Problem 5

In [1]:
```
import numpy as np
import sys

def round_sigfig(x, sig=12):
    ndigits=sig-int(np.floor(np.log10(abs(x))))-1
    return round(x, ndigits)

n=1 #starting value of n
counter=0;
sequence = [round_sigfig(1., 12)] #first value in sequence is 1
e_prev=np.exp(1) #initial values of e in sequence for while loop, this doesnt matter
e_curr=1.

while(round_sigfig(e_prev, 12) != round_sigfig(e_curr, 12)):
    counter += 1
    n = n*10
    e_prev = e_curr
    e_curr = (1+1/n)**n
    sequence.append(round_sigfig(e_curr, 13))



print('n_stop = {} \n'.format(counter-1))
print('converged e = {} \n'.format(round_sigfig(e_curr, 12)))
print('Terms in sequence: \n {} \n'.format(sequence))
print('value of e in numpy: {} '.format(np.exp(1)))
print('value of e in numpy: {} '.format(np.exp(1)))
```

```
n_stop = 13

converged e = 2.71611003409

Terms in sequence:
 [1.0, 2.5937424601, 2.704813829422, 2.716923932236, 2.718145926825, 2.718268237192, 2.718280469096, 2.718281694132, 2.718281798347, 2.718282052012, 2.718282053235, 2.718282053357, 2.718523496037, 2.716110034087, 2.716110034087]

value of e in numpy: 2.718281828459045
value of e in numpy: 2.718281828459045
```

The above converged to something somewhat close to e, but this isn't actually what I expected to see. I increased the tolerance below and redid the problem

In [2]:
```
tol=14 #number of sigfigs
n=1 #starting value of n
counter=0;
sequence = [round_sigfig(1., tol)] #first value in sequence is 1
e_prev=np.exp(1) #initial values of e in sequence for while loop, thi
s doesnt matter
e_curr=1.

while(round_sigfig(e_prev, tol) != round_sigfig(e_curr, tol)):
    counter += 1
    n = n*10
    e_prev = e_curr
    e_curr = (1+1/n)**n
    sequence.append(round_sigfig(e_curr, tol))



print('n_stop = {} \n'.format(counter-1))
print('converged e = {} \n'.format(round_sigfig(e_curr, tol)))
print('Terms in sequence: \n {} \n'.format(sequence))
print('machine epsilon: {} '.format(sys.float_info.epsilon))
print('Note that  1+1e-16={} '.format(1+1/10**16))
```

```
n_stop = 16

converged e = 1.0

Terms in sequence:
 [1.0, 2.5937424601, 2.7048138294215, 2.7169239322356, 2.718145926824
9, 2.7182682371923, 2.7182804690958, 2.7182816941321, 2.718281798347
4, 2.7182820520116, 2.7182820532348, 2.7182820533571, 2.718523496037
2, 2.7161100340869, 2.716110034087, 3.0350352065493, 1.0, 1.0]

machine epsilon: 2.220446049250313e-16
Note that  1+1e-16=1.0
```

As expected, once $n = 10^{16}$, $1/n$ is smaller than $eps/2$ and so $1 + 1/n = 1.0$, which is found for both $n = 10^{16}$ and $n = 10^{17}$, causing the loop to stop. For any $n < eps/2$ we have $(1 + 1/n)^n = 1.0$

In [3]:
```
1+1/10**16
1+1e-16
```

Out[3]: 1.0