

Review of Basic Statistical Procedures

Basic Statistical Inference

Let X_1, \dots, X_n be a random sample with pdf $f(\theta)$, where either f or θ or both may be unknown.

Inference:

- Estimation
- Hypothesis Testing

Approaches:

- Frequentist
- Bayesian

Parameter Estimation

- The Maximum Likelihood Method

Let X_1, \dots, X_n be a random sample from $f(x, \theta)$. The *likelihood function* of θ is defined to be

$$L(\theta) = \prod_i^n f(x_i; \theta)$$

Under regularity conditions, $\hat{\theta}$, the MLE, is approximately normal, with mean θ and variance $\frac{1}{nI(\theta)}$, where

$$I(\theta) = E \left[\frac{\partial}{\partial \theta} \log f(x, \theta) \right]^2$$

- The Method of Moments

Define the k 'th *moment* of $f(x, \theta)$ as

$$\mu_k = E[X^k].$$

The corresponding *sample moments* are given as

$$\hat{\mu}_k = \frac{1}{n} \sum_{i=1}^n X_i^k$$

Let $\theta_j = h_j(\mu_1, \dots, \mu_s)$ be parameters of interest.

The method of moments estimates of θ_j

$$\hat{\theta}_j = h_j(\hat{\mu}_1, \dots, \hat{\mu}_s)$$

Hypothesis Testing

Suppose we are interested in determining whether $\theta \in \Theta_1$ versus $\theta \in \Theta_2$, where Θ_1 and Θ_2 are disjoint subsets of the parameter space.

Suppose X_1, \dots, X_n is a random sample from $f(x, \theta)$, the hypotheses may be written as

$$H_0 : \theta \in \Theta_0$$

$$H_1 : \theta \in \Theta_1.$$

- Type I error occurs when H_0 is falsely rejected
Type II: When H_0 is not rejected when it should be.

The probability of Type I error is known as *level of significance*.

Suppose H_0 specifies that the pdf of the data is $f_0(x)$, and H_1 claims $f_1(x)$.

Neyman-Pearson Lemma. Let ϕ^* be a test that accepts when

$$\frac{f_0(x)}{f_1(x)} > c$$

with significance level α^* . Let ϕ be another test of level $\alpha \leq \alpha^*$. Then the power of ϕ^* is greater than or equal to that of ϕ .

For testing composite hypotheses, $H_o : \theta \in \Theta_o$ vs $H_1 : \theta \in \Theta_1$, the *generalized likelihood ratio tests* may be constructed:

$$\Lambda = \frac{\sup_{\theta \in \Theta_o} [L(\theta)]}{\sup_{\theta \in \Theta_1} [L(\theta)]}$$

where Θ_o and Θ_1 are partitions of the parameter space Θ .

In application, one uses the following formulation

$$\Lambda = \frac{\sup_{\theta \in \Theta_0} [L(\theta)]}{\sup_{\theta \in \Theta} [L(\theta)]}$$

The likelihood ratio test rejects for small values of Λ , or large values of $-\log \Lambda$.

Given a test statistic $T = T(X_1, \dots, X_n; \theta_o)$, and the observed value t_{obs} , the *p-value* is defined as the probability of observing a value of T as extreme as or more extreme than t_{obs} . Small p-values are evidence against H_o .

Confidence Intervals

Let $L = L(X_1, \dots, X_n)$ and $U = U(X_1, \dots, X_n)$ be statistics, such that $Pr[L \leq \theta \leq U] = 1 - \alpha$. Then the interval $[L, U]$ is called a $100(1 - \alpha)\%$ confidence interval for θ .

Assumptions of Classical Statistical Inference

- Independence
- Normality (or other specified distribution)

Validation of Assumptions:

- Histograms, qqnorm, etc., for normality
- Box-plots, stem-and-leaf diagrams for outlier detection
- Simple time series plot for serial correlations: including trends and cycles.

When distribution of data skewed, confidence intervals tend to be large, on the average, and p-values may be inflated.

Nonparametric methods do not make explicit assumptions about underlying distributions.

Most commonly used nonparametric methods are based on rank transformations of the observations.

Example: Cedar-apple rust data:

- Disease that affects apple trees, with symptom, rust-colored spots on apple leaves. Study conducted to study cause.
- In the first year of experiment the number of affected leaves on a random sample of 8 trees was counted on Variety 1.
- Normally, the average number of affected leaves is at most 25.

Data:

Tree	Count
1	38
2	10
3	84
4	36
5	50
6	35
7	73
8	48

Does the data suggest the apple trees are affected?

Standard Tests for Location

One-Sample Problem

Let X_1, \dots, X_n be a random sample from a distribution with mean μ and variance σ^2 . We wish to test the hypothesis

$$H_0 : \mu = \mu_0$$

vs.

$$H_1 : \mu > \mu_0$$

When the distribution is normal, and σ^2 unknown (the usual case), a procedure that is commonly used is the Student's t-test, given by

$$T = \frac{\sqrt{n}(\bar{X} - \mu_0)}{S}$$

which under the null has a t_{n-1} distribution.

Example: Cedar-apple rust data:

```
> variety1
```

```
[1] 38 10 84 36 50 35 73 48
```

```
> t.test(variety1,mu=25)
```

One Sample t-test

data: variety1

t = 2.651, df = 7, p-value = 0.03289

alternative hypothesis: true mean is not equal to 25

95 percent confidence interval:

27.34963 66.15037

sample estimates:

mean of x: 46.75

If EDA shows that the data is non-normal, then the *Wilcoxon signed-rank* test may be used.

- Let $D_i = X_i - \mu_0$
- Rank $|D_1|, \dots, |D_n|$
- Let T_+ and T_- be the sums of the ranks assigned to the positive and negative d_i 's, respectively. Clearly, if H_0 is true, $T_+ \approx T_-$.

Further, under H_0 ,

$$E[T_+] = \frac{n(n+1)}{4}$$

and

$$Var(T_+) = \frac{n(n+1)(2n+1)}{24}$$

- The Wilcoxon signed-rank test rejects for large values of T_+ , or when

$$Z = \frac{T_+ - E[T_+ | H_0]}{\sqrt{Var(T_+ | H_0)}}$$

exceeds a standard normal critical point.

- The test requires the underlying distribution to be symmetric.

Example: Cedar-apple rust data (cont'd):

- Disease that affects apple trees, with symptom, rust-colored spots on apple leaves. Study conducted to study cause.
- In the first year of experiment the number of affected leaves on a random sample of 8 trees from Variety 1 and a random sample of 7 trees from Variety 2 counted.

Data:

Variety 1		Variety 2	
Tree	Count	Tree	Count
1	38	1	27
2	10	2	28
3	84	3	57
4	36	4	66
5	50	5	77
6	35	6	49
7	73	7	62
8	48		

Does the data suggest there is a difference between the two varieties in the mean number of affected leaves?

Two-Sample Problem

Let X_1, \dots, X_n , and Y_1, \dots, Y_m , be independent random samples from distributions with respective means, μ_1 and μ_2 , and variances, σ_1^2 and σ_2^2 .

We are interested in the hypothesis

$$H_0 : \mu_1 - \mu_2 = \Delta_0$$

vs

$$H_1 : \mu_1 - \mu_2 > \Delta_0$$

When both distributions are normal, with unknown, but equal, variance σ^2 , the two-sample t-test is given by

$$T = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{\sqrt{S_p^2(\frac{1}{n} + \frac{1}{m})}}$$

where the pooled variance S_p^2 is defined as

$$S_p^2 = \frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$$

It is easy to show that T has a t_{n+m-2} distribution.

Example: Cedar-apple rust data (cont'd):

```
> variety2 <- c(27,28,57,66,77,49,62)
```

```
> t.test(variety1,variety2,var.equal=T)
```

Two Sample t-test

data: variety1 and variety2

$t = -0.501$, $df = 13$, $p\text{-value} = 0.6247$

alternative hypothesis:

true difference in means is not equal to 0

95 percent confidence interval:

-29.40555 18.33412

sample estimates:

mean of x mean of y

46.75000 52.28571

Example: Cedar-apple rust data (cont'd):

```
> var.test(variety1,variety2)
```

F test to compare two variances

data: variety1 and variety2

F = 1.499, num df = 7, denom df = 6,

p-value = 0.638

alternative hypothesis:

true ratio of variances is not equal to 1

95 percent confidence interval:

0.2631926 7.6728062

sample estimates:

ratio of variances

1.499006

When the variances are not equal, an approximate test is *Welch's modified two-sample t test*, given by

$$T = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{S_w}$$

where

$$S_w^2 = \frac{S_1^2}{n} + \frac{S_2^2}{m}$$

T has an approximate t_ν distribution, with

$$\nu \approx \left[\frac{c^2}{n-1} + \frac{(1-c)^2}{m-1} \right]^{-1}$$

where

$$c = \frac{S_1^2}{nS_w^2}$$

Example: Cedar-apple rust data (cont'd):

```
> t.test(variety1,variety2)
```

Welch Two Sample t-test

data: variety1 and variety2

t = -0.5082, df = 12.956,

p-value = 0.6198

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

-29.07417 18.00275

sample estimates:

mean of x mean of y

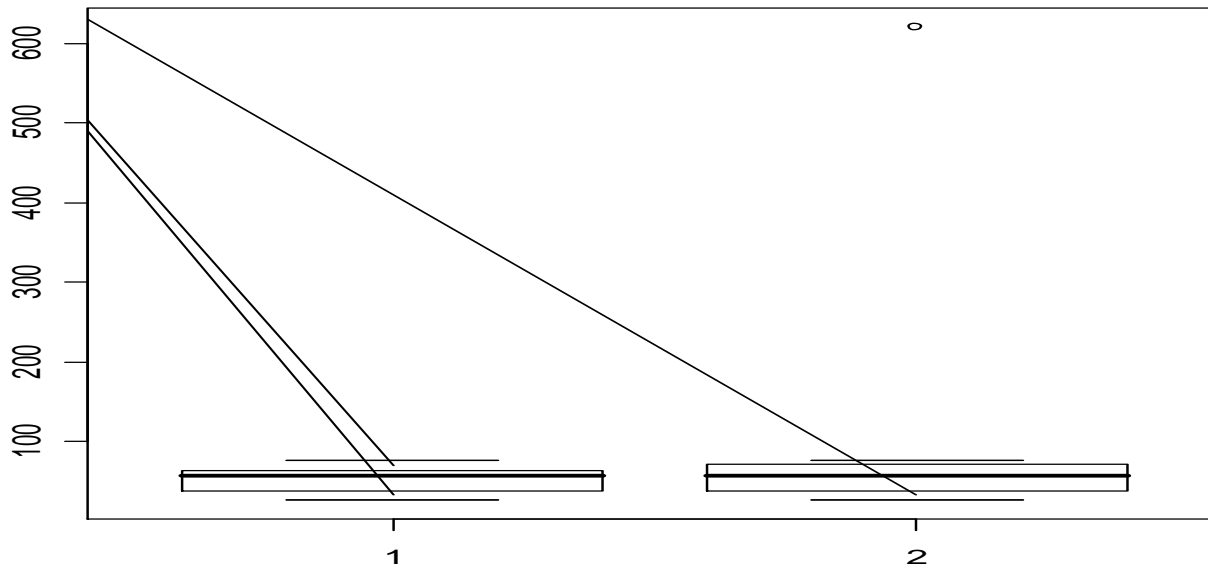
46.75000 52.28571

Example: Cedar-apple rust data (cont'd):

```
> variety2 <- c(27,28,57,66,77,49,62)
```

```
> variety3 <- c(27,28,57,66,77,49,6200)
```

```
> boxplot(variety2,variety3)
```



Example: Cedar-apple rust data (cont'd):

```
t.test(variety1,variety2,var.equal=T)
```

Two Sample t-test

data: variety1 and variety2

t = -0.501, df = 13,

p-value = 0.6247

alternative hypothesis:

true difference in means is not
equal to 0

95 percent confidence interval:

-29.40555 18.33412

sample estimates:

mean of x mean of y

46.75000 52.28571

```
> variety3 <- c(27,28,57,66,77,49,620)
```

```
> t.test(variety1,variety3,var.equal=T)
```

Two Sample t-test

data: variety1 and variety3

t = -1.1151, df = 13, p-value = 0.285

alternative hypothesis: true difference in
means is not equal to 0

95 percent confidence interval:

-250.4077 79.9077

sample estimates:

mean of x mean of y

46.75 132.00

Wilcoxon Rank-Sum Test

- Combine the two samples, and rank the observations
- Let T_1 be the sum of the ranks for the observations in the 1st group, and T_2 for the second
- The Wilcoxon rank-sum test is then based on T_W , which has an asymptotic standard normal distribution,

$$T_W = \frac{T_1 - E(T_1 \mid H_o)}{\sqrt{\text{Var}(T_1 \mid H_o)}}$$

where

$$E(T_1 \mid H_o) = \frac{n(n + m + 1)}{2}$$

and

$$\text{Var}(T_1 \mid H_o) = \frac{nm(n + m + 1)}{12}$$

In some applications, the Mann-Whitney (U) form of the Wilcoxon rank-sum test is used, due to the relative ease of computing critical values using the latter. The two are related by the equation

$$U_1 = T_1 - \frac{n(n+1)}{2}$$

- The test is most appropriate when the populations have the same shape and differ only in location (e.g., same dispersion) However, the distributions do not have to be symmetric.

Example: Cedar-apple rust data (cont'd):

```
> wilcox.test(variety1,variety2)
```

Wilcoxon rank sum test

data: variety1 and variety2

W = 24, p-value = 0.6943

alternative hypothesis: true mu
is not equal to 0

```
> variety3 <-  
  c(27,28,57,66,77,49,620)
```

```
> wilcox.test(variety1,variety3)
```

Wilcoxon rank sum test

data: variety1 and variety3

W = 22, p-value = 0.5358

alternative hypothesis: true mu
is not equal to 0

Example: Cedar-apple rust data (cont'd):

- Disease that affects apple trees, with symptom, rust-colored spots on apple leaves. Study conducted to study cause.
- In the first year of experiment the number of affected leaves on a random sample of 8 trees from Variety 1
- All red cedar trees within 100 yards of the orchard were then removed and the following year the same Variety 1 trees were examined for affected leaves.

Data:

<u>Year 1</u>		<u>Year 2</u>
Tree	Count	Count
1	38	32
2	10	16
3	84	57
4	36	28
5	50	55
6	35	12
7	73	61
8	48	29

Does the data suggest there is a difference between the mean number of affected leaves in the two periods?

Example: Cedar-apple rust data (cont'd):

Apply 2-sample t-test?

```
> t.test(year1, year2, var.equal=T)
```

Two Sample t-test

data: variety1 and year2

t = 0.9899, df = 14, p-value = 0.3390

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

-12.25070 33.25070

sample estimates:

mean of x mean of y

46.75 36.25

Paired Samples

Consider a study comparing weight gains in rats before and after receiving a new dietary regime. Let $(X_1, Y_1), \dots, (X_n, Y_n)$ be the respective weights of n rats. Clearly, there is dependence between observations on the same rat, hence, the two-sample procedures discussed above will not be appropriate.

Let $D_i = X_i - Y_i$, $i = 1, \dots, n$. Then the problem may be handled based on the D_i 's using the one-sample t-test (paired t-test) or the Wilcoxon signed-rank test.

Example: Cedar-apple rust data (cont'd):

```
> t.test(year1,year2,paired=T)
```

Paired t-test

data: variety1 and year2

$t = 2.4342$, $df = 7$,

p-value = 0.04514

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

0.2999574 20.7000426

sample estimates:

mean of the differences

10.5

```
> wilcox.test(year1,year2,paired=T)
```

Wilcoxon signed rank test with continuity correction

data: year1 and year2

$V = 32.5$, $p\text{-value} = 0.04967$

alternative hypothesis: true μ is not equal to 0

Measures of Associations

Given a random sample $(X_1, Y_1), \dots, (X_n, Y_n)$ on n individuals, we are often interested in the degree of association or relationship between X and Y .

1. *Pearson's product-moment correlation coefficient*

The population correlation coefficient is defined as

$$\rho = \frac{E(X - \mu_x)(Y - \mu_y)}{\sigma_x \sigma_y}$$

and has the property $-1 \leq \rho \leq 1$.

The corresponding sample quantity is given by

$$r = \frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{(n - 1)S_x S_y}$$

with similar properties as ρ .

For the problem of testing $H_0 : \rho = 0$ vs. $H_1 : \rho > 0$, a suitable test statistic is given by

$$T = \sqrt{\frac{n-2}{1-r^2}}$$

which has a t_{n-2} null distribution.

The test of the more general hypothesis $H_0 : \rho = \rho_0$ vs. $H_1 : \rho > \rho_0$ involves Fisher's Z -transformation. Let

$$Q(r) = \frac{1}{2} \ln \left(\frac{1+r}{1-r} \right)$$

Then a test statistic is given by

$$Z = \frac{Q(r) - Q(\rho)}{\sqrt{\frac{1}{n-3}}}$$

which has an approximate standard normal null distribution.

For purposes of confidence interval construction, one often uses the following inverse hyperbolic tangent transformation. Let $H(r) = \operatorname{atanh}(r)$. Then $H(r) - H(\rho)$ has an approximate $N(0, 1/(n - 3))$ distribution. A

100(1 - α)% confidence interval for ρ is

$$\operatorname{tanh} \left[(H(r) \pm Z_{\alpha/2} \frac{1}{\sqrt{n - 3}}) \right].$$

Remarks

- The product moment correlation coefficient may not be appropriate for ordinal data
- The sample correlation coefficient r is extremely sensitive to outliers

2. *Spearman's rank correlation coefficient*

Spearman's rho is a measure of association based on ranks.

Given $(X_1, Y_1), \dots, (X_n, Y_n)$, let d_i denote the difference between the ranks of X_i and Y_i , when the X 's and Y 's are ranked separately among themselves.

For tied observation, the mean of the ranks are taken.

When there are no ties, Spearman's rho r_s coincides with the Pearson product-moment correlation coefficient applied to the ranks, and is given by

$$r_s = 1 - \frac{6 \sum_i d_i^2}{n(n^2 - 1)}$$

3. *Kendall's tau*

Kendall's tau is another rank-based measure of association. Let R_i and S_i denote the ranks of X_i and Y_i , respectively. Then

$$\tau = \frac{1}{\binom{n}{2}} \sum_{i < j} \text{sgn}(R_i - R_j)(S_i - S_j)$$

where

$$\text{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

```
> cor.test(variety1,year2)
```

Pearson's product-moment
correlation

data: variety1 and year2

t = 3.973, df = 6,

p-value = 0.007342

alternative hypothesis:

true correlation is not equal to 0

95 percent confidence interval:

0.3662168 0.9725356

sample estimates:

cor

0.851221

```
> cor.test(variety1,year2,method="sp")
```

Spearman's rank correlation rho

data: variety1 and year2

p-value = 0.002232

alternative hypothesis:

true rho is not equal to 0

sample estimates:

rho

0.9285714

Goodness-of-Fit Tests

CDF Test

Let X_1, \dots, X_n be a random sample from F . The empirical distribution function, F_n , is defined as

$$F_n(x) = \frac{\sum_i I[X_i \leq x]}{n}$$

for $x \in R$. As an estimator of F , F_n has several desirable properties.

One way of assessing goodness-of-fit is by looking at a plot of F_n and F , superimposed. The

R function `cdf.compare()` provides such plots.

Chi-Square Test of Goodness of Fit

Let X_1, \dots, X_n be a random sample from F . Group the observations x_1, \dots, x_n into k classes. Let m_i and M_i be the observed and expected, under F , counts in the i 'th class, respectively. The chi-square test of goodness-of-fit is given by:

$$\chi_G^2 = \sum_i \frac{(m_i - M_i)^2}{M_i}$$

and has an approximate χ_{k-1}^2 distribution under F . In `R`, the function `chisq.gof` may be used.

The Kolmogorov-Smirnov Test

Let X_1, \dots, X_n be iid F. Then the one-sample K-S test is given by

$$T_{KS} = \sup_x | F_n(x) - F(x) |$$

In the case of two samples, let X_1, \dots, X_n and Y_1, \dots, Y_m be independent random samples from F and G, respectively. Then the K-S test for comparing the two distributions is given by:

$$T_{KS} = \sup_x | F_n(x) - G_m(x) |$$

In `R`, the procedures are implemented using `ks.gof()`.

Bootstrap Tests

The bootstrap may be used to construct approximate confidence intervals and test critical points, when the associated distributions are not tractable.

Consider the problem of comparing two means μ_1 and μ_2 , when sampling X_1, \dots, X_n and Y_1, \dots, Y_m from F and G , respectively.

To implement the bootstrap, one draws pseudo-random samples from F_n and G_m under $H_0 : \mu_1 = \mu_2$.

- Draw with replacement, B pseudo-samples: X_1^*, \dots, X_n^* from F_n and Y_1^*, \dots, Y_m^* from \tilde{G}_m , where \tilde{G}_m is the edf of $\tilde{Y}_1, \dots, \tilde{Y}_m$, and

$$\tilde{Y}_i = Y_i + \bar{X} - \bar{Y}$$

For the b'th pseudo-sample, compute

$$Z_b^* = \frac{(\bar{X}^* - \bar{Y}^*)}{\sqrt{\frac{s_1^*}{n} + \frac{s_2^*}{m}}}$$

- Calculate the p-value as

$$p_B = \frac{\sum_b^B I[|Z_b^*| \geq z_{obs}]}{B}$$

where z_{obs} is the observed value of the test statistic based on the original sample.

Example. Suppose the manufacturer of a certain product claimed that less than 15%, the industry standard, of the items manufactured by his factory were defective.

To test whether his claim was true, a random sample of 100 items was taken, of which 13 turned out to be defective.

Test the relevant hypothesis.

Let p denote the probability of success

The hypothesis of interest is

$$H_0 : p = 0.15$$

vs

$$H_1 : p < 0.15$$

A reasonable test is one which rejects H_0 for small values of X .

An *exact test* is obtained computing the p-value as $Pr[X \leq 13 \mid p = 0.15]$, or

$$= \sum_{j=1}^{13} \binom{100}{j} 0.15^j 0.85^{n-j}$$

```
binom.test(13, 100, 0.15, alt = "l")
```

Exact binomial test

data: 13 out of 100

number of successes = 13, n = 100,

p-value = 0.3474

Let X be the number of successes in n trials.

For large n , such that $\min(np, nq) \geq 5$,

$$Z = \frac{x - np - 0.5}{\sqrt{npq}}$$

approximately Standard Normal.

```
> prop.test(13,100,0.15,alt="l")
```

1-sample prop test with continuity correction

X-square = 0.1765, df = 1, p-value = 0.3372

95 percent confidence interval:

0.0000000 0.2009056

Denote $\hat{p} = X/n$.

An approximate $100(1 - \alpha)\%$ confidence interval for p may be constructed based on the pivot

$$\frac{|\hat{p} - p| - \frac{1}{2n}}{\sqrt{pq/n}} \leq Z_{\alpha/2}$$

which gives (P_L, P_U) , where

$$P_L = \frac{(2n\hat{p} + Z_{\alpha/2}^2 - 1) - Z_{\alpha/2}\sqrt{Z_{\alpha/2}^2 - (2 + 1/n) + 4\hat{p}(n\hat{q} + 1)}}{2(n + Z_{\alpha/2}^2)}$$

and

$$P_U = \frac{(2n\hat{p} + Z_{\alpha/2}^2 + 1) + Z_{\alpha/2}\sqrt{Z_{\alpha/2}^2 - (2 + 1/n) + 4\hat{p}(n\hat{q} + 1)}}{2(n + Z_{\alpha/2}^2)}$$

Exact values for P_L and P_U may be obtained by solving the equations

$$Pr[X \geq x \mid P_L] = \alpha/2$$

and

$$Pr[X \leq x \mid P_U] = \alpha/2$$

Example. Suppose the manufacturer of a certain product claimed that the percentage of defective items manufactured by his factory was less than that for the competitor.

To test whether his claim was true, random samples of 131 and 281 items were taken (from each company) of which 161 and 271, respectively, turned out to be non-defective.

Test the relevant hypothesis.

Let p_1 and p_2 denote the respective proportions of defective items.

The hypotheses of interest are

$$H_0 : p_1 = p_2$$

vs.

$$H_1 : p_1 \neq p_2$$

A large sample test, with Yates' correction for continuity, is given by

$$Z = \frac{|\hat{p}_1 - \hat{p}_2| - \frac{1}{2}\left(\frac{1}{n} + \frac{1}{m}\right)}{\sqrt{\hat{p}_c \hat{q}_c \left(\frac{1}{n} + \frac{1}{m}\right)}} \quad \hat{p}_c = \frac{n\hat{p}_1 + m\hat{p}_2}{n + m}$$

An approximate $100(1 - \alpha)\%$ confidence interval for $p_1 - p_2$ is

$$\hat{p}_1 - \hat{p}_2 \pm \left(Z_{\alpha/2} \sqrt{\hat{p}_1 \hat{q}_1 / n + \hat{p}_2 \hat{q}_2 / m} + \frac{1}{2} \left(\frac{1}{n} + \frac{1}{m} \right) \right)$$

Example (cont'd):

2-sample test for equality of proportions with continuity correction

```
> x <- c(161,131)
```

```
> n <- c(271,281)
```

```
> prop.test(x,n)
```

X-square = 8.5518, df = 1, p-value = 0.0035

alternative hypothesis: two.sided

95 percent confidence interval:

0.04169418 0.21411336

Example: Suppose random samples of sizes $n=100$ and $m=150$ gave $x=5$ and $y=7$, respectively.

```
X<-c(5,7); n <- c(100,150)  
prop.test(x, n)
```

X-square = 0.0146, df = 1, p-value = 0.9039

Warning messages:

```
Expected counts < 5. Chi-square/normal  
approximation may not be  
appropriate. in: prop.test(x, n)
```

Problem Set 2

Reading Assignment:

Chapter 2,3,4. The Statistical Sleuth: A Course in Methods of Data Analysis. Ramsey & Schafer

Consider the data on **Chicken Weights by Feed Type** (chickwts) in R library MASS

1. Determine whether there is a significant difference in the mean weights of chicks fed meatmeal vs. those fed casein using each of the following procedures:
 - a) A parametric procedure
 - b) A non-parametric procedure
 - c) A re-sampling procedureDiscuss the assumption underlying each of the analyses, their validity, and any remedial measures to be taken
2. Using the data for chicks fed soybean and those on sunflower, compute the following, based on a suitable bootstrap method:
 - a) A 95% confidence interval for the difference in median weight for the two groups
 - b) A 95% bootstrap confidence interval for the ratio of the variances of soybean fed to sunflower fed chicks
 - c) A 95% confidence interval for the ratio of the variances of soybean fed to sunflower fed chicksDiscuss the assumption underlying each of the analyses, their validity, and any remedial measures to be taken
3. Assume that if the weight of a chick is below 258, that chick is classified under “LOW WEIGHT”. For chicks fed meatmeal vs. those fed soybean,
 - a) Determine whether there is a significant difference in the proportions of the chicks classified under “LOW WEIGHT”.
 - b) Construct a 95% confidence interval for the difference in the proportions of the chicks classified under “LOW WEIGHT”.Discuss the assumption underlying each of the analyses, their validity, and any remedial measures to be taken

