P2 - ELT 330 - 8/4/21

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1 a)

$$F = M\ddot{z} + B\dot{z} + Kz$$

$x_1 = z$

$x_2 = \dot{z}$

$\dot{x}_2 = \ddot{z}$

$u_1 = F$

$$\ddot{z} = \frac{F}{m} - \frac{B}{m}\dot{z} - \frac{K}{m}z$$

$$\dot{x}_2 = \frac{u}{m} - \frac{B}{m}x_2 - \frac{K}{m}x_1$$

e $\dot{x}_1 = \dot{z} = x_2$

$y_1 = z = x_1$

$y_2 = f_K = Kz = Kx_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{B}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ K & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix}$$

b) Para condições nulas:

$$y(s) = \begin{bmatrix} 1 & 0 \\ K & 0 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{B}{m} \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ K & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ \frac{K}{m} & s+\frac{B}{m} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s+1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+s+1} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} s+1 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s^2+s+1} \begin{bmatrix} s+1 & 1 \\ s+1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$y(s) = \begin{pmatrix} \frac{1}{s^2+s+1} \\ \frac{1}{s^2+s+1} \end{pmatrix}$$

c) $x[(K+1)T] = (TA + I)\,x(KT) + TBu(KT)$

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(K)$$

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} u(K) \end{bmatrix}$$

$$\begin{bmatrix} y_1(K) \\ y_2(K) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix}$$

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$$X(s) = [sI - A]^{-1} X(0) + [sI - A]^{-1} B\, U(s)$$

$$X(s) = \left(\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \left(\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/s \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ +1 & 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ +1 & 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/s \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s^2}{(s+1)(s^2-s+1)} \\ -\frac{1}{(s+1)(s^2-s+1)} \\ \frac{s}{-1-s^3} \end{bmatrix} + \begin{bmatrix} \frac{s}{(s+1)(s^2-s+1)} \\ -\frac{1}{s(s+1)(s^2-s+1)} \\ \frac{1}{-s^3-1} \end{bmatrix} = \begin{bmatrix} \frac{s^2+s}{(s+1)(s^2-s+1)} \\ \frac{-1-s}{s(s+1)(s^2-s+1)} \\ \frac{1+s}{-s^3-1} \end{bmatrix}$$

$$\frac{s\cancel{(s+1)}}{\cancel{(s+1)}(s^2-s+1)}$$

$$\frac{s}{s^2-s+1}$$

$$Y(s) = C\,X(s) + D\,U(s)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} [X(s)] + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/s \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{s^2+s}{(s+1)(s^2-s+1)} \\ \frac{-1-s}{s(s+1)(s^2-s+1)} \\ \frac{1+s}{-s^3-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/s \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2-s+1} \\ \frac{s-1}{s^2-s+1} \\ \frac{-1}{s^2-s+1} \end{bmatrix}$$

$$Y_1 = \frac{s}{s^2-s+1} = \frac{s-0,5+0,5}{\underbrace{(s-0,5)^2}_{s^2-s+0,25}+0,75} = \frac{s-0,5}{(s-0,5)^2+0,75} + \frac{0,5}{(s-0,5)^2+0,75}\,\frac{\sqrt{3}}{\sqrt{3}}$$

$$Y_1(s) = \frac{s-\frac{1}{2}}{(s-\frac{1}{2})^2+\frac{3}{4}} + \frac{\frac{\sqrt{3}}{2}}{(s-\frac{1}{2})^2+\frac{3}{4}} \cdot \frac{1}{\sqrt{3}} \Rightarrow y_1(t) = \mathcal{L}^{-1}\{Y(s)\} \Rightarrow$$ OBS: Foi utilizado a seguinte inversa:

$$\frac{s-b}{(s+b)^2+a^2} = e^{-bt}\cos(at)$$

$$\frac{a}{(s+b)^2+a^2} = e^{-bt}\,\mathrm{sen}(at)$$

$$y_1(t) = e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{e^{-\frac{1}{2}t}}{\sqrt{3}}\,\mathrm{sen}\left(\frac{\sqrt{3}}{2}t\right)$$

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3) Realimentação negativa: $\dfrac{g_1}{1+g_1 g_2} \rightarrow \begin{cases} g_1 = \dfrac{1}{0.5s^2+\sqrt{2}s+1} \\ g_2 = 1 \end{cases}$

$$\frac{\left(\dfrac{1}{0.5s^2+\sqrt{2}s+1}\right)}{1+\left(\dfrac{1}{0.5s^2+\sqrt{2}s+1}\right)\cdot 1} = \frac{1}{0.5s^2+\sqrt{2}s+2} = \frac{2}{s^2+2\sqrt{2}s+4} = \boxed{\frac{1}{2}\cdot\frac{4}{s^2+2\sqrt{2}s+4}}$$

$$g(s) = \frac{1}{2}\cdot\frac{4}{s^2+2\sqrt{2}s+4} \rightarrow \begin{cases} \xi = \dfrac{2\sqrt{2}}{2.2} = \dfrac{\sqrt{2}}{2} \\ \omega_n = 2 \end{cases}$$

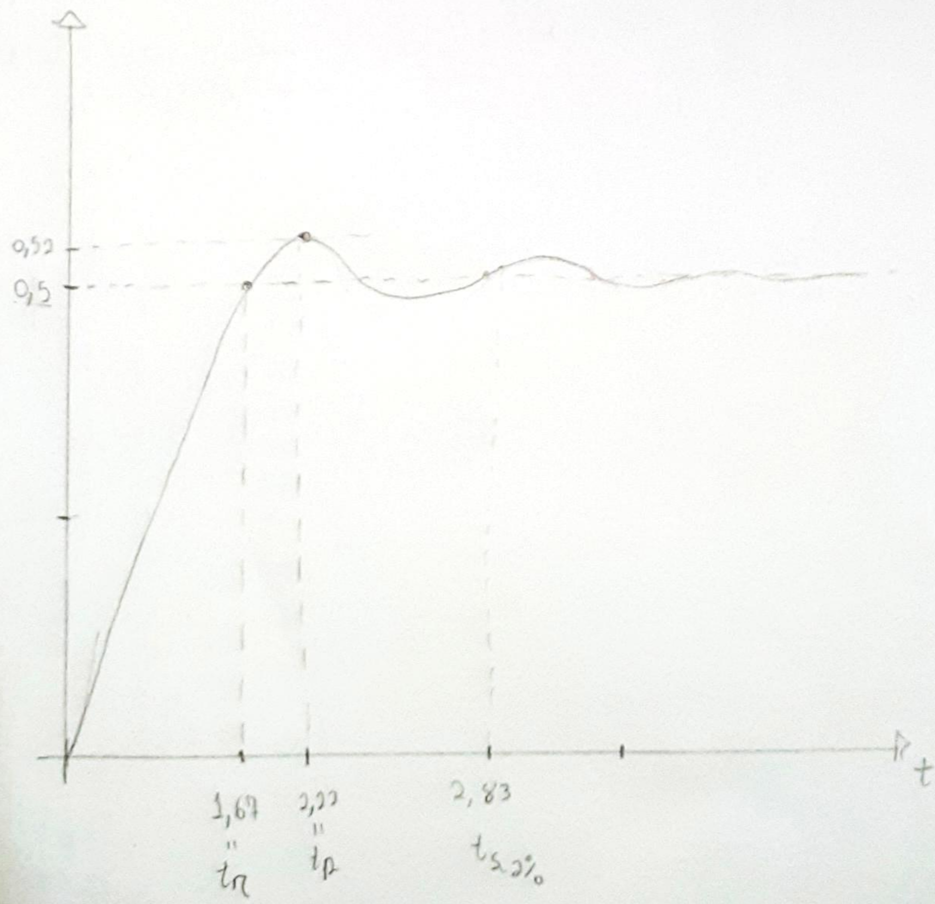
a) $t_r = \dfrac{\pi - \cos^{-1}(\xi)}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi - \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)}{2\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} = 1.67\ s$

b) $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi}{2\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} = 2{,}22\ s$

c) $M_p(\%) = e^{-\left(\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)} = 4{,}32\%$

d) $t_{s_{2\%}} = \dfrac{4}{\xi\omega_n} = \dfrac{4}{\frac{\sqrt{2}}{2}\cdot 2} = 2{,}83\ s$

e) como ξ está entre 0 e 1 a curva é subamortecida.



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4 $h_0 = 2m$

$$q_i = A\frac{dh}{dt} + q_0 \quad \begin{cases} q_i = 2\, m^3/2 \\ A = 1\, m^2 \\ q_0 = K h^{0,5} = \sqrt{2}\, h^{0,5} = \sqrt{2h} \end{cases}$$

$$q_i = A\dot{h} + K h^{0,5} \Rightarrow \boxed{\dot{h} = \frac{q_i}{A} - \frac{K}{A} h^{0.5}}$$

$$\dot{h} = \frac{2}{1} - \frac{\sqrt{2}}{1} h^{0,5}$$

$$\boxed{\dot{h} = 2 - \sqrt{2}\, h^{0,5}}$$

$$g(h,\dot{h}) = 2 - \sqrt{2}\, h^{0,5} - \dot{h} = 0$$

$$\dot{h}|_{p,o} = 0$$

Como não é linear, devemos fazer a linearização, portanto:

$$g(h) \cong g(h)|_{h=h_0} + \frac{dg}{dh}\Big|_{h=h_0} (h - h_0)$$

$$g(h) = 0 + \left(0 - \frac{\sqrt{2}}{2} h^{-0,5}\right)\Big|_{h=2} (h-2)$$

$$g(h) = (-0,5)(h-2)$$

$$\boxed{g(h) = 1 - 0,5h}$$

$$\dot{h} = 1 - 0,5h \Rightarrow sH(s) = \frac{1}{s} - 0.5H(s) \Rightarrow H(s) = \frac{1}{s} \cdot \frac{1}{s+0,5}$$

$$H(s) = \frac{1}{s(s+0,5)} = \frac{A}{s} + \frac{B}{s+0,5} = \frac{As + A0{,}5 + Bs}{s(s+0,5)} \quad \begin{cases} A + B = 0 \Rightarrow B = -2 \\ 0,5A = 1 \Rightarrow A = 2 \end{cases}$$

$$H(s) = \frac{2}{s} - \frac{2}{s+0,5} \Rightarrow \boxed{h(t) = (2 - 2e^{-0.5t})}$$

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