Universidade Federal de Viçosa
Centro de Ciências Exatas
Departamento de Matemática

Gabarito 1ª Lista - MAT 135 - Geometria Analítica e Álgebra Linear 2017/II

1. (a) Não, (b) Não, (c) Sim, ordem 4x5, (d) Não.

2. (a) $A_{4\times 5}$, (b) $a_{21} = -4$, $a_{34} = 1$ e $a_{44} = 0$.

3. $c_{23} = 5$, $d_{41} = 5$.

4. $A = \begin{bmatrix} -2 & 1 & 0 & -1 \\ 1 & -2 & 3 & 2 \\ 0 & 3 & 0 & 5 \\ -1 & 2 & 5 & 4 \end{bmatrix}$.

5. (a) $A^2 = I$, (b) $A^3 = A$, (c) $A^{31} = A$, (d) $A^{42} = I$.

6. $x = -1$, $y = 1$.

7. (a) $x = -1$, (b) $x = -4$, $y = -4$, $z = -2$, (c) $\begin{array}{llll} x = 2, & y = -7, & z = -2 \\ x = -2, & y = -3, & z = 10 \end{array}$

8. (a) $\begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix}$,

   (b) $\begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$,

   (c) $\begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}$,

   (d) $\begin{bmatrix} 10 & -6 \\ -14 & 2 \\ -6 & -8 \end{bmatrix}$.

9. $\begin{bmatrix} a & b \\ 0 & a-b \end{bmatrix}$.

10. (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$.

    (b) 4 matrizes: $\begin{bmatrix} \sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} -\sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} \sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}$, $\begin{bmatrix} -\sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}$.

    (c) Não, $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

11. (b) A matriz identidade.

12. $A^3 = 0$.

13.

14.

15.

16. (a) ± 1, (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ e $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

17. Ortogonais: A, C e D.
    Não ortogonais: B.

18. (a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (c) $T_{-\alpha} = (T_\alpha)^T$.

19.

20.

21.

22.

23. Sim

24.

25. (a) 4^5, (b) P é invertível, (c) -9, (d) Q é invertível.

26. -5

27. (a) 576,

    (b) $A^{-1} = \begin{bmatrix} 1 & \frac{5}{2} & \frac{17}{8} & -\frac{31}{12} \\ 0 & \frac{1}{2} & \frac{3}{8} & -\frac{5}{12} \\ 0 & 0 & \frac{1}{4} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{3} \end{bmatrix}$,

(c) $B^{-1} = \begin{bmatrix} -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ -\frac{7}{6} & -\frac{1}{2} & -1 & 0 \\ -\frac{25}{24} & -\frac{3}{8} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$,

(d) $(AB)^{-1} = \begin{bmatrix} -\frac{215}{288} & -\frac{37}{288} & -\frac{67}{144} & -\frac{25}{72} \\ -\frac{23}{32} & -\frac{5}{32} & -\frac{3}{16} & -\frac{1}{8} \\ -\frac{5}{6} & -\frac{1}{6} & -\frac{1}{3} & -\frac{1}{6} \\ \frac{31}{24} & \frac{5}{24} & -\frac{1}{12} & -\frac{1}{6} \end{bmatrix}$,

(e) $\det C = 0$ ou $\det C = \frac{1}{16}$.

28. $\det Q = (-2)^n$.

29. (a) 58, (b) 58, (c) 3364, (d) $A^{-1} = \begin{bmatrix} \frac{25}{29} & -\frac{32}{29} & -\frac{13}{29} & \frac{10}{29} \\ \frac{7}{29} & -\frac{2}{29} & \frac{1}{29} & -\frac{3}{29} \\ -\frac{157}{58} & \frac{165}{58} & \frac{77}{58} & -\frac{14}{29} \\ -\frac{73}{58} & \frac{83}{58} & \frac{31}{58} & -\frac{3}{58} \end{bmatrix}$,

(e) 58, (f) 3. $\begin{bmatrix} 36 & 23 & 35 & 32 \\ 23 & 25 & -2 & 17 \\ 35 & -2 & 95 & 47 \\ 32 & 17 & 47 & 50 \end{bmatrix}$.

30. $p(x) = x^3 - 2x^2 - x + 3$ e $A^{-1} = -\frac{1}{3}(A^2 - 2A - I)$.

31. (a) -123, (b) $1 + a + b + c$, (c) $-c^4 + c^3 - 16c^2 + 8c - 2$, (d) -5 (e) -120, (f) -120.

32. (a) $x = 0, -1, 1/2$, (b) $x = 40/11$, (c) $x = \frac{3}{4} \pm \frac{1}{4}\sqrt{33}$.

33. $\det(A) = a_{41}\, a_{32}\, a_{23}\, a_{14}$.

34.

35. (a) $A^{-1} = \begin{bmatrix} \frac{29}{152} & \frac{11}{152} & -\frac{1}{8} \\ -\frac{21}{152} & \frac{13}{152} & \frac{1}{8} \\ \frac{27}{152} & \frac{5}{132} & \frac{1}{8} \end{bmatrix}$,

(b) $A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

(c) $A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & -\frac{5}{2} & 1 & -\frac{1}{2} \\ -1 & 2 & -1 & 0 \end{bmatrix}$,

(d) $A^{-1} = \begin{bmatrix} 0 & 0 & \frac{1}{4} & 0 \\ 0 & -1 & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{5}{6} & -\frac{13}{24} & 0 \\ \frac{2}{9} & \frac{16}{9} & -\frac{53}{36} & \frac{1}{3} \end{bmatrix}$,

36.

37. (a) $A^{-1} = \begin{bmatrix} -\frac{1}{8} & \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} & 0 \end{bmatrix}$,

(b) $A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{1}{14} \\ -\frac{1}{7} & \frac{3}{14} \end{bmatrix}$,

(c) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$,

(d) $A^{-1} = \frac{1}{51} \begin{bmatrix} -5 & -28 & -6 \\ -2 & 16 & 1 \\ -2 & -1 & 1 \end{bmatrix}$.

38. (a) $X = A^{-1}B^{-1}$, (b) $X = A^{-1}B^T$, (c) $X = A^{-1}$,
(d) $X = B^T - A$, (e) $X = A^{-1}BAB^{-1}$, (f) $X = (B^T)^{-1}A$.

39. **Correção:** A matriz A a ser considerada neste exercício é

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 2 & 3 & 5 \end{bmatrix}.$$

Assim, teremos:

$$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}.$$

40. Por exemplo, consideramos as matrizes

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix},$$

$$E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix}, \quad E_6 = \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{4}{3} \\ 0 & 0 & 1 \end{bmatrix}.$$

41. (a) $A = \begin{bmatrix} 20 & 15 & 30 \end{bmatrix}. \begin{bmatrix} 50 & 15 & 6 & 70 & 25 \\ 500 & 1 & 5 & 20 & 30 \\ 200 & 8 & 7 & 50 & 40 \end{bmatrix}$.

(b) Os elementos de AB representam o valor total de compra e o preço total de transporte de todos os materiais utilizados na construção de todos os estabelecimentos.

42. Faça os produtos AB e AC, onde $A = \begin{bmatrix} 6 & 7 & 5 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 25 & 15 & 70 \\ 30 & 25 & 40 \\ 60 & 10 & 55 \\ 15 & 30 & 60 \end{bmatrix}$ e $C = \begin{bmatrix} 7,5 & 5 & 4,5 & 6,5 \end{bmatrix}$.

43. (a) F, (b) V, (c) V, (d) V, (e) F, (f) V, (g) V, (h) F, (i) V, (j) F, (k) F, (l) V , (m) F, (n) F , (o) F, (p) V, (q) V, (r) V, (s) F, (t) V, (u) F, (v) F , (w) V, (x) F, (y) V, (z) F.