3ai. $x \cdot x \cdot x \cdot \ldots$

$x \cdot x = x^2(1+\varepsilon_{R1})$

$x^2(1+\varepsilon_{R1}) \cdot x = x^3(1+\varepsilon_{R1})(1+\varepsilon_{R2}) = x^3(1+\varepsilon_{R1}+\varepsilon_{R2}+\cancel{\varepsilon_{R1}\varepsilon_{R2}}^{\,0})$

$x^3(1+\varepsilon_{R1}+\varepsilon_{R2}) \cdot x = x^4(1+\varepsilon_{R1}+\varepsilon_{R2}+\varepsilon_{R3})$

$x^n(1+\varepsilon_{tot}) = x^n(1+(n-1)\varepsilon_R)$

$$\boxed{\varepsilon_{tot} = (n-1)\varepsilon}$$

aii. $e^{n\ln x}$

$\left[e^{n\ln x(1+\varepsilon_{ln})(1+\varepsilon_{rd})}\right](1+\varepsilon_e)$

$\left[e^{n\ln x(1+\varepsilon_{ln}+\varepsilon_{rd}+\cancel{\varepsilon_{ln}\varepsilon_{rd}}^{\,0})}\right](1+\varepsilon_e)$

$\left[e^{n\ln x}\, e^{n\ln x[\varepsilon_{ln}+\varepsilon_{rd}]}\right](1+\varepsilon_e)$

$e^{n\ln x}\left[1+n\ln x(\varepsilon_{ln}+\varepsilon_{rd})\right](1+\varepsilon_e)$

$\cancel{e^{n\ln x}}(1+\varepsilon_{tot}) = \cancel{e^{n\ln x}}(1+\varepsilon_e+n\ln x(\varepsilon_{ln}+\varepsilon_{rd}) + n\varepsilon_e\ln x(\varepsilon_{ln}+\varepsilon_{rd}))$

$\varepsilon_{tot} = \varepsilon + n\ln x(2\varepsilon) + \underline{n\varepsilon\ln x(2\varepsilon)} \approx 0$

$$\boxed{\varepsilon_{tot} = \varepsilon(1+2n\ln x)}$$

$\cancel{\varepsilon}(n-1) = \cancel{\varepsilon}(1+2n\ln x)$

$\frac{n-2}{2n} = \ln x$

$\frac{1}{2} - \frac{1}{n} = \ln x$

Repeated multiplication is better when x is large

log-exponential method is better when $x < 1$

3bi. $x^{a(1+\varepsilon_a)}$

$e^{a\ln x(1+\varepsilon_a)}$
$e^{a\ln x + a\varepsilon_a \ln x}$

$e^{a\ln x}\, e^{a\varepsilon_a \ln x}$

$e^{a\ln x}(a\varepsilon_a \ln x)$

$\cancel{e^{a\ln x}}(1+\varepsilon_{tot}) = \cancel{e^{a\ln x}}(a\varepsilon_a \ln x)$

$\boxed{\varepsilon_{tot} = a\varepsilon_a \ln x - 1}$

3bii. $x \cdot x \cdot x \cdot \ldots$

$x(1+\varepsilon_x)\cdot x(1+\varepsilon_x) = x^2(1+\varepsilon_x)(1+\varepsilon_x) =$
$= x^2(1+\varepsilon_x+\varepsilon_x+\cancel{\varepsilon_x^2}\approx 0)$
$= x^2(1+2\varepsilon_x)$

$x^2(1+2\varepsilon_x)\cdot x(1+\varepsilon_x) = x^3(1+3\varepsilon_x)$

$\cancel{x^a}(1+\varepsilon_{tot}) = \cancel{x^a}(1+n\varepsilon_x)$

$\boxed{\varepsilon_{tot} = a\varepsilon_x}$

For ε_a, a very large a and $\ln x$ could create substantial error.
For ε_x, a very large a could create substantial error.