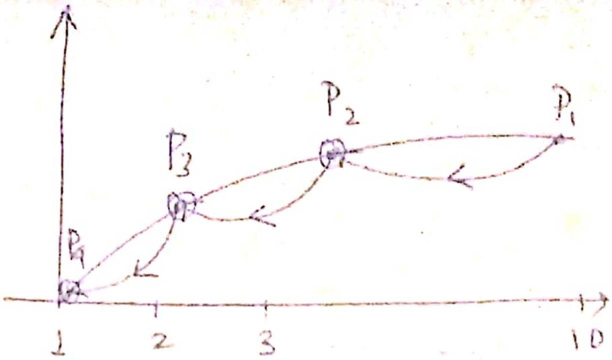
6. On starting off with a random number in [1,10] and carrying out successive square root operations; we find ourselves traversing the points $P_1 \longrightarrow P_2 \longrightarrow P_3 \longrightarrow P_4 \longrightarrow \dots \longrightarrow P_n$.



If n is large; we find $P_n \rightarrow 1$.

so; carrying out square roots very large number of times leaves us with a number of the form $(1+d)$ where $d \ll 1$.

Now; in representing this number as a machine number it is fair to assume it would look like:-

$\boxed{+}$ | 0 | 0 | ... | b_{-p+1} | b_{-p} | $\times\ 2^0$ → this is because we have an addition by 1 so; 2^0 is the exponent.

some final bits in mantissa would be filled

Now; each successive square root operation takes off one bit from the left in the mantissa. so; after 53 iterations (starting from a number in the range of 1 - 10;) we would be left with either a zero or 1 at the b_{-52} bit.

The fact that we have a 2-point degeneracy (all the numbers collapse to either of two values i.e; e^0 or e^2) is also a proof that only the last bit in the mantissa was different at the end of 53 square roots.

So, all the numbers we started of with will have a mantissa that looks like either of the following:-

$A = \boxed{0\,|\,0\,|\,0\,|\cdots|\,0\,|\,0}$ (or) $B = \boxed{0\,|\,0\,|\,0\,|\cdots|\,0\,|\,1}$

raised to the exponent 2^0.

Now; A can be represented as:-

$(1+0) = 1$ and squaring that 53 times will give 1.

so, all numbers that have mantissa A collapse to 1.

For B; we can write it as:- $\left(1+\frac{1}{2^{52}}\right)$ and squaring it 53 times :-

$$\left(1+\frac{1}{2^{52}}\right)^{2^{53}} = \left\{\left(1+\frac{1}{2^{52}}\right)^{2^{52}}\right\}^2$$

Now; from the relation:- $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$;

we can say that the number $\left\{\left(1+\frac{1}{2^{52}}\right)^{2^{52}}\right\}^2$ will be very close to e^2.

Therefore; all the numbers that had mantissa B will collapse to e^2.

Therefore; only e^0 and e^2 yeild correct results after square-rooting and squaring 53 times.

Now; carrying out a similar treatment for $n = 52$:-

However; this time the last 2 bits will be filled (because we start off with a range of 0--10 and the numbers closer to 10 survive upto 2 bits in mantissa after 52 square roots)

$A_1 = \boxed{0|0|0|\ldots|0|1|1}$ 0.5 $A_2 = \boxed{0|0|\ldots|0|1|0}$

0.5   0.5

$B_1 = \boxed{0|0|\ldots|0|0|1}$ 0.5 $B_2 = \boxed{0|0|\ldots|0|0|0}$

$$A_1 = \left(1 + \frac{1}{2^{51}} + \frac{1}{2^{52}}\right) = \left(1 + \frac{1}{2^{52}\left(\frac{2}{3}\right)}\right)$$

and; $$A_1^{52} = \left\{\left(\left(1 + \frac{1}{2^{52}\left(\frac{2}{3}\right)}\right)^{2^{52}\left(\frac{2}{3}\right)}\right)\right\}^{3/2} \cong e^{3/2}$$

~~B_1 and B_2 are e^2 and e^0 res~~

$$A_2^{52} = \left(1 + \frac{1}{2^{51}}\right)^{2^{52}} \cong e^2$$

$$B_1 = \left(1 + \frac{1}{2^{52}}\right)^{2^{52}} \simeq e \quad \text{and} \quad B_2 = (1+0)^{2^{52}} = e^0$$

so; we have 4-point degeneracy $= e^0, e, e^2, e^3$

However; e^3 falls outside our domain and so we cannot see it.

so; we have 3-point degeneracy $= e^0, e^1, e^2$ for 52 square roots followed by 52 squares.

For 51 square roots and 51 squares;

The range of mantissa configurations:-

A_1 | 0 | 0 | 0 | .... | 0 | 1 | 1 | 1 | (or) B_1 | 0 | 0 | . . | 0 | 1 | 1 | 0 |

(or)

A_2 | 0 | 0 | 0 | ... | 0 | 1 | 0 | 1 | (or) B_2 | 0 | 0 | ... | 0 | 0 | 1 | 1 |

(or)

A_3 | 0 | 0 | .... | 0 | 0 | 1 | 0 | (or) B_3 | 0 | 0 | ... | 0 | 0 | 0 | 1 |

(or)

A_4 | 0 | 0 | .... | 0 | 0 | 0 | 0 | (or) B_4 | 0 | 0 | .... | 0 | 1 | 0 | 0 |

$$A_1^{2^{51}} = \left(1 + \frac{1}{2^{52}}(4+2+1)\right)^{2^{51}} = \left\{\left(1 + \frac{1}{2^{52} \times (1/7)}\right)^{2^{52} \times \frac{1}{7}}\right\}^{7 \times \frac{1}{2}} \approx e^{3.5}$$

$$A_2^{2^{51}} = \left(1 + \frac{1}{2^{52}}\left(\times \frac{1}{1/5}\right)\right)^{2^{51}} = \left\{\left(1 + \frac{1}{2^{52} \times \frac{1}{5}}\right)^{2^{52} \times \frac{1}{5}}\right\}^{5/2} \simeq e^{2.5}$$

$$A_3^{2^{51}} = \left(1 + \frac{1}{2^{51}}\right)^{2^{51}} \simeq e^{1}; \quad A_4^{2^{51}} = e^0 = 1$$

$$B_1^{2^{51}} = \left\{\left(1 + \frac{1}{2^{51} \times \frac{1}{3}}\right)^{2^{51} \times \frac{1}{3}}\right\}^{3} = e^3; \quad B_2^{2^{52}} = \left\{\left(1 + \frac{1}{2^{52} \times \frac{1}{3}}\right)^{2^{52} \times \frac{1}{3}}\right\}^{3/2} \simeq e^{1.5}$$

$$B_3^{2^{51}} = \left\{\left(1 + \frac{1}{2^{52}}\right)^{2^{52}}\right\}^{1/2} \simeq e^{0.5}$$

$$B_4^{2^{51}} = \left\{\left(1 + \frac{1}{2^{50}}\right)^{2^{50}}\right\}^{2} \simeq e^{2}$$

However; we can go till e^2 in our domain. So; the values are :-

$e^0, e^{0.5}, e^1, e^{1.5}, e^2$

which remain the same and other values collapse to these!

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