1}{2}\left[\frac{1}{\sigma_1^2}\sum_{i=1}^{n}(x_i^2 - 2\mu_1 \cdot x_i + \mu_1^2) + \frac{1}{\sigma_2^2}\sum_{i=1}^{n}(x_i^2 - 2\mu_2 \cdot x_i + \mu_2^2)\right)$$

$$\frac{\partial L}{\partial \sigma_1} = \frac{1}{\sigma_1} - \frac{1}{2} \times 2\left(\sum_{i=1}^{n} x_i^2 - 2\mu_1 \sum x_i + n\mu_1^2\right) \cdot \sigma_1^3 = 0$$

$$\Rightarrow \frac{1}{\sigma_1} - \frac{\sum_{i=1}^{n}(x_i - \mu_1)^3}{\sigma_1^3} = 0$$

$$1 - \frac{\sum_{i=1}^{n}(x_i - \mu_1)^2}{\sigma_1^2} = 0$$

$$1 = \frac{\sum_{i=1}^{n}(x_i - \mu_1)^2}{\sigma_1^2}$$

$$\sigma_1^2 = \sum_{i=1}^{n}(x_i - \mu_1)^2$$

$$\sigma_1^2 = \sum_{i=1}^{n}(x_i - \bar{x})$$ by $\mu_1 = \bar{x}$

$$\sigma_1 = \sqrt{\sum_{i=1}^{n}(x_i - \bar{x})}$$

in the same way $\sigma_2 = \sqrt{\sum_{i=1}^{n}(x_i - \bar{x})}$