Problem A:

1. False.
2. True
3. True.
4. True.
5. False

Problem B:

1. mean $= 0$, covariance matrix $= \frac{2\Sigma}{n}$
2. $N(0, 2\Sigma)$
3. $\frac{1}{2}S_x + \frac{1}{2}S_y$
4. We can use Hotelling-T^2 test statistic:

$$T^2 = (\bar{X})' \left(\frac{S}{n}\right)^{-1} (\bar{X}), \text{ where } S = \frac{1}{n-1}\sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})'$$

This test is also equivalent to the likelihood ratio test

$$T^2 = \frac{(n-1)|\hat{\Sigma}_0|}{|\hat{\Sigma}|} - (n-1)$$

5. ~~True~~.

Problem C:

1. True.
2. True.
3. True
4. True
5. False
6. False.
7. True
8. False

Problem D.

1. By calculating the eigen values and eigen vectors.

The linear combination with the maximum variance is the eigen vector with biggest eigen value.

$$\begin{pmatrix} -0.116 \\ -0.932 \\ 0.343 \end{pmatrix}$$

2. Likewise: the eigenvector with smallest eigen value is

$$\begin{pmatrix} -0.632 \\ 0.336 \\ 0.699 \end{pmatrix}$$

3. $\lambda_{max} = 3.222$.

4. $\lambda_{min} = 1.398$

Problem E.

1. $Var(\bar{x}) = \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i) = \frac{1}{n^2} n \cdot \sigma^2 = \frac{\sigma^2}{n}$.

2. $X_t = \phi_0 + \phi_1 X_{t-1} + a_t$

$Var(X_t) = \frac{\sigma^2}{1-\phi_1^2}$, $cov(X_t, X_{t-1}) = \phi_1 \sigma^2$

$$Var(\bar{x}) = \frac{1}{n^2} Var\left(\sum_{t=1}^{n} X_t\right)$$

$$= \frac{1}{n^2}\left[\sum_{i=1}^{n} Var(X_t) + 2\sum_{t=2}^{n} Cov(X_t, X_{t-1})\right]$$

$$= \frac{1}{n^2}\left[n \cdot \frac{\sigma^2}{1-\phi_1^2} + 2\phi_1\sigma^2\right]$$

Then $\sqrt{n}\,\bar{x} \to \frac{\sigma^2}{1-\phi_1^2} = \frac{1}{1-\phi_1^2}$ as $n \to \infty$

3. According to CLT, $\sqrt{n}(\bar{x}-\mu)/s \to N(0,1)$

4. $\sqrt{n}(\bar{x}-\mu)/s \longrightarrow N(0, \frac{\sqrt{2}}{S}) = N(0, \frac{\frac{2\phi_1^2}{(1-\phi_1)} - \phi_1^2}{\phi_1^2})$

$= N(0, \frac{2}{1-\phi_1} - 1)$

5. With the series correlation, we can no longer refer to t-test. more specifically, can cannot estimate variance ~~covariance~~ with s.