6.3: Dominors

Question:
There is an 8*8 chessboard in which two diagonally opposite corners have been cut off. You are given 31 dominos, and a single domino can cover exactly two squares. Can you use the 31 dominos to cover the entire board? Prove your answer by providing an example or showing why it's impossible.
Answer:
We cannot cover the entire board by the 31 dominors.
Proof:
Since the chessboard originally have 32 white squares and 32 black squares. When we cut off the two suqares in diagonally opposite corners, we can see there are 30 white squares and 32 black squares, or there are 30 black squares and 32 white squares, which is equivalent.
Then suppose there are 30 black squares and 32 white squares. Since every dominor can cover exactly 1 black and 1 white square, we can find that only if there is 31 black squares and 31 white squares can the dominors cover the entire board.
Therefore, we cannot cover the entire board by the 31 dominors.

6.4: Ants on a Triangle

Question:
There are three ants on different vertices of a triangle. What is the probability of collision (between any two or all of them) if they start walking on the sides of the sides if the triangle? Assume that each ant randomly picks a direction, with either direction being equally likely to be chosen, and that they walk at the same speed. Similarly, find the probability of collision with n ants on an n-vertex polygon.
Answer:
When in a triangle, for every ant, there are two directions, the clockwise and the counter clockwise.
Only when the three ants walk in the same direction won't they have collisions.
The probability of walking in the same direction is $2 \times (\frac{1}{2})^3 = \frac{1}{4}$.
Thus, the probability of collision is $1 - \frac{1}{4} = \frac{3}{4}$.
Follow up:
For the n-vertex polygon, every ant still just has two directions, the clockwise and the counter clockwise, and only when the three ants walk in the same direction won't they have collisions.
Thus, the probability of walking in the same direction is $2 \times (\frac{1}{2})^n = (\frac{1}{2})^{n-1}$.
Thus, the probability of collision is $1 - (\frac{1}{2})^{n-1}$.

6.5: Jugs of Water

Question:
You have a five-quart jug, a three-quart jug, and an unlimited supply of water (but no measuring cups). How would you come up with exactly four quarts of water. Note that the jugs are oddly shaped, such that filling exactly "half" of the jug would be impossible.
Answer:
The following is one method: Let the quarts of water in five-quart jug be quart5;

Let the quarts of water in three-quart jug be quart3;
1)Filld the 5-quart jug. quart5=5,quart3=0;
2)Filled the 3-quart jug by water in 5-quart.quart5=2, quart3=3;
3)Pulled out the water in 3-quart jug. quart5=2, quart3=0;
4)Put the water in 5-quart jug to 3-quart jug.quart5=0; quart3=2; 5)Filled the 5-quart jug. quart5=5; quart3=2; 6)Filled the 3-quart jug by water in 5-quart.quart5=4, quart3=3;
Then we have the 4 quarts water.

6.6: Blue-Eyed Island

Question:
A bunch of people are living on an island, when a visior comes with a strange order: all blue-eyed people must leave the island as soon as possible. There will be a flight out at 8:00pm every evening. Each person can see everyone else's eye color, but they do not know their own (nor is anyone allowed to tell them). Additionally, they do not know how many people have blue eyes, although they do knowthat at least one person does. How many days will it take the blue-eyed people to leave?
Answer:
If there is x people, it will take x days for leaving.
Proof:
When there is just 1 blue-eyed person, then when somebody does not see any person with blue eyes, he can realize that he is the one in blue eyes and leave that night.
When there are 2 blue-eyed people, then they can both see one person in blue eyes.Then they can realize that there is 1 or 2 people with blue eyes, one is the person they see, the other may be heself.
Then if on the first night, nobody leaves, then they can be sure that the other person also see a blue-eyed person, that is heself. Therefore, they both leave on the second night.
Then for three people. If there are only two eyes in blue, they will leave on the second night. If not, there are three blue-eyed people
Then we can see if there are x blue-eyed people, then they can all see $x-1$ people in blue eyes. They need $x-1$ nights to ensure if there is $x-1$ blue-eyed people or heself also is blue-eyed.Then on the xth day, all blue-eyed people will leave out.

6.9: 100 Lockers

Question:
There are 100 closed lockers in a hallway. A man begins by opening all 100 lockers. Next, he closes every second locker. Then on his thirs pass, he toggles every third locker (closes it if it is open or opens it if it is closed). This process continues for 100 pases, such that on each pass i, the man toggles every ith locker. After his 100th pass in the hallway, in which he toggles only locker No.100, how many locker are open?
Answer:
Firstly, to transfer this problem into a math problem we can find that when the locker n is in the mth turn and m is a factor of n, it will change the status.
Then if after some times of operations some locker is still on, it means it is operated by odd times. So the factor number of this locker number is odd, which means it is a complete square number. Then we can search the complete square number from integer 1 to integer 100, since $1^2 = 1$,

$10^2 = 100$, we can see there are 10 complete square numbers.
Therefore, we can see there are 10 lockers still open.