Approximation of π and e

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1 Differences in output of program vs. output from math.h

```
pi_bbp() = 3.141592653589791, M_PI = 3.141592653589793, diff = 0.000000000000002, terms = 10
e() = 2.718281828459043, M_E = 2.718281828459045, diff = 0.000000000000002, e terms = 16
pi_madhava() = 3.141592653589800, M_PI = 3.141592653589793, diff = 0.000000000000007, terms = 26
pi_euler() = 3.141572046272299, M_PI = 3.141592653589793, diff = 0.000020607317494, terms = 46341
pi_viete() = 3.141592653589789, M_PI = 3.141592653589793, diff = 0.000000000000004, terms = 23
```

Figure 1: Program Output

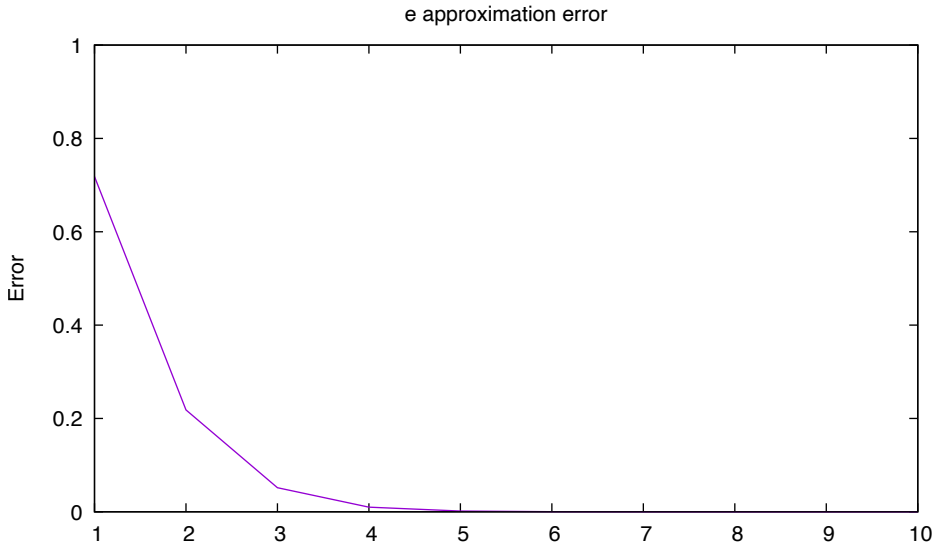
Bailey-Borwein-Plouffe formula uses only 10 terms to approximate π and achieves only 2e-15 difference compared to π in math.lib. Madhava series and Viète's formula use similar number of terms to approximate π. Madhava series uses 26 terms and Viète's formula uses 23 terms. However, Euler sequence takes much more time to converge. It takes 46341 iterations and still has 0.000020607317494 difference compared to M_PI in math.lib. As for the approximation of the value of e, the formula takes 16 terms and leaves only 2e-15 difference compared to M_E in math.lib.

2 Reasoning for Differences

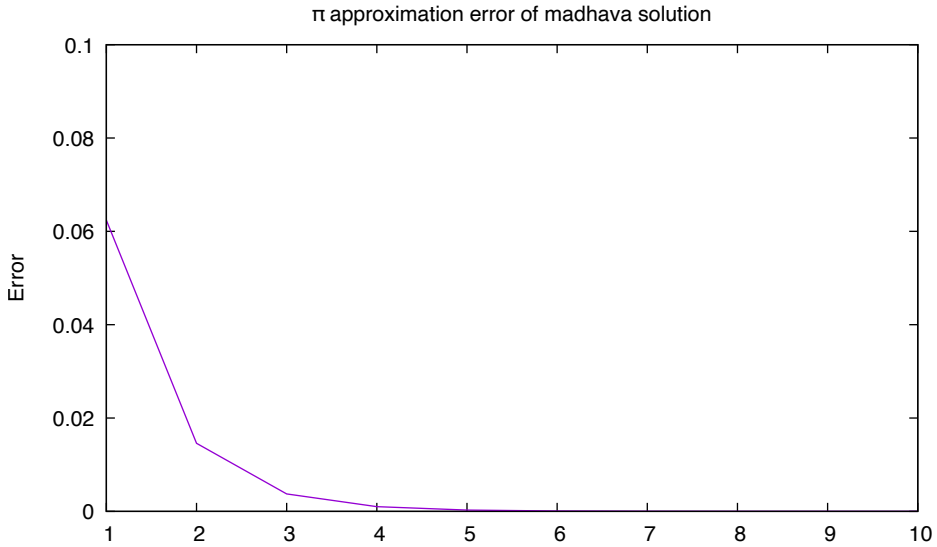
1. **Difference between program's output and math.lib**: Both e and π are irrational number which means we can't never actually calculate these two numbers. Even in the math.lib, these two numbers are just approximation with very high precision. The program's outputs still have error because we need more terms to approximate. More terms, more accurate.

2. **Difference between the approximations between formulas**: Different formulas need different terms to converge. So for example, within 100 terms, different formulas will converge to different degree. Consequently, these formula will have distinct results.

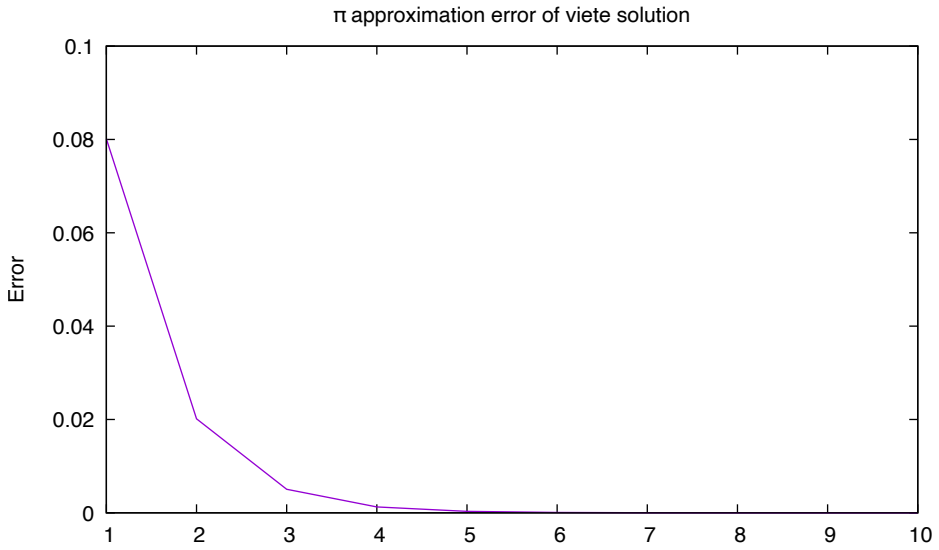
3 Graphs



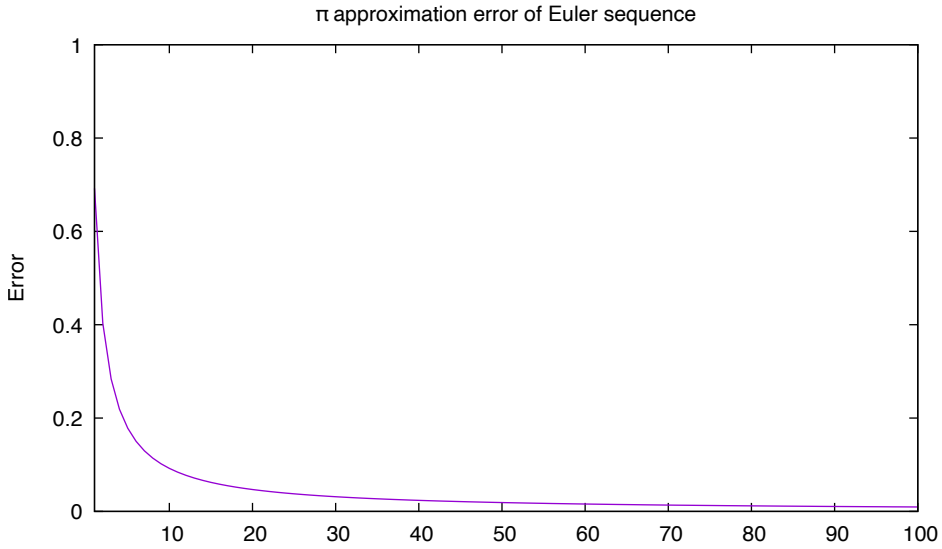
The formula takes almost 5 terms to converge, making the difference between the approximation and M_E very small.



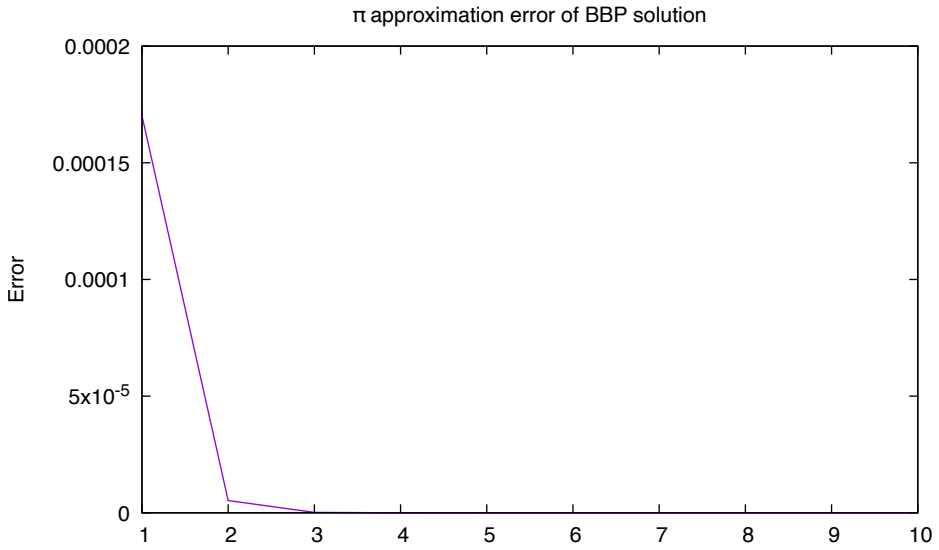
Madhava series takes 6 terms to achieve high accuracy, making the purple line almost invisible.



Viète's formula performs almost the same as Madhava series, but it starts with a relatively higher error 0.08. Madhava series starts at 0.06.



Euler sequence doesn't reach decent accuracy even with 100 terms. We can see the curve is approaching the error=0 slowly.



Bailey-Borwein-Plouffe formula converges very fast, using only 3 terms to obtain error close to 0.