Nome: Merirson Alves - 96708

P3 - ELT331 - controle 2 - 26/10/2021 - PER3

(1) $x(k-1) - 4x(k-2) + 4x(k-3) = 2u(k-1) + u(k-3)$

Como $x(k)=0$ e $U(k)=0$ para $k \leq 0$, temos

$$X(z)z^{-1} - 4X(z)z^{-2} + 4X(z)z^{-3} = 2U(z)z^{-1} + U(z)z^{-3}$$

$$X(z)\left[\frac{1}{z} - \frac{4}{z^2} + \frac{4}{z^3}\right] = U(z)\left[\frac{2}{z} + \frac{1}{z^3}\right] \Rightarrow X(z)\left[\frac{z^2-4z+4}{z^3}\right] = U(z)\left[\frac{2z^2+1}{z^3}\right]$$

$$X(z) = \left(\frac{z}{z-1}\right)\left(\frac{2z^2+1}{\cancel{z^3}}\right)\left(\frac{\cancel{z^3}}{z^2-4z+4}\right) = \frac{2z^3+z}{(z-1)(z^2-4z+4)} \Rightarrow \boxed{X(z) = \frac{2z^3+z}{(z-1)(z-2)^2}}$$

Como o numerador e denominador são de grau 3, temos que dividir os polinômios:

$$\begin{array}{l|l} 2z^3 + 0z^2 + 1z - 0 & z^3 - 5z^2 + 8z - 4 \\ \hline -2z^3 + 10z^2 - 16z + 8 & 2 \\ 0 + 10z^2 - 15z + 8 & \end{array}$$

$$X(z) = 2 + \underbrace{\frac{10z^2 - 15z + 8}{(z-1)(z-2)^2}}_{\star}$$

$$X(z) = 2 + \star, \text{ onde } \star = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{(z-2)^2}$$

$$\star = Az^2 - 4zA + 4A + Bz^2 - 3zB + 2B + Cz - C = 10z^2 - 15z + 8$$

$$\begin{cases} A + B + 0 = 10 \Rightarrow A = 10 - B \Rightarrow A = -3 \\ -4A - 3B + C = -15 \Rightarrow \begin{cases} B + C = 25 \\ -2B - C = -36 \end{cases} \Rightarrow \begin{matrix} B = 7 \\ C = 18 \end{matrix} \\ 4A + 2B - C = 4 \end{cases} \quad X(z) = 2 + \frac{3}{z-1} + \frac{7}{z-2} + \frac{18}{(z-2)^2}$$

multiplicando por z e calculando a inversa temos:

$$x(k) = 2\delta[k] + 3u[k-1] + 7 \times 2^{(k-1)}u[k-1] + 9(k-1)2^{(k-1)}u[k-1]$$

$x(0) = [2] + [3u(-1)] + [7 \times 2^{-1}u(-1)] + [9(-1)2^{-1}u(-1)] \Rightarrow x(0) = 2$

$x(1) = [0] + [3 \times u(0)] + [7 \times 2^0 u(0)] + [9(0)2^0 u(0)] \Rightarrow x(1) = 10$

$x(2) = [0] + [3 \times u(1)] + [7 \times 2^1 u(1)] + [9(1)2^1 u(1)] \Rightarrow x(2) = 35$

$x(3) = [0] + [3 \times u(2)] + [7 \times 2^2 u(2)] + [9(2)2^2 u(2)] \Rightarrow x(3) = 103$

② $G_c(s) = 2\frac{(s-6.93)}{(s-1)}$ $T = 0.1s$ $G_p(s) = \frac{1}{s+1}$ $G_h(s) = \frac{1-e^{-sT}}{s}$

→ Sem controlador:

a) $G_1(s) = G_h(s)\,G_p(s) = (1-e^{-sT})\left(\frac{1}{s(s+1)}\right) \xrightarrow{Z\{\cdot\}} G_1(z) = \left(\frac{z-1}{z}\right)\left(\frac{(1-e^{-0.1})\,z}{(z-1)(z-e^{-0.1})}\right)$

$G_1(z) = \frac{1-e^{-0.1}}{z-e^{-0.1}}$. Agora, obtendo FT em malha fechada temos $F_1(z) = \frac{G_1(z)}{1+G_1(z)}$

$F_1(z) = \frac{1-e^{-0.1}}{z-e^{-0.1}+1-e^{-0.1}} \Rightarrow F_1(z) = \frac{C(z)}{R(z)} = \frac{1-e^{-0.1}}{z-(2e^{-0.1}-1)}$ Isolando $C(z)$ e aplicando a entrada degrau, temos:

$C(z) = \frac{(1-e^{-0.1})\,z}{(z-1)(z-(2e^{-0.1}-1))} = \frac{0.095163\,z}{(z-1)(z-0.809675)}$ Aplicando frações parciais:

$\frac{A}{z-1} + \frac{B}{z-0.809675} = \frac{0.095163\,z}{(z-1)(z-0.809675)} \Rightarrow A = \left.\frac{0.095163\,z}{z-0.809675}\right|_{z=1} = 0.5$

$B = \left.\frac{0.095163\,z}{z-1}\right|_{z=0.809675} = -0.404837$

$C(z) = \frac{0.5}{z-1} - \frac{0.404837}{z-0.809675}$

$$\boxed{C(k) = \frac{1}{2}u[k-1] - 0.404837 \times 0.809675^{(k-1)}\,u[k-1]}$$

→ Controlador $D(z)$:

b) $A = e^{+6.93\times 0.1} = 1.999706$

$B = e^{+1\times 0.1} = 1.105171$

$C = \frac{2\times(-6.93)(1-B)}{(-1)(1-A)}$

$C = 1.458098$

$$\boxed{D(z) = 1.458098\,\frac{(z-1.999706)}{(z-1.105171)}}$$

c) com o controlador: $G_2(s) = G_c(s)\,G_h(s)\,G_p(s) = (1-e^{-sT})\left[\frac{2(s-6.93)}{(s+1)(s-1)s}\right]$ ☆

Aplicando frações parciais, encontramos:

☆ $= \frac{2(s-6.93)}{s(s+1)(s-1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} \Rightarrow A = \left.\frac{2(s-6.93)}{(s+1)(s-1)}\right|_{s=0} = +13.86$

$B = \left.\frac{2(s-6.93)}{s(s-1)}\right|_{s=-1} = -7.93$

$C = \left.\frac{2(s-6.93)}{s(s+1)}\right|_{s=1} = -5.93$

$$\boxed{G_2(s) = \left[\frac{13.86}{s} - \frac{7.93}{s+1} - \frac{5.93}{s-1}\right](1-e^{-sT})}$$

2- continuação

Fazendo a transformada Z de $g_2(s)$, temos:

$$g_2(z) = \left[\frac{13.86\,z}{z-1} - \frac{7.93\,z}{z-0.904837} - \frac{5.93\,z}{z-1.105171}\right]\left(\frac{z-1}{z}\right) = \frac{0.130979\,z - 0.26969}{(z-1.105171)(z-0.904837)}$$

$$F(z) = \frac{g_2(z)}{1+g_2(z)} = \frac{0.130979\,z - 0.26969}{z^2 - 1.879029\,z + 0.73030}$$

OBS:

$$\boxed{z^2 - 1.879029\,z + 0.7303 = (z-1.329883)(z-0.549146)}$$

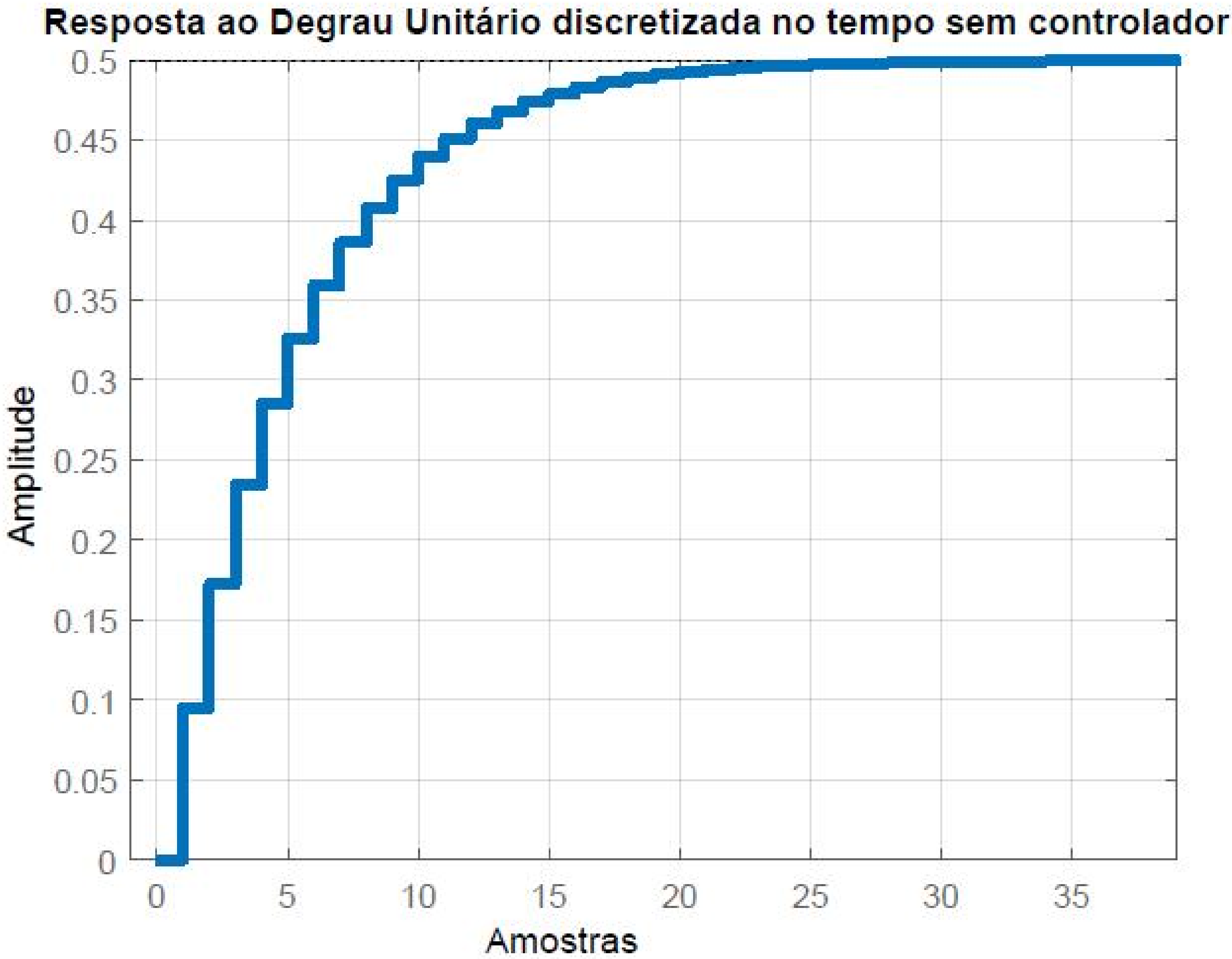
$$C(z) = F(z)\,R(z) = F(z)\,\frac{z}{z-1} \Rightarrow \boxed{C(z) = \frac{0.130979\,z^2 - 0.26969\,z}{(z-1)(z^2 - 1.979029\,z + 0.73030)}} \quad ☆☆$$

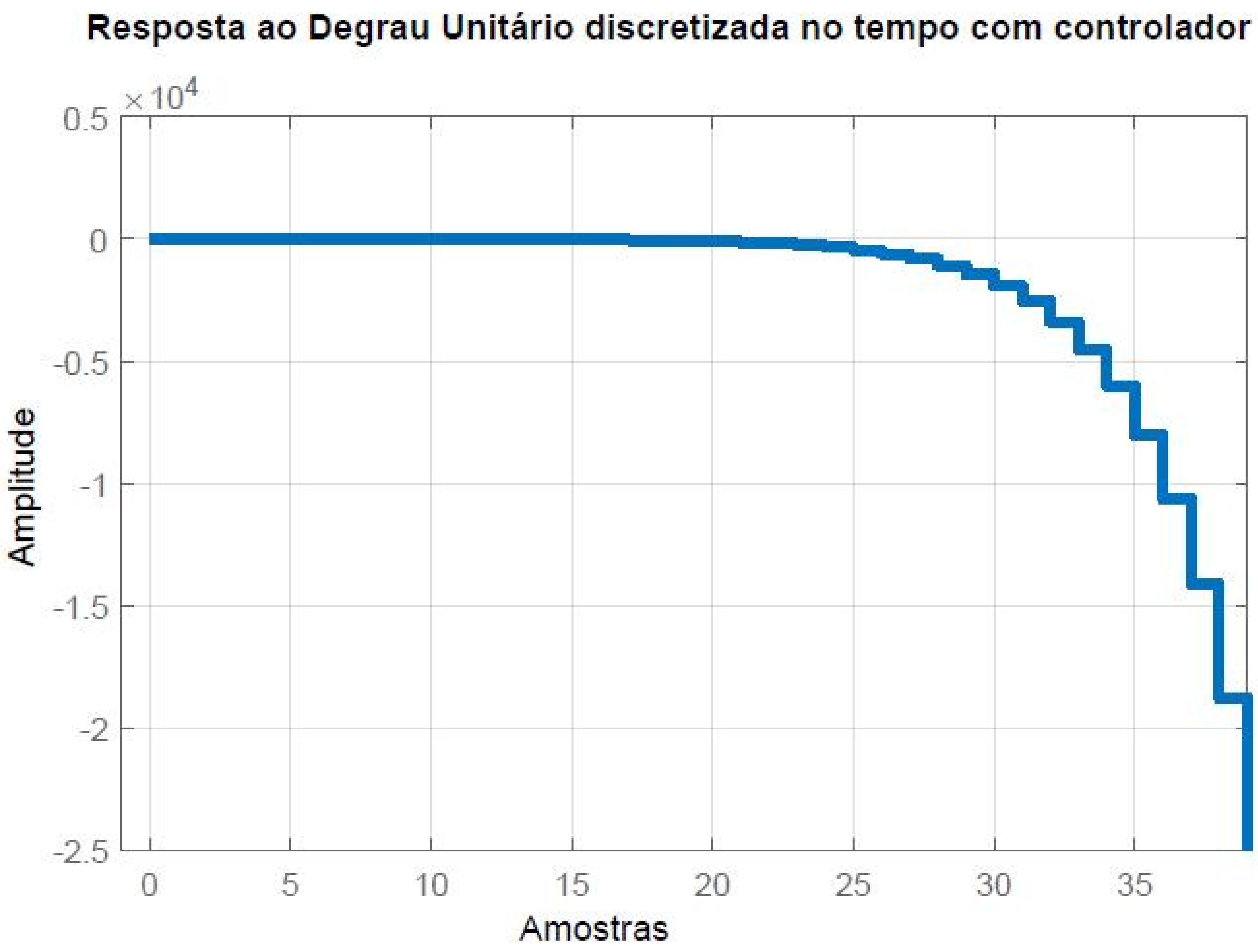
Aplicando Frações parciais, temos

$$☆☆ = \frac{A}{z-1} + \frac{B}{z-1.329883} + \frac{C}{z-0.549146}$$

$$C(z) = \frac{0.93264}{z-1} + \frac{0.30852}{z-0.549146} - \frac{0.49313}{z-1.329883}$$

$$\boxed{c(k) = 0.93264\,u[k-1] - 0.30852 \times 0.549146^{(k-1)}\,u[k-1] - 0.49313 \times 1.329883^{(k-1)}\,u[k-1]}$$


Resposta ao Degrau Unitário discretizada no tempo sem controlador
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Amplitude
0
5
10
15
20
25
30
35
Amostras



Resposta ao Degrau Unitário discretizada no tempo com controlador
×10^4
0.5
0
-0.5
-1
-1.5
-2
-2.5
Amplitude
0
5
10
15
20
25
30
35
Amostras

③ $G_h(s) = \dfrac{(1-e^{sT})}{s}$. $G_p(s) = \dfrac{K}{s(s+1)}$ $\quad T = 1\,\Omega$

$$G(s) = G_h(s)\,G_p(s) = (1-e^{-sT})\underbrace{\left[\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}\right]}_{☆}$$

$$☆ = As(s+1) + B(s+1) + C(s^2) = 1$$

$$\begin{cases} A + 0 + C = 0 \Rightarrow C = 1 \\ A + B + 0 = 0 \Rightarrow A = -1 \\ 0 + B + 0 = 1 \Rightarrow B = 1 \end{cases}$$

$$G(s) = (1 - e^{+sT})\left(\frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}\right)K \xRightarrow{Z\{\cdot\}} G(z) = \left(\frac{z-1}{z}\right)\left[\frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-T}}\right]K$$

Para $T=1$, temos: $G(z) = K\left[\dfrac{e^{-1}z + \cancel{z} - \cancel{z} - e^{-1} - e^{-1} + 1}{(z-1)(z-e^{-1})}\right] = K\left[\dfrac{e^{-1}z - 2e^{-1} + 1}{(z-1)(z-e^{-1})}\right]$

Aplicando a transformação bilinear, temos: $z = \dfrac{1 + (Tw/2)}{1 - (Tw/2)} = \dfrac{2+Tw}{2-Tw} = \dfrac{2+w}{2-w}$

Portanto:

$$G(w) = \frac{\dfrac{e^{-1}(2+w)}{2-w} + (1-2e^{-1})}{\left(\dfrac{2+w}{2-w} - 1\right)\left(\dfrac{2+w}{2-w} - e^{-1}\right)} = \frac{\dfrac{(3e^{-1}-1)w + (2-2e^{-1})}{\cancel{(2-w)}}}{\dfrac{(2+2e^{-1})w^2 + (4-4e^{-1})w}{(2-w)^2}}$$

$$G(w) = \frac{[(3e^{-1}-1)w + (2-2e^{-1})](2-w)}{(2+2e^{-1})w^2 + (4-4e^{-1})w} = \frac{-0.103638w^2 - 1.056965w + 2.528482}{2.735759w^2 + 2.528482w}$$

Agora fazendo $1 + KG(w) = 0$, temos:

$$2{,}735759w^2 + 2{,}528482w - 0{,}103638Kw^2 - 1{,}056965Kw + 2{,}528482K = 0$$

$$(2{,}735759 - 0{,}103638K)w^2 + (2{,}528482 - 1{,}056965K)w + 2{,}528482K = 0$$

$$\begin{array}{c|ll|l}
w^2 & (2.735759 - 0.103638K) & 2.528482K & \Rightarrow K < \dfrac{2.735759}{0.103638} \Rightarrow K < 26.397258 \\
w^1 & (2.528482 - 1.056965K) & 0 & \Rightarrow K < \dfrac{2.528482}{1.056965} \Rightarrow K < 2.392210 \\
w^0 & 2.528482K & 0 & \Rightarrow K > 0
\end{array}$$

Portanto, os valores do ganho K para o sistema ser estável se encontra de: $0 < K < 2.392210$