

$$1. \quad \left| \frac{x - \text{rd}(x)}{x} \right| \leq 2^{-p}$$

1st discarded bit is 0:

$$|x - \text{rd}(x)| < (0.0111\dots)_2 \times 2^{t-p} = \frac{1}{2} 2^{t-p} = 2^{t-p-1}$$

$$(0.b_1 b_2 \dots)_2 > (0.b_1 00\dots)_2 = (0.1)_2 \quad \checkmark$$

1st discarded bit is 1:

$$x - \text{rd}(x) = (0.1b_{p+2}\dots)_2 \times 2^{t-p} - 2^{t-p}$$

$$x - \text{rd}(x) = \frac{1}{2} + \frac{1}{2} (0.b_{p+2}\dots)_2 - 1 \times 2^{t-p}$$

$$|x - \text{rd}(x)| = \frac{1}{2} [1 - (0.b_{p+2}\dots)_2] \times 2^{t-p} < \frac{1}{2} \cdot 2^{t-p} = 2^{t-p-1}$$

$$(0.b_1 b_2 \dots)_2 \geq (0.b_1 00\dots)_2 = (0.1)_2$$

$$\left| \frac{x - \text{rd}(x)}{x} \right| < \frac{0.1\dots 2^{-k} 2^t}{0.b_1 b_2 \dots 2^t} < \frac{2^{-k-1} 2^t}{2^t \cdot \frac{1}{2}} = 2^{-k}$$