Introduction to Machine Learning

Homework 2

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In [33]:

```
import pandas as pd
import numpy as np
from sklearn.linear_model import LinearRegression
import statsmodels.api as sm
```

Import data and process

In [2]:

```
bidenraw=pd.read_csv('problem-set-2/nes2008.csv')
bidenraw.describe()
```

Out[2]:

	biden	female	age	educ	dem	rep
count	1807.000000	1807.000000	1807.000000	1807.000000	1807.000000	1807.000000
mean	62.163807	0.552850	47.535141	13.360266	0.431655	0.205313
std	23.462034	0.497337	16.887444	2.440257	0.495444	0.404042
min	0.000000	0.000000	18.000000	0.000000	0.000000	0.000000
25%	50.000000	0.000000	34.000000	12.000000	0.000000	0.000000
50%	60.000000	1.000000	47.000000	13.000000	0.000000	0.000000
75%	85.000000	1.000000	59.500000	16.000000	1.000000	0.000000
max	100.000000	1.000000	93.000000	17.000000	1.000000	1.000000

In [34]:

```
x = np.array(bidenraw[['female','age','educ','dem','rep']])
y = np.array(bidenraw[['biden']])
```

In [35]:

```
bidenraw[['female','age','educ','dem','rep']].columns
```

Out[35]:

```
Index(['female', 'age', 'educ', 'dem', 'rep'], dtype='object')
```

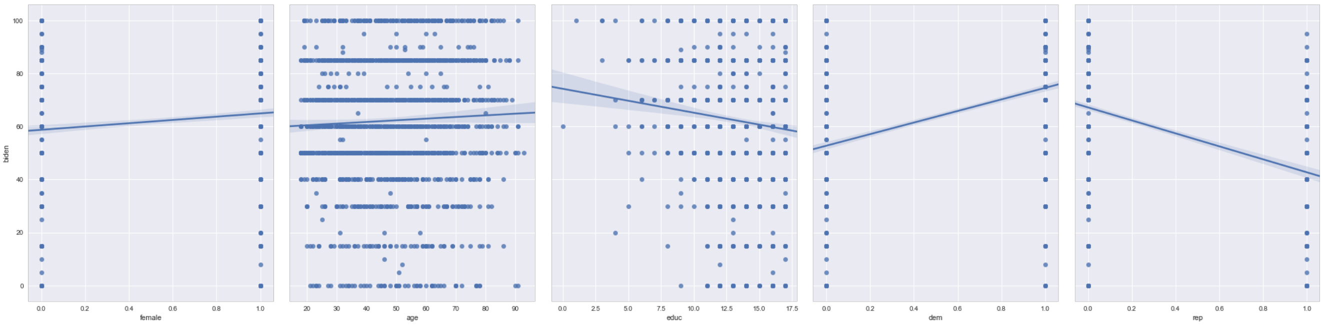
Plot the scatter graph for biden against each variable

In [45]:

```
import seaborn as sns
%matplotlib inline
sns.pairplot(bidenraw, x_vars=['female','age','educ','dem','rep'], y_vars='biden', size=7, aspect=0.8,
kind='reg')
```

Out[45]:

<seaborn.axisgrid.PairGrid at 0x240cda36788>



In [46]:

```
# Alternatively this is a simple way to fit linear model:
# SKlearn with simple but not enough statistic summary
#     model = LinearRegression()
#     model.fit(x, y)
#     r_sq = model.score(x, y)
#     r_sq
```

1. Estimate the MSE of the model using the traditional approach.

- Typically, this is desirable when there is a need for more detailed results.
- Need to add the column of ones to the inputs if you want statsmodels to calculate the intercept □ .

In [83]:

```
x = bidenraw[['female','age','educ','dem','rep']]
y = bidenraw[['biden']]
```

In [84]:

```
# Add the column of ones to the inputs
x = sm.add_constant(x)
# Create a model and fit it
model = sm.OLS(y, x)
results = model.fit()
results.summary()
```

Out[84]:

OLS Regression Results

Dep. Variable:	biden	**R-squared:**	0.282
Model:	OLS	**Adj. R-squared:**	0.280
Method:	Least Squares	**F-statistic:**	141.1
Date:	Sat, 01 Feb 2020	**Prob (F-statistic):**	1.50e-126
Time:	15:40:16	**Log-Likelihood:**	-7966.6
No. Observations:	1807	**AIC:**	1.595e+04
Df Residuals:	1801	**BIC:**	1.598e+04
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	58.8113	3.124	18.823	0.000	52.683	64.939
female	4.1032	0.948	4.327	0.000	2.243	5.963
age	0.0483	0.028	1.708	0.088	-0.007	0.104
educ	0.3453	0.195	1.773	0.076	0.727	0.037

educ	-0.3453	0.195	-1.773	0.076	-0.727	0.037
dem	15.4243	1.068	14.442	0.000	13.330	17.519
rep	-15.8495	1.311	-12.086	0.000	-18.421	-13.278

Omnibus:	87.979	**Durbin-Watson:**	1.996
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	101.940
Skew:	-0.533	**Prob(JB):**	7.31e-23
Kurtosis:	3.466	**Cond. No.**	348.

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Calculate MSE

In [43]:

```
# results.mse_total
y_pred = results.predict(x)
metrics.mean_squared_error(y,y_pred)
```

Out[43]:

```
395.2701692786484
```

Discussion

One biggest caveat is that this calculation is not appropriate in the first place in that the model is based on the whole sample set and therefore is a biased and poor estimate of error.

2. Calculate the test MSE of the model using the simple holdout validation approach

Split the sample set into a training set (50%) and a holdout set (50%)

In [78]:

```
X = bidenraw[['female','age','educ','dem','rep']]
y = bidenraw[['biden']]
from sklearn.model_selection import train_test_split
```

Be sure to set your seed prior to this part of your code to guarantee reproducibility of results

In [79]:

```
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=1, test_size=0.5)
print (X_train.shape)
print (y_train.shape)
print (X_test.shape)
print (y_test.shape)
```

```
(903, 5)
(903, 1)
(904, 5)
(904, 1)
```

Fit the linear regression model using only the training observations

In [82]:

```
# Add the column of ones to the inputs
X_train = sm.add_constant(X_train)
# Create a model and fit it
mlmodel2 = sm.OLS(y_train, X_train)
mlresults2 = mlmodel2.fit()
mlresults2.summary()
```

Out[82]:

OLS Regression Results

Dep. Variable:	biden	**R-squared:**	0.282
Model:	OLS	**Adj. R-squared:**	0.278
Method:	Least Squares	**F-statistic:**	70.40
Date:	Sat, 01 Feb 2020	**Prob (F-statistic):**	3.61e-62
Time:	15:40:08	**Log-Likelihood:**	-3963.2
No. Observations:	903	**AIC:**	7938.
Df Residuals:	897	**BIC:**	7967.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	56.0910	4.370	12.836	0.000	47.515	64.667
female	4.1864	1.309	3.197	0.001	1.617	6.756
age	0.0921	0.040	2.310	0.021	0.014	0.170
educ	-0.3883	0.268	-1.448	0.148	-0.914	0.138
dem	16.8127	1.485	11.320	0.000	13.898	19.728
rep	-12.7175	1.810	-7.024	0.000	-16.271	-9.164

Omnibus:	28.751	**Durbin-Watson:**	2.016
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	31.229
Skew:	-0.414	**Prob(JB):**	1.65e-07
Kurtosis:	3.382	**Cond. No.**	349.

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Calculate the MSE using only the test set observation

In [81]:

```
# Use model created from training set to predict based on test set
X_test = sm.add_constant(X_test)
y_pred = mlresults.predict(X_test)
metrics.mean_squared_error(y_test,y_pred)
```

Out[81]:

```
411.85958298595153
```

How does this value compare to the training MSE from question 1?

Clearly there the MSE from question 1 is a determined number, as long as the data set is given. However, the MSE using simple random split in question 2 varies from different random seeds. The random characteristic of MSE in question 2 is the major reason to implement approach in question 3.

3. Repeat the simple validation set approach from the previous question 1000 times, using 1000 different splits of the observations into a training set and a test/validation set.

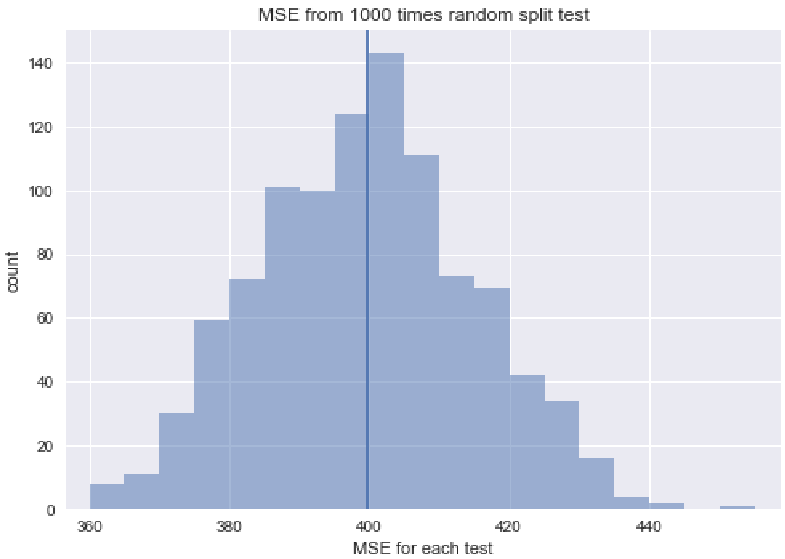
In [16]:

```
import warnings
warnings.filterwarnings('ignore')

X = bidenraw[['female','age','educ','dem','rep']]
y = bidenraw[['biden']]
from sklearn.model_selection import train_test_split
from sklearn import metrics
ml1000mse = []
for i in range(1000):
    X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=i, test_size=0.5)
    # Add the column of ones to the inputs
    X_train = sm.add_constant(X_train)
    # Create a model and fit it
    mlmodel = sm.OLS(y_train, X_train)
    mlresults = mlmodel.fit()
    X_test = sm.add_constant(X_test)
    y_pred = mlresults.predict(X_test)
    ml1000mse.append(metrics.mean_squared_error(y_test, y_pred))
```

In [29]:

```
# fixed bin size
from matplotlib import pyplot as plt
plt.style.use('seaborn')
bins = np.arange(300, 500, 5) # fixed bin size
plt.xlim([min(ml1000mse)-5, max(ml1000mse)+5])
plt.hist(ml1000mse, bins=bins, alpha=0.5)
plt.axvline(np.asarray(ml1000mse).mean())
plt.title('MSE from 1000 times random split test')
plt.xlabel('MSE for each test')
plt.ylabel('count')
plt.show()
```



Description of histogram and comments

First the MSE ditribution of 1000 times tests takes a form of normal distribution like shape, exhiniting the effect of central limit theorem. The result also expalins the point of repeated randomized split test, which is to provide a consistent estimate of the model and therefore are more reliable when making inference about the population. The mean of MSE is 399.6, which is quite stable regardless which randomization seed we set.

In [18]:

```
np.asarray(ml1000mse).mean()
```

```
np.asarray(ml1000mse).mean()
```

Out[18]:

```
399.6615061066422
```

4. Compare the estimated parameters and standard errors from the original model in question 1 (the model estimated using all of the available data) to parameters and standard errors estimated using the bootstrap (B =1000).

In [73]:

```
# scikit-learn bootstrap
from sklearn.utils import resample
# data sample
# prepare bootstrap sample
boot = resample(bidenraw, replace=True, n_samples=1000, random_state=1)

# out of bag observations
oob=pd.concat([bidenraw, boot]).drop_duplicates(keep=False)
# population = np.random.normal(loc=mean, scale=stdev, size=50000)
```

In [69]:

```
# Add the colum of ones to the inputs
X_train = boot[['female','age','educ','dem','rep']]
y_train = boot[['biden']]
X_train = sm.add_constant(X_train)
# Create a model and fit it
mlmodel = sm.OLS(y_train, X_train)
mlresults = mlmodel.fit()
mlresults.summary()
```

Out[69]:

OLS Regression Results

Dep. Variable:	biden	**R-squared:**	0.275
Model:	OLS	**Adj. R-squared:**	0.271
Method:	Least Squares	**F-statistic:**	75.23
Date:	Sat, 01 Feb 2020	**Prob (F-statistic):**	6.49e-67
Time:	15:25:56	**Log-Likelihood:**	-4423.9
No. Observations:	1000	**AIC:**	8860.
Df Residuals:	994	**BIC:**	8889.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	57.2631	4.323	13.245	0.000	48.779	65.747
female	3.1547	1.287	2.450	0.014	0.628	5.681
age	0.0977	0.039	2.478	0.013	0.020	0.175
educ	-0.4160	0.263	-1.584	0.114	-0.931	0.099
dem	15.7901	1.453	10.871	0.000	12.940	18.641
rep	-15.0100	1.802	-8.328	0.000	-18.547	-11.473

Omnibus:	19.051	**Durbin-Watson:**	1.963
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	19.695
Skew:	-0.324	**Prob(JB):**	5.29e-05
Kurtosis:	3.231	**Cond. No.**	346.

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [72]:

```
X_test = oob[['female','age','educ','dem','rep']]
y_test = oob[['biden']]
# Use model created from training set to predict based on test set
X_test = sm.add_constant(X_test)
y_pred = mlresults.predict(X_test)
metrics.mean_squared_error(y_test,y_pred)
```

Out[72]:

```
422.4466985286168
```

Description of coefficient and std

In [99]:

```
Comparision = pd.DataFrame()
Comparision['Q1_coefficient'] = results.params
Comparision['Q4_coefficient'] = mlresults.params
Comparision['Q1_se'] = results.bse
Comparision['Q4_se'] = mlresults.bse
Comparision['Q1_p'] = results.pvalues
Comparision['Q4_p'] = mlresults.pvalues
Comparision
```

Out[99]:

	Q1_coefficient	Q4_coefficient	Q1_se	Q4_se	Q1_p	Q4_p
const	58.811259	57.263108	3.124437	4.323418	2.694143e-72	5.384052e-37
female	4.103230	3.154657	0.948229	1.287427	1.592601e-05	1.444293e-02
age	0.048259	0.097724	0.028247	0.039430	8.772744e-02	1.336097e-02
educ	-0.345335	-0.416020	0.194780	0.262651	7.640571e-02	1.135282e-01
dem	15.424256	15.790141	1.068033	1.452559	8.144928e-45	4.370008e-26
rep	-15.849506	-15.009987	1.311362	1.802393	2.157309e-32	2.706188e-16

Discussion

The bootstrap method is a resampling technique for estimating a sampling distribution. The idea of the bootstrap method is to generate new data from a “population” by repeated sampling from data with replacement. According to the result we can see that in a large sense (sign and magnitude) coefficients from Q1 and Q4 remain compariable. However, when taking a closer look, some coefficient do show different confidence interval such as age. It is understandable to see some small difference are the assumption for bootstrap is that every observation in the sample set has equal posibility to be chosen. This is an implicit generalization of the greater population. In bootstrapping, we take the sample we have and sample from it, whereas Q1 use the whole sample set directly. We can also calculate the MSE of Q4 which is 422, greater (or more precisely less stable) than Q1's MSE.

In [ ]: