Homework set 2.

1. a) Ideal gas equation:

$$P = \rho \cdot R \cdot T$$

Hydrostatic Equation:

$$\frac{dP}{dz} + g \cdot \rho = 0$$

$$\frac{dz}{dp} = -\frac{RT}{gP}.$$

$$Z(p) = R \cdot \int_{P}^{P_s} \frac{T}{g} \cdot \frac{dp}{P}$$

$$\Delta Z = \frac{R}{g} \cdot \int_{P}^{P_s} T \cdot \frac{1}{P} dp.$$

$$= \frac{R}{g} \cdot \int_{500}^{1000} \langle T \rangle \cdot \frac{1}{P} dp.$$

$$= \frac{R \langle T \rangle}{g} \cdot \ln 1000 - \ln 500 = \frac{R \cdot \langle T \rangle}{g} \ln 2.$$

b) Equator: $T_{e500mbar} = -10\,°C$, $T_{e1000mbar} = 25\,°C$

$$\bar{T} = 7.5\,°C$$

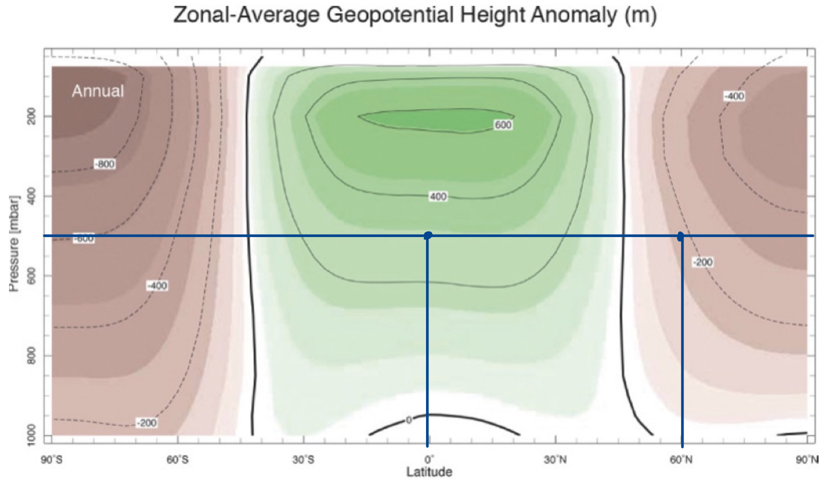
$$\Delta Z_e = \frac{287 \cdot (7.5 + 273)}{9.8} \cdot \ln(2) = 5694 \text{ (m)}.$$

$T_{60°N\,500} = -30\,°C$. $T_{60°\,1000} = -2.5\,°C$.

$\bar{T} = -16.25\,°C$.

$$\Delta Z_{60°N} = \frac{287 \cdot (273 - 16.25)}{9.8} \cdot \ln(2) = 5212 \text{ (m)}$$

c) $Z_e - Z_{60°N} = 5694 - 5212 = 482 (m)$



$\Delta Z' \approx 300 - (-200) \approx 500$ m.

Similar to the calculated ΔZ.

2. a). $f = 2 \cdot \Omega \cdot \sin\varphi = 7.3 \times 10^{-5} /s$

b). $F_c = f \cdot \hat{z} \times u = 7.3 \times 10^{-5} \cdot 36 = 2.7 \times 10^{-3}$ N.

southward. Because jet stream is from west.

c). $F_x = \nu \cdot \nabla^2 u = \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$

$\approx 1.34 \times 10^{-5}$

$\frac{Dv}{Dy} = \frac{36 - 0}{3300 \times 10^3} \approx 10^{-5}$, $\frac{D^2 v}{Dy^2} \approx \frac{10^{-5}}{10^6} \approx 10^{-11}$

$\frac{Dw}{Dz} = \frac{36 - 0}{10 \times 10^3} \approx 10^{-4}$ $\frac{D^2 w}{Dz^2} \approx \frac{10^{-4}}{10^4} \approx 10^{-8}$.

$\therefore F_x \approx 10^{-5} \times 10^{-8} \approx 10^{-13}$, $\ll F_c \sim 10^{-5}$

$\therefore$ Friction is negligible.

3. $f = 2 \cdot \Omega \cdot \sin\varphi = 1.03 \times 10^{-4} / s$

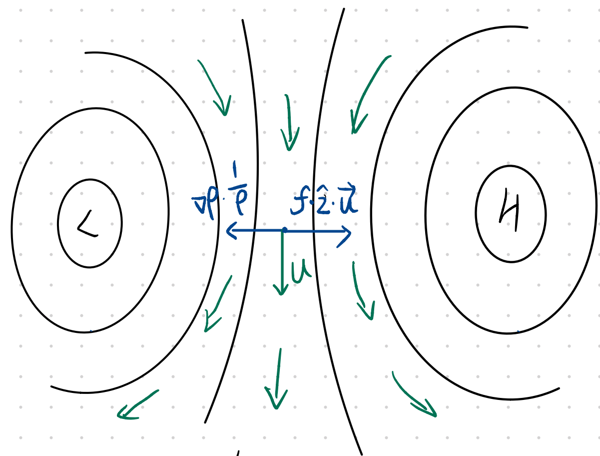
$F_c = f \cdot \hat{z} \cdot \vec{u} = 1.03 \times 10^{-4} \cdot 15 = 1.55 \times 10^{-3}$ N.

$S = \frac{1}{2}(2 \cdot \Omega \cdot v \cdot \sin\varphi) \cdot t^2 = 1.24 \times 10^{-2}$ m.

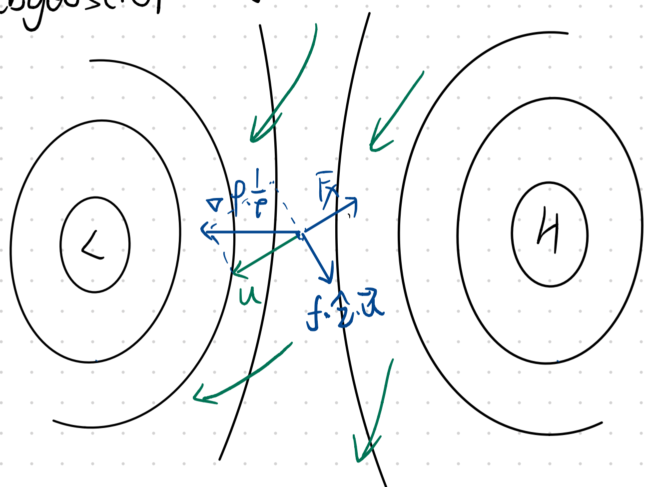
$$R_o = \frac{U \cdot \tau_{rot}}{L} \simeq \frac{10 \cdot 10^5}{10} \simeq 10^5$$

Ro is very large, so the influence of coriolis can be neglected.

4. a) geostrophic flow



b) subgeostrophic flow.



5. $f = 2\Omega \cdot \sin\varphi = 7.3\times10^{-5}$.

$$R_o = \frac{U}{f\cdot L} = \frac{50}{7.3\times10^{-5}\cdot 50\times10^{3}} = 13.7$$

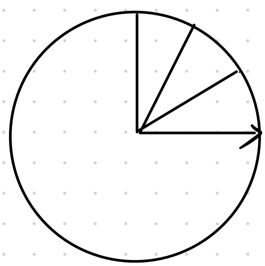
Ro is large, so the coriolis is neglectable, it isn't geostrophic flow.

6. a)

$$\Delta Z_1 = \frac{R}{g}\cdot\int_{p}^{p_s} T\cdot\frac{1}{p}\,dp$$

$$= \frac{287}{9.8}\cdot 265\cdot\ln\frac{1000}{200} = 12490 \text{ m}$$

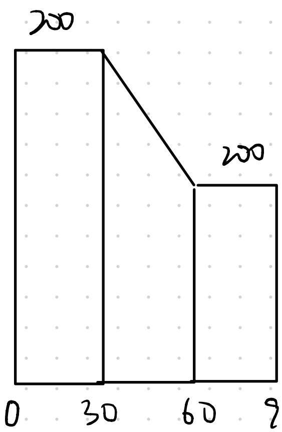
$$\Delta Z_2 = \frac{287}{9.8}\cdot 235\cdot\ln 5 = 11076 \text{ m}$$



$$\Delta Z = \Delta Z_1 - \Delta Z_2 = 1414 \text{ m}.$$

b)

$$\frac{\Delta Z}{\Delta y} = \frac{\Delta Z}{a\cdot\pi\cdot d\varphi} = \frac{1414}{6.38\times10^{6}\cdot 2\cdot 3.14\cdot\frac{30}{360}} = 4.23\times10^{-4}$$



$$f = 2\Omega\cdot\sin\varphi = 1.03\times10^{-4}$$

$$u = -\frac{g}{f}\cdot\frac{\Delta Z}{\Delta y}$$

$$= \frac{-9.8}{1.03\times10^{-4}}\cdot 4.23\times10^{-4}$$

$$= 40.25 \text{ m/s}.$$