HomeWork 2
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Problem 1.

a.

```
input_data = 'The hat on that cat';
input_data = abs(input_data);
len = length(input_data);
matrix = [];
it = 1;

for i=1:len
    matrix_data = [input_data(1:end),it];
    matrix = [matrix;matrix_data];
    input_data = [input_data(2:end),input_data(1)];
    it = it+1;
end

matrix = sortrows(matrix);
for i=1:len
    if matrix(i,len+1) == 1
        result_X = i;
    end
end
result_y = char(matrix(:,len));
```

b. Algorithm
   a. If the length of each row is less than the length of the input string, add a column y on the left of the original column;
   b. It sorts the rows of A lexicographically where every row is treated as a word;
   c. Take row L as the final output;

c. The code will be:
```
input_data = 'cbabbaaa';
input_data = abs(input_data);
input_data = input_data';

len = length(input_data);
matrix = [];
it = 1;

for i=1:len
    matrix = [input_data(1:len),matrix];
    matrix = sortrows(matrix);
end

matrix = char(matrix);
```

d. Reconstruct:
The input is 'babacaab' and the string after the transform is 'cbabbaaa'. The reconstruct progress will be:

```
input_data = 'cbabbaaa';
input_data = abs(input_data);
input_data = input_data';

len = length(input_data);
matrix = [];
it = 1;

for i=1:len
    matrix = [input_data(1:len),matrix];
    matrix = sortrows(matrix);
end

matrix = char(matrix);
```

the output is:

val =

        'aabbabac'
        'abacaabb'
        'abbabaca'
        'acaabbab'
        'babacaab'
        'bacaabba'
        'bbabacaa'
        'caabbaba'

Take the 5th line as the result which is 'babacaab'.
Problem 2.

Code:
```
X=[];
Y=[];

for i = 0:31
    X = [X;i^2/3];
    Y = [Y;sin((2*i+1)*pi/32)]
end

fft_X = fft(X);
fft_Y = fft(Y);
abs_X = abs(fft_X);
abs_Y = abs(fft_Y);


[sort_X,index_X] = sortrows(abs_X,1);
[sort_Y,index_Y] = sortrows(abs_Y,1);

Xsmallest = index_X(1:17);
Ysmallest = index_Y(1:17);

for i = 1:17
    fft_X(index_X(i)) = 0;
    fft_Y(index_Y(i)) = 0;
end

i_X = ifft(fft_X);
i_Y = ifft(fft_Y);
```

a.   X = 3472.00000000000 + 0.00000000000000i
           384.463300905693 + 1732.80774615191i
           -30.5379073648631 + 857.999273322811i
           -107.374421200434 + 562.612600992140i
           -134.248388668020 + 412.025781311675i
           -146.665874057433 + 319.294875612056i
           -153.387579686168 + 255.420716828244i

-157.414662176692 + 207.957935033681i
-160.000000000000 + 170.666666666667i
-161.741260385515 + 140.062513634758i
-162.952198975084 + 114.035820871560i
-163.809588509571 + 91.2232338689351i
-164.418277998646 + 70.6924479783416i
-164.842564210904 + 51.7711673356531i
-165.122313973885 + 33.9477106994615i
-165.281597031812 + 16.8091995062894i
-165.333333333333 + 0.00000000000000i
-165.281597031812 - 16.8091995062894i
-165.122313973885 - 33.9477106994615i
-164.842564210904 - 51.7711673356531i
-164.418277998646 - 70.6924479783416i
-163.809588509571 - 91.2232338689351i
-162.952198975084 - 114.035820871560i
-161.741260385515 - 140.062513634758i
-160.000000000000 - 170.666666666667i
-157.414662176692 - 207.957935033681i
-153.387579686168 - 255.420716828244i
-146.665874057433 - 319.294875612056i
-134.248388668020 - 412.025781311675i
-107.374421200434 - 562.612600992140i
-30.5379073648631 - 857.999273322811i
384.463300905693 - 1732.80774615191i

Y=6.10622663543836e-16 + 0.00000000000000i
1.56827424527297 - 15.9229556267552i
4.38231337323921e-16 - 7.94766685013237e-16i
4.93430825863106e-16 - 4.25378578172991e-16i
2.65361626333167e-16 - 3.63083881070623e-16i
1.07876927046680e-15 - 7.31823213308418e-17i
4.71780225305545e-16 + 7.22688828477049e-16i
5.62430208535118e-16 + 6.08831174113916e-16i
-2.08166817117217e-16 + 4.85722573273506e-16i
-1.00651941838518e-15 + 6.08831174113916e-16i
-1.30006157953161e-16 - 2.95560309336384e-16i
3.14537541583566e-17 - 2.95226926255873e-16i
2.06483159132525e-16 - 3.63083881070623e-16i
6.16792198762051e-16 + 2.40755236602103e-16i
-7.24494253445047e-16 + 8.51519436273706e-16i
4.44089209850063e-16 + -0.00000000000000i
-1.02695629777827e-15 + 0.00000000000000i
4.44089209850063e-16 + 0.00000000000000i
-7.24494253445047e-16 - 8.51519436273706e-16i
6.16792198762051e-16 - 2.40755236602103e-16i
2.06483159132525e-16 + 3.63083881070623e-16i

3.14537541583566e-17 + 2.95226926255873e-16i
-1.30006157953161e-16 + 2.95560309336384e-16i
-1.00651941838518e-15 - 6.08831174113916e-16i
-2.08166817117217e-16 - 4.85722573273506e-16i
5.62430208535118e-16 - 6.08831174113916e-16i
4.71780225305545e-16 - 7.22688828477049e-16i
1.07876927046680e-15 + 7.31823213308418e-17i
2.65361626333167e-16 + 3.63083881070623e-16i
4.93430825863106e-16 + 4.25378578172991e-16i
4.38231337323921e-16 + 7.94766685013237e-16i
1.56827424527297 + 15.9229556267552i

b. abs_X = 3472
1774.94639774484
858.542553871791
572.767147384755
433.344983039444
351.368889068382
297.938738983426
260.817711460849
233.938263460921
213.957339291695
198.890893689646
187.497359141886
178.971484715333
172.781146954528
168.575875005365
166.134149124822
165.333333333333
166.134149124822
168.575875005365
172.781146954528
178.971484715333
187.497359141886
198.890893689646
213.957339291695
233.938263460921
260.817711460849
297.938738983426
351.368889068382
433.344983039444
572.767147384755
858.542553871791
1774.94639774484

abs_y = 6.10622663543836e-16
16

9.07579632109294e-16
6.51475950960910e-16
4.49718464623690e-16
1.08124871840795e-15
8.63050244073240e-16
8.28856524403220e-16
5.28450415778197e-16
1.17633189966071e-15
3.22889296138415e-16
2.96897754516846e-16
4.17690315543291e-16
6.62114567431543e-16
1.11802382516062e-15
4.44089209850063e-16
1.02695629777827e-15
4.44089209850063e-16
1.11802382516062e-15
6.62114567431543e-16
4.17690315543291e-16
2.96897754516846e-16
3.22889296138415e-16
1.17633189966071e-15
5.28450415778197e-16
8.28856524403220e-16
8.63050244073240e-16
1.08124871840795e-15
4.49718464623690e-16
6.51475950960910e-16
9.07579632109294e-16
16

c.   X = 3472.00000000000 + 0.00000000000000i
        384.463300905693 + 1732.80774615191i
        -30.5379073648631 + 857.999273322811i
        -107.374421200434 + 562.612600992140i
        -134.248388668020 + 412.025781311675i
        -146.665874057433 + 319.294875612056i
        -153.387579686168 + 255.420716828244i
        -157.414662176692 + 207.957935033681i
        0.00000000000000 + 0.00000000000000i
        0.00000000000000 + 0.00000000000000i
        0.00000000000000 + 0.00000000000000i
        0.00000000000000 + 0.00000000000000i
        0.00000000000000 + 0.00000000000000i
        0.00000000000000 + 0.00000000000000i
        0.00000000000000 + 0.00000000000000i
        0.00000000000000 + 0.00000000000000i

0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
-157.414662176692 - 207.957935033681i
-153.387579686168 - 255.420716828244i
-146.665874057433 - 319.294875612056i
-134.248388668020 - 412.025781311675i
-107.374421200434 - 562.612600992140i
-30.5379073648631 - 857.999273322811i
384.463300905693 - 1732.80774615191i

Y = 0.00000000000000 + 0.00000000000000i
1.56827424527297 - 15.9229556267552i
4.38231337323921e-16 - 7.94766685013237e-16i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
1.07876927046680e-15 - 7.31823213308418e-17i
4.71780225305545e-16 + 7.22688828477049e-16i
5.62430208535118e-16 + 6.08831174113916e-16i
0.00000000000000 + 0.00000000000000i
-1.00651941838518e-15 + 6.08831174113916e-16i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
-7.24494253445047e-16 + 8.51519436273706e-16i
0.00000000000000 + 0.00000000000000i
-1.02695629777827e-15 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
-7.24494253445047e-16 - 8.51519436273706e-16i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i
-1.00651941838518e-15 - 6.08831174113916e-16i
0.00000000000000 + 0.00000000000000i
5.62430208535118e-16 - 6.08831174113916e-16i
4.71780225305545e-16 - 7.22688828477049e-16i
1.07876927046680e-15 + 7.31823213308418e-17i
0.00000000000000 + 0.00000000000000i
0.00000000000000 + 0.00000000000000i

4.38231337323921e-16 + 7.94766685013237e-16i

1.56827424527297 + 15.9229556267552i

d. $\hat{X}$ = 86.9271542345052

-15.3733298999893

-26.1421101583952

8.91906381187432

22.3479890260940

6.48765118122680

-0.664002545746513

15.8483813335520

31.5090632903040

29.0825048115503

24.8627342410832

37.0110824540913

55.1476526409247

60.7151915359465

59.3161972670417

69.6344446339467

90.3011113006134

102.685019603090

104.084013871995

112.913976849088

136.110739995588

155.675621925563

159.895392496030

166.027922630991

191.700574007572

221.037262990425

228.188916717399

226.051307385285

253.955715504399

307.690077852188

318.458858110594

231.593820901172

$\hat{y}$ = 0.0980171403295605

0.290284677254462

0.471396736825998

0.634393284163646

0.773010453362737

0.881921264348355

0.956940335732209

0.995184726672197

0.995184726672197

0.956940335732209

0.881921264348355

0.773010453362737
0.634393284163646
0.471396736825998
0.290284677254462
0.0980171403295607
-0.0980171403295606
-0.290284677254462
-0.471396736825998
-0.634393284163645
-0.773010453362737
-0.881921264348355
-0.956940335732209
-0.995184726672197
-0.995184726672197
-0.956940335732209
-0.881921264348355
-0.773010453362737
-0.634393284163646
-0.471396736825998
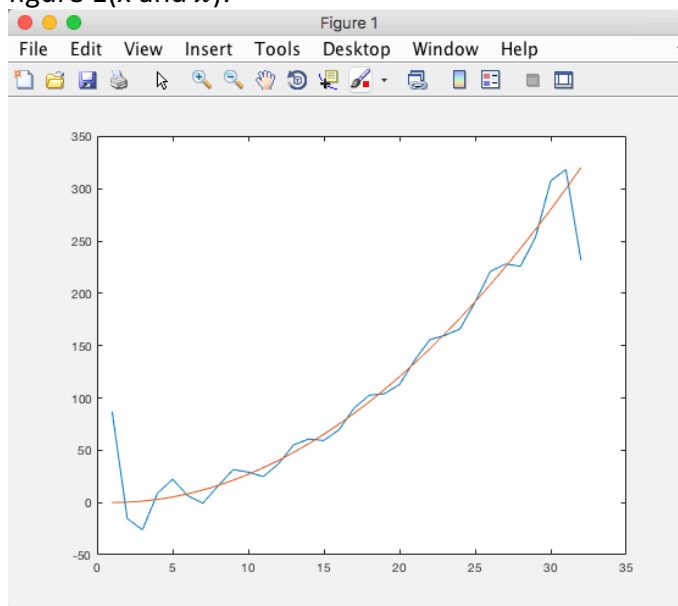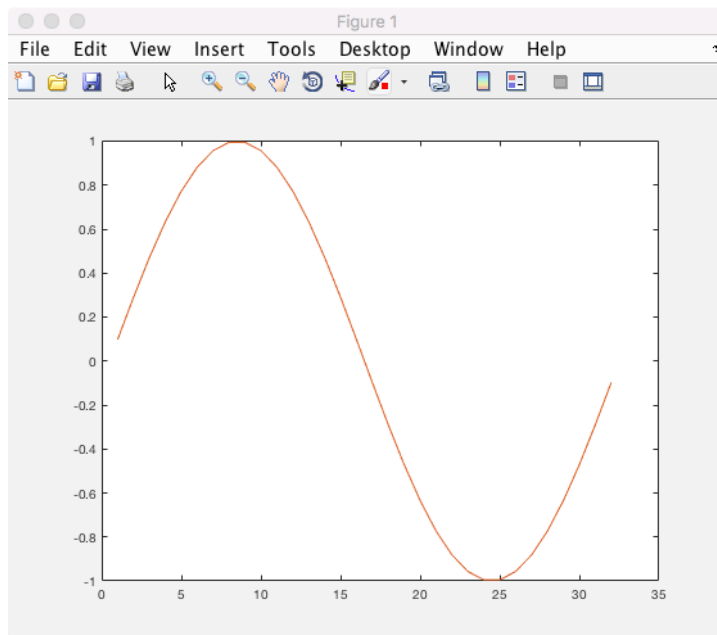-0.290284677254463
-0.0980171403295604

e. figure 1(x and $\hat{x}$):

figure 2(y and $\hat{y}$):

Problem 3.

Code:

```
x=[];
y=[];

for i = 0:31
    x = [x;i^2/3];
    y = [y;sin((2*i+1)*pi/32)]
end

X = dct(x);
Y = dct(y);

for i = 16:32
    X(i) = 0;
    Y(i) = 0;
end

i_X = idct(X);
i_Y = idct(Y);
```

a.   X = 613.768686069923
        -535.841149514534
        138.114216261511
        -59.3449274209456
        34.3590561210941
        -21.2223992484599
        15.1417256564255
        -10.7155690842292
        8.41152541306387
        -6.38748024242396
        5.29170234082456
        -4.19179952818034
        3.59175893815435
        -2.92364096301509

2.56098449969019

0

0

0

0

0

0

0

0

0

0

0

0

0

0

0

0

Y = 1.07943856534510e-16

3.39940413550419

-4.87030390708512e-16

-2.03305677839866

6.86898891379504e-17

-0.480860950538638

1.45297784207247e-16

-0.222085269130132

2.65539973596442e-17

-0.127899094613052

1.70738134463007e-16

-0.0825225720043399

1.98443830466117e-16

-0.0570079917149157

4.41996116577035e-17

0

0

0

0

0

0

0

0

0

0

0

0

0

0

0
0
0
0

$\hat{x} =$ 0.187533777572611
0.221832728143558
1.06968778954588
3.05821451730264
5.62398355307324
8.35003486650247
11.7033647802826
16.2338824622476
21.6145906582690
27.1835692784543
33.0896305611452
40.0694241253843
48.1836611456106
56.6690559432202
65.2320874012509
74.6055228403487
85.3300330942114
96.7691294688807
108.130149464721
119.877968344402
133.052398324295
147.448725605501
161.794328029126
175.922395263026
191.315072878877
208.669895664396
226.326533032123
242.775312858306
259.816564798971
280.478279904247
302.863695482461
318.333441358105

$\hat{y}=$ 0.119128386685838
0.261614427407037
0.467362573481054
0.649619900889913
0.778046137971353
0.872472948766605
0.950778351823799
1.00084925438537
1.00191060246243

0.954259433317885

0.875197453420438

0.773168197804488

0.640596975896597

0.473396437605441

0.285070972245645

0.0942180961420271

-0.0942180961420269

-0.285070972245645

-0.473396437605441

-0.640596975896597

-0.773168197804488

-0.875197453420438

-0.954259433317885

-1.00191060246243

-1.00084925438537

-0.950778351823799

-0.872472948766605

-0.778046137971353

-0.649619900889914

-0.467362573481054

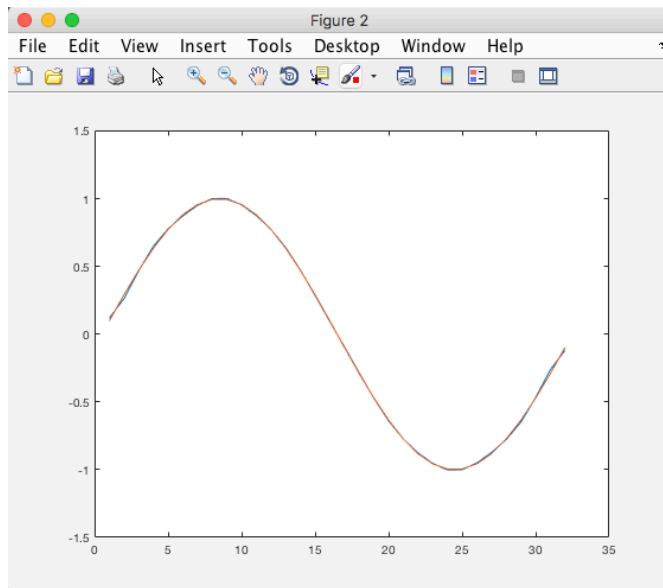-0.261614427407038

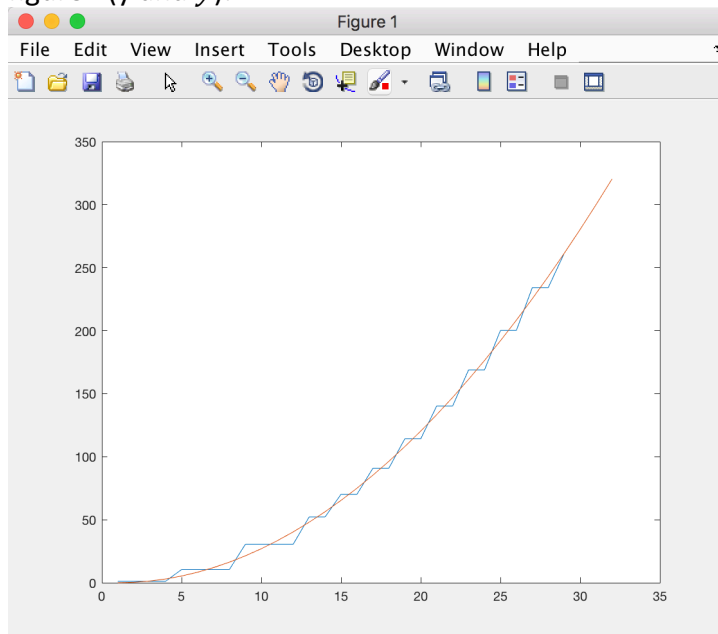-0.119128386685837

b. figure 1(x and $\hat{x}$):
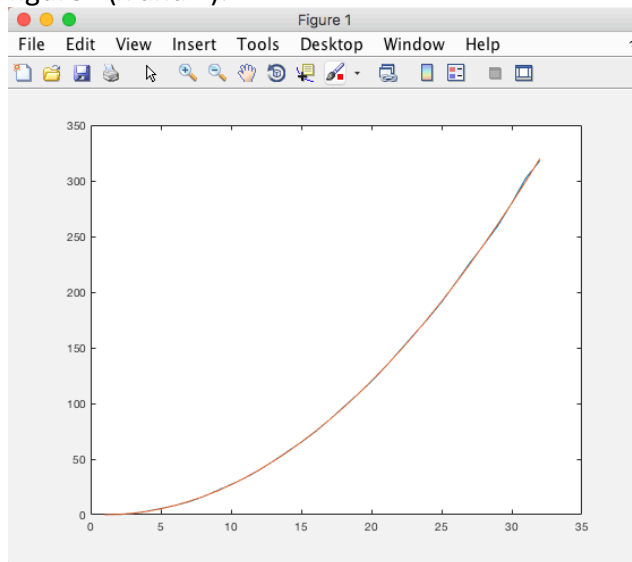
figure 2(y and $\hat{y}$):

Problem 4:

Code:

```
x=[];
y=[];

for i = 0:31
    x = [x;i^2/3];
    y = [y;sin((2*i+1)*pi/32)];
end

Had = hadamard(32);

Had_X = Had * x;
Had_Y = Had * y;

Had_X_abs = abs(Had_X);
Had_Y_abs = abs(Had_Y);

[sort_X,index_X] = sortrows(Had_X_abs,1);
[sort_Y,index_Y] = sortrows(Had_Y_abs,1);

Xsmallest = index_X(1:17);
Ysmallest = index_Y(1:17);

for i = 1:17
    Had_X(index_X(i)) = 0;
    Had_Y(index_Y(i)) = 0;
end

Had_x = Had\Had_X;
Had_y = Had\Had_Y;
```
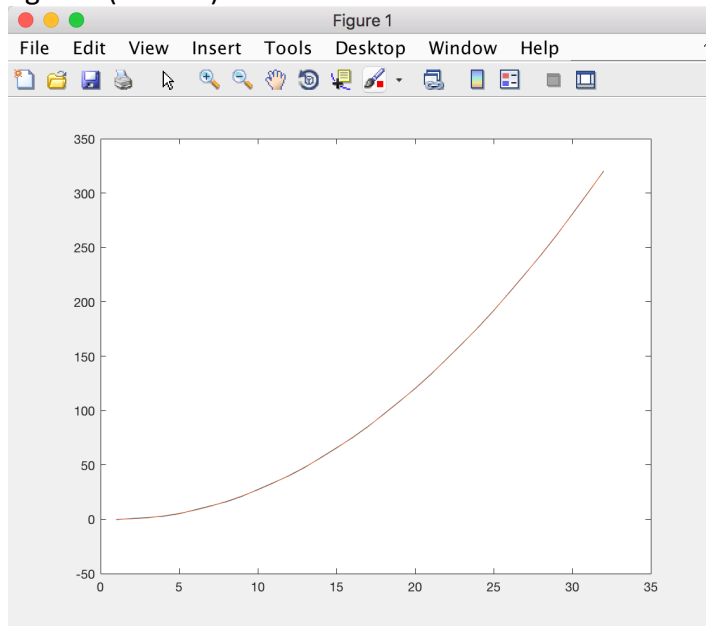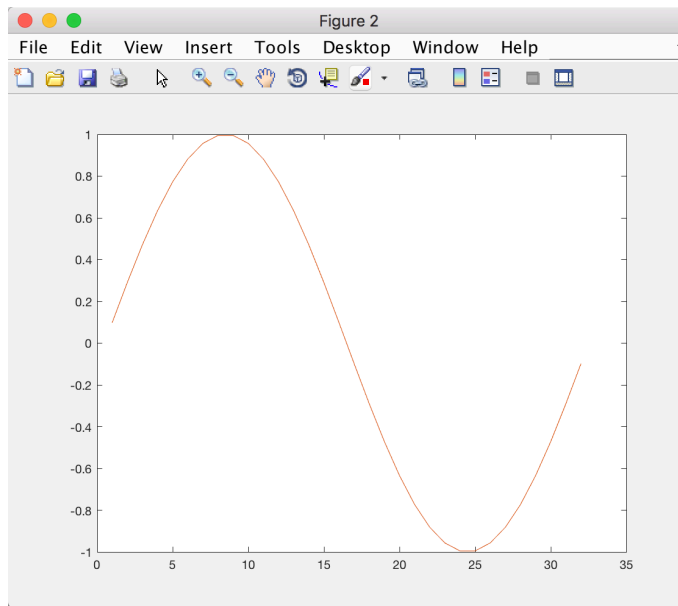
a. figure 1(x and $\hat{x}$):
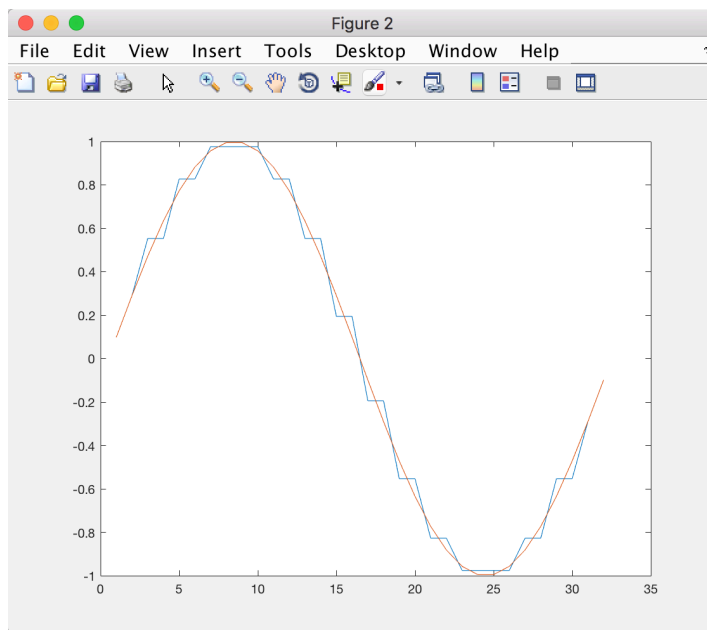
figure 2(y and $\hat{y}$):

b. Code:

```matlab
  x=[];
  y=[];

  for i = 0:31
      x = [x;i^2/3];
      y = [y;sin((2*i+1)*pi/32)];
  end

  Had = hadamard(32);

  Had_X = Had * x;
  Had_Y = Had * y;

  Had_X_abs = abs(Had_X);
  Had_Y_abs = abs(Had_Y);

  [sort_X,index_X] = sortrows(Had_X_abs,1);
  [sort_Y,index_Y] = sortrows(Had_Y_abs,1);

  Xsmallest = index_X(1:17);
  Ysmallest = index_Y(1:17);

  for i = 1:17
      Had_X(index_X(i)) = 0;
      Had_Y(index_Y(i)) = 0;
  end

  Had_x = Had\Had_X;
  Had_y = Had\Had_Y;
```
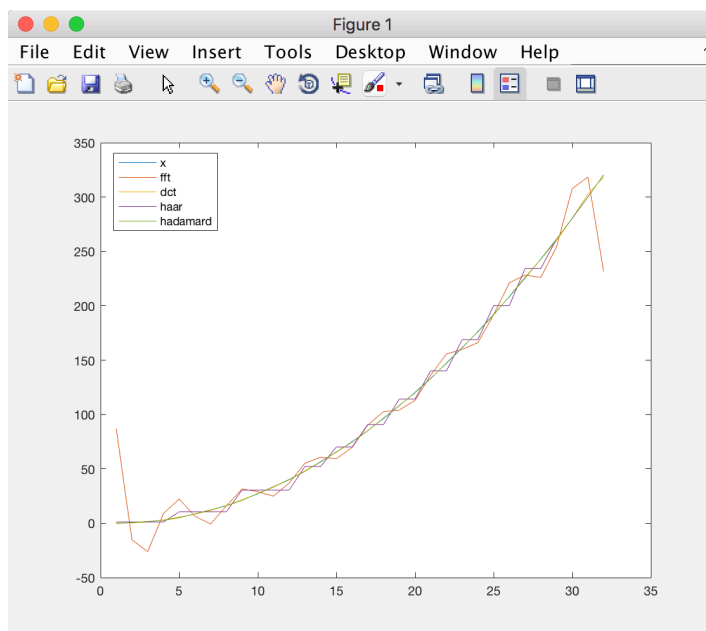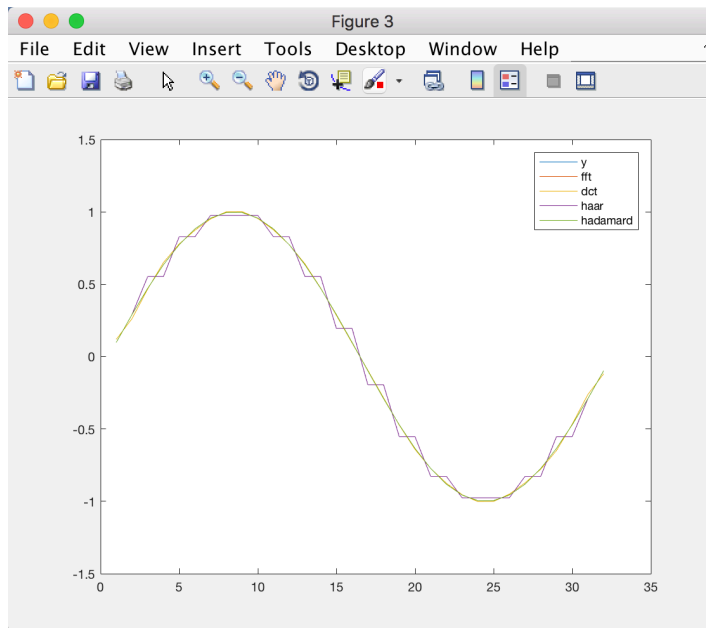
figure 1(x and $\hat{x}$):

figure 2(y and $\hat{y}$):

Problem 5:

a.

b. MSE of $\hat{x}$ of problem 2 = 599.1948
MSE of $\hat{x}$ of problem 3 = 0.5648
MSE of $\hat{x}$ of problem 4(a) = 30.8576
MSE of $\hat{x}$ of problem 4(b) = 0.1111

c. The $\hat{x}$ of Hadamard is the best reconstruction of x;

d. For all of the $\hat{y}$s:

MSE of $\hat{y}$ of problem 2 = 9.5213e-33
MSE of $\hat{y}$ of problem 3 = 1.1764e-04
MSE of $\hat{y}$ of problem 4(a) = 0.0036
MSE of $\hat{y}$ of problem 4(b) = 8.7696e-32

The $\hat{y}$ of Fourier transform is the best reconstruction of y;