(b) Then when A is symmetric, calculate

$$\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{x}^T A \mathbf{x}\right)$$

to show what we had in the class.

a) $\frac{\partial}{\partial x} = \begin{bmatrix} \frac{\partial (x^T A x)}{\partial x_1} \\ \frac{\partial x^T A x}{\partial x_2} \\ \vdots \\ \frac{\partial (x^T A x)}{\partial x_p} \end{bmatrix}$

$$x^T A x = [x_1 \; x_2 \cdots x_p] \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & & & \\ \vdots & & & \\ a_{p1} & \cdots & & a_{pp} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix}$$

$$= [x_1 \cdots x_p] \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots a_{1p}x_p \\ \vdots \\ a_{p1}x_1 + a_{p2}x_2 + \cdots a_{pp}x_p \end{bmatrix}$$

$$= \sum_{i=1}^{p} \sum_{j=1}^{p} x_i a_{ij} x_j$$

$k = 1 \cdots p$ $\quad \frac{\partial x^T A x}{\partial x_k} = \frac{\partial}{\partial x_k} \Big((a_{11}x_1 + a_{12}x_2 + \cdots a_{1p}x_p)x_1 + (a_{21}x_1 + a_{22}x_2 + \cdots a_{2p}x_p)x_2 + \cdots (a_{p1}x_1 + a_{p2}x_2 + \cdots a_{pp}x_p)x_p \Big)$

$$= \frac{\partial}{\partial x_k} \Big(a_{11}x_1^2 + a_{12}x_2x_1 + \cdots a_{1p}x_px_1 + a_{21}x_1x_2 + a_{22}x_2^2 + \cdots a_{2p}x_1x_2 + \vdots + a_{p1}x_1x_p + a_{p2}x_2x_p + \cdots a_{pp}x_p^2 \Big)$$

$$= \sum_{j=1}^{p} \left(a_{kj}x_j + a_{jk}x_j \right)$$

$$\frac{\partial x^T A x}{\partial x} = \begin{bmatrix} \sum_{j=1}^{p} a_{1j}x_j + a_{j1}x_j \\ \vdots \\ \sum_{j=1}^{p} a_{kj}x_j + a_{jp}x_j \end{bmatrix} = \begin{bmatrix} 2a_{11} & a_{12}+a_{21} & a_{13}+a_{31} \cdots a_{1n}a_{p} \\ \vdots \\ a_{p1}+a_{1p} & a_{p2}a_{2p} \cdots & + 2a_{pp} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_p \end{bmatrix} \quad ①$$

$$= (A + A^T) \cdot X$$

b) If A is symmetry, then $A = A^T$

$$\frac{\partial x^T A x}{\partial x} = (A + A^T) \cdot X = 2A \cdot X.$$

In [1]:
```
import numpy as np
import pandas as pd
%pylab inline
df = pd.read_csv('ESLmixture.csv')
df.head()
```

```
%pylab is deprecated, use %matplotlib inline and import the required libr
aries.
Populating the interactive namespace from numpy and matplotlib
```

Out[1]:

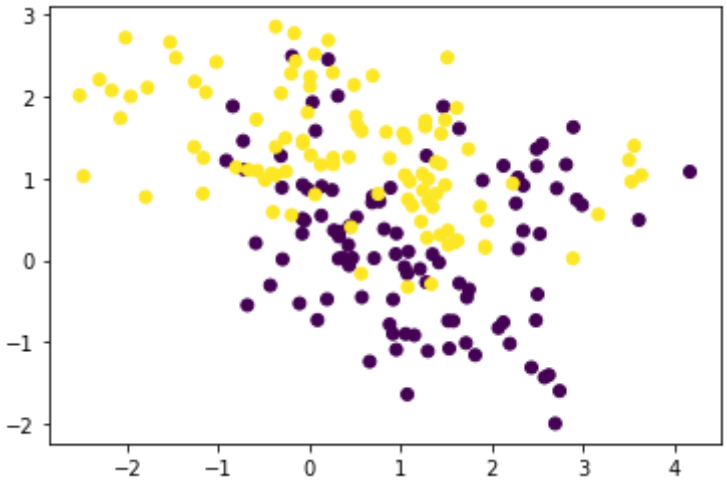
	Unnamed: 0	x.1	x.2	y
0	1	2.526093	0.321050	0
1	2	0.366954	0.031462	0
2	3	0.768219	0.717486	0
3	4	0.693436	0.777194	0
4	5	-0.019837	0.867254	0

In [58]:
```
X = df[['x.1','x.2']].to_numpy()
y = df[['y']].to_numpy()
y = y.reshape(1,-1)
int(y[:,101])
```

Out[58]: 1

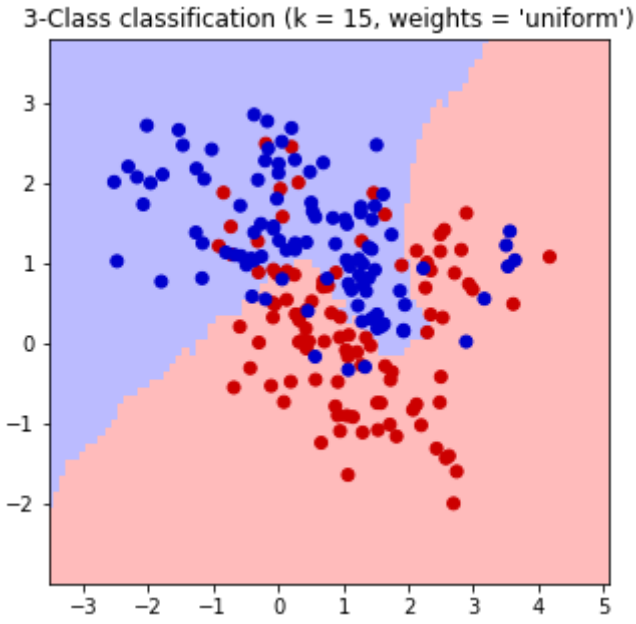
In [8]:
```
scatter(X[:,0],X[:,1],c = y)
```

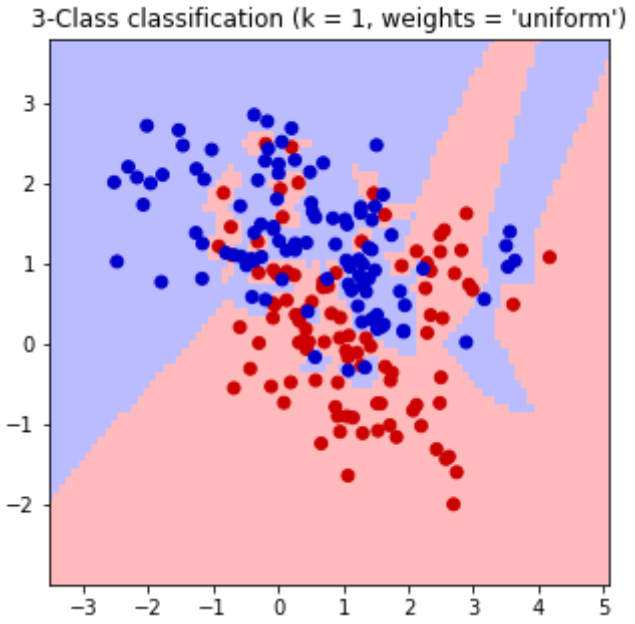
Out[8]: <matplotlib.collections.PathCollection at 0x7ff189675fd0>



In [68]:
```
import pandas as pd
import numpy as np
from sklearn import neighbors
import matplotlib.pyplot as plt
import matplotlib.lines as lines
from matplotlib.colors import ListedColormap
import math
#given the coordinate of x, output a value
def knn_strategy(x_2d, k,y):
    y = y.reshape(1,-1)
    distance = []
    for i in range(0,200):
        distance.append(math.sqrt((x_2d[0]-X[i,0])**2+(x_2d[1]-X[i,1])**2))
    df = pd.DataFrame(data = {'distance':distance})
    df = df.sort_values(by = 'distance')
    index_list = df.index.values.tolist()[:k]
    value_of_y = []
    for i in index_list:
        value_of_y.append(int(y[:,i]))
    if np.mean(value_of_y)>=0.5:
        return 1
    else:
        return 0

    #index_no = df.columns.get_loc('distance')
    #print(index_no)

knn_strategy([2,2], 15, y)
# Create color maps
cmap_light = ListedColormap(['#FFBBBB', '#BBFFBB', '#BBBBFF'])
cmap_bold = ListedColormap(['#CC0000', '#00AA00', '#0000CC'])
# import some data to play with
#iris = datasets.load_iris()
#X = iris.data[:,:2]  # we only take the first two features
#y = iris.target

h = 0.1 # step size in the mesh

# Points in a mesh of [x_min, m_max] x [y_min, y_max]
x_min, x_max = X[:,0].min()-1, X[:,0].max()+1
y_min, y_max = X[:,1].min()-1, X[:,1].max()+1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))
grid = np.c_[xx.ravel(), yy.ravel()]

weights = 'uniform'
for n_neighbors in [15,1]:
    # we create an instance of Neighbours Classifier and fit the data.
    Z = []
    for i in grid:
        Z.append(knn_strategy(i,n_neighbors,y))
    # Put the result into a color plot
    Z=np.array(Z)
    Z = Z.reshape(xx.shape)
    plt.figure(figsize=(5,5))
    if False:
```

```
        plt.scatter(xx, yy, c=Z, cmap=cmap_light, edgecolor='none')
    else:
        plt.pcolormesh(xx, yy, Z, cmap=cmap_light, shading='auto')

    # Plot also the training points
    plt.scatter(X[:,0], X[:,1], c=y, cmap=cmap_bold)
    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())
    plt.title("3-Class classification (k = %i, weights = '%s')"
              % (n_neighbors, weights))
```





In [34]:

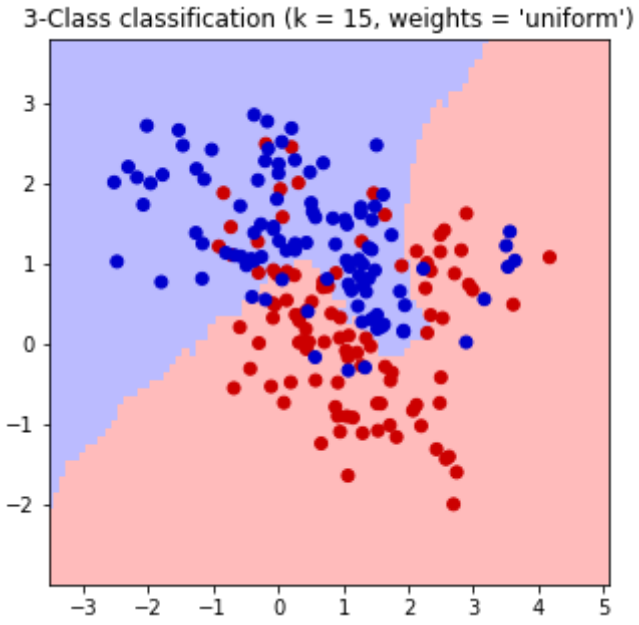
```
```

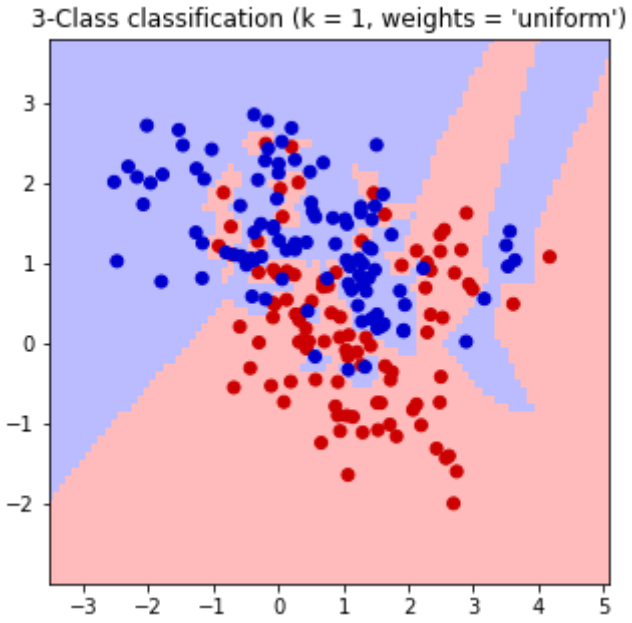
Out[34]: 25

In [3]:
```
# Create color maps
import pandas as pd
import numpy as np
from sklearn import neighbors
import matplotlib.pyplot as plt
import matplotlib.lines as lines
from matplotlib.colors import ListedColormap
cmap_light = ListedColormap(['#FFBBBB', '#BBFFBB', '#BBBBFF'])
cmap_bold = ListedColormap(['#CC0000', '#00AA00', '#0000CC'])
df = pd.read_csv('ESLmixture.csv')
X = df[['x.1','x.2']].to_numpy()
y = df[['y']].to_numpy()
# import some data to play with
#iris = datasets.load_iris()
#X = iris.data[:,:2]  # we only take the first two features
#y = iris.target

h = 0.1 # step size in the mesh

# Points in a mesh of [x_min, m_max] x [y_min, y_max]
x_min, x_max = X[:,0].min()-1, X[:,0].max()+1
y_min, y_max = X[:,1].min()-1, X[:,1].max()+1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))
grid = np.c_[xx.ravel(), yy.ravel()]

weights = 'uniform'
for n_neighbors in [15,1]:
    # we create an instance of Neighbours Classifier and fit the data.
    clf = neighbors.KNeighborsClassifier(n_neighbors, weights=weights)
    clf.fit(X, y)

    Z = clf.predict(grid)

    # Put the result into a color plot
    Z = Z.reshape(xx.shape)
    plt.figure(figsize=(5,5))
    if False:
        plt.scatter(xx, yy, c=Z, cmap=cmap_light, edgecolor='none')
    else:
        plt.pcolormesh(xx, yy, Z, cmap=cmap_light, shading='auto')

    # Plot also the training points
    plt.scatter(X[:,0], X[:,1], c=y, cmap=cmap_bold)
    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())
    plt.title("3-Class classification (k = %i, weights = '%s')"
              % (n_neighbors, weights))
```

```
/Users/shoufumisakai/opt/anaconda3/lib/python3.9/site-packages/sklearn/ne
ighbors/_classification.py:198: DataConversionWarning: A column-vector y
was passed when a 1d array was expected. Please change the shape of y to
(n_samples,), for example using ravel().
  return self._fit(X, y)
/Users/shoufumisakai/opt/anaconda3/lib/python3.9/site-packages/sklearn/ne
ighbors/_classification.py:198: DataConversionWarning: A column-vector y
was passed when a 1d array was expected. Please change the shape of y to
```

```
(n_samples,), for example using ravel().
  return self._fit(X, y)
```





###Question3

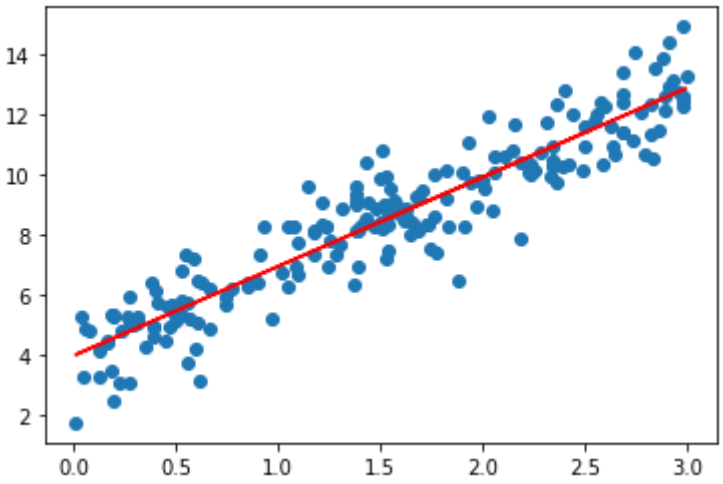
In [13]:
```
##a)
from numpy.linalg import inv
x = np.random.uniform(0,3,200)
mu, sigma = 0, 1 # mean and standard deviation
s = np.random.normal(mu, sigma, 200)
Y = 4+3*x+s
X = np.zeros((200,2))
X[:,0]=1
X[:,1]=x
```

In [14]:
```
##b
beta_hat = np.matmul(inv(np.matmul(X.T, X)),np.matmul(X.T, Y))
beta_hat
```

Out[14]:
```
array([3.98799664, 2.96617678])
```

In [15]:
```
##c)
f = beta_hat[0]+beta_hat[1]*x
scatter(x, Y)
plt.plot(x, f, color ='r')
```

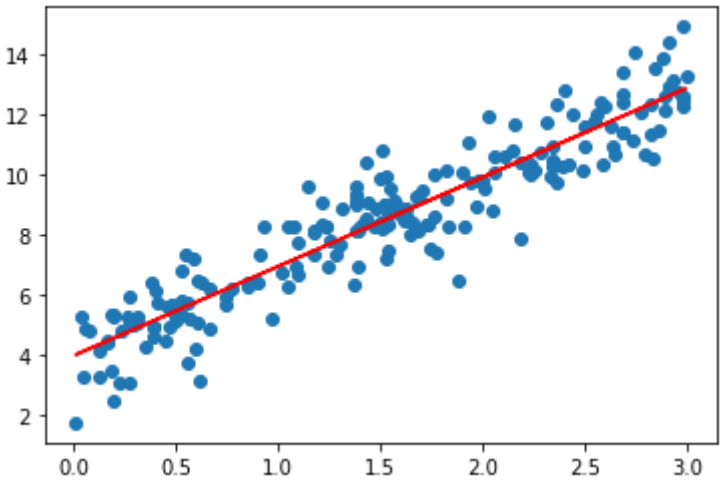
Out[15]:
```
[<matplotlib.lines.Line2D at 0x7fb409b234f0>]
```



In [16]:
```
from sklearn.linear_model import LinearRegression
reg = LinearRegression().fit(X, Y)
beta0=reg.intercept_
beta1=reg.coef_[1]
print("beta0",reg.intercept_)
print("beta1",reg.coef_[1])
f2 = beta0+beta1*x
scatter(x, Y)
plt.plot(x, f2, color ='r')
```

```
beta0 3.987996639082673
beta1 2.9661767780968376
```

Out[16]:
```
[<matplotlib.lines.Line2D at 0x7fb3f8293910>]
```



In [ ]: