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Modelling and Control of a PVTOL Quadrotor Carrying a Suspended Load

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***Abstract*— This work proposes the representation of the high level dynamic model of a quadrotor UAV with a suspended load, through Euler-Lagrange equations. The rotorcraft movement is restricted to the XZ plane of the Cartesian space, so that it performs like a PVTOL machine. A nonlinear controller is proposed to stabilize the quadrotor and the load, during positioning and trajectory tracking tasks. Finally, simulated results are presented in order to validate the proposal.**

I. Introduction

Researches using unmanned aerial vehicles (UAVs) have experienced a meaningful growth along the last few years, due to the high applicability of such vehicles and the increasing technology associated to them (sensors and embedded processors, for instance).

Rotorcraft UAVs, like a quadrotor, for instance, have great three-dimensional mobility when compared with ground vehicles and even other unmanned aerial vehicles (airplanes, gliders and balloons). Their ability to take off and land vertically, to hover with the possibility of changing its orientation, to move ahead or laterally while keeping the height, to completely change their direction of flight and to stop their movement abruptly, certainly have motivated several researches in the area. However, from the point of view of control, these aircrafts are inherently unstable, non-linear, multi-variable, with complex and highly coupled dynamics [1], what introduces some complexity when designing proper controllers for autonomous navigation.

Despite of this, their mobility has motivated a wide range of rotorcraft UAV applications, so that such aircrafts are performing tasks in places of difficult access and high risk, or even over large areas. Amongst such tasks, one can cite those related to public safety, management of natural hazards, environmental management, intervention in hostile environments, infrastructure maintenance, precision agriculture and cargo transportation [2], [3].

When regarding load transportation, one can choose between two approaches. The first one consists in using an UAV equipped with graspers [4], which will carry the load closer to its center of gravity, thus increasing the total mass and the rotational inertia, slowing down the attitude dynamics and response of the vehicle. The second solution is a cable-suspended load [5], [6], [7], [8], [9], which preserves the agility of the vehicle but creates an additional degree of freedom, due to the swinging motion of the cargo. Since there is no actuator to perform the control of the suspended mass, the related new degrees of freedom are non-actuated.

To overcome such characteristics, a dynamic compensation module can be included in the control system, or even a single solution typically based on nonlinear techniques can be adopted, including both kinematic and dynamic models of the rotorcraft.

In [10], a quadrotor with a cable suspended load is considered to be a differentially-flat hybrid system, to deal with the case when the tension on the cable goes to zero. The model and controller used are for the planar case, which consists of a three-tier inner-outer loop based control scheme, regulating the quadrotor and load attitude or the load position in the plane. In turn, in [6] a quadrotor with a load using a flexible cable is taken into account and modeled as a system of serially-connected links. A geometric nonlinear control system is used, with a controller designed starting from a simplified dynamic model, which asymptotically stabilize the position of the vehicle and keeps the cable in its vertical position. In [5], the nonlinear model of the system is decoupled into two independent dynamic systems: an inner loop related to the attitude, and an outer one related to the remaining states. A nested saturation controller is implemented to stabilize the system and to control the oscillations of the suspended load.

The concept of Planar Vertical Take off and Landing (PVTOL) [11] is associated to an aircraft capable to perform movements only in the vertical plane, and has been an important benchmark for control design. Unfortunately, some nonlinear control techniques are not possible to apply straightforwardly, such as feedback linearization or sliding mode control [12]. In contrast, many control strategies have been proposed in the literature, such as Liouvillian systems [13] and robust controllers based on

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classical and adaptive pole placement techniques [14], for instance.

In such context, this work proposes a nonlinear high-level modelling and control strategy based on feedback linearization for a quadrotor with a suspended load, considered as a PVTOL aircraft. Both the underactuated dynamic model of the vehicle and the cargo are regarded. The effects of the load on the vehicle are here considered as a modeled disturbance applied to the system. It is used the same high level platform used in other works of our research group [15], [16], [17], [18], [19], to simulate experiments considering this UAV-cargo system.

To give details about such development, the paper is hereinafter split in some sections. Section II presents an overview of the aircraft model with the suspended mass, emphasizing the control variables and the degrees of freedom available for control. In the sequel, the development of the high-level controller is shown in Section III. Following, Section IV shows some simulated results and, finally, Section V highlights the main conclusions of the work.

II. Dynamic Model of a Quadrotor with a Suspended Load

According to [20], [21], a rotary wing machine is represented by four interconnected dynamic subsystems, as shown in Figure 1. Actuator Dynamics and Rotary-Wings Dynamics can be understood as the low-level dynamic model, whereas Torque and Force Generation and Rigid Body Dynamics correspond to the high-level dynamic model. This work focuses on the high-level modelling of a UAV with a suspended load, thus just the last two blocks are hereinafter considered. The model correspondent to them is obtained using Euler-Lagrange equations, in a way similar to [22], [23].

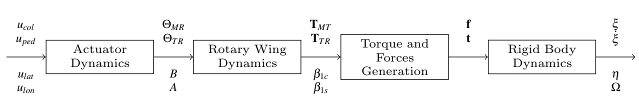

Fig. 1. Block representation of the Helicopter dynamic model.

In this work, the modelling is based on an AR.Drone quadrotor, from Parrot Inc. An important detail is that its motors are not aligned with the ${}^bx$ and ${}^by$ axis, as it can be seen in Figure 2: they are turned 45° around the axis ${}^bz$. Thereby, to perform a lateral or a longitudinal maneuver the combined action of all four engines is required. However, considering the vehicle as a PVTOL one, as in this case, its movement is restricted to the **XZ** plane, i.e, there are neither displacements in the y axis nor variations in the roll (ϕ) and yaw (ψ) angles, as illustrated in Figure 3. In such vehicle, the forces f_3 and f_4 act together to realize the movements, as well as the forces f_4 and f_1. Notice that the quadrotor is an underactuated system [23], even when flying like a PVTOL aircraft.

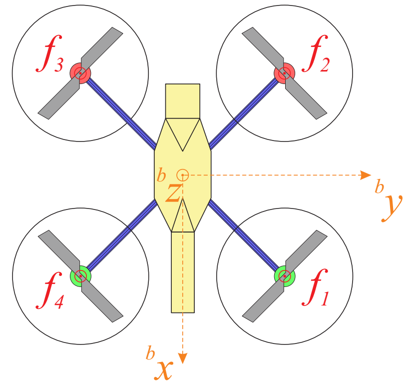

Fig. 2. Top view of the Ar.Drone, from Parrot, Inc.,the UAV considered in this work.

In order to introduce the modelling, the attitude of the UAV in the 2D generalized coordinate space, i.e, ${}^e\mathbf{q} = \begin{bmatrix} {}^e\xi & {}^e\eta & {}^e\eta_c \end{bmatrix}^T$, is firstly defined. As there is no movement in the y axis, the coordinates vector becomes ${}^e\xi = \begin{bmatrix} x & z \end{bmatrix}^T \in \mathbb{R}^2$, correspondent to longitudinal and normal displacements, according to the inertial reference system $\langle e \rangle$, ${}^s\eta = \boldsymbol{\theta} \in \mathbb{R}$, correspondent only to the pitch angle, due to the movement restriction, according to the spatial reference system $\langle s \rangle$, and ${}^b\eta = \boldsymbol{\theta}_c \in \mathbb{R}$, correspondent to the pitch angle of the load in its own reference system $\langle l \rangle$.

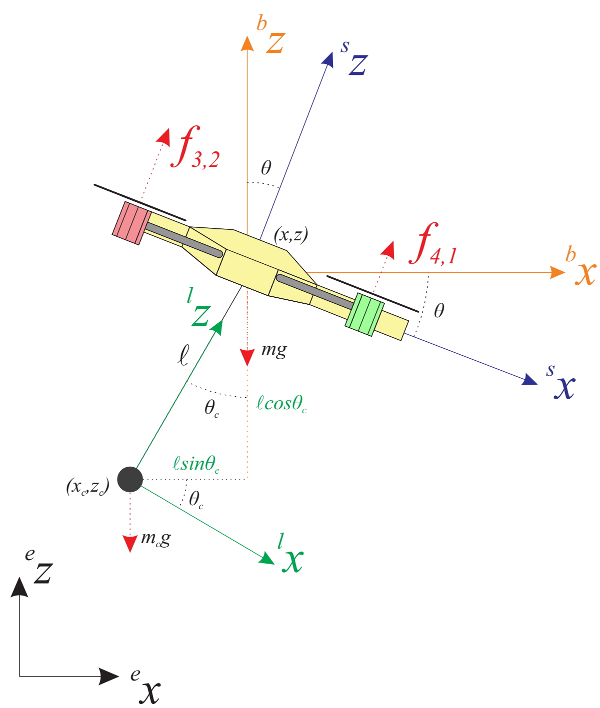

Fig. 3. The reference systems adopted for the UAV moving in the **XZ** plane while carrying a load.

Considering the quadrotor as a free body in the space under the action of external forces and torques, the total energy in $\langle e \rangle$ can be expressed by the Lagrangian function L, which represents the total kinetic energy K

minus the potential energy U, i.e,

$$L = K - U, \tag{1}$$

subject to the Euler-Lagrange constraint

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial L}{\partial \mathbf{q}} = \begin{bmatrix}\mathbf{f} \\ \tau\end{bmatrix}. \tag{2}$$

Assumption 1: The cable is rigid, inelastic, mass-less, and the aerodynamic effects on the load are negligible.

Since there are two bodies in the free space, the aircraft and the load, there are two kinetic and potential energies, associated to each one of these two bodies, so that the total kinetic and potential energies are

$$K = \underbrace{\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{z}^2 + \frac{1}{2}I_{yy}\dot{\theta}^2}_{K_1} + \underbrace{\frac{1}{2}m_c\dot{x}_c^2 + \frac{1}{2}m_c\dot{z}_c{}^2 + \frac{1}{2}I\dot{\theta}_c{}^2}_{K_2} \tag{3}$$

$$U = \underbrace{mgz}_{U_1} + \underbrace{m_c g z_c}_{U_2}, \tag{4}$$

where K_1 and U_1 are associated to the UAV and K_2 and U_2 are associated to the suspended mass. m and m_c are, respectively, the aircraft and the suspended load masses, whereas g is the gravity acceleration. (x_c, z_c) is the load position, which can be expressed in terms of the angle θ_c, the cable length l and the position of the quadrotor, as illustrated in Figure 3, as being

$$x_c = x - l\sin\theta_c \tag{5}$$

$$z_c = z - l\cos\theta_c \tag{6}$$

As we are considering just movements in the **XZ** plane, the matrix of inertia is represented only by its I_{yy} term, and the same reasoning is valid for the cargo. The angular velocity Ω is represented by $\dot{\theta}$, according to the same assumption.

The vector of forces $\mathbf{f}$, which represents the propulsion generated by the motors, is given by

$$\mathbf{f} = \begin{bmatrix} f_x & f_z \end{bmatrix}^T = \mathscr{R}\mathscr{A}_t \begin{bmatrix} f_1 & f_2 \end{bmatrix}^T, \tag{7}$$

$$\mathscr{R} = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix}. \tag{8}$$

In addition, the $\mathscr{A}_t$ matrix relates the propulsion generated and the total force actuating on the vehicle. For a quadrotor this matrix is given by

$$\mathscr{A}_t = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

, because all the propulsion forces generated in a quadrotor face upwards, in the direction of the $^b z$ axis.

In a similar way, the torque vector τ is given by

$$\tau = \mathscr{A}_r \begin{bmatrix} f_1 & f_2 \end{bmatrix}^T, \tag{9}$$

where the matrix $\mathscr{A}_r$ is given by

$$\mathscr{A}_r = \begin{bmatrix} k_1 & k_1 \\ 0 & 0 \end{bmatrix},$$

where k_1 represents the distance between the reference axis and the location where the force is applied.

Applying (3) and (4) in (1) and then in (2), the high level model, in the inertial reference system, is given by

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau, \tag{10}$$

where

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m+m_c & 0 & 0 & -m_c l c_{\theta_c} \\ 0 & m+m_c & 0 & m_c l s_{\theta_c} \\ 0 & 0 & I_{yy} & 0 \\ -m_c l c_{\theta_c} & m_c l s_{\theta_c} & 0 & I_c + m_c l^2, \end{bmatrix} \tag{11}$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} 0 & 0 & 0 & m_c l s_{\theta_c}\dot{\theta}_c \\ 0 & 0 & 0 & m_c l c_{\theta_c}\dot{\theta}_c \\ 0 & 0 & 0 & 0 \\ m_c l s_{\theta_c}\dot{\theta} & m_c l c_{\theta_c}\dot{\theta}_c & 0 & m_c l s_{\theta_c}\dot{x} + m_c l c_{\theta_c}\dot{z} \end{bmatrix}, \tag{12}$$

and

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} 0 \\ (m+m_c)g \\ 0 \\ m_c l s_{\theta_c} g \end{bmatrix}. \tag{13}$$

Notice that c, s and t are the $\cos, \sin$ and $\tan$ functions, respectively.

The dynamic model described has some typical properties that deserves mention, such as

i. $\mathbf{M}(\mathbf{q})$ is symmetrical and positive definite;
ii. $\mathbf{M}(\mathbf{q})^{-1}$ exists and is also positive definite;
iii. $\mathbf{C}(\mathbf{q},\mathbf{0}) = \mathbf{0}\ \forall q \in \mathbb{R}^3$;
iv. $\mathbf{N} = \dot{\mathbf{M}} - 2\mathbf{C} = \underline{\dot{\mathbf{q}}}^T \frac{\partial \mathbf{M}}{\partial \mathbf{q}} - \dot{\mathbf{M}}$ is anti-symmetrical if $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is obtained via Christopher symbols.

III. The Proposed Controller

Considering that the behavior of θ_c is a modeled disturbance which is applied to the system, Eq. 10 can be split into the systems described by

$$\bar{\mathbf{M}}(\mathbf{q}) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_{yy} \end{bmatrix}, \bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \bar{\mathbf{G}}(\mathbf{q}) = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix}$$

and

$$\bar{\mathbf{D}} = \begin{bmatrix} m_c\ddot{x} - m_c l c_{\theta_c}\ddot{\theta}_c + m_c l s_{\theta_c}\dot{\theta}_c^2 \\ m_c\ddot{z} + m_c l s_{\theta_c}\ddot{\theta}_c + m_c l c_{\theta_c}\dot{\theta}_c^2 + m_c g \\ 0 \end{bmatrix}. \tag{14}$$

Therefore, the system is now represented as

$$\bar{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}} + \bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \bar{\mathbf{G}}(\mathbf{q}) + \bar{\mathbf{D}} = \tau, \tag{15}$$

where $\bar{\mathbf{D}}$ is the modeled disturbance matrix.

Applying the concept of feedback linearization for nonlinear systems, one can propose the control signal

$$\tau = \bar{\mathbf{M}}(\mathbf{q})\nu + \bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \bar{\mathbf{G}}(\mathbf{q}) + \bar{\mathbf{D}}, \tag{16}$$

where $\nu = \ddot{\mathbf{q}}_d + \kappa_1 \tanh(\kappa_2 \dot{\tilde{\mathbf{q}}}) + \kappa_3 \tanh(\kappa_4 \tilde{\mathbf{q}})$, and $\kappa_i, i = 1, \cdots, 4$ are positive diagonal gain matrices.

Adopting such a control law, the resulting closed loop system equation is

$$\ddot{\tilde{\mathbf{q}}} + \kappa_1 \tanh(\kappa_2 \dot{\tilde{\mathbf{q}}}) + \kappa_3 \tanh(\kappa_4 \tilde{\mathbf{q}}) = 0,$$

which is asymptotically stable.

The behavior of the load angle θ_c is governed by

$$(I_c + m_c l^2)\ddot{\theta}_c + m_c l[c_{\theta_c}(-\ddot{x} + 2\dot{\theta}_c \dot{z}) + s_{\theta_c}(\ddot{z} + 2\dot{\theta}_c \dot{x} + g)] = 0,$$

which comes from (10).

IV. Simulation Results

The simulation results were obtained using the *AuRoRA Platform* (Autonomous Rotorcraft Research and Development) as a ground station, which is connected to a virtual UAV, represented by its model, in this case an AR.Drone quadrotor [18]. The platform, in this case, is configured as a software-in-loop one, but in the sequel of this research our idea is to use it as a real ground station, as described in [18]. In this platform one can add and select models and controllers in order to study the UAV behavior. It is also possible to see the current position of the vehicle, thus allowing analyzing the aircraft performance along the task accomplishment, as Figures 4 and 9 illustrate. In such figures the UAV is shown in a certain time instant, with the suspended load exhibiting a certain angular displacement with respect to its natural position (with the cable in the vertical).

This section presents two simulations, in which the model dealt with in Section II and the controller proposed in Section III were implemented in the platform and simulated. The first simulation is a positioning task, which complements the hovering task, i.e, the quadrotor is programmed to reach a given altitude without having horizontal displacements and, in the sequence, it should perform frontal rise and fall displacements, i.e, the aircraft should reach a desired location by performing upward (or downward) and forward (or backward) movements simultaneously. The second simulation is a trajectory tracking task, in which the vehicle should follow an 8 shape trajectory.

The parameters are the same used in [24] with the addition of the load mass $m_c = 100g$ and the cable length $l = 0.50\ m$.

A. Positioning task

The positioning task is accomplished using 6 different points in the XZ plane, as shown in Figure 4. The first positioning change, p_1 to p_2, is a hovering task, in which the vehicle should move itself from the coordinates $(0\ m, 0\ m)$ to $(0\ m, 1\ m)$, just ascending 1 m. Is worth mentioning that even when the vehicle is in the position $(0\ m, 0\ m)$ it is still subject to all the effects of the load along the task (the task is not just a take-off one). Such movement is similar to the one correspondent to go from the position $(1\ m, 0\ m)$ to the position $(2\ m, 0\ m)$, an in-flight mission.

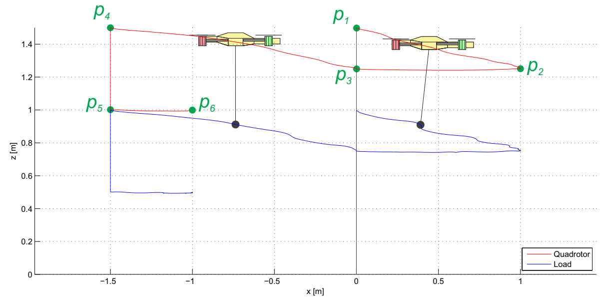

Fig. 4. Positioning task in the XZ plane.

Figures 5 and 6 show the variation of the x and z coordinates along time, during the task accomplishment. All target-point changes, except for the first one, were set with 10 s of time interval in between them, and the controller achieved all points before such time had elapsed, as one can see in the figures. In Figure 12 one can check the variations in the angles θ and θ_c, which show the effort of the controller to reject the disturbance caused by the load. It can also be seen that descent movements are harder to perform than rising movements, due to the characteristic of the quadrotor (when performing a descending movement it looses sustaining).

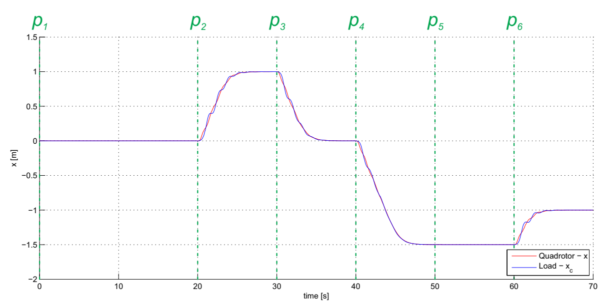

Fig. 5. Variation of the x coordinate along time.

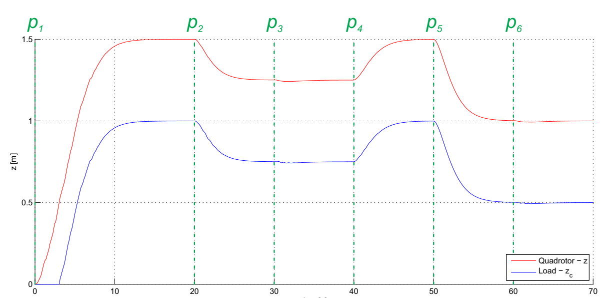

Fig. 6. Variation of the z coordinate along time.

Even the load having an oscillation, it is considered as a disturbance. Thus, the objective of the controller is to stabilize and guide the aircraft, rejecting the disturbance, not being intended to guide the load. Indeed, the angular oscillations are caused by the load dynamics and the effort of the controller to reject the disturbance. Notice that such oscillations are lower than 9° for the load and less then 7° for the vehicle. In order to start its movement, the aircraft needs to generate an angle, and, with a certain delay, it causes an oscillation in the load, which forces the controller to compensate for them. In all cases, the controller managed to reject the oscillations before the reference changing.

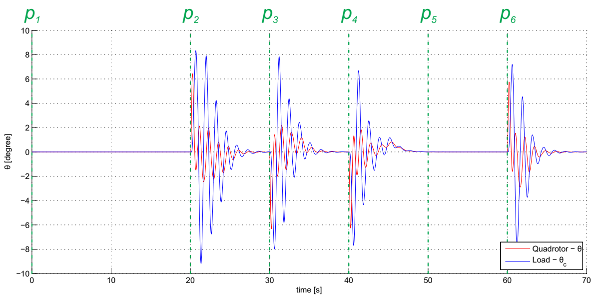

Fig. 7. Angular variations along time.

The forces f_x and f_z and the torque τ applied by the controller in the aircraft are shown in Figure 8. The force f_z is almost constant along the simulation, because it compensates the gravity and generates the z axis movement. Thus, one can see the force variations along the task accomplishment, when the target-point changes, as in the instant marked as p_1, f_z grows a little, or in the instant p_2, the force decreases for a while. These variations corresponds to the acceleration imposed to the UAV to go up or down. The same occurs with the force f_x when it should generate a longitudinal movement forward or backward.

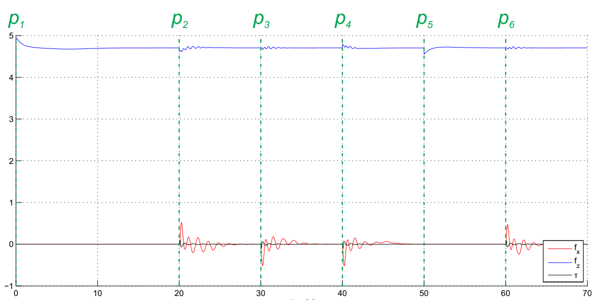

Fig. 8. The forces f_z and f_x and the torque τ imposed to the UAV by the controller along time.

It is worth mentioning that the target-points p_i placed in such figures are associated to the time instants in which they were set as references for the controller (p_2 becomes the target-point at the time instant 20 s, and will be reached within the next 10 s, or before the time instant 30 s, when the new setup p_3 is established.

B. Trajectory Tracking task

Figure 9 illustrates the accomplishment of an 8 shape-trajectory tracking mission. Before the beginning of the trajectory, the vehicle increases its altitude (a hovering task), and only after that it starts tracking the trajectory. It can be seen that the vehicle and the load accomplish the task without position oscillations. A detailed view of the behavior of the variables x and z along time can be seen in Figures 10 and 11, respectively. In all these figures the black dashed line shows the UAV reference. As for the load, it just follows the movement according to its dynamics, i.e., there is no reference for the load to follow.

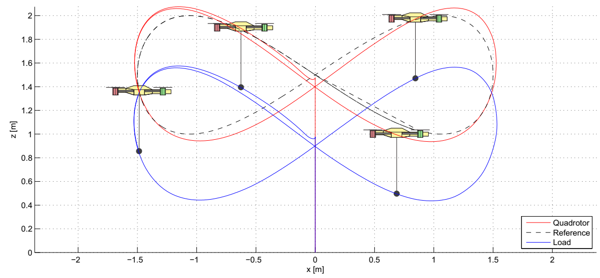

Fig. 9. Accomplishing an 8 shape 2D trajectory tracking.

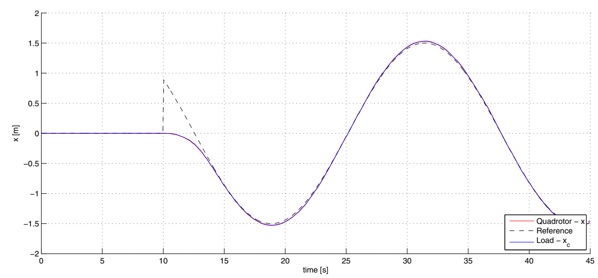

Fig. 10. Trajectory task x variations.

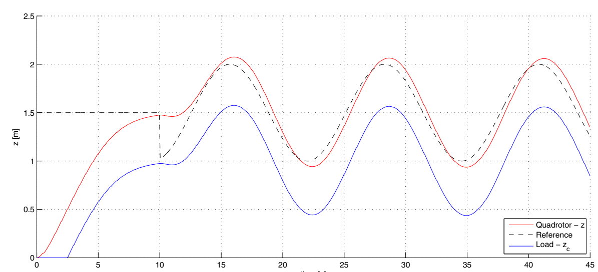

Fig. 11. Trajectory task z variations.

As one can see in Figure 12, small angular oscillations occur, mainly upon a direction change: due to the load inertia, it tends to sway either when the UAV starts a movement or when it changes its direction of movement, i.e., when accelerations are imposed to the UAV.

As Figure 8, Figure 13 shows the forces acting in the vehicle along time. Differently of the positioning task, the trajectory tracking does not have abrupt reference changes, thus causing a smoother movement of the UAV.

V. Concluding Remarks

The possibility of load transportation using UAVs is useful in many applications. In this work it is consi-

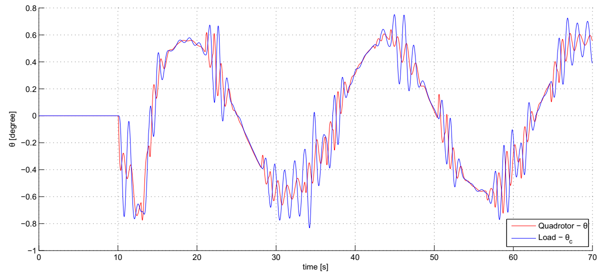

Fig. 12. Angular variations during the trajectory tracking task.

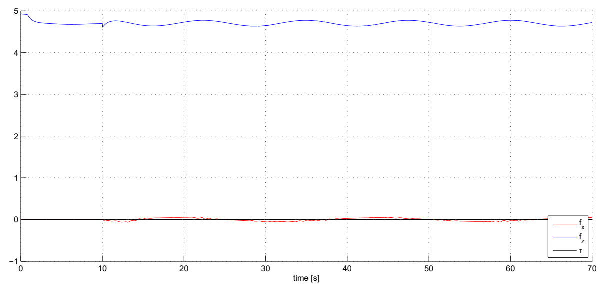

Fig. 13. The forces f_z and f_x and the torque τ imposed to the UAV by the controller along time.

dered the problem of a UAV with a suspended load accomplishing a task in the XZ plane, i.e., performing as a PVTOL aircraft. It is presented the modeling of the aircraft with the suspended mass, using Euler-Lagrange equations, upon considering their movements limited to a plane. Adopting the proposed model, a high-level nonlinear controller using feedback linearization is proposed to guide the UAV in positioning and trajectory tracking tasks. Simulation results were also presented, which validate both the model and the controller.

It is worth mentioning that the load is regarded as a modeled disturbance, i.e., the controller aims at guiding the vehicle through the desired path rejecting the noise generated by the load, not guiding it.

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