Nota = 19,0

P1 - ELT 221 - 6/10/2020 / Meritson F. de O. Alves - 96708

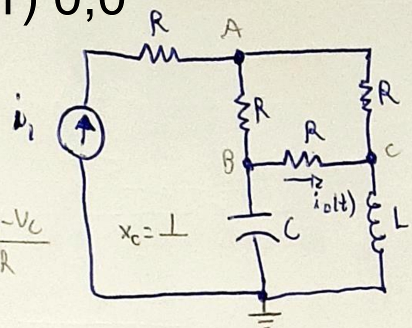
① Seja o circuito a seguir:

Q1) 0,0

a) $F(s) = \dfrac{I_o(s)}{I_1(s)} = ?$

b) $R = 1, L = 1, C = 1$ (Ω, H, F). Quais os zeros de $F(s)$?

c) // // //. Quais os polos de $F(s)$?



$I_o = \dfrac{V_B - V_C}{R}$

$LKC(A) \Rightarrow -I_1 + \dfrac{V_A - V_B}{R} + \dfrac{V_A - V_C}{R} = 0 \Rightarrow V_A\left(\dfrac{2}{R}\right) + V_B\left(\dfrac{-1}{R}\right) + V_C\left(\dfrac{-1}{R}\right) = I_1$

$LKC(B) \Rightarrow \dfrac{V_B - V_A}{R} + I_o + \dfrac{V_B}{1/sC} = 0 \Rightarrow V_A\left(\dfrac{-1}{R}\right) + V_B\left(\dfrac{2}{R} + sC\right) + V_C\left(\dfrac{-1}{R}\right) = 0$

$LKC(C) \Rightarrow \dfrac{V_C - V_A}{R} + \dfrac{V_C - 0}{sL} - I_o = 0 \Rightarrow V_A\left(\dfrac{-1}{R}\right) + V_B\left(\dfrac{-1}{R}\right) + V_C\left(\dfrac{2}{R} + \dfrac{1}{sL}\right) = 0$

$2V_A - V_B - V_C = RI_1$

$-V_A + V_B(2 + RsC) - V_C = 0$

$-V_A - V_B + V_C\left(2 + \dfrac{R}{sL}\right) = 0$

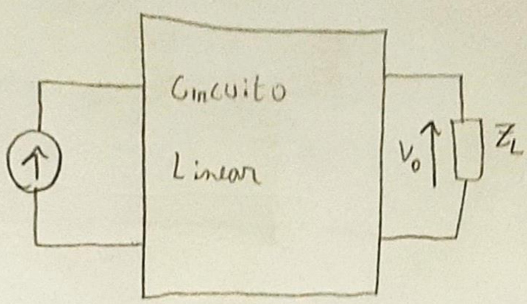
$$\begin{bmatrix} \frac{2}{R} & -\frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{2}{R}+sC & -\frac{1}{R} \\ -\frac{1}{R} & -\frac{1}{R} & \frac{2}{R}+\frac{1}{sL} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ 0 \end{bmatrix}$$

0,0

Questão 2 - 96708

Q2) 8,0

2



condições iniciais nulas

$V_i(t) = 4u(t)$ [V]

$V_0(t) = (1 - 2e^{-t} + e^{-2t})u(t)$ [V]

Aplicando a transformada de Laplace

a) $\dot{V}_i(S) = \frac{4}{S}$

$$\dot{V}_0(S) = \left(\frac{1}{S} - \frac{2}{S+1} + \frac{1}{S+2}\right) = \frac{\cancel{S^2} + \cancel{3S} + 2 - \cancel{2S^2} - \cancel{4S} + \cancel{S^2} + \cancel{S}}{S(S+1)(S+2)} = \frac{2}{S(S+1)(S+2)}$$

$$F(S) = \frac{V_0(S)}{V_i(S)} = \frac{\cancel{2}}{\cancel{S}(S+1)(S+2)} \cdot \frac{\cancel{S}}{\cancel{4}_2} = \frac{1}{2(S+1)(S+2)}$$

3,0

b) Resposta natural:

$$F(S) = 0,5\left[\frac{A}{S+1} + \frac{B}{S+2}\right] = \left(Ae^{-t} + Be^{-2t}\right)$$

Resposta Forçada: $\dot{V}_i = 2\angle{-45°}$, $s = (-1 + j)$

$$V_0 = \frac{V_i}{2(S+1)(S+2)} \Rightarrow V_0 = \frac{\cancel{2}\angle{-45°}}{\cancel{2}(-1+j+1)(-1+j+2)} = \frac{\sqrt{2}}{2}\angle{-180} \Rightarrow V_0 = \frac{\sqrt{2}}{2} e^{-t} sen(t - 180°) \text{ [V]}$$

Resposta completa:

$$V_0 = V_n + V_f = Ae^{-t} + Be^{-2t} + \frac{\sqrt{2}}{2} e^{-t} sen(t + 180°) \text{ em } t=0: \boxed{V_0 = 0 = A + B}$$

$$\frac{dV_0}{dt} = -Ae^{-t} - 2Be^{-2t} \pm \frac{\sqrt{2}}{2} e^{-t} sen(t - 180°) + \frac{\sqrt{2}}{2} e^{-t} cos(t - 180°) = 0$$

$$\text{em } t=0 \Rightarrow 0 = -A - 2B + \frac{\sqrt{2}}{2} \Rightarrow A + 2B = -\frac{\sqrt{2}}{2}$$

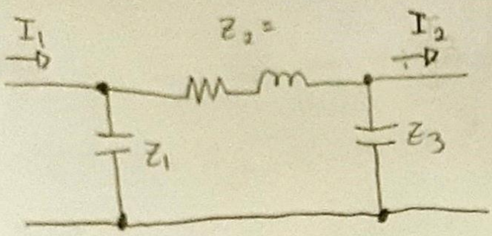
$$(-1)\begin{cases} A + B = 0 \\ A + 2B = -\frac{\sqrt{2}}{2} \end{cases} \Rightarrow B = -\frac{\sqrt{2}}{2} \quad A = \frac{\sqrt{2}}{2} \Rightarrow \boxed{V_0 = \frac{\sqrt{2}}{2}\left[e^{-t} - e^{-2t} + e^{-t} sen(t + 180°)\right] \text{ [V]}}$$

5,0

Q3) 0,0

Questão 3 - 96708

* como I_2 está para fora, logo:



$$y = \begin{bmatrix} 1,1428 & -0,1428 \\ 0.1428 & -1,1428 \end{bmatrix}; \Delta y = -1,2856$$

$$LKC \Rightarrow -I_1 + \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} = 0 \Rightarrow I_1 = V_1\left(\frac{2}{X_C} + \frac{1}{R+X_L}\right) + V_2\left(\frac{-1}{R+X_L}\right)$$

$$LKC \Rightarrow I_2 + \frac{V_2}{Z_3} + \frac{V_2 - V_1}{Z_2} = 0 \Rightarrow I_2 = V_1\left(\frac{+1}{R+X_L}\right) + V_2\left(\frac{-2}{X_C} - \frac{1}{R+X_L}\right)$$

$$T = \begin{bmatrix} 8.0028 & -7.0028 \\ 9.0028 & -8.0028 \end{bmatrix}$$

0,0

Questão 4 - 96708

Q4) 5,0

a) Para $I_2 = 0$ A ions medição 2:

$$Z_{11} = \frac{V_1}{I_1} = \frac{20}{2} = 10\,\Omega \quad e \quad Z_{21} = \frac{V_2}{I_1} = \frac{10}{2} = 5\,\Omega$$

$$Z = \begin{bmatrix} 10 & 12 \\ 5 & 10 \end{bmatrix}$$

2,0

Para medição 1:

$$\begin{cases} V_1 = I_1 Z_{11} + I_2 Z_{12} \\ V_2 = I_1 Z_{21} + I_2 Z_{22} \end{cases} \Rightarrow \begin{cases} 4 = 20 + 0{,}5 Z_{12} \\ 0 = 5 - 0{,}5 Z_{22} \end{cases} \Rightarrow \begin{matrix} Z_{12} = 12\,\Omega \\ Z_{22} = 10\,\Omega \end{matrix}$$

b) LKT $\Rightarrow \begin{cases} 100 = 25 I_1 + V_1 & (1) \\ V_2 = -R_o I_2 & (2) \end{cases}$ e $\begin{cases} V_1 = 10 I_1 + 12 I_2 & (3) \\ V_2 = 5 I_1 + 10 I_2 & (4) \end{cases}$

De (1) $\Rightarrow V_1 = 100 - 25 I_1$ (5)

Substituindo (5) em (3):

$100 - 25 I_1 = 10 I_1 + 12 I_2$

$\boxed{100 = 35 I_1 + 12 I_2}$ (6)

Substituindo (2) em (4):

$-R_o I_2 = 5 I_1 + 10 I_2$

$\boxed{I_1 = \frac{-I_2 (R_o + 10)}{5}}$ (7)

Substituindo (7) em (6):

$100 = (-7R_o - 70 + 12) I_2$

$\boxed{I_2 = \frac{-100}{7R_o + 58}}$

$$\frac{dP}{dR_o} = \frac{d(R_o I^2)}{dR_o} = \frac{d}{dR_o}\left(\frac{(-100)^2 R_o}{(7R_o + 58)^2}\right) = \frac{\cancel{(-100)^2}(7R_o+58)^{\cancel{2}} - 2\cancel{(7R_o+58)}7\cancel{(-100)^2}R_o}{\cancel{(7R_o+58)^4}} = 0 \Rightarrow 7R_o + 58 - 14R_o = 0$$

$$58 = 7R_o \Rightarrow \boxed{R_o = \frac{58}{7}}$$

2,0

c) $P_{max} = R I^2 = R_o \cdot \left(\frac{-100}{7R_o + 58}\right)^2 = \frac{58}{7}\left(\frac{-100}{116}\right)^2 = 6{,}16\,W$

1,0

Q5) 6,0

Questão 5 - 96708

a) $h_{11} = 1{,}5k$, $h_{12} = 1$, $h_{21} = -0.5$ e $h_{22} = 1m$

$$h = \begin{bmatrix} 1.5k & 1 \\ -0.5 & 1m \end{bmatrix} \quad \Delta h = 1.5 + 0.5 = 2 \quad T = \begin{bmatrix} +4 & 3K \\ 2m & 2 \end{bmatrix} \Rightarrow T^2 = \begin{bmatrix} 22 & 18000 \\ 0.012 & 10 \end{bmatrix}$$

3,0

b)
$$\begin{cases} V_1 = AV_2 - BI_2 & (1) \\ I_1 = CV_2 - DI_2 & (2) \end{cases} \quad e \quad \begin{cases} V_g = 100 I_1 + V_1 & (3) \\ V_2 = -R_0 I_2 & (4) \end{cases}$$

De (4): $I_2 = \dfrac{-V_2}{R_0}$

Substituindo em (2):

$I_1 = CV_2 - D\left(-\dfrac{V_2}{R_0}\right)$

$I_1 = V_2\left(C + \dfrac{D}{R_0}\right)$ (5)

De (4): $I_2 = \dfrac{-V_2}{R_0}$

Substituindo em (1):

$V_1 = AV_2 - B\left(-\dfrac{V_2}{R_0}\right)$

$V_1 = V_2\left(A + \dfrac{B}{R_0}\right)$ (6)

Substituindo (5) e (6) em (3), temos:

$V_g = 100 V_2 \left[C + \dfrac{D}{R_0}\right] + V_2 \left[A + \dfrac{B}{R_0}\right]$

$\dfrac{V_g}{V_1} = 100C + \dfrac{100D}{R_0} + A + \dfrac{B}{R_0}$

$\dfrac{V_g}{V_1} = \dfrac{211}{5} \Rightarrow \boxed{\dfrac{V_2}{V_g} = \dfrac{5}{211} \cong 0.024}$

3,0