We evaluate:

$$J = \int_0^{\frac{\pi}{2}} \ln(\sin x)\, dx = \int_0^{\frac{\pi}{2}} \ln(\cos x)\, dx$$

$$K = \int_0^{\frac{\pi}{2}} \left(\frac{x}{\sin x}\right)^2 dx \qquad\qquad L = \int_0^{\infty} \frac{\ln(1+x^2)}{1+x^2}\, dx$$

The equality $\int_0^{\frac{\pi}{2}} \ln(\sin x)\, dx = \int_0^{\frac{\pi}{2}} \ln(\cos x)\, dx$ follows from the substitution $x \mapsto \pi/2 - x$. Then, adding the two integrals together,

$$2J = \int_0^{\frac{\pi}{2}} \ln(\sin x) + \ln(\cos x)\, dx = \int_0^{\frac{\pi}{2}} \ln(\sin x \cos x)\, dx = \int_0^{\frac{\pi}{2}} \ln\left(\frac{1}{2}\sin 2x\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \ln\left(\frac{1}{2}\right) + \ln(\sin 2x)\, dx = -\frac{\pi}{2}\ln 2 + \frac{1}{2}\int_0^{\pi} \ln(\sin x)\, dx$$

Since $\sin x = \sin(\pi - x)$, the integrand is symmetric across the line $x = \pi/2$ on the interval $[0, \pi]$ (this is also readily apparent from the graph of $\sin x$) so we can write

$$2J = -\frac{\pi}{2}\ln 2 + \int_0^{\frac{\pi}{2}} \ln(\sin x)\, dx = -\frac{\pi}{2}\ln 2 + J$$

$$\Rightarrow J = -\frac{\pi}{2}\ln 2$$

With this, we can evaluate K and L.
K is evaluated using two applications of integration by parts.

$$K = \int_0^{\frac{\pi}{2}} x^2 \csc^2 x\, dx = [-x^2 \cot x]_0^{\frac{\pi}{2}} + 2\int_0^{\frac{\pi}{2}} x \cot x\, dx$$

Note $\lim_{x\to 0} x^2 \cot x = \lim_{x\to 0} \frac{x^2 \cos x}{\sin x} = \lim_{x\to 0} \frac{x^2(1 - x^2/2 + \cdots)}{x - x^3/6 + \cdots} = 0$ by L'Hopital's rule,

$$= 0 + 2\left([x \ln|\sin x|]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \ln|\sin x|\, dx\right)$$

Note $\lim_{x\to 0} x \ln|\sin x| = \lim_{x\to 0} \frac{\ln|\sin x|}{1/x} = \lim_{x\to 0} \frac{\cot x}{-1/x^2} = 0$ by above,

$$\Rightarrow K = \pi \ln 2$$

L is evaluated using the trigonometric substitution $x = \tan\theta$,

$$L = \int_0^{\infty} \frac{\ln(1+x^2)}{1+x^2}\, dx = \int_0^{\frac{\pi}{2}} \ln(\sec^2\theta)\, d\theta = -2\int_0^{\frac{\pi}{2}} \ln(\cos\theta)\, d\theta = \pi \ln 2$$

Incidentally, $\int_0^{\infty} \frac{\ln(1+x^2)}{x^2}\, dx = \left[-\frac{1}{x}\ln(1+x^2)\right]_0^{\infty} + \int_0^{\infty} \frac{1}{x}\frac{2x}{1+x^2}\, dx = \pi$ (integration by parts)