Homework 1: Problem 5

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March 12, 2019

The value of n_{stop} was 10^{13}, and the final value was 2.71611003409. We can write the floating point error for the function as follows:

$$\text{fl}(v^n) = v^n(1 + n\epsilon_v) \quad |\epsilon_v| < \text{eps}$$

for some value v (note that the errors from computing v are neglected if n is very large). As n grows to 10^{12} or 10^{13} then the quantity $n\cdot$ eps becomes noticeably large. When $n = 10^{13}$ and eps $= 2^{-53}$ then $n\cdot$ eps $= 0.0011$ which is substantial. At $n = 10^{13}$ it is the 4th significant digit that differs from the previous value in the sequence.

$$
\begin{gathered}
n = 10^0, s = 2.00000000000 \\
n = 10^1, s = 2.59374246010 \\
n = 10^2, s = 2.70481382942 \\
n = 10^3, s = 2.71692393224 \\
n = 10^4, s = 2.71814592682 \\
n = 10^5, s = 2.71826823719 \\
n = 10^6, s = 2.71828046910 \\
n = 10^7, s = 2.71828169413 \\
n = 10^8, s = 2.71828179835 \\
n = 10^9, s = 2.71828205201 \\
n = 10^{10}, s = 2.71828205323 \\
n = 10^{11}, s = 2.71828205336 \\
n = 10^{12}, s = 2.71852349604 \\
n = 10^{13}, s = 2.71611003409
\end{gathered}
$$