EXERCÍCIO 8 DA LISTA V

COM RUNGE-KUTTA 2 E 4

MAT 271 – Cálculo Numérico – PER3/2021/UFV

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EXERCÍCIO DA LISTA V COM PVI DE SEGUNDA ORDEM

Exercício 8: Seja o PVI de segunda ordem:

$$\begin{cases} y'' = -3y' - 2y + e^x \\ y(0) = 1 \\ y'(0) = 2 \end{cases}$$

Use os Métodos de Runge-Kutta de ordens 2 e 4, com $h = 0.2$, para encontrar uma aproximação de $y(0.4)$.

Fazendo $z = y'$, obtemos:

$$\begin{cases} y' = \mathrm{z} \\ y(0) = 1 \end{cases} \qquad \begin{cases} z' = -3z - 2y + e^x \\ z(0) = 2 \end{cases}$$

$$\begin{cases} y' = z = f(x, y, z) \\ y(0) = 1 \end{cases}$$

$$\begin{cases} z' = -3z - 2y + e^{x} = g(x, y, z) \\ z(0) = 2 \end{cases}$$

$h = 0.2;\ \ x_0 = 0, x_1 = 0.2, x_2 = 0.4.$

$y_0 = y(0) = 1, z_0 = \mathrm{z}(0) = 2;\ y(0.4) \cong y_2 =?$

$$\begin{cases} y' = z = f(x,y,z) \\ y(0) = 1 \end{cases} \qquad \begin{cases} z' = -3z - 2y + e^x = g(x,y,z) \\ z(0) = 2 \end{cases}$$

Método de Runge-Kutta de ordem 2:

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2), n = 0, \dots, N-1$$

$$k_1 = f(x_n, y_n, z_n)$$

$$k_2 = f(x_n + h, y_n + hk_1, z_n + hk_1^*)$$

$$z_{n+1} = z_n + \frac{h}{6}(k_1^* + k_2^*), n = 0, \dots, N-1$$

$$k_1^* = g(x_n, y_n, z_n)$$

$$k_2^* = g(x_n + h, y_n + hk_1, z_n + hk_1^*)$$

$$f(x,y,z) = z;\ g(x,y,z) = -3z - 2y + e^{x}$$

$$h = 0.2;\quad x_0 = 0, x_1 = 0.2, x_2 = 0.4. \qquad y_0 = y(0) = 1, z_0 = \mathrm{z}(0) = 2;\ y(0.4) \cong y_2 = ?$$

$$\boxed{\begin{aligned} y_{n+1} &= y_n + \frac{h}{2}(k_1 + k_2), n = 0,1 \\ k_1 &= f(x_n, y_n, z_n) \\ k_2 &= f(x_n + h, y_n + hk_1, z_n + hk_1^*) \end{aligned}}$$

$$\boxed{\begin{aligned} z_{n+1} &= z_n + \frac{h}{6}(k_1^* + k_2^*), n = 0,1 \\ k_1^* &= g(x_n, y_n, z_n) \\ k_2^* &= g(x_n + h, y_n + hk_1, z_n + hk_1^*) \end{aligned}}$$

$\boxed{n = 0}$

$$k_1 = f(x_0, y_0, z_0) = z_0 \quad = 2$$

$$k_1^* = g(x_0, y_0, z_0) = -3z_0 - 2y_0 + e^{x_0} = -7$$

$$k_2^* = g(x_0 + 0.2, y_0 + 0.2k_1, z_0 + 0.2k_1^*)$$

$$k_2 = f(x_0 + h, y_0 + 0.2k_1, z_0 + 0.2k_1^*) = z_0 + 0.2k_1^* \quad = 0.6$$

$$k_2^* = g(0.2, 1 + 0.2(2), 2 + 0.2(-7))$$

$$k_2^* = g(0.2, 1.4, 0.6) = -1.5779$$

$$y_1 = y_0 + \frac{h}{2}(k_1 + k_2) \quad = 1 + \frac{0.2}{2}(2 + 0.6) = 1.26$$

$$z_1 = z_0 + \frac{h}{2}(k_1^* + k_2^*) \quad = 2 + \frac{0.2}{2}(-7 - 1.57789) = 1.1422$$

$$f(x,y,z)=z;\ g(x,y,z)=-3z-2y+e^{x}$$

$$h=0.2;\ \ x_0=0,\ x_1=0.2,\ x_2=0.4. \qquad y_0=y(0)=1,\ z_0=z(0)=2;\ y(0.4)\cong y_2=?$$

$\boxed{y_1=1.26}$ $\boxed{z_1=1.1422}$

$\boxed{n=1}$

$$k_1=f(x_1,y_1,z_1)=z_1 \quad =1.1422$$

$$k_1^*=g(x_1,y_1,z_1)\ =g(0.2,1.26,1.1422)=-4.7252$$

$$k_2=f(x_1+h,y_1+0.2k_1,z_1+0.2k_1^*)\ \ =z_1+0.2k_1^*$$

$$k_2=0.1972$$

$$k_2^*=g(x_1+0.2,y_1+0.2k_1,z_1+0.2k_1^*)$$

$$k_2^*=g(0.4,1.26+0.2(1.1422),1.1422+0.2(-4.7252))$$

$$k_2^*=g(0.4,1.4884,0.1972)\quad =-2.0766$$

$$y_2=y_1+\frac{h}{2}(k_1+k_2)\quad =1.26+\frac{0.2}{2}(1.1422+0.1972)\quad =1.3939$$

$$z_2=z_1+\frac{h}{2}(k_1^*+k_2^*)\quad =1.1422+\frac{0.2}{2}(-4.7252-2.0766)\quad =0.4620$$

$\boxed{y(0.4)\cong y_2=1.3939}$ $\boxed{y'(0.4)=z(0.4)\cong z_2=0.4620}$

Usando um Método de Runge-Kutta de ordem 4

$$f(x,y,z) = z;\ g(x,y,z) = -3z - 2y + e^x \qquad y_0 = 1, z_0 = 2;\ y(0.4) \cong y_2 =?$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1^* + 2k_2^* + 2k_3^* + k_4^*), n = 0,1$$

$$k_1^* = f(x_n, y_n, z_n)$$

$$k_2^* = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1^*, z_n + \frac{h}{2}k_1^{**}\right)$$

$$k_3^* = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2^*, z_n + \frac{h}{2}k_2^{**}\right)$$

$$k_4^* = f(x_n + h, y_n + hk_3^*, z_n + hk_3^{**})$$

$$z_{n+1} = z_n + \frac{h}{6}(k_1^{**} + 2k_2^{**} + 2k_3^{**} + k_4^{**}), n = 0,1$$

$$k_1^{**} = g(x_n, y_n, z_n)$$

$$k_2^{**} = g\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1^*, z_n + \frac{h}{2}k_1^{**}\right)$$

$$k_3^{**} = g\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2^*, z_n + \frac{h}{2}k_2^{**}\right)$$

$$k_4^{**} = g(x_n + h, y_n + hk_3^*, z_n + hk_3^{**})$$

$$f(x,y,z) = z$$

$$g(x,y,z) = -3z - 2y + e^{x}$$

$$y(0) = 1, z(0) = 2$$

$$x_0 = 0, y_0 = 1, z_0 = 2$$

$$\boxed{n = 0}$$

$$k_1^* = f(x_0, y_0, z_0) \quad = 2$$

$$k_1^{**} = g(x_0, y_0, z_0) \quad = -7$$

$$k_2^* = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_1^*, z_0 + \frac{h}{2}k_1^{**}\right) = 1.3$$

$$k_2^{**} = g\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_1^*, z_0 + \frac{h}{2}k_1^{**}\right) \quad = -5.1948$$

$$k_3^* = f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_2^*, z_0 + \frac{h}{2}k_2^{**}\right) \quad = 1.4805$$

$$k_3^{**} = g\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_2^*, z_0 + \frac{h}{2}k_2^{**}\right) \quad = -5.5963$$

$$k_4^* = f(x_0 + h, y_0 + hk_3^*, z_0 + hk_3^{**}) \quad = 0.8807$$

$$k_4^{**} = g(x_0 + h, y_0 + hk_3^*, z_0 + hk_3^{**}) \quad = -4.0129$$

$$y_1 = y_0 + \frac{h}{6}(k_1^* + 2k_2^* + 2k_3^* + k_4^*) \quad = 1.2814$$

$$z_1 = z_0 + \frac{h}{6}(k_1^{**} + 2k_2^{**} + 2k_3^{**} + k_4^{**}) \quad = 0.9135$$

Façam o passo $n = 1$.