P4 - Elt 441 - SEP - 30/3/22

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$A = 9+6 = 15$ $B = 7+8 = 15$

Dados dos ramos do sistema:

K	m	$\overline{Z}_{km}$
1	2	0.015 + j0.3
2	3	0.015 + j0.8

$\varepsilon_p = 0.001$ $\varepsilon_q : 0.001$

Dados (Pot)	P_2	P_3	Q_3
Dados (est.)	V_1	V_2	θ_1
Incognita (est)	θ_2	θ_3	V_3
Incognita (Pot)	P_1	Q_1	Q_2

* Definições das variaveis a serem encontradas

Para esse sistema as admitâncias das linhas são:

$$\rightarrow y_{12} = \frac{+1}{Z_{12}} = \frac{1}{0.015 + j0.3} = 0.1663 - 3.3250j$$

$$\rightarrow y_{23} = \frac{1}{Z_{23}} = \frac{1}{0.015 + j0.8} = 0.0234 - 1.2496j$$

$\rightarrow y_{11} = y_{21} = y_{12}$

$\rightarrow y_{23} = y_{32} = y_{33}$

$\rightarrow y_{13} = y_{31} = 0$

$\rightarrow y_{22} = y_{11} + y_{33}$

$$Y = \begin{bmatrix} y_{11} & -y_{12} & -y_{13} \\ -y_{21} & y_{22} & -y_{23} \\ -y_{31} & -y_{32} & y_{33} \end{bmatrix} =$$

$$Y = \begin{bmatrix} 0.1663 - 3.325j & -0.1663 + 3.325j & 0 + 0j \\ -0.1663 + 3.325j & 0.1897 - 4.5746j & -0.0234 + 1.2496j \\ 0 + 0j & -0.0234 + 1.2496j & 0.0234 - 1.2496j \end{bmatrix}$$

matriz admitância

$Y = G + jB$

Parte real

$$G = \begin{bmatrix} 0.1663 & -0.1663 & 0 \\ -0.1663 & 0.1897 & -0.0234 \\ 0 & -0.0234 & 0.0234 \end{bmatrix}$$

Parte imaginária

$$B = \begin{bmatrix} -3.325 & 3.325 & 0 \\ 3.325 & -4.5746 & 1.2496 \\ 0 & 1.2496 & -1.2496 \end{bmatrix}$$

Com os dados fornecidos de G e B, temos as seguintes equações:

$$P_2 = V_2 \left[V_1 (G_{21} \cos(\theta_2 - \theta_1) + B_{21} \operatorname{sen}(\theta_2 - \theta_1)) + V_3 (G_{23} \cos(\theta_2 - \theta_3) + B_{23} \operatorname{sen}(\theta_2 - \theta_3)) + V_2 G_{22} \right]$$

$$P_3 = V_3 \left[V_2 (G_{32} \cos(\theta_3 - \theta_2) + B_{32} \operatorname{sen}(\theta_3 - \theta_2)) + V_3 G_{33} \right]$$

$$Q_3 = V_3 \left[V_2 (G_{32} \operatorname{sen}(\theta_3 - \theta_2) - B_{32} \cos(\theta_3 - \theta_2)) - V_3 B_{32} \right]$$

$$\star \begin{cases} P_{2_{esp}} = 0.24 \; ; \; P_{3_{esp}} = -0.23 \; ; \; Q_{3_{esp}} = 0.06 \; ; \; V_1 = 1 \; ; \; V_2 = 1 \; ; \; \theta_1 = 0 \\ \Delta P_2 = P_{2_{esp}} - P_2 \qquad \Delta P_3 = P_{3_{esp}} - P_3 \qquad \Delta Q_3 = Q_{3_{esp}} - Q_3 \end{cases}$$

Substituindo os valores nas equações encontramos:

$$\Delta P_2 = -V_3 \left[1.2496 \operatorname{sen}(\theta_2 - \theta_3) - 0.0234 \cos(\theta_2 - \theta_3) \right] - 3.3250 \operatorname{sen}(\theta_2) + 0.1663 \cos(\theta_2) + 0.0503$$

$$\Delta P_3 = -V_3 \left[0.0234 V_3 - 1.2496 \operatorname{sen}(\theta_2 - \theta_3) - 0.0234 \cos(\theta_2 - \theta_3) \right] - 0.23$$

$$\Delta Q_3 = -V_3 \left[1.2496 V_3 + 0.0234 \operatorname{sen}(\theta_2 - \theta_3) - 1.2496 \cos(\theta_2 - \theta_3) \right] + 0.06$$

Partindo destas equações, podemos calcular a matriz jacobiana do sistema:

$$\frac{\partial \Delta P_2}{\partial \theta_2} = -V_3 \left[0.0234 \operatorname{sen}(\theta_2 - \theta_3) + 1.2496 \cos(\theta_2 - \theta_3) \right] - 0.1663 \operatorname{sen}(\theta_2) - 3.3250 \cos(\theta_2)$$

$$\frac{\partial \Delta P_2}{\partial \theta_3} = -V_3 \left[-0.0234 \operatorname{sen}(\theta_2 - \theta_3) - 1.2496 \cos(\theta_2 - \theta_3) \right]$$

$$\frac{\partial \Delta P_2}{\partial V_3} = -1.2496 \operatorname{sen}(\theta_2 - \theta_3) + 0.0234 \cos(\theta_2 - \theta_3)$$

$$\frac{\partial \Delta P_3}{\partial \theta_2} = -V_3 \left[0.0234 \operatorname{sen}(\theta_2 - \theta_3) - 1.2496 \cos(\theta_2 - \theta_3) \right]$$

$$\frac{\partial \Delta P_3}{\partial \theta_3} = -V_3 \left[-0.0234 \operatorname{sen}(\theta_2 - \theta_3) + 1.2496 \cos(\theta_2 - \theta_3) \right]$$

$$\frac{\partial \Delta P_3}{\partial V_3} = -0.0469 V_3 + 1.2496 \operatorname{sen}(\theta_2 - \theta_3) + 0.0234 \cos(\theta_2 - \theta_3)$$

$$\frac{\partial \Delta Q_3}{\partial \theta_2} = -V_3 \left[1.2496 \operatorname{sen}(\theta_2 - \theta_3) + 0.0234 \cos(\theta_2 - \theta_3) \right]$$

$$\frac{\partial \Delta Q_3}{\partial \theta_3} = -V_3 \left[-1.2496 \operatorname{sen}(\theta_2 - \theta_3) - 0.0234 \cos(\theta_2 - \theta_3) \right]$$

$$\frac{\partial \Delta Q_3}{\partial V_3} = -2.4991 V_3 - 0.0234 \operatorname{sen}(\theta_2 - \theta_3) + 1.2496 \cos(\theta_2 - \theta_3)$$

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}$$

Para a realização do método de Newton - Raphson, definimos uma solução inicial com $[\theta_2^0, \theta_3^0, V_3^0] = [0, 0, 1]$. O algoritimo para resolução é $g(x^m) = -J(x^m)\,\Delta X^m$, portanto:

m	X^m	$g(X^m)$	$J(X^m)$	$-J(X^m)^{-1}$	ΔX^m
0	$\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$	$\begin{matrix} 0.2400 \\ -0.2300 \\ 0.0600 \end{matrix}$	$\begin{matrix} -4.5746 & 1.2496 & 0.0234 \\ 1.2496 & -1.2496 & -0.0234 \\ -0.0234 & 0.0234 & -1.2496 \end{matrix}$	$\begin{matrix} +0.3007 & +0.3007 & 0 \\ +0.3008 & +1.1008 & -0.0150 \\ 0 & +0.0150 & +0.8000 \end{matrix}$	$\begin{matrix} +0.0030 \\ -0.1819 \\ +0.0446 \end{matrix}$
1	$\begin{matrix} 0.0030 \\ -0.1819 \\ 1.0446 \end{matrix}$	$\begin{matrix} -0.0093 \\ 0.0085 \\ -0.0249 \end{matrix}$	$\begin{matrix} -4.6130 & 1.2975 & -0.2067 \\ 1.2785 & -1.2785 & 0.2038 \\ -0.2640 & 0.2640 & -1.3865 \end{matrix}$	$\begin{matrix} +0.3007 & +0.3028 & -0.0003 \\ 0.3007 & 1.1094 & 0.1183 \\ -4.4017 & 0.1536 & 0.7438 \end{matrix}$	$\begin{matrix} -0.0002 \\ 0.0036 \\ -0.0172 \end{matrix}$
2	$\begin{matrix} 0.0028 \\ -0.1783 \\ 1.0273 \end{matrix}$	$\begin{pmatrix} -8.0551 \\ 7.2672 \\ -39.6729 \end{pmatrix} \times 10^{-5}$	—	—	—

Agora, com θ_2, θ_3 e V_3, determinados, podemos calcular o valor de P_1, Q_1 e Q_2, portanto:

$$\begin{cases} P_1 = V_1\left[V_2\left(G_{12}\cos(\theta_1-\theta_2) + B_{12}\,\text{sen}(\theta_1-\theta_2)\right) + V_1 G_{11}\right] \\ Q_1 = V_1\left[V_2\left(G_{12}\,\text{sen}(\theta_1-\theta_2) - B_{12}\cos(\theta_1-\theta_2)\right) - V_1 B_{11}\right] \\ Q_2 = V_2\left[V_1\left(G_{21}\,\text{sen}(\theta_2-\theta_1) - B_{21}\cos(\theta_2-\theta_1)\right) + V_3\left(G_{23}\,\text{sen}(\theta_2-\theta_3) - B_{23}\cos(\theta_2-\theta_3)\right) - V_2 B_{22}\right] \end{cases}$$

Substituindo os valores, temos:

$$\begin{cases} P_2 = -0.1663\cos(\theta_2) + 0.1663 - 3.3250\,\text{sen}(\theta_2) \\ Q_1 = 0.1663\,\text{sen}\,\theta_2 - 3.3250\cos(\theta_2) + 3.3250 \\ Q_3 = V_3\left[-0.0234\,\text{sen}(\theta_2-\theta_3) - 1.2496\cos(\theta_2-\theta_3)\right] - 0.1663\,\text{sen}(\theta_2) - 3.3250\cos(\theta_2) + 4.5746 \end{cases}$$

Substituindo os valores de θ_2, θ_3 e V_3, temos: $P_1 = -0.0092$

$Q_1 = 0.0005$

$Q_2 = -0.0180$

Respostas			
$P_1 = -0.0092$	$Q_1 = 0.0005$ $Q_2 = -0.0180$	$V_3 = 1.0273$	$\theta_2 = 0.0028$ $\theta_3 = -0.1783$