4b. $f(x) = 1 - e^{-x}$

$$(1-e^{-x})\left(1 + \frac{e^{-x}}{1-e^{-x}}\varepsilon_{exp} \cdot \varepsilon_{rd}\right)$$

$$(1-e^{-x})(1+\varepsilon_{rd}) - e^{-x}\varepsilon_{exp} = 1 - e^{-x_A}$$

$$\cancel{1} - e^{-x} + \varepsilon_{rd}(+1-e^{-x}) - e^{-x}\varepsilon_{exp} = \cancel{1} - e^{-x_A}$$

$$e^{-x} - \varepsilon_{rd}(1-e^{-x}) + e^{-x}\varepsilon_{exp} = e^{-x_A}$$

$$-\ln\left(e^{-x} - (1-e^{-x})\varepsilon_{rd} + e^{-x}\varepsilon_{exp}\right) = x_A$$

$$(e^{-x} - \varepsilon_{rd} - e^{-x}\varepsilon_{rd} + e^{-x}\varepsilon_{exp})$$

$$-\ln\left(e^{-x}(1 - \varepsilon_{rd} + \varepsilon_{exp} - \varepsilon_{rd}e^{x})\right) = x_A$$

$$x - \ln(1 + (1+e^{x})\varepsilon_{rd} + \varepsilon_{exp}) = x_A$$

$$-\ln(1 - (1+e^{x})\varepsilon_{rd} + \varepsilon_{exp}) = x_A - x$$

$$|x_A - x| \leq -(1+e^{x})\varepsilon_{rd} + \varepsilon_{xp} \approx -e^{x}\varepsilon$$

$$\frac{|x_A - x|}{x} = \left|\frac{2 - e^{x} \cdot \varepsilon}{1 \cdot x}\right| = \boxed{\frac{e^{x}}{x}}$$

c. (Plots included in parts a and b)
As x gets smaller, the relative size of ε becomes larger and more significant. When x is large, ε is insignificant. As x decreases, ε can become almost equal in size.

4d. $2^{-b} = 1 - e^{-x}$

$2^{-1} \leq 1 - e^{-x}$

$\cancel{0.5 - x \leq -e^{-x}}$

$\cancel{-0.5 \geq e^{-x}}$

$\cancel{\ln(-0.5) \geq -x}$

$\cancel{-0.5 \leq -e^{-x}}$

$e^{-x} \leq 0.5$

$\ln e^{-x} \leq \ln 0.5$

$-x \leq -0.6931$

$x \geq 0.6931$

$$\boxed{\begin{array}{l} 2^{-1} \to x = 0.6931 \\ 2^{-2} \to x = 0.2877 \\ 2^{-3} \to x = 0.1335 \\ 2^{-4} \to x = 0.0645 \end{array}}$$

$2^{-2} \leq 1 - e^{-x}$

$e^{-x} \leq 0.75$

$-x \leq -0.2877$

$x \geq 0.2877$

$2^{-3} \leq 1 - e^{-x}$

$e^{-x} \leq 0.875$

$-x \leq -0.1335$

$x \geq 0.1335$

$2^{-4} \leq 1 - e^{-x}$

$e^{-x} \leq 0.9375$

$-x \leq -0.0645$

$x \geq 0.0645$

4e. 1 bit: $x = 0.6931 \to \frac{e^x}{x} = 2.88545 \to 2^1$
2 bit: $x = 0.2877 \to \frac{e^x}{e} = 4.63454 \to 2^2$
3 bit: $x = 0.1335 \to \frac{e^x}{e} = 8.56046 \to 2^3$
4 bit: $x = 0.0645 \to \frac{e^x}{e} = 16.53683 \to 2^4$

Upper bound on the error is appx $\boxed{2^n}$

4f. Use a taylor series up to the 9^{th} term. Without accounting for ε, the taylor series up to the 9^{th} term will be $\sim 10^{15}$ accurate. Any more terms past this will just add to the error (due to rep. multiplication of x) without adding to the accuracy significantly.