Universidade Federal de Viçosa
Departamento de Matemática

MAT 141 (Turma 1) – Cálculo Diferencial e Integral I – 2017/II
1ª Lista de Integrais (07/11/2017)

1) Faça a antidiferenciação. Verifique o resultado, calculando a derivada de sua resposta.

(a) $\displaystyle\int y^3(2y^2-3)\,dy$

(b) $\displaystyle\int \sqrt{x}(x+1)\,dx$

(c) $\displaystyle\int \frac{x^2+4x-4}{\sqrt{x}}\,dx$

(d) $\displaystyle\int \sqrt[3]{x}+\frac{1}{\sqrt[3]{x}}\,dx$

(e) $\displaystyle\int e^{-x}\,dx$

(f) $\displaystyle\int x^2\ln(100)\,dx$

2) Determine a função $y=y(x), y\in\mathbb{R}$ tal que

(a) $\dfrac{dy}{dx}=x^3-x+1$, $y(1)=1$

(b) $\dfrac{d^2y}{dx^2}=e^{-x}$, $y(0)=0$ e $y'(0)=-1$

3) Determine a função $y=f(x)$, definida num intervalo aberto I, com $1\in I$ e tal que:

(a) $f(1)=-1$ e $\dfrac{dy}{dx}=2y^2, \forall x\in I.$

(b) $f(1)=1$ e $\dfrac{dy}{dx}=xy, \forall x\in I.$

4) Determine a função f contínua tal que:

(a) $f(\pi)=2$ e $\displaystyle\int f'(x)\text{tg}\,x\,dx=\text{sen}^3 x-\cos x+c,\ c\in\mathbb{R}$

(b) $f(0)=5$ e $\displaystyle\int \text{arctg}\left(\frac{f'(x)}{x}\right)\,dx=x^3+c,\ c\in\mathbb{R}$

(c) $f(0)=1$ e $\displaystyle\int (1+x^2)f'(x)\,dx=x+c,\ c\in\mathbb{R}$

5) Obtenha a equação de uma curva $y=f(x)$, sabendo que o coeficiente angular da reta tangente em um de seus pontos é igual ao simétrico do dobro de sua abscissa e que o ponto $(1,1)$ pertence a curva.

6) Em todos os pontos de uma curva $y=f(x)$ tem-se que y” $=x^2-1$. Obtenha a equação desta curva que passa pelo ponto $(1,1)$ e a reta tangente neste ponto é paralela à reta $x+12y-13=0$.

7) A velocidade de uma partícula, no instante t, em movimento numa linha reta é dada por $v(t)=\sqrt[3]{t^2}$. Determine a função horária do movimento $s(t)$ sabendo que $s(0)=3$.

8) Uma partícula move-se ao longo de um eixo x. Determine a função posição da partícula, sabendo-se que:

(a) $v(t)=t^3-2t^2+1$ e $x(0)=1$

(b) $a(t)=4\cos(2t), v(0)=-1$ e $x(0)=-3.$

9) Estima-se que daqui a t meses a população de uma certa cidade estará variando segundo uma taxa de $2+6\sqrt{t}$ pessoas por mês. A população atual é de 5000 pessoas. Qual a população daqui a 9 meses?

10) Um tanque de armazenamento de petróleo sofre uma ruptura no instante $t=0$ e o petróleo vaza do tanque a uma taxa de $100\,e^{-0{,}01\,t}$ litros/min. Determine a quantidade de litros de petróleo que vazou do tanque nos primeiros sessenta minutos.

11) Mostre que:

(a) Se F é uma primitiva de f, então $\int f(ax+b)\ dx = \frac{1}{a}F(ax+b)+c,\ c\in\mathbb{R}$

(b) Para $n \neq -1$, $\int (ax+b)^n\ dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1}+c,\ c\in\mathbb{R}$

12) Dado $a>0$, mostre que:

(a) $\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \operatorname{arcsen}\left(\frac{x}{a}\right)+c$

(b) $\int \frac{1}{a^2+x^2}\ dx = \frac{1}{a}\operatorname{arctg}\left(\frac{x}{a}\right)+c$

(c) $\int \frac{1}{\sqrt{x^2\pm a}}\ dx = \ln\left|x+\sqrt{x^2\pm a}\right|+c$

(d) $\int \frac{1}{x^2-a^2}\ dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$

13) Resolva (usando o método de substituição)

(a) $\int \frac{1}{3x-2}\ dx$

(b) $\int \frac{1}{(3x-2)^2}\ dx$

(c) $\int x\operatorname{sen}(x^2)\ dx$

(d) $\int x^2e^{x^3}\ dx$

(e) $\int x^3\cos(x^4)\ dx$

(f) $\int \operatorname{sen}^5(x)\cos(x)\ dx$

(g) $\int \frac{3x}{5+6x^2}\ dx$

(h) $\int \frac{x}{(1+4x^2)^2}\ dx$

(i) $\int x\sqrt{1+3x^2}\ dx$

(j) $\int \frac{\operatorname{sen}(x)}{\cos^2(x)}\ dx$

(k) $\int \frac{1-2x}{\sqrt{3x+2}}\ dx$

(l) $\int \frac{1}{\sqrt{9-x^2}}\ dx$

(m) $\int \frac{x^2}{9+x^2}\ dx$

(n) $\int \frac{x}{1+x^4}\ dx$

14) Complete o quadrado e use as integrais do exercício (12) para calcular as integrais:

(a) $\int \frac{1}{\sqrt{2-x-x^2}}\ dx$

(b) $\int \frac{3-4x}{\sqrt{3x-x^2-2}}\ dx$

(c) $\int \frac{3x-1}{x^2+8x+25}\ dx$

(d) $\int \frac{1}{2x^2+3x+1}\ dx$

(e) $\int \frac{x-1}{\sqrt{1-x-x^2}}\ dx$

15) Calcule (usando o método de integração por partes)

(a) $\int x^2e^x\ dx$

(b) $\int x^2\ln(x)\ dx$

(c) $\int x\sec^2(x)\ dx$

(d) $\int x(\ln(x))^2\ dx$

(e) $\int (ln(x))^2\ dx$

(f) $\int xe^{2x}\ dx$

(g) $\int e^{-2x}\operatorname{sen}(x)\ dx$

(h) $\int x^3e^{x^2}\ dx$

(i) $\int x^3\cos(x^2)\ dx$

(j) $\int e^{-x}\cos(2x)\ dx$

(k) $\int \sqrt{16-x^2}\ dx$

(l) $\int \frac{x^2}{\sqrt{16-x^2}}\ dx$

(m) $\int x^2\sqrt{16-x^2}\ dx$

(n) $\int \operatorname{arcsen}(x/4)\ dx$

(o) $\int x\operatorname{arcsen}x\ dx$

16) Resolva as integrais

(a) $\int \operatorname{sen}^2x\ \cos^3x\ dx$

(b) $\int \cos^3x\ \operatorname{sen}^3x\ dx$

(c) $\int \operatorname{sen}2x\ \sqrt{1+\cos^2x}\ dx$

(d) $\int \operatorname{sen}2x\ \sqrt{5+\operatorname{sen}^2x}\ dx$

(e) $\int \cos^5x\ dx$

(f) $\int \operatorname{tg}^3x\ \sec^2x\ dx$

(g) $\int \operatorname{tg} x \sec^3 x \, dx$

(h) $\int \operatorname{tg}^3 x \sec^4 x \, dx$

(i) $\int \operatorname{sen} x \sqrt{3+\cos x} \, dx$

(j) $\int \operatorname{sen} x \sec^2 x \, dx$

17) Calcule

(i) $\int 2x^3\sqrt{x} + e^{2x+3} \, dx$

(ii) $\int e^{\sqrt{y}} \, dx$

(iii) $\int 8x^2\sqrt{6x^3+5} \, dx$

(iv) $\int \dfrac{\operatorname{sen} \theta}{(7-\cos\theta)^3} \, d\theta$

(v) $\int \sec^2(5x+3) \, dx$

(vi) $\int \dfrac{x^3+x+1}{1+x^2} \, dx$

(vii) $\int \dfrac{a^3-1}{a-1} \, da$

(viii) $\int \dfrac{3x}{(4-3x^2)^8} \, dx$

(ix) $\int \dfrac{\operatorname{sen} x}{\cos^5 x} \, dx$

(x) $\int \operatorname{sen}(2x) \operatorname{sen} x \, dx$

(xi) $\int \operatorname{tg} x \, dx$

(xii) $\int \dfrac{x^3+1}{x-2} \, dx$

(xiii) $\int \dfrac{1}{\sqrt{16-(x-1)^2}} \, dx$

(xiv) $\int x + \sec^2(5x) \, dx$

(xv) $\int \operatorname{tg} x \sec^2 x \, dx$

(xvi) $\int \dfrac{1}{(x+3)^2+4} \, dx$

(xvii) $\int \dfrac{1}{x^2+6x+13} \, dx$

(xviii) $\int \dfrac{x^2}{x^2+1} \, dx$

(xix) $\int \cos\theta \sqrt[3]{\operatorname{sen}\theta} \, dx$

(xx) $\int \dfrac{\cos x}{3-\operatorname{sen} x} \, dx$

(xxi) $\int x\, 2^{x^2} \, dx$

(xxii) $\int x\sqrt{x+1} \, dx$

(xxiii) $\int \operatorname{sen}^3 x \, dx$

(xxiv) $\int \operatorname{sen}^2 x \, dx$

(xxv) $\int \sec x \, dx$

(xxvi) $\int \operatorname{tg}^4 x \, dx$

(xxvii) $\int \dfrac{x^3}{\sqrt{4+x^2}} \, dx$

(xxviii) $\int \dfrac{1}{\sqrt{8x-x^2}} \, dx$

(xxix) $\int \dfrac{x}{\sqrt{1-x^4}} \, dx$

(xxx) $\int \dfrac{\cos^3 x}{\operatorname{sen}^4 x} \, dx$

18) Calcule as integrais

(i) $\int \sqrt{1-\cos(4x)} \, dx$

(ii) $\int \dfrac{3x^2-7x}{3x+2} \, dx$

(iii) $\int \dfrac{3x+2}{\sqrt{1-x^2}} \, dx$

(iv) $\int \dfrac{1}{\sqrt{x}(\sqrt{x}+1)} \, dx$

(v) $\int \dfrac{6x+7}{x^2+4x+4} \, dx$

(vi) $\int \dfrac{x-1}{x^2} \, dx$

(vii) $\int \operatorname{sen}(\sqrt{x}) \, dx$

(viii) $\int \sqrt{a^2-x^2} \, dx$

(ix) $\int \sqrt{t}\ln t \, dx$

(x) $\int \cos(\ln t) \, dx$

(xi) $\int \sec 3x \, dx$

(xii) $\int \operatorname{tg}^5 x \sec^7 x \, dx$

(xiii) $\int x \operatorname{arctg} x \, dx$

(xiv) $\int \ln(1+x^2) \, dx$

(xv) $\int \dfrac{x^3+x}{x-1} \, dx$

(xvi) $\int \dfrac{x+9}{x^2+2x+3} \, dx$

(xvii) $\int x\cos^2 x \, dx$

(xviii) $\int \dfrac{\sqrt{9-x^2}}{x^2} \, dx$

(xxi) $\int \dfrac{x^2}{3\sqrt{x^2-4}} \, dx$

(xxiii) $\int \dfrac{2x^4+4x^4+2x^3+2x-1}{2x^3} \, dx$

(xxiv) $\int \dfrac{x^3+x+1}{1+x^2} \, dx$

Gabarito

1) (a) $\frac{1}{3}y^6 - \frac{3}{4}y^4 + c$
   (b) $\frac{2}{3}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} + c$
   (c) $\frac{2}{5}x^{\frac{5}{2}}\frac{8}{3}x^{\frac{1}{2}} + c$
   (d) $\frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{\frac{2}{3}} + c$
   (e) $-e^{-x}$
   (f) $\frac{x^3}{3}\ln(100)$

2) (a) $y = \frac{x^4}{4} - \frac{x^2}{2} + x + \frac{1}{4}$
   (b) $y = e^{-x} - 1$

3) (a) A função $f(x) = \frac{-1}{2x-1}$, $x > \frac{1}{2}$ satisfaz as condições dadas (A condição $x > \frac{1}{2}$ é para garantir que 1 pertença ao domínio de f.)
   (b) A função $y = \frac{1}{\sqrt{e}}e^{\frac{x^2}{2}}$, $x \in \mathbb{R}$ satisfaz as condições dadas.

4) (a) $f(x) = -\cos^3 x + \operatorname{sen} x + 1$
   (b) $f(x) = -1/6\ln|\cos(3x^2)| + 5$
   (c) $f(x) = \operatorname{arctg} x + 1$

5) $y = 2 - x^2$

6) $12y = x^4 - 6x^2 + 7x + 10$

7) $\frac{3}{5}\sqrt[3]{t^5} + 3$

8) (a) $x(t) = t^4/4 - 2t^3/3 + t + 1$
   (b) $-\cos(2t) - t - 2$

9) 5126 pessoas

10) $10.000(1 - e^{-0,6})$

13) (a) $\frac{1}{3}\ln|3x-2| + c$
   (b) $-\frac{1}{3(3x-2)} + c$
   (c) $-\frac{1}{2}\cos(x^2) + c$
   (d) $\frac{1}{3}e^{x^3} + c$
   (e) $\frac{1}{4}\operatorname{sen}(x^4) + c$
   (f) $\frac{1}{6}\operatorname{sen}^6(x) + c$
   (g) $\frac{1}{4}\ln(5 + 6x^2) + c$
   (h) $-\frac{1}{8(1+4x^2)} + c$
   (i) $\frac{1}{9}\sqrt{(1+3x^2)^3} + c$
   (j) $\frac{1}{\cos(x)} + c$
   (k) $-\frac{2\sqrt{3x+2}\cdot(6x-17)}{27}$
   (l) $\operatorname{arcsen}\left(\frac{x}{3}\right) + c$
   (m) $x - 3\operatorname{arctg}\left(\frac{x}{3}\right) + c$
   (n) $\frac{1}{2}\operatorname{arctg}(x^2) + c$

14) (a) $\operatorname{arcsen}\left(\frac{2x+1}{3}\right) + c$
   (b) $4\sqrt{3x - x^2 - 2} - 3\operatorname{arcsen}(2x-3) + c$
   (c) $\frac{3}{2}\ln|x^2 + 8x + 25| - \frac{13}{3}\operatorname{arctg}\left(\frac{x+4}{3}\right) + c$
   (d) $\ln\left|\frac{2x+1}{2x+2}\right| + c$
   (e) $-\frac{3}{2}\operatorname{arcsen}\left(\frac{2x+1}{\sqrt{5}}\right) - \sqrt{-x^2 - x + 1} + c$

15) (a) $e^x(x^2 - 2x + 2) + c$
   (b) $\frac{1}{3}x^3(\ln(x) - \frac{1}{3}) + c$
   (c) $x\operatorname{tg}(x) + \ln|\cos(x))| + c$
   (d) $\frac{x^2}{2}\left[(\ln x)^2 - \ln x + \frac{1}{2}\right] + c$
   (e) $x(\ln x)^2 - 2x(\ln x - 1) + c$
   (f) $\frac{1}{2}e^{2x}\left(x - \frac{1}{2}\right) + c$
   (g) $-\frac{1}{5}e^{-2x}(\cos x + 2\operatorname{sen} x) + c$
   (h) $\frac{1}{2}(x^2 - 1)e^{x^2} + c$
   (i) $\frac{1}{2}(x^2\operatorname{sen} x^2 + \cos x^2) + c$
   (j) $\frac{e^{-x}}{5}(2\operatorname{sen} 2x - \cos 2x) + c$
   (k) $8\,\mathrm{asin}\left(\frac{x}{4}\right) + \frac{x\sqrt{16-x^2}}{2} + c$
   (l) $8\operatorname{arcsen}\left(\frac{x}{4}\right) - \frac{x\sqrt{16-x^2}}{2} + c$
   (m) $\frac{1}{4}[8\operatorname{arcsen}(x/4) - x\sqrt{(16-x^2)^2} + 2x\sqrt{16-x^2}] + c$
   (n) $\frac{x}{4}[\operatorname{arcsen}(x/4) + \sqrt{16-x^2}] + c$
   (o) $\frac{1}{2}[x^2\operatorname{arcsen}(x) + \frac{\operatorname{arcsen}(x)}{2} + \frac{x\sqrt{1-x^2}}{2}] + c$

16) (a) $\frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5} + c$
   (b) $\frac{\operatorname{sen}^4 x}{4} - \frac{\operatorname{sen}^6 x}{6} + c$
   (c) $-\frac{2}{3}\sqrt{(1+\cos^2 x)^3} + c$
   (d) $\frac{2}{3}\sqrt{(5+\operatorname{sen}^2 x)^3} + c$

(e) $\operatorname{sen} x - \frac{2}{3}\operatorname{sen}^3 x + \frac{1}{5}\cos^5 x + c$

(f) $\frac{1}{4}\operatorname{tg}^4 x + c$

(g) $\frac{1}{3}sec^3(x) + c$

(h) $\frac{1}{6}sec^6(x) - \frac{1}{4}sec^4(x) + c$

(i) $-\frac{2}{3}\sqrt{(3+cos(x))^3} + c$

(j) $sec(x) + c$

17) (i) $\frac{4}{9}x^{9/2} + \frac{1}{2}e^{2x+3} + C$

(ii) $x\,e^{\sqrt{y}} + C$

(iii) $\frac{8}{27}\sqrt{(6x^3+5)^3} + C$

(iv) $-\frac{1}{2(7-\cos\theta)^2} + C$

(v) $\frac{1}{5}\operatorname{tg}(5x+3) + C$

(vi) $x^2/2 + \operatorname{arctg} x + C$

(vii) $a^3/3 + a^2/2 + a + C$

(viii) $\frac{1}{14(4-3x^2)^7} + C$

(ix) $\frac{1}{4\cos^4 x} + C$

(x) $\frac{\operatorname{sen} x}{2} - \frac{\operatorname{sen} 3x}{6} + C$

(xi) $\ln(\sec\ x) + C$

(xii) $9\ln(x-2) + (x^3 + 3x^2 + 12x)/3 + C$

(xiii) $\frac{(x-1)\sqrt{-x^2+2x+15}}{2} - 8\operatorname{arcsen}\left(\frac{1-x}{4}\right) + C$

(xiv) $\frac{x^2}{2} + \frac{1}{5}\operatorname{tg}(5x) + C$

(xv) $\frac{1}{2}\operatorname{tg}^2 x + C$

(xvi) $\frac{1}{2}\operatorname{arctg}((x+3)/2) + C$

(xvii) $\frac{1}{2}\operatorname{arctg}((x+3)/2) + C$

(xviii) $x - \operatorname{arctg} x + C$

(xix) $\frac{3}{4}\sqrt[3]{\operatorname{sen}^4 x} + C$

(xx) $-\ln(3 - \operatorname{sen} x) + C$

(xxi) $2^{(x^2-1)}/\ln 2 + C$

(xxii) $\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$

(xxiii) $\frac{1}{3}\cos(x^3) - \cos x + C$

(xxiv) $x/2 - \operatorname{sen}(2x)/4 + C$

(xxv) $\ln|\operatorname{tg} x + \sec x| + C$

(xxvi) $\frac{1}{3}\operatorname{tg}^3 x – \operatorname{tg} x + x + C$

(xxvii) $\frac{x^2}{3}\sqrt{x^2+4} - \frac{8}{3}\sqrt{x^2+4} + C$

(xxviii) $-\operatorname{arcsen}(1 - x/4) + C$

(xxix) $-\frac{1}{2}\operatorname{arctg}(\sqrt{x^{-2} - x^2} + C$

(xxx) $\frac{1}{\operatorname{sen} x} - \frac{1}{3\operatorname{sen}^3 x} + C$

Algumas dicas.

(a) Use divisão de polinômios em, por exemplo, (17vi) e (17vii).

(b) Complete quadrado em, por exemplo, (17xvii) e (17xxviii).

(c) Multiplique/divida por $\sec x + \operatorname{tg} x$ em (17xxv).

(d) Mostre que $\operatorname{sen}^2 x = \frac{1-\cos(2x)}{2}$ e $\cos^2 x = \frac{1+\cos(2x)}{2}$. Use uma dessas relações em, por exemplo, (17xxiv).

(e) Use uma relação trigonométrica apropriada em, por exemplo, (17x), (17xi), (17xxiii), (17xxvi) e (17xxx).

18) (i) $\frac{1}{2}\sqrt{2}\operatorname{sen}(2x) + C$

(ii) $\frac{1}{2}x^2 - 3x + 2\ln(3x+2) + C$

(iii) $-3\sqrt{1-x^2} + 2\operatorname{arcsen} x + C$

(iv) $2\ln(\sqrt{x}+1) + C$

(v) $\frac{5}{x+2} + 6\ln(x+2) + C$

(vi) $\ln x + \frac{1}{x} + C$

(vii) $2\operatorname{sen}\sqrt{x} - 2\sqrt{x}\cos\sqrt{x} + C$

(viii) $\frac{1}{2}a^2\operatorname{arcsen}(x/a) + x\sqrt{a^2-x^2} + C$

(ix) $\frac{2}{3}x^{3/2}\ln x - 4/9x^{3/2} + C$

(x) $\frac{1}{2}x[\cos(\ln x) + \operatorname{sen}(\ln x)] + C$

(xi) $\frac{\operatorname{sen} x}{2\cos^2 x} + \frac{1}{2}\ln(\sec x) + \operatorname{tg} x + C$

(xii) $\frac{1}{11}\sec^{11} x - \frac{2}{9}\sec^9 x + \frac{1}{7}\sec^7 x + C$

(xiii) $\frac{1}{2}[x^2\operatorname{arctg} x - x + \operatorname{arctg} x] + C$

(xiv) $x\ln(x^2+1) - 2x + 2\operatorname{arctg} x + C$

(xv) $\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + 4\ln(x-1) + C$

(xvi) $\frac{1}{2}\ln(x^2+2x+3) + 4\sqrt{2}\operatorname{arctg}((x+1)/\sqrt{2}) + C$

(xvii) $x\operatorname{sen}(2x)/4 + \cos(2x) + 2x^2 + C$

(xviii) $-\operatorname{arcsen}(x/3) - \frac{\sqrt{9-x^2}}{x} + C$

(xxi) $\frac{1}{6}x\sqrt{x^2-4} + \frac{2}{3}\ln(x + \sqrt{x^2-4}) + C$

(xxiii) $\frac{x^3}{3} + x^2 + x - \frac{1}{x} + \frac{1}{4x^2} + C$

(xxiv) $\frac{x^2}{2} + \operatorname{arctg} x + C$