8a. $Y_{N+1} = e - (n+1) Y_N$

$Y_{N+1} - e = -(n+1) Y_N$

$$\frac{-Y_{N+1} + e}{(N+1)} = Y_N$$

$$\boxed{Y_{N-1} = \frac{-Y_N + e}{N}}$$

8a. $Y_{N-1} = \dfrac{-Y_N + e}{N}$

$$Y_{N-2} = \frac{-Y_{N-1} + e}{N-1} = \frac{\frac{-Y_N + e}{N} + e}{N-1} = \frac{1}{N-1}\left(\frac{-Y_N}{N} + \frac{e}{N} + e\right)$$

$$= \frac{1}{N(N-1)}\left(-Y_N + e + Ne\right)$$

$$Y_{N-3} = \frac{-Y_{N-2} + e}{N-2} = \frac{1}{N-2}\left(\frac{1}{N(N-1)}\left(-Y_N + e + Ne\right) + e\right)$$

$$= \frac{1}{N(N-1)(N-2)}\left(-Y_N + 2e + Ne\right)$$

$$Y_{n-m} = (n-m)!\left(\frac{e(1-n)^{m-1} - Y_N}{n!}\right)$$

$$Y_k = \frac{k!}{n!} \frac{\left(e(1-n)^{\cancel{m-1}\, n-k-1} - Y_N\right)}{n!}$$

$$(\text{cond } g_k)(Y_N) = g'(Y_N)\frac{Y_n}{Y_k} = \frac{k!}{n!}\frac{Y_N}{Y_k}$$

$\dfrac{Y_k}{Y_N}$ can't be greater than 1 so

$$(\text{cond } g_k)(Y_N) = \boxed{\frac{k!}{n!}}$$

b. $\cancel{\frac{k!}{N!}}$ $\quad \varepsilon < \frac{k!}{N!}$

$$N! < \frac{k!}{\varepsilon}$$

$$N(N-1)! < \frac{k!}{\varepsilon}$$

$$N < \frac{k!}{\varepsilon (N-1)!}$$

~~N = k - m~~

$k = N - m$

$N = k + m$

$$N < \frac{k!}{\varepsilon (k+m)!}$$

$$\boxed{N < \frac{1}{\varepsilon (k+1)(k+2)\cdots(k+m)}}$$

c. $\quad \varepsilon = \frac{k!}{N!} \rightarrow \quad 5 \times 10^{-16} = \frac{20!}{N!}$

$N = 30 - \frac{20!}{30!} = 9.17 \times 10^{-5}$

$\boxed{N = 31} \rightarrow \frac{20!}{31!} = 2.9587 \times 10^{-16} \approx 5 \times 10^{-16}$

$N = 32 \rightarrow \frac{20!}{32!} = 9.246 \times 10^{-18}$