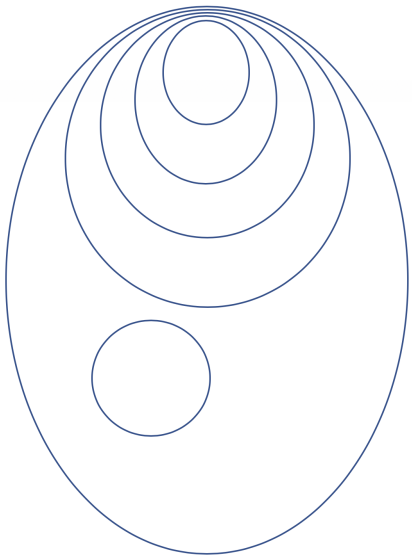
Irrational Numbers - AN2 I

The Number Systems

In this course we will talk about six different number systems:

1. Natural Numbers: $\mathbb{N}$
2. Whole Numbers: $\mathbb{W}$
3. Integers: $\mathbb{Z}$
4. Rational Numbers: $\mathbb{Q}$
5. Irrational Numbers $\mathbb{I}$
6. Real Numbers: $\mathbb{R}$



What Is A Rational Number?

- A **rational number** is any number that can be expressed as the quotient of two integers.
- A *repeating*, or *terminating* integer.
  i.e. $1.\overline{56721343}$, or 1.12, or $\sqrt{.49}$

What Is An Irrational Number?

- An **irrational number** is a number that cannot be expressed as the quotient of two integers.
- A *non-repeating*, and *non-terminating* decimal.
  i.e 1.12349123639..., π, and $\sqrt{2}$

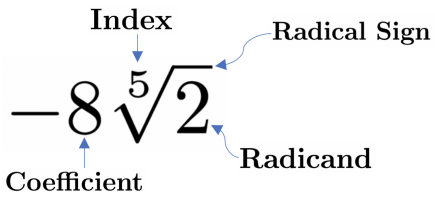
Practice:

Identify Whether each number is Rational, or Irrational:

i. $\sqrt{34}$
ii. $\frac{1}{2}$
iii. $\frac{36}{2}$
iv. $\frac{\pi}{36}$
v. $0.\overline{654}$
vi. $\sqrt{0.75}$
vii. $\sqrt{9}$
viii. $\sqrt{0.25}$
ix. $0.\overline{333}$

What Is A Radical?

- A **radical** is a number that involves a root
- A **mixed radical** is a number that is part radical, and part integer. *Mixed*:



- An **entire radical** An entire radical is a number that only has a root. The *entire* number is under the radical.

Expressing An *Entire Radical* As A *Mixed Radical* In *Simplest Form*

In order to express an entire radical as a mixed radical, we use prime factorization:
Example: $\sqrt{48}$

1. Prime factorization: $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
2. Identify groups based on index (in this case 2): $48 = 2^2 \cdot 2^2 \cdot 3$
3. Write with radical: $\sqrt{48} = \sqrt{2^2 \cdot 2^2 \cdot 3}$
4. Note that $\sqrt{a}\sqrt{b} = \sqrt{ab}$, so $\sqrt{48} = \sqrt{2^2}\sqrt{2^2}\sqrt{3}$
5. Simplifying: $\sqrt{48} = 2 \cdot 2 \cdot \sqrt{3}$
6. So $\sqrt{48} = 4\sqrt{3}$

i. $\sqrt{32}$

ii. $\sqrt[3]{40}$

Practice:

Convert The Following To Mixed Radicals In Simplest Form:

i. $\sqrt{27}$

iii. $\sqrt{75}$

ii. $\sqrt[3]{80}$

iv. $\sqrt[3]{128}$

Expressing A *Mixed Radical* as an *Entire Radical.*

Example: $2\sqrt{5}$:

1. Begin by rewriting coefficient as as a radical: $2\sqrt{5} = \sqrt{2^2}\sqrt{5} = \sqrt{4}\sqrt{5}$
2. Next, remember $\sqrt{a}\sqrt{b} = \sqrt{ab}$, so $\sqrt{4}\sqrt{5} = \sqrt{4 \cdot 5}$
3. Next, simplify: $\sqrt{4 \cdot 5} = \sqrt{20}$
4. So $2\sqrt{5} = \sqrt{20}$.

i. $5\sqrt{5}$

ii. $2\sqrt{7}$

Practice:

Convert The Following Into Entire Radicals:

i. $5\sqrt{7}$

iv. $10\sqrt{2}$

ii. $2\sqrt{6}$

v. $3\sqrt[3]{2}$

iii. $3\sqrt{13}$

vi. $2\sqrt[3]{5}$