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A Multi-Layer Control Scheme for Multi-Robot Formations with Adaptive Dynamic Compensation

Alexandre Santos Brandão*, Felipe Nascimento Martins*, Vinicius Thiago Lecco Rampinelli*,
Mário Sarcinelli-Filho*, Teodiano Freire Bastos-Filho*, and Ricardo Carelli†
*Graduate Program on Electrical Engineering
Federal University of Espírito Santo, Vitória, ES, BRAZIL 29075–910
Email: {abrandao, felipem, vinicius.rampinelli, mario.sarcinelli, tfbastos}@ele.ufes.br
†Institute of Automatics, National University of San Juan, San Juan, ARGENTINA 5400
Email: rcarelli@inaut.unsj.edu.ar

***Abstract*—This paper presents a multi-layer scheme to control a formation of three mobile robots. Each layer works as an independent module, dealing with a specific part of the problem of formation control, thus giving to the system more flexibility. In order to reduce formation errors, the proposed architecture includes a layer which performs an adaptive dynamic compensation, using a robust updating law, which compensates for each robot dynamics. The controller is able to guide the robots to the desired formation, including the possibility of time-varying position and/or shape. Stability analysis is performed for the closed-loop system, and the result is that the formation errors are ultimately bounded. Finally, simulation results for a group of three unicycle-like mobile robots are presented, which show that system performance is improved when the adaptive dynamic compensation layer is included in the formation control scheme.**

I. Introduction

The interest in multi-robot control systems has increased in recent years, after the perception that a group of mobile robots can execute certain tasks much more efficiently than a single specialized robot [1]. Surveillance in a large area [2], search and rescue [3], and large-objects transportation [4] are some examples of tasks that are better tailored for a group of robots. Other tasks are simply not accomplishable by a single mobile robot, demanding a group of coordinated robots to perform it, like the problem of sensors and actuators positioning [5], and the entrapment/escorting mission, in which some mobile robots should position themselves around a specified agent in order to entrap or to escort it [6].

While executing certain tasks it is important that the controller adopted takes the robot dynamics into account. As an example, the trajectory tracking task can be severely affected by the change imposed to the robot dynamics when it is carrying an object, as shown in [7]. In [8] it is mentioned that an acceptable error associated to the accomplishment of a task by a single robot might be unacceptable when the same robot is part of a formation. Hence, some formation control architectures yet proposed in the literature have considered the dynamics of the mobile robots. Examples of proposals considering the dynamics of the robots are [9] and [10]. In [9] a behavior-based decentralized formation maneuver control is proposed for a group of robots. The control strategy is based on two conflicting objectives, namely to keep the formation and to move the robots to the destination point, considering the dynamics of the individual robots. In [10] a centralized formation controller is presented, in which the so called "second order kinematic model" of the formation considers the robots dynamics to generate the control signals. In both cases, the control errors are shown to be ultimately bounded, but the results are restricted to the case in which the dynamics of the robots are exactly known and do not change from task to task.

If the parameters characterizing the dynamics of the robots are assumed unknown or time-varying, the use of an adaptive controller should be considered. In [11] an adaptive dynamic compensator was applied to the follower robot of a decentralized leader-follower formation architecture. There, it has been shown that the adaptive dynamic compensation was responsible for a meaningful reduction in the formation errors during task accomplishment. The formation control architecture presented in [8] also adopts an adaptive dynamic compensator. The architecture is divided in four modules. The upper module is responsible for planning the trajectory to be followed by the team of robots. The next module controls the formation, whose shape is determined by the distance between a robot and two other robots, or by the distance and angle between two robots. Another module performs the kinematic control of the robots, while the lower module is responsible for the adaptive dynamic controller that generates torque reference signals to the robots.

The present paper discusses the application of a multi-layer scheme to control the formation of a team of three unicycle-like mobile robots. The proposed structure is organized in five layers, in a hierarchical way. The first two are planning layers, namely the Off-line Planning Layer and the On-line Planning Layer. The next one is the Control Layer, which is responsible for generating the control signals to be sent to the robots of the formation. The Dynamic Compensation Layer compensates for the

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dynamics of each robot to reduce the velocity tracking error. Finally, the Robot Layer represents the unicycle-like mobile vehicles, which have internal servo controllers that receive reference signals for the robots linear and angular velocities. For the Control Layer we propose a formation controller that has individual gains for each one of the formation variables, which allows the establishment of higher priority to the formation pose or shape, to improve system performance during the execution of a specific task. For the Dynamic Compensation Layer we use a controller designed using a new dynamic model that has linear and angular velocities as input signals, instead of torques. This characteristic makes it appropriate to be applied in commercial mobile robots, that usually accept velocity commands. Also, it uses a robust updating law, which makes the dynamic compensation system robust to parameter variations and guarantees that no parameter drift will occur. Stability analysis of the closed-loop control system is performed, showing that the formation errors are ultimately bounded, and this bound depends on the velocity tracking errors, which are reduced by the dynamic compensation. Finally, simulation results for a group of three unicycle-like mobile robots show that the system works properly and that its performance is improved when the adaptive dynamic compensation layer is included in the formation control scheme.

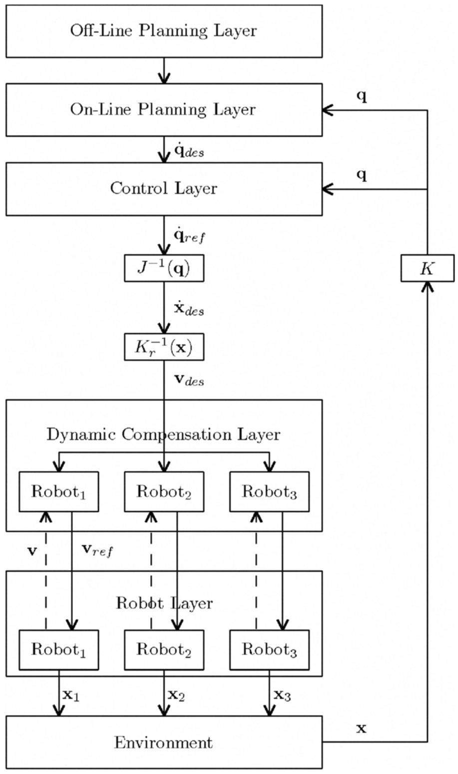

Fig. 1. The proposed Multi-layer Scheme.

II. The Multi-Layer Scheme Adopted

This section describes the multi-layer scheme adopted for controlling a formation of multiple mobile robots, which is shown in Fig. 1. Each layer works as an independent module, dealing with a specific part of the problem of formation control, as it can be observed in the figure.

Such control scheme includes a basic structure defined by the Robot Layer, the Control Layer, and the Environment, which are embedded in an hierarchical layered structure. Its main characteristics are: (a) the presence of two planning layers, namely the Off-line Planning Layer and the On-line Planning Layer. The first one is responsible for setting up the initial conditions, thus generating the trajectory to be tracked, and for establishing the desired formation structure (if a formation is considered). The other one has the objective of changing the references in order to make the formation to react to the environment according to the sensorial data currently available, to adjust the trajectory to avoid obstacles, for example; (b) the Control Layer, which is responsible for generating the control signals to be sent to the robot (or the formation) in order to reach the desired values established by the Off-line or On-line Planning Layer; (c) the Dynamic Compensation Layer, which compensates for the dynamics of each robot to reduce the velocity tracking error; (d) the Robot Layer, which represents the unicycle-like mobile vehicle or the formation; (e) the Environment, which represents all the objects surrounding the robot or the formation, including the relationships between the robots, like the distance between robots to avoid intra-formation collision, for example. It is not considered as a layer in this scheme, because its states cannot be totally controlled or foreseen during navigation.

One advantage of the proposed scheme is the independence of each layer, i.e., changes within a layer do not cause structural changes in the other layers. As an example, several dynamic compensation approaches can be tested using the same formation control strategy and vice-versa. It is worth mentioning that a simpler structure can be obtained from the presented scheme, that is, some layers can be eliminated whenever the basic structure is maintained and the absence of the eliminated layers do not affect the remaining layers. For example, the On-line Planning Layer could be discarded in the case of trajectory tracking by a multi-robot formation in a known environment free of obstacles, because the entire task accomplishment is controlled by the Control Layer. Moreover, autonomous navigation with obstacle avoidance for one or more robots, considering its non-compensated dynamic model, could be structured using the On-line Planning Layer and the basic structure.

It is important to stress that some additional blocks are necessary to complete the multi-layer scheme, such as $J^{-1}(\cdot)$, $\mathbf{K_r^{-1}}$ and K, which represents the inverse Jacobian matrix, the inverse kinematic model of the robot (or robots), and the forward kinematic transformation for the formation (if a formation is considered), respectively.

In this paper we deal with a multi-layer scheme without the On-line Planning Layer, to guide the formation structure aforementioned, i.e., we consider that the Off-line Planning Layer generates a reference trajectory that is directly sent to the Control Layer.

A. Robot Kinematic Model

The i-th robot kinematic model is

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\psi}_i \end{bmatrix} = \begin{bmatrix} \cos\psi_i & -a_i \sin\psi_i \\ \sin\psi_i & a_i \cos\psi_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_i \\ \omega_i \end{bmatrix}, \qquad (1)$$

where u_i and ω_i are, respectively, the linear and angular velocities, $\mathbf{h_i} = [x_i \;\; y_i]^T$ is the vector containing the coordinates of the point of interest, ψ_i is the orientation, and a_i is the distance from the point of interest to the point in the middle of the virtual axis linking the traction wheels of the i-th robot.

Considering only the coordinates of the point of interest $\mathbf{h_i}$, the inverse kinematics is

$$\mathbf{v_i} = \mathbf{K_{ri}^{-1}} \dot{\mathbf{h}}_\mathbf{i}, \qquad (2)$$

where $\mathbf{v_i} = [u_i \; \omega_i]^T$, and

$$\mathbf{K_{ri}^{-1}} = \begin{bmatrix} \cos\psi_i & \sin\psi_i \\ -\frac{1}{a_i} sin\psi_i & \frac{1}{a_i}\cos\psi_i \end{bmatrix}, \quad \text{with} \quad a_i > 0.$$

Hereinafter a formation of three mobile robots is considered. The forward kinematics of such structure is

$$\mathbf{K_r} = \begin{bmatrix} \mathbf{K_{r1}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_{r2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K_{r3}} \end{bmatrix},$$

where the numeric subscript stands for the i-th robot. It should be noticed that robots with different kinematic models can be used (heterogeneous formation), just by updating $\mathbf{K_{ri}}$ in the $\mathbf{K_r}$ matrix.

III. The Formation-Control Layer

This section implements the Control Layer for a centralized formation control considering three unicycle-like mobile robots. The state variables used to represent the whole formation are shown in Fig. 2, as proposed in [12]. The formation pose is defined by $\mathbf{P}_F = [x_F \; y_F \; \psi_F]$ and the structure shape is defined by $\mathbf{S}_F = [p_F \; q_F \; \beta_F]$, which represent the distance between R_1 and R_2, the distance between R_1 and R_3, and the angle $R_2\widehat{R_1}R_3$, respectively. It is worthy mentioning that (x_F, y_F) represents the position of the centroid of the formation, as well as that the whole formation is controlled by taking the global frame xy as the reference.

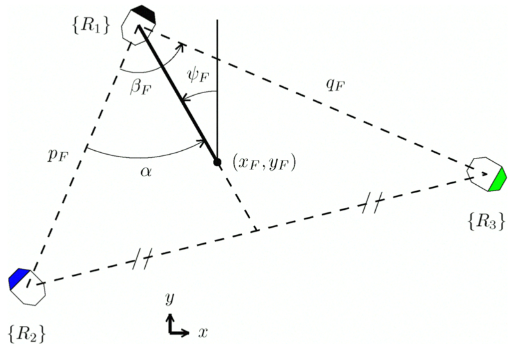

Fig. 2. Formation variables.

A. Forward and Inverse Kinematics Transformation

Before introducing the formation control law, it is necessary to express the relationship between the formation pose-shape and the robot positions, which is given by the forward kinematics transformation (K), as follows

$$\mathbf{P}_F = \begin{bmatrix} \dfrac{x_1 + x_2 + x_3}{3} \\ \dfrac{y_1 + y_2 + y_3}{3} \\ \arctan \dfrac{\frac{2}{3}x_1 - \frac{1}{3}(x_2 + x_3)}{\frac{2}{3}y_1 - \frac{1}{3}(y_2 + y_3)} \end{bmatrix}^T, \qquad (3)$$

$$\mathbf{S}_F = \begin{bmatrix} \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} \\ \arccos \dfrac{p_F^2 + q_F^2 - r_F^2}{2 p_F q_F} \end{bmatrix}^T, \qquad (4)$$

where $r_F = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$.

Hereinafter, the whole formation will be represented by $\mathbf{q} = [\mathbf{P}_F \; \mathbf{S}_F]^T$ and the robot position by $\mathbf{x} = [\mathbf{h_1} \; \mathbf{h_2} \; \mathbf{h_3}]^T$. It is important to stress that the orientation of the robots is not considered in this proposal. Therefore, the inverse kinematics transformation (K^{-1}) is given by

$$\mathbf{x} = \begin{bmatrix} \mathbf{h_1} \\ \mathbf{h_2} \\ \mathbf{h_3} \end{bmatrix} = \begin{bmatrix} x_F + \frac{2}{3}h_F \sin\psi_F \\ y_F + \frac{2}{3}h_F \cos\psi_F \\ x_F + \frac{2}{3}h_F \sin\psi_F - p_F \sin(\alpha + \psi_F) \\ y_F + \frac{2}{3}h_F \cos\psi_F - p_F \cos(\alpha + \psi_F) \\ x_F + \frac{2}{3}h_F \sin\psi_F + q_F \sin(\beta_F - \alpha - \psi_F) \\ y_F + \frac{2}{3}h_F \cos\psi_F - q_F \cos(\beta_F - \alpha - \psi_F) \end{bmatrix}, \qquad (5)$$

where $h_F = \sqrt{\frac{1}{2}\left(p_F^2 + q_F^2 - \frac{1}{2}r_F^2\right)}$ is the distance between $\{R_1\}$ and the central point of the segment $\overline{\{R_2\}\{R_3\}}$, passing through (x_F, y_F), and $\alpha = \arccos \dfrac{p_F^2 + h_F^2 - \frac{1}{4}r_F^2}{2 p_F h_F}$.

Taking the time derivative of the forward and the inverse kinematics transformations we can obtain the relationship between the $\mathbf{x}$ and $\mathbf{q}$ velocities, represented by the Jacobian matrix, which is given by $\dot{\mathbf{q}} = J(\mathbf{x})\dot{\mathbf{x}}$ in the forward way, and by $\dot{\mathbf{x}} = J^{-1}(\mathbf{q})\dot{\mathbf{q}}$ in the inverse way, where

$$J(\mathbf{x}) = \frac{\partial \mathbf{q}_{n\times 1}}{\partial \mathbf{x}_{m\times 1}} \quad \text{and} \quad J^{-1}(\mathbf{q}) = \frac{\partial \mathbf{x}_{m\times 1}}{\partial \mathbf{q}_{n\times 1}},$$

for $m, n = 1,\ 2,\ \cdots,\ 6$.

B. The Formation Controller

The Control Layer receives from the upper layer the desired formation pose and shape $\mathbf{q_{des}} = [\mathbf{P}_{Fd}\ \mathbf{S}_{Fd}]^T$, and its desired variations $\dot{\mathbf{q}}_{\mathbf{des}} = [\dot{\mathbf{P}}_{Fd}\ \dot{\mathbf{S}}_{Fd}]^T$. It generates the pose and shape variation references $\dot{\mathbf{q}}_{\mathbf{ref}} = [\dot{\mathbf{P}}_{Fr}\ \dot{\mathbf{S}}_{Fr}]^T$, where the subscripts d and r represent the desired and reference signals, respectively. Defining the formation error as $\tilde{\mathbf{q}} = \mathbf{q_{des}} - \mathbf{q}$, the proposed formation control law is

$$\dot{\mathbf{q}}_{\mathbf{ref}} = \dot{\mathbf{q}}_{\mathbf{des}} + \boldsymbol{\kappa}\tilde{\mathbf{q}}, \tag{6}$$

where $\boldsymbol{\kappa}$ is a positive definite diagonal gain matrix. Let us consider a difference $\boldsymbol{\delta}_{\mathbf{v}}$ between the desired and the real formation variations, such as $\dot{\mathbf{q}} = \dot{\mathbf{q}}_{\mathbf{ref}} + \boldsymbol{\delta}_{\mathbf{v}}$. Then, the closed loop system equation can be written as

$$\dot{\tilde{\mathbf{q}}} + \boldsymbol{\kappa}\tilde{\mathbf{q}} = -\boldsymbol{\delta}_{\mathbf{v}}. \tag{7}$$

Considering the Lyapunov candidate function $V = \frac{1}{2}\tilde{\mathbf{q}}^{\mathbf{T}}\tilde{\mathbf{q}} > 0$, its first time derivative is $\dot{V} = \tilde{\mathbf{q}}^{\mathbf{T}}\dot{\tilde{\mathbf{q}}} = -\tilde{\mathbf{q}}^{\mathbf{T}}\boldsymbol{\kappa}\tilde{\mathbf{q}} - \tilde{\mathbf{q}}^{\mathbf{T}}\boldsymbol{\delta}_{\mathbf{v}}$. Assuming perfect velocity tracking, $\boldsymbol{\delta}_{\mathbf{v}} = \mathbf{0}$, and one can conclude that $\dot{V} < 0$, which means that the equilibrium is globally asymptotically stable, i.e., $\tilde{\mathbf{q}} \to \mathbf{0}$ when $t \to \infty$. On the other hand, if $\boldsymbol{\delta}_{\mathbf{v}}$ is not null, the equilibrium will be asymptotically stable if $\tilde{\mathbf{q}}^{\mathbf{T}}\boldsymbol{\kappa}\tilde{\mathbf{q}} > |\tilde{\mathbf{q}}^{\mathbf{T}}\boldsymbol{\delta}_{\mathbf{v}}|$. A sufficient condition for that is $\|\tilde{\mathbf{q}}\| > \|\boldsymbol{\delta}_{\mathbf{v}}\|/\lambda_{\min}(\boldsymbol{\kappa})$, where $\lambda_{\min}(\boldsymbol{\kappa})$ represents the minimum eigenvalues of $\boldsymbol{\kappa}$. That means that the formation error $\tilde{\mathbf{q}}$ is ultimately bounded, and its bound depends on the formation velocity tracking error $\boldsymbol{\delta}_{\mathbf{v}}$ (in Section IV it is shown that $\boldsymbol{\delta}_{\mathbf{v}}$ can be reduced by the Dynamic Compensation Layer).

IV. The Dynamic Compensation Layer

The objective of the Dynamic Compensation Layer is to compensate for the dynamics of each robot, thus reducing the velocity tracking error. This layer receives, after proper conversion, the desired velocities $\mathbf{v_{des}} = [\mathbf{v_{d1}}\ \mathbf{v_{d2}}\ \mathbf{v_{d3}}]^T$ for all robots, and generates velocity references $\mathbf{v_{ref}} = [\mathbf{v_{r1}}\ \mathbf{v_{r2}}\ \mathbf{v_{r3}}]^T$ to be sent to the robots. Here, $\mathbf{v_{di}} = [u_{di}\ \omega_{di}]^T$ are the desired linear and angular velocities, and $\mathbf{v_{ri}} = [u_{refi}\ \omega_{refi}]^T$ are the reference velocities, both for the i-th robot.

The design of the dynamic compensation controller is based on a new mobile robot dynamic model we have proposed [13], [14], which has linear and angular velocities as inputs, instead of torques. Thus, before presenting the proposed dynamic controller, a brief description of the dynamic model considered is now introduced.

A. Robot Dynamic Model

The robot dynamic model here used is [14]

$$\mathbf{H}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{F}(\mathbf{v})\mathbf{v} + \boldsymbol{\Delta} = \mathbf{v_r}, \tag{8}$$

where $\mathbf{v_r} = [u_{ref}\ \omega_{ref}]^T$ is the vector of the references for linear and angular velocities, $\boldsymbol{\Delta}$ is an uncertainty vector, and the matrices $\mathbf{H}$, $\mathbf{C}(\mathbf{v})$ and $\mathbf{F}(\mathbf{v})$ are defined as

$$\mathbf{H} = \begin{bmatrix}\theta_1 & 0\\ 0 & \theta_2\end{bmatrix}, \quad \mathbf{C}(\mathbf{v}) = \begin{bmatrix}0 & -\theta_3\omega\\ \theta_3\omega & 0\end{bmatrix},$$

$$\mathbf{F}(\mathbf{v}) = \begin{bmatrix}\theta_4 & 0\\ 0 & \theta_6 + (\theta_5 - I\theta_3)u\end{bmatrix},$$

where $I = 1rad^2/m^2$, $\boldsymbol{\theta} = [\theta_1\ \ \theta_2\ \ \theta_3\ \ \theta_4\ \ \theta_5\ \ \theta_6]^T$ is the vector of model parameters, which are functions of some physical parameters of the robot, such as its mass, its moment of inertia, the electrical characteristics of its motors, etc. The model here presented has some important properties, namely:

1. $\mathbf{H}$ is a symmetric and positive definite matrix ($\mathbf{H} = \mathbf{H}^T > 0$);
2. The inverse of $\mathbf{H}$ exists and is also positive definite ($\exists\ \mathbf{H}^{-1} > 0$);
3. $\mathbf{F}$ is a symmetric and positive definite matrix ($\mathbf{F} = \mathbf{F}^T > 0$) if $\theta_6 > -(\theta_5 - I\theta_3)u$;
4. $\mathbf{H}$ is a constant matrix if the robot parameters do not change;
5. $\mathbf{C}(\mathbf{v})$ is a skew symmetric matrix;
6. $\mathbf{F}(\mathbf{v})$ can be considered a constant matrix if $\theta_6 \gg |(\theta_5 - I\theta_3)u|$ and the robot parameters do not change;
7. The mapping $\mathbf{v_r} \to \mathbf{v}$ is strictly output passive if $\theta_6 > -(\theta_5 - I\theta_3)u$ and $\boldsymbol{\Delta} = \mathbf{0}$.

Parameter identification performed for four different unicycle-like mobile robots (three Pioneer robots, from Mobile Robots, and a robotic wheelchair), has shown that the conditions $\theta_6 > -(\theta_5 - I\theta_3)u$ and $\theta_6 \gg |(\theta_5 - I\theta_3)u|$ are verified. For more details about the dynamic model and its parameters, see [13] and [14].

B. The Adaptive Dynamic Compensation

The adaptive dynamic compensation is performed separately for each robot. Each one of the three controllers receive the references for linear and angular velocities $\mathbf{v_{di}}$ from the Control Layer, and generates another pair of linear and angular velocities commands $\mathbf{v_{ri}}$ for the servos of the i-th robot. The dynamic compensation control law for the i-th robot is

$$\mathbf{v_{ri}} = \hat{\mathbf{H}}_{\mathbf{i}}(\dot{\mathbf{v}}_{\mathbf{di}} + \mathbf{T}(\tilde{\mathbf{v}}_{\mathbf{i}})) + \hat{\mathbf{C}}_{\mathbf{i}}\mathbf{v_{di}} + \hat{\mathbf{F}}_{\mathbf{i}}\mathbf{v_{di}}, \tag{9}$$

where $\hat{\mathbf{H}}_{\mathbf{i}}$, $\hat{\mathbf{C}}_{\mathbf{i}}$, and $\hat{\mathbf{F}}_{\mathbf{i}}$ are the estimates of $\mathbf{H_i}$, $\mathbf{C_i}$, and $\mathbf{F_i}$, respectively, $\tilde{\mathbf{v}}_{\mathbf{i}} = \mathbf{v_{di}} - \mathbf{v_i}$ is the vector of velocity errors, $\mathbf{T}(\tilde{\mathbf{v_i}}) = \begin{bmatrix}l_{ui} & 0\\ 0 & l_{\omega i}\end{bmatrix}\begin{bmatrix}\tanh(\frac{k_{ui}}{l_{ui}}\tilde{u_i})\\ \tanh(\frac{k_{\omega i}}{l_{\omega i}}\tilde{\omega_i})\end{bmatrix}$, $k_{ui} > 0$ and $k_{\omega i} > 0$ are gain constants, $l_{ui} \in \mathbb{R}$ and $l_{\omega i} \in \mathbb{R}$ are saturation constants, and $\tilde{\omega_i} = \omega_{di} - \omega_i$, $\tilde{u_i} = u_{di} - u_i$ are the current velocity errors. The term $\mathbf{T}(\tilde{\mathbf{v}}_\mathbf{i})$ provides a saturation in order to guarantee that the commands to be sent to the robot are always below its physical limits.

The robust updating law

$$\dot{\hat{\boldsymbol{\theta}}}_\mathbf{i} = \gamma_\mathbf{i}\mathbf{G}_\mathbf{i}^\mathbf{T}\tilde{\mathbf{v}}_\mathbf{i} - \gamma_\mathbf{i}\boldsymbol{\Gamma}_\mathbf{i}\hat{\boldsymbol{\theta}}_\mathbf{i} \qquad (10)$$

is adopted to update the parameter estimates, where

$$\mathbf{G}_\mathbf{i} = \begin{bmatrix} \sigma_{1i} & 0 & -\omega_{di}\omega_i & u_{di} & 0 & 0 \\ 0 & \sigma_{2i} & u_{di}\omega_i - u_i\omega_{di} & 0 & u_i\omega_{di} & \omega_{di} \end{bmatrix},$$

$\sigma_{1i} = \dot{u}_{di} + l_{ui}\tanh(\frac{k_{ui}}{l_{ui}}\tilde{u}_i)$, $\sigma_{2i} = \dot{\omega}_{di} + l_{\omega i}\tanh(\frac{k_{\omega i}}{l_{\omega i}}\tilde{\omega}_i)$, and $\boldsymbol{\gamma_i}$ and $\boldsymbol{\Gamma_i}$ are diagonal positive definite gain matrices. Considering $V = \frac{1}{2}\tilde{\mathbf{v}}_\mathbf{i}^\mathbf{T}\mathbf{H}_\mathbf{i}\tilde{\mathbf{v}}_\mathbf{i} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_\mathbf{i}^\mathbf{T}\gamma_\mathbf{i}^{-1}\tilde{\boldsymbol{\theta}}_\mathbf{i} > 0$ as a Lyapunov candidate function, in [14] it has been shown that $\tilde{\mathbf{v}}_\mathbf{i}$ and $\tilde{\boldsymbol{\theta}}_{\boldsymbol{i}}$ are ultimately bounded, even if the dynamic parameters vary smoothly ($\dot{\boldsymbol{\theta}}_{\boldsymbol{i}}$ is bounded).

The velocity tracking error $\tilde{\mathbf{v}}_\mathbf{i}$ for the robots of the formation are related to the formation velocity tracking error $\boldsymbol{\delta}_\mathbf{v}$ through

$$\boldsymbol{\delta}_\mathbf{v} = [\boldsymbol{\delta}_\mathbf{v1}\ \boldsymbol{\delta}_\mathbf{v2}\ \boldsymbol{\delta}_\mathbf{v3}]^\mathbf{T} = J(\mathbf{x})\mathbf{K}_\mathbf{r}(\mathbf{x})[\tilde{\mathbf{v}}_\mathbf{1}\ \tilde{\mathbf{v}}_\mathbf{2}\ \tilde{\mathbf{v}}_\mathbf{3}]^\mathbf{T}.$$

As the dynamic compensation controller is able to reduce $\tilde{\mathbf{v}}_\mathbf{i}$, $\boldsymbol{\delta}_\mathbf{v}$ is also reduced. Therefore, according to this result and to the conclusion presented in Section III, the Dynamic Compensation Layer is able to reduce the bound of the formation control error $\tilde{\mathbf{q}}$.

V. Simulation Results and Discussion

Three simulations were executed using the MRSiM platform [15] in order to evaluate the performance of the proposed scheme, always using the model of the robot Pioneer of Mobile Robots, Inc. In every simulation the initial pose and dynamic characteristics of each robot, the desired trajectory and the desired formation shape are the same, and the robots should maintain the desired formation while its centroid tracks the desired trajectory. The difference between each simulation is that in the first one the dynamic compensation is deactivated. In the second and in the third simulations the dynamic compensation is activated, but with wrong initial parameter estimates (an error of 40%). Parameter updating is performed only in the third simulation.

The desired trajectory is described by

$$\begin{aligned} x_{Fd} &= 0.2t \\ y_{Fd} &= 4\cos(\pi t/30), \quad \text{where } \theta = \arctan\left(\frac{\frac{dy_{Fd}}{dt}}{\frac{dx_{Fd}}{dt}}\right). \\ \psi_{Fd} &= \pi/2 - \theta \end{aligned}$$

It is important to mention that this trajectory was chosen in order to excitate the dynamics of the robots by changing their acceleration. The values of the gain matrix $\boldsymbol{\kappa}$ were adjusted considering the system performance with the dynamic compensation deactivated. In this case, higher values increase oscillations, and lower values cause a delay in the formation tracking. After this gain setting, the values obtained were used in all simulations. The resulting robot velocities are $0.1 \le u \le 0.7\ m/s$ and $-1.0 \le \omega \le 1.0\ rad/s$.

To compare the results, IAE and ITAE performance indexes were calculated considering the pose $\mathbf{P}_F$ and shape $\mathbf{S}_F$ of the formation, and the results are shown in Table I. It can be seen that the best performance was obtained in the third simulation, in which the dynamic compensation with parameter updating is activated.

TABLE I
Performance indexes

	IAE		ITAE	
	$\mathbf{P}_F$	$\mathbf{S}_F$	$\mathbf{P}_F$	$\mathbf{S}_F$
No Dyn. Comp.	37.2	28.6	1077.7	701.1
Dyn. Comp.	18.0	13.5	540.6	310.7
Adapt. Dyn. Comp.	16.8	13.3	476.9	302.0

Fig. 3 (a) and (c) show the tracked trajectory for the first and third simulations, respectively. The shaded and dashed triangles indicate the instantaneous pose and shape for the desired and current formations, respectively. The initial and final desired shapes are $\mathbf{S}_F = [1m,\ 1m,\ \pi/3rad]$. In the interval $14s < t < 46s$ it changes to $\mathbf{S}_F = [0.75m,\ 1.5m,\ \pi/6rad]$ to simulate, for example, an obstacle avoidance situation. Formation pose and shape errors for these cases are shown in Fig. 3 (b) and (d). It can be seen that the overall behavior of the third simulation is much better than the first one, which illustrates the importance of the adaptive dynamic compensation proposed in this paper.

VI. Concluding Remarks

It was presented a multi-layer scheme to control a formation of three mobile robots. The formation controller proposed to act as the Control Layer is able to guide the robots while keeping a time-varying desired formation, including the possibility of priorizing formation pose and/or shape. Stability analysis for the closed-loop system was also presented, and the result is that the formation errors are ultimately bounded. It was shown, via simulation results, that the adaptive dynamic compensation layer is able to reduce the formation errors, thus improving system performance. In future works we will present some experimental results, and address the scalability of the proposed scheme to show that it can control a group of n robots, with different formation shapes.

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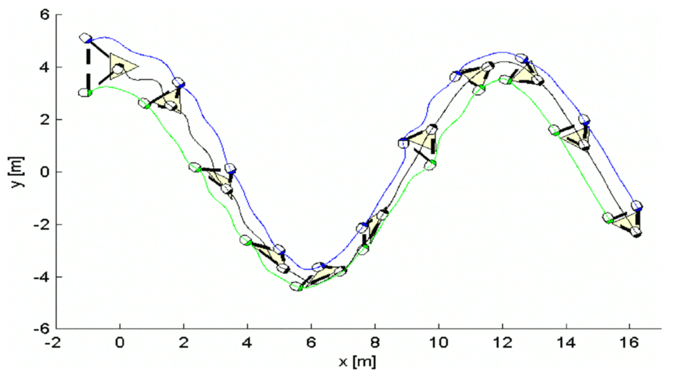
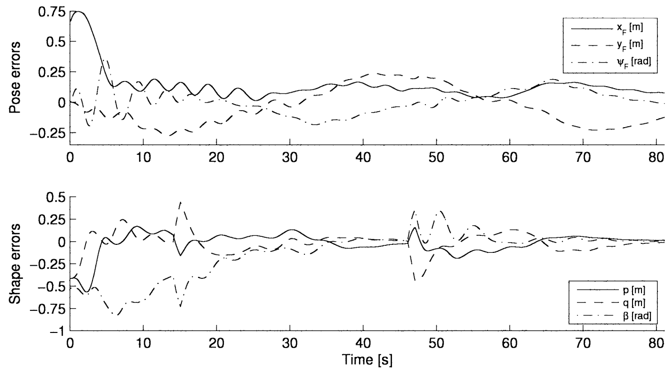
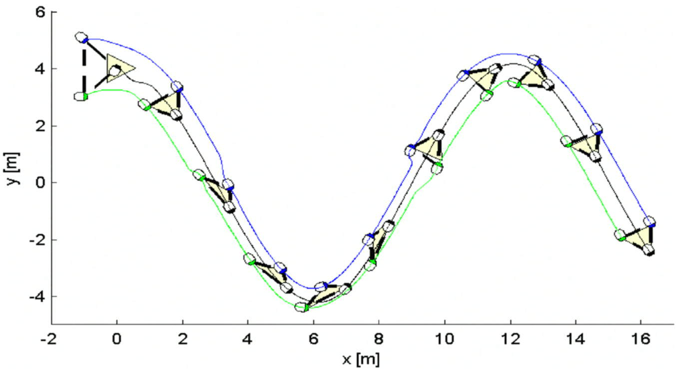


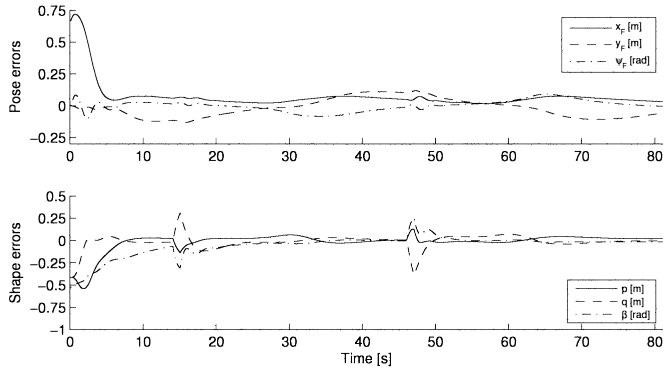
(a) Trajectory followed without dynamic compensation.



(b) Formation errors without dynamic compensation.



(c) Trajectory followed with adaptive dynamic compensation.



(d) Formation errors with adaptive dynamic compensation.

Fig. 3. Simulation results.

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