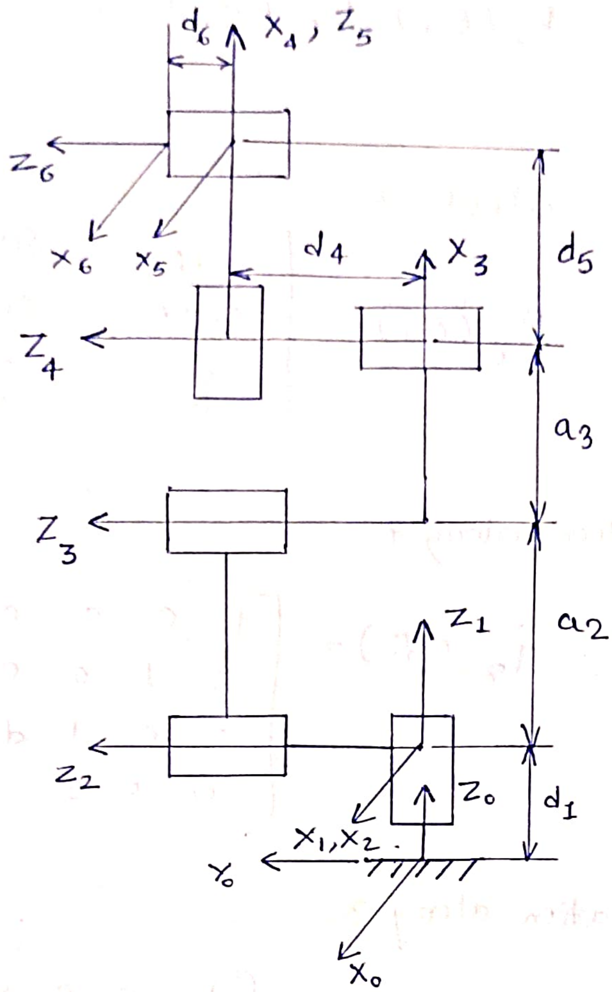

DH - Table (general)

i	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1
2	0	$+90°$	0	θ_2
3	a_3	0	0	θ_3
4	a_4	0	d_4	θ_4
5	0	$-90°$	d_5	θ_5
6	0	$+90°$	d_6	θ_6

For the Current Diagram configuration, DH - Table is

i	a_i	α_i	d_i	θ_i
1	0	0	d_1	0
2	0	$+90°$	0	0
3	a_3	0	0	$90°$
4	a_4	0	d_4	0
5	0	$-90°$	d_5	$-90°$
6	0	$+90°$	d_6	0

(Vertically Fully Stretched case)

Transformation Matrix

$$T_i = R_z(\theta_i)\, T_z(d_i)\, T_x(a_i)\, R_x(\alpha_i) \quad \rightarrow \text{Eq } ①$$

Now,

Rotation about z:

$$R_z(\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation along z:

$$T_z(d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation along x:

$$T_x(a_i) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation about x:

$$R_x(\alpha_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, after multiplying all the four like in Eqn 1, we get

$$T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example:

let

$a_3 = a_4 = 1$ & $d_1 = d_4 = d_5 = d_6 = 1$

& $\theta_i = [0°, 0°, 90°, 0°, -90°, 0°]$

$\hookrightarrow$ Mean vertically fully stretched case

$$T_1 = \begin{bmatrix} \cancel{\cos} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ since $a_1 = 0, \alpha_1 = 0, d_1 = 1, \theta_1 = 0$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ since $a_2 = 0, \alpha_2 = \pi/2, d_2 = 0, \theta_2 = 0$

$$T_3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ since $a_3 = 1, \alpha_3 = 0, d_3 = 0, \theta_3 = \pi/2$

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ since $a_4 = 1, \alpha_4 = 0, d_4 = 1, \theta_4 = 0$

$$T_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ since $a_5 = 0, \alpha_5 = -\pi/2, d_5 = 1, \theta_5 = -\pi/2$

$$T_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ since $a_6 = 0, \alpha_6 = \pi/2, d_6 = 1, \theta_6 = \pi/2$

for transformation from some p to q $(p < q)$ index (Joint).

$$T_{pq} = T_{pa} T_{ab} T_{bc} T_{cd} \dots T_{-q}$$

where $p < a < b < \dots < q$ in joint index.

Similarly $T_{06} = T_{01} T_{12} T_{23} T_{34} T_{45} T_{56}$

& $\cancel{T_{06} =} \begin{bmatrix} 0 & 0 & 1 & 1 \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{1} \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$T_{06} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

from this position of end-effector $p = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$

i.e. $(x, y, z) = (0, -2, 4)$

Rotation matrix is $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

& Roll $= 90^\circ$, pitch $= 0^\circ$, yaw $= 0^\circ$

$\det(R) = 1$.

Jacobian column / Matrix

for revolute joint

$$J_{vi} = z_{i-1} \times (O_n - O_{i-1})$$

$$J_{wi} = z_{i-1}$$

for Joint 1

1) $z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; $O_n = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\rightarrow$ & $O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

end-effector origin of frame 0

$$\Rightarrow J_{v1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$\Rightarrow J_{v1} = \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix}$$

Now,

$$J_{w1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow J_1 = \begin{bmatrix} \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{matrix} \rightarrow J_{v1} \\ \\ \rightarrow J_{w1} \end{matrix}$$

similarly we have to do for all joints then the Jacobian matrix be

$$J = \Big[[J_1] \; [J_2] \; [J_3] \; [J_4] \; [J_5] \; [J_6] \Big]_{6\times 6}$$

Example:

again let $a_3 = a_4 = 1$; $d_1 = d_4 = d_5 = d_6 = 1$

& $\theta_i : [0^\circ, 0^\circ, 90^\circ, 0^\circ, -90^\circ, 0^\circ]$.

from result in Transformation Example,

$$T_{06} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow O_6 = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$$

for Joint 1)

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \& \quad O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow J_{v11} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$\Rightarrow J_{v1} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$J_{\omega 1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow J_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

2) $z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ & $O_1 = [0, 0, 1]^T$

~~Since z_1 is same as z_2~~

$J_1 = J_2$

$\rightarrow J_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

3) $z_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ & $O = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$J_{v3} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right)$

$J_{v3} = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

Similarly we get all J_i upto J_6 from J_1

at last we will get

$$J = \begin{bmatrix} 2 & 2 & -3 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow \det(J) = 0$

$\rightarrow$ Robot is in a singular configuration!