# Empirical Asset Pricing A HW3

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#### February 2, 2021

#### 1. Data Processing

I use Python to help analyze the data (see Appendix B the full code). The Data range of this exercise is 01.1969-12.2018, a total span of 50 years. I get the monthly FF three-factor data, and returns of 25 portfolios formed on size and book-to-market from Kenneth French; monthly labor compensation from BEA Table 2.6; and monthly time-series of the default spread ("Baa - Aaa") from FRED. I first discuss the test of conditional CAPM and then come to the return prediction practice. Note that I also go through the practice using unconditional CAPM as a benchmark, the result of which is given in the Appendix A.

Figure 1: Time series of monthly default spread and the short rate (%)

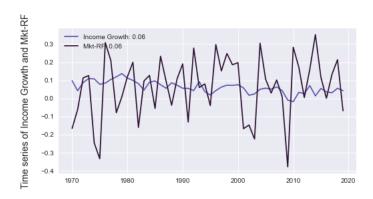


Spread is the difference of Moody's Seasoned Aaa Corporate Bond Yield [AAA] and Baa Corporate Bond Yield [BAA] (not seasonally adjusted), retrieved from FRED, Federal Reserve Bank of St. Louis. RF is the from Fama-French, proxied by the yield of 3-month US Treasury bill, not adjusted by inflation.

I plot this graph because I hope to get a sense of the moves of the bond yield spread, denoted as  $R^{prem}$ , which is used by Jagannathan and Wang (1996) as a proxy of market risk premium. In particular, they assume that the market risk premium, denoted as  $E_t(R^{Mkt-RF}_{t+1})$ , is a linear function of  $R^{prem}_t$ . And their argument says: "stock prices vary over the business cycle, and market risk premium will also vary over the business cycle...," "..., interest-rate variables are likely to be most helpful in predicting future business conditions." As suggested by the literature, I do find a strong association between the pattern of Spread and the business cycle. Interestingly, another observation is that the risk-free rate has been pushing to the ground for a decade, now it is just 0.03%, due to the effect of QE.

The next graph gives the time series of labor income growth:

Figure 2: Time series of annual labor income growth and the market return



In terms of volatility, labor income growth is much more stable than the market return. The inclusion of labor income is to better capture the risk of the market portfolio. Note that I annualise the growth and return to get a better sense of the patterns.

### 2. Time Series Regression

In this part, I use the same technique to estimate the factor loading of 25 portfolios. I report their betas as follows:

Table 1: Factor loading estimates of 25 testing portfolios

Panel A: Mkt-RF								
	BM1	BM2	BM3	BM4	BM5			
ME1	1.41	1.23	1.10	1.01	1.04			
ME2	1.39	1.17	1.04	1.00	1.11			
ME3	1.32	1.12	1.00	0.96	1.04			
ME4	1.23	1.08	1.00	0.94	1.06			
ME5	0.98	0.94	0.87	0.88	0.94			
Panel	Panel B: Spread							
	BM1	BM2	BM3	BM4	BM5			
ME1	0.60	0.57	0.77	0.46	0.43			
ME2	0.45	0.69	0.58	0.49	0.20			
ME3	0.42	0.65	0.46	0.43	0.40			
ME4	0.18	0.37	0.23	0.23	0.06			
ME5	-0.16	0.01	-0.27	-0.46	-0.18			
Panel	Panel C: Labor							
	BM1	BM2	BM3	BM4	BM5			
ME1	0.57	0.33	0.32	0.38	0.41			
ME2	0.08	0.12	0.06	0.34	0.31			
ME3	0.08	0.10	-0.05	-0.06	-0.13			
ME4	-0.12	-0.16	-0.19	-0.05	0.27			
ME5	-0.19	0.03	-0.02	0.10	0.20			

Here, row index 'ME' represents the size dimension, measured using market value. The larger the suffix, the bigger the company. Column index 'BM' represents the book-to-market dimension, measured by the book-to-market ratio. The larger the suffix, the lower the market value relative to the book value of the company.

Overall I notice a strong tendency for small stocks to have high loadings on all three risk factors. Second, the loadings by large fail to explain the risks premium associated with high book-to-market

characteristics, as it shows that high ratios correspond to low loadings. This raises a warning for the choice of testing portfolios, which I will get back in the next section.

#### 3. Cross Sectional Regression

For the second stage, I use Fama-Macbeth to estimate the factor loadings, I add a constant in the regression and test whether it is significantly different from 0:

$$R_i^e = c + c_{vw}\hat{\beta}_i + c_{labor}\hat{\beta}_i^{labor} + c_{prem}\hat{\beta}_i^{prem} + \varepsilon_i, \forall i$$

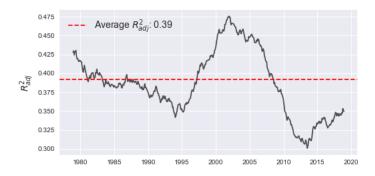
If we assume that these estimations are independent from each other, we can simple use their means as the best estimate of their true value, and use the sample standard error to get the t-statistics. The detailed procedure is documented in the code. I report the result below.

Table 2: Fama-Macbeth estimates of price of risk

Price of risk estimated from 25 testing portfolios					
	c	$c_{vw}$	$c_{prem}$	$c_{labor}$	
FM coef	1.65	-1.04	0.34	-0.00	
t-stats	4.39	-2.65	2.85	-0.02	

Apparently this model is not well supported by the test. More specifically, Its constant is significantly positive, indicating a large portion of unexplained premiums. Second, the market risk premium  $c_{vw}$  is negative and significant, violating our assumption that it should deliver positive risk premium. Thirdly,  $c_{prem}$  is not significant at all, contradicting with the estimate of Jagannathan and Wang (1996)<sup>1</sup>. My guess is that this has to do with the test portfolio. In the paper they choose 100 portfolios sorted on size, which is less convincing than using the double-sorting portfolios in my view. Next, I report the average  $R^2$  and plot it using a rolling window of 10 years.

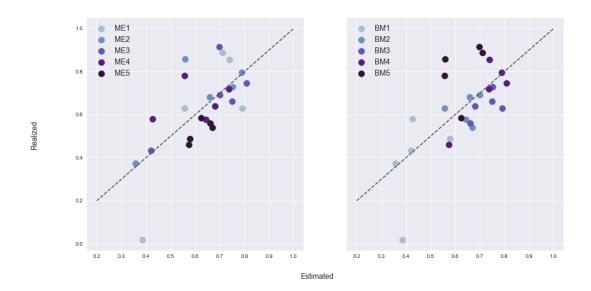
Figure 3: Moving average of  $R_{adj}^2$  from Fama-Macbeth



There is quite a bit variation of power of explanation across time. Again, this result is not as strong as the result from the original paper (0.55). Lastly, I show the scatter plots group by ME and BM.

<sup>&</sup>lt;sup>1</sup>I tried to use the same time period but this does not help get any closer.

Figure 4: Scatter plots between realized and estimated excess return



From this graph we can evaluate the fitting condition. First of all, the overall fitting situation is captured by  $R^2$ , and there is a moderate linear association between the estimation and real excess returns. However, there is also sizable deviation from the diagonal, suggesting that the conditional CAPM model is still insufficient to capture the entire risk structure. Thirdly, compared with the left scatter plot, the right one classified by group of BM ratios shows less explaining power within each subgroups (the estimated returns do not move accordingly as size changes). Lastly, the least fitted point is the portfolio with smallest size and lowest book to market value, where the estimated value is much higher than realized excess return.

I use the 25 market cap and book to market ratio sorted portfolios to study the difference (see Appendix A) between the unconditional CAPM and the conditional CAPM (market portfolio adjusted) advocated by Jagannathan and Wang (1996), the estimated betas of market excess returns remain almost the same. The price of risk from Fama-Macbeth shows a decrease in market risk and an increase in the unexplained intercept, which seems to make things worse. By looking at the  $R^2$  and scattered plot, the fitting is indeed better in conditional CAPM. Overall the attempt to use conditional CAPM delivers a mix change to the original model.

For the purpose of completeness, I also use the sorting originally used by Jagannathan and Wang (1996), but with a coarser grid<sup>2</sup>. The result aligns largely with their findings, and it has a better fit than the previous testing portfolio (average adjusted  $R^2$  is 0.52, see fig. 8). Except that I don't get significant price of risk on labor in come growth (see table 6). The simple comparison raise questions on the credibility of the model used in Jagannathan and Wang (1996). The change of testing portfolios fundamentally impacts the power of explanation.

#### 4. Return predictability

Below I run predictability regressions for the one-year ahead market excess return using (a) the default spread and (b) the short rate as predictor.

 $<sup>^225</sup>$  market cap and beta sorted portfolios

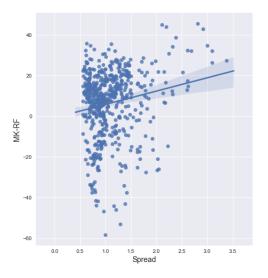
Table 3: Predictability regressions

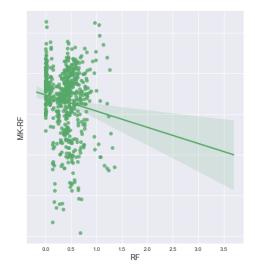
Univariate regression coefficients and t-values				
	RF	Spread		
coef	NaN	6.55		
$\mathbf{t}$	NaN	4.32		
coef	-7.96	NaN		
$\mathbf{t}$	-3.27	NaN		

I conduct two regression tests, both using 1 year ahead market excess return as the dependent variable, and short rate and yield spread as regressor respectively. Note that I calculate the 1 year ahead market excess return using rolling window sum.

Note that both regressions show significant prediction power, the sign of the coefficients aligns with intuition: worse economic environment is associated with low risk free rate and high yield spread, which is also the time when risk premium is high. However, I cannot find strong short term statistical correlation between this Spread and next month market excess return (very small correlation coefficient: -0.02). This also coincides with the classical observation that return predictability is mostly seen in the longer horizon. Lastly, I show the scatter plots using regressors and the 1 year ahead market excess return:

Figure 5: Scatter plot from predictive regressions





# References

 $\label{eq:local_problem} \begin{array}{l} {\rm Jagannathan,\ R.\ and\ Wang,\ Z.\ (1996).\ The\ conditional\ capm\ and\ the\ cross-section\ of\ expected\ returns.} \\ \underline{\rm The\ Journal\ of\ finance,\ 51(1):3-53.} \end{array}$ 

# **Appendices**

# Appendix A

The following results are counterpart benchmarks for the second practice question.

Table 4: Factor loading estimates of 25 testing portfolios, unconditional CAPM

Panel	Panel A: Mkt-RF						
	BM1	BM2	BM3	BM4	BM5		
ME1	1.42	1.23	1.10	1.02	1.04		
ME2	1.39	1.18	1.05	1.00	1.11		
ME3	1.32	1.12	1.00	0.96	1.04		
ME4	1.23	1.09	1.00	0.95	1.07		
ME5	0.98	0.94	0.86	0.88	0.94		

Here, row index 'ME' represents the size dimension, measured using market value. The larger the suffix, the bigger the company. Column index 'BM' represents the book-to-market dimension, measured by the book-to-market ratio. The larger the suffix, the lower the market value relative to the book value of the company.

Table 5: Fama-Macbeth estimates of price of risk, unconditional CAPM

Price of risk estimated from 25 testing portfolios				
		c	$c_{vw}$	
	FM coef	1.30	-0.62	
	t-stats	3.26	-1.43	

Figure 6: Moving average of  $R^2_{adj}$  from Fama-Macbeth, unconditional CAPM



Figure 7: Scatter plots between realized and estimated excess return, unconditional CAPM

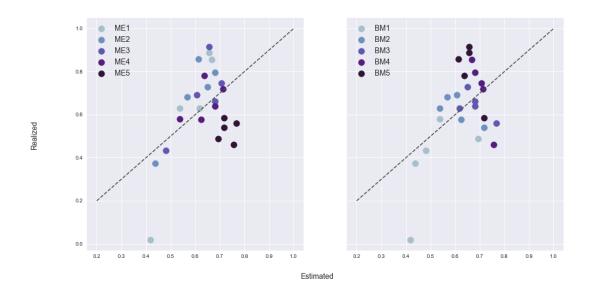
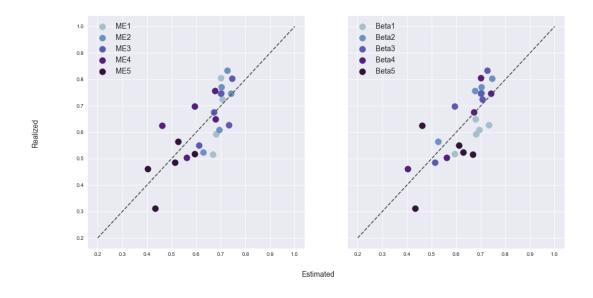


Table 6: Fama-Macbeth estimates of price of risk, 25 ME&Beta sorted portfolios

Price of risk estimated from 25 testing portfolios					
	c	$c_{vw}$	$c_{prem}$	$c_{labor}$	
FM coef	0.77	-0.22	0.35	0.09	
t-stats	4.53	-0.89	2.48	0.45	

Figure 8: Scatter plots between realized and estimated excess return,  $25~\mathrm{ME\&Beta}$  sorted portfolios



## Appendix B

```
#!/usr/bin/env python
2 # coding: utf-8
# # Empirical Asset Pricing A 2021
5 # ## Homework 3&4: on empirical tests for conditional CAPM, return predictability
6 # **Xinyu Liu, INSEAD**
8 # **02.02.2021**
10 # ## Overview
11 #
_{12} # The goal of this exercise is to get a sense of the testing procedures in conditional
      CAPM, and check predictability of the market return.
14 # ## Preparation: Import packages and access data
15 #
16
17 # In[1]:
19
10 import pandas_datareader.data as web # module for reading datasets directly from the
^{21} #pip install pandas-datareader (in case you haven't install this package)
22 from pandas_datareader.famafrench import get_available_datasets
23 import pandas as pd
24 import numpy as np
25 import datetime as dt
26 import matplotlib.pyplot as plt
plt.style.use('seaborn')
28 from matplotlib.dates import DateFormatter
29 import matplotlib.dates as mdates
30 import statsmodels.api as sm
import scipy as sp
from dateutil.relativedelta import relativedelta
33 import datapungibea as dpb
34 import os
35 # print latex
36 # from IPython.display import display, Math
37
39 # In[2]:
40
42 ###########################
43 # Fama French Factor Grabber
44 ###########################
{\tt \#https://randlow.github.io/posts/finance-economics/pandas-datareader-KF/}
46 #Please refer to this link if you have any further questions.
48 #You can extract all the available datasets from Ken French's website and find that
      there are 297 of them. We can opt to see all the datasets available.
datasets = get_available_datasets()
print('No. of datasets:{}'.format(len(datasets)))
51 #datasets # comment out if you want to see all the datasets
52
53
54 # In[7]:
55
57 ###########################
58 #Customize your data selection
59 ##########################
60 #It is important to check the description of the dataset we access by using the
      following codes
61 sdate = '1969-01-01'
62 edate='2018-12-31'
63 dir = os.path.realpath('.')
_{\rm 66} # #### For $M kt-Rf, SMB, HML$ Factors:
```

```
68 # In[4]:
70
71 Datatoread='F-F_Research_Data_Factors'
72 ds_factors = web.DataReader(Datatoread, 'famafrench', start=sdate, end=edate) # Taking [0]
       as extracting 1F-F-Research_Data_Factors_2x3')
73 print('\nKEYS\n{}'.format(ds_factors.keys()))
74 print('DATASET DESCRIPTION \n {}'.format(ds_factors['DESCR']))
_{75} #From the printed information we know that we need to select the "O" name in the
       dictionary
76 #copy the right dict for later examination
dfFactor = ds_factors[0].copy()
78 # 0 for monthly data and 1 for yearly data
79 dfFactor.reset_index(inplace=True)
81 #Date format adjustment
82 # dfFactor['Date']=dfFactor['Date'].dt.strftime('%Y-%m')
83 dfFactor = dfFactor.set_index(['Date'])
84 dfFactor.index=dfFactor.index.to_timestamp()
85 # dfFactor['Date']=dfFactor['Date'].dt.to_timestamp(freq='M').dt.strftime('%Y-%m')
86 #Obtained object dtype
# dfFactor.index=pd.to_datetime(dfFactor.index)
88 #Obtained dt64, which is needed for the plotting
90 RF = dfFactor['RF']
91 # dfFactor=dfFactor.drop(columns = ['RF'])
92 # I check the scale of the data by printing out the head:
93 dfFactor.head()
95
_{96} # #### For 25 portfolios formed on size and book-to-market (5 x 5)
98 # In[16]:
100
_{101} # I searched for the exact name for this portfolio set by methods mentioned above
102 #It is important to check the description of the dataset we access by using the
       following codes
103 Datatoread_PORT='25_Portfolios_5x5'
Datatoread_PORT='25_Portfolios_ME_BETA_5x5'
105 ds_PORT = web.DataReader(Datatoread_PORT,'famafrench',start=sdate,end=edate) # Taking
       [0] as extracting 1F-F-Research_Data_Factors_2x3')
print('\nKEYS\n{}'.format(ds_PORT.keys()))
print('DATASET DESCRIPTION \n {}'.format(ds_PORT['DESCR']))
108 #From the printed information we know that we need to select the "O" name in the
      dictionary
109 #copy the right dict for later examination
dfPORT = ds_PORT[0].copy()
dfPORT.reset_index(inplace=True)
112
dfPORT = dfPORT.set_index(['Date'])
_{\rm 114} # I check the scale of the data by printing out the head:
dfPORT.head()
116
117
##### For monthly time-series of the default spread ( Baa - Aaa )
119
120 # In [53]:
121
122
123 # from fredapi import Fred
124 # fred = Fred(api_key='867c31a2baca3a69effa928b9b294289')
# Aaa = fred.get_series_latest_release('AAA')
# Baa = fred.get_series_latest_release('BAA')
127 ####
_{128} # The API above is not stable so I make a local copy and access them below
129 ####
filename = os.path.join(dir, 'Data', 'AAA.csv')
131 Aaa = pd.read_csv(filename,index_col='DATE',parse_dates=True)
filename = os.path.join(dir, 'Data', 'BAA.csv')
Baa = pd.read_csv(filename,index_col='DATE',parse_dates=True)
135 Bond_spread = pd.DataFrame({'Aaa':Aaa.iloc[:,0].values,'Baa':Baa.iloc[:,0].values},index
```

```
= Aaa.index)
136 Bond_spread = Bond_spread[(Bond_spread.index<=pd.to_datetime(edate)) & (Bond_spread.
      index>=pd.to_datetime(sdate))]
Bond_spread['Spread'] = Bond_spread['Baa']- Bond_spread['Aaa']
   dfFactor = dfFactor.merge(Bond_spread[['Spread']], how='inner', left_index=True,
       right_index=True)
140
141 # In [56]:
142
143
   def portfolio_plot(df, num_subplot, plot_name='testing',figsize=(8,8), cmap ='twilight'
144
       n = num_subplot
145
       fig, axes = plt.subplots(n,1,figsize=figsize,sharex=True,sharey=True)
years_fmt = mdates.DateFormatter('%Y')
146
147
       # fig.suptitle('Time series of relevant variables',fontsize=16)
148
       # Add an origin point at the top of the dataframe
149
150
       dfcopy = df.copy()
         dfcopy.index = dfcopy.index.to_timestamp()
151 #
152 #
         origin = dfcopy.index[0]-relativedelta(months=1)
153 #
         dfcopy.loc[origin,:] = [1]*len(dfcopy.columns)
154 #
         dfcopy=dfcopy.sort_index()
       dfFactor_cum = dfcopy
156
       for k,factortitle in enumerate(dfcopy.columns):
           if n==1:
158
               ax = axes
160
           else:
               ax = axes[k//n]
161
           ax.plot(dfFactor_cum.index,dfFactor_cum[factortitle], label='{}: {:.2f}'.format(
162
       factortitle, dfFactor_cum[factortitle].mean()))
           ax.xaxis.set_major_formatter(years_fmt)
163
164
           colormap = plt.cm.get_cmap(cmap)
           colors = [colormap(i) for i in np.linspace(0.3, 0.5,len(ax.lines))]
165
166
           for i, j in enumerate(ax.lines):
               j.set_color(colors[i])
167
           ax.legend(fontsize = 10,loc=2)
168
       fig.text(0.04, 0.5, 'Time series of ' +plot_name, va='center', ha='center',rotation=
169
       'vertical', fontsize = 14)
       plt.savefig("Time series of "+plot_name)
170
       plt.show()
171
172 portfolio_plot(dfFactor[['Spread', 'RF']], 1, plot_name='Spread and RF', figsize=(8,4),
       cmap = 'twilight')
174
# #### For monthly time-series of labor income growth (BEA)
176
177 # In [57]:
178
179
180 BEA_data = dpb.data('FDA2D756-CCOA-4AAA-A1D5-980FA23F31BB') #or data = dpb.data("API Key
NIPA_cons=BEA_data.NIPA('T20600', frequency='M')
_{\rm 182} #Download annual consumption data on nondurable goods from Table 2.6.
         Personal
                   Income and Its Disposition, Monthly
NIPA_cons.reset_index(inplace=True)
185 Compensation_data=NIPA_cons[NIPA_cons['LineDescription'] == '-Compensation of employees']
186 Compensation_data = Compensation_data.T.iloc[4:,:]
Compensation_data.columns=['Compensation']
188 Compensation_data.index = pd.to_datetime(Compensation_data.index.values, format='%YM%m')
189 Compensation_data['Income Growth'] = (Compensation_data['Compensation']-
       Compensation_data['Compensation'].shift(1))/Compensation_data['Compensation'].shift
       (1)
_{\rm 190} # Convert strings to datetime
191 Compensation_data = Compensation_data[(Compensation_data.index<=pd.to_datetime(edate)) &
        (Compensation_data.index>=pd.to_datetime(sdate))]
192 Compensation_data['Mkt-RF'] = dfFactor['Mkt-RF']/100
193 Compensation_data['Income Growth'] = Compensation_data['Income Growth']
labor_market = (Compensation_data[['Income Growth','Mkt-RF']]+1).astype('f').resample('Y
       ').prod()-1
195 portfolio_plot(labor_market, 1, plot_name='Income Growth and Mkt-RF (monthly)', figsize
     =(8,4), cmap ='twilight')
```

```
dfFactor['Labor'] = Compensation_data['Income Growth'].astype('f')*100
197 # I don't know why but the api is not stable so I kept a copy of data
# Compensation_data.to_pickle('compensation')
199 #or [All just for saving the intermediary data]
200 # Compensation_data.to_csv(os.path.join(dir, 'Data','Compensation.csv'))
# Compensation_data = pd.read_pickle('compensation')
# dfFactor.to_csv(os.path.join(dir, 'Data','dfFactor.csv'))
203
204
205 # ## Test functions
206 # #### Define the function for conducting cross-sectional test, where the first stage is
        a time series regression
208 # In[10]:
209
210
_{\rm 211} # I can import directly the saved dfFactor
filename = os.path.join(dir, 'Data', 'dfFactor.csv')
dfFactor = pd.read_csv(filename,index_col='Date',parse_dates=True)
214
215
216 # In[17]:
217
218
def FamaMacbeth_Test(factor_matrix, test_assets, RF):
220
           test_assets.index = test_assets.index.to_timestamp()
221
       except Exception:
222
           pass
       # Step one, time series regression, obtain estimated beta for each portfolio
224
       X = sm.add_constant(factor_matrix)
225
       beta_matrix = pd.DataFrame()
226
       for i in range(len(test_assets.columns)):
227
228
           y= test_assets.iloc[:,i]-RF
           model = sm.OLS(y, X)
229
           results = model.fit()
230
           beta_i = pd.DataFrame(results.params[1:]).T
231
232
           beta_matrix= pd.concat([beta_matrix, beta_i])
       beta_matrix.index = test_assets.columns
234
       # Step two, cross sectional regression, obtain estimated intercept and factor risk
235
       premium period by period
       X = sm.add_constant(beta_matrix)
236
       premium_matrix = pd.DataFrame()
237
       rsquare_matrix = []
238
       for i in range(len(test_assets.index)):
239
           # Note to be consisitent we should still use the excess return
240
           y= test_assets.iloc[i,:]-RF[i]
241
           model = sm.OLS(y, X)
242
           results = model.fit()
243
           premium_i = pd.DataFrame(results.params).T
244
           premium_matrix= pd.concat([premium_matrix, premium_i])
245
246
           rsquare_matrix.append(results.rsquared_adj)
247
248
       premium_matrix.index = factor_matrix.index
249
       ## Key formula to calculate the statistics
250
251
       point_estimate = premium_matrix.mean()
       N = len(test_assets.index)
252
       std = premium_matrix.std()/np.sqrt(N)
253
       df = N-1
254
       significant_level = 0.975
255
       critical_value = sp.stats.t.ppf(significant_level, df)
256
       CI = [point_estimate-std*critical_value, point_estimate+std*critical_value]
257
       reports = pd.DataFrame(point_estimate).T
258
       reports = reports.rename(index={0:'FM coef'})
259
260
       reports.loc['t-stats',:]= reports.iloc[0,:]/std
261
       print(reports.round(2).to_latex())
262
       return beta_matrix, premium_matrix, point_estimate, rsquare_matrix
263
264
266 # In[18]:
```

```
267
beta_matrix, premium_matrix, point_estimate, rsquare_mean = FamaMacbeth_Test(dfFactor[[])
       Mkt-RF', 'Spread', 'Labor']], dfPORT, RF)
271
272 # In [73]:
273
274
275 beta_matrix, premium_matrix, point_estimate, rsquare_mean = FamaMacbeth_Test(dfFactor[['
       Mkt-RF']], dfPORT, RF)
278 # In[286]:
279
281 # Sensitivity check for the parameters
282 \text{ cut} = 240
283 beta_matrix, premium_matrix, point_estimate, rsquare_mean = FamaMacbeth_Test(dfFactor[[])
       Mkt-RF', 'Spread', 'Labor']].iloc[:cut,:], dfPORT.iloc[:cut,:], RF[:cut])
285
286 # In [21]:
287
288
# Rolling average calcualtion for list data
290 numbers = rsquare_mean
291 window_size = 120
292 numbers_series = pd.Series(numbers)
293 windows = numbers_series.rolling(window_size)
294 moving_averages = windows.mean()
295 moving_averages_list = moving_averages.tolist()
296 without_nans = moving_averages_list[window_size - 1:]
297
298
299 # In[22]:
301
_{\rm 302} # plot time series of rolling average
fig, axes = plt.subplots(1,1,figsize=(8,4),sharex=True,sharey=True)
304 fig.text(0.04, 0.5, r'$R^2_{adj}$', va='center', ha='center',rotation='vertical',
       fontsize = 14)
305 colormap = plt.cm.get_cmap('twilight')
axes.plot(dfPORT.index[window_size - 1:], without_nans, c=".3")
307 axes.axhline(y=np.mean(rsquare_mean),color='r', linestyle='--',label='Average '+r'$R^2_{{{1}}}
      adj}$'+': {}'.format(np.round(np.mean(rsquare_mean),2)))
308 axes.legend(fontsize = 14)
309 plt.plot()
plt.savefig('Rsquared')
311 plt.show()
312
313
314 # In[23]:
315
316
317 # Make the output table more readable
318 beta matrix = beta matrix.round(2)
319 for content in beta_matrix.T.index:
       print_report = pd.DataFrame(beta_matrix.T.loc[content,:].values.reshape(5,5),columns
320
       = ["BM" + str(i+1) for i in range(5)], index= ["ME" + str(i+1) for i in range(5)])
       print_report = pd.concat([print_report], axis=1, keys=[content])
321
       print(print_report.to_latex())
322
323
325 # In [24]:
326
327
328 # Process result from regressions to plot scatter plot
329 X = sm.add_constant(beta_matrix)
330 Estimated = X @ point_estimate
Realized = (dfPORT.sub(RF,axis = 'index')).mean()
332
333
```

```
334 # In[26]:
335
336
337 # Make the scatter plot
fig, axes = plt.subplots(1,2,figsize=(16,8),sharex=True,sharey=True)
fig.text(0.04, 0.5, 'Realized', va='center', ha='center', rotation='vertical', fontsize =
       14)
340 fig.text(0.5,0.04, 'Estimated', va='center', ha='center',rotation='horizontal',fontsize
       = 14)
341 colormap = plt.cm.get_cmap('twilight')
342 colors = [colormap(i) for i in np.linspace(0.1, 0.5,5)]
343 axes[0].plot([0.2, 1], [0.2, 1], ls="--", c=".3")
344 for i in range(0,5):
       axes[0].scatter(Estimated[i*5:(i+1)*5], Realized[i*5:(i+1)*5], c=colors[i], label = 'ME
345
        '+str(i+1), s=140)
axes[0].legend(fontsize = 14)
axes[1].plot([0.2, 1], [0.2, 1], ls="--", c=".3")
348 for i in range (0,5):
       axes[1].scatter(Estimated[i::5],Realized[i::5],c=colors[i],label = 'BM'+str(i+1), s
349
       =140)
axes[1].legend(fontsize = 14)
351 plt.plot()
plt.savefig('Scatter_mebetaCAPM')
353 plt.show()
354
355
356 # ### Return predictability test
357 # 1. Default spread
358 # 2. Short rate
359
360 # In[144]:
362
363 to_predict = dfFactor[['Mkt-RF']].rolling(12).sum().shift(-12)
364
365
366 # In[145]:
367
368
369 # Make the scatter plot
370 import seaborn as sns
fig, axes = plt.subplots(1,2,figsize=(16,8),sharex=True,sharey=True)
372 colormap = plt.cm.get_cmap('twilight')
colors = [colormap(i) for i in np.linspace(0.3, 0.5,5)]
374 \text{ \# axes}[0].plot([0.2, 1], [0.2, 1], ls="--", c=".3")
for i, k in enumerate(dfFactor[['Spread','RF']].columns):
       print(i,k)
376
       sns.regplot(dfFactor[[k]],to_predict['Mkt-RF'],ax= axes[i])
377
       axes[i].set_xlabel(k, fontsize = 14)
378
       axes[i].set_ylabel('MK-RF', fontsize = 14)
379
380 # plt.plot()
plt.savefig('Return_predictability')
382 plt.show()
383
384
385 # In[150]:
386
387
388 # Output the regression test result in latex
389 beta_matrix = pd.DataFrame()
390 for i in range(len(dfFactor[['Spread','RF']].columns)):
       y = to_predict[:-12]
X = sm.add_constant(dfFactor[['Spread','RF']].iloc[:-12,i])
391
392
       model = sm.OLS(y, X)
       results = model.fit()
394
       beta_i = pd.DataFrame(results.params[1:]).T
395
396
       beta_i = beta_i.rename(index = {0:'coef'})
       beta_matrix= pd.concat([beta_matrix, beta_i])
397
       t_i = pd.DataFrame(results.tvalues[1:]).T
398
       t_i = t_i.rename(index = {0:'t'})
399
       beta_matrix= pd.concat([beta_matrix, t_i])
400
401 print(beta_matrix.round(2).to_latex())
```