
Algebra And Number - AN3 - Exponent Laws

Proving Exponent Laws

1. Prove That $(a^m)(a^n) = a^{m+n}$:

m	n	$(a^m)(a^n)$	a^{m+n}
8	2	$(a^8)(a^2) = (a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a) \cdot (a \cdot a)$	a^{10}
5	4		
3	1		
4	3		
3	2		
5	3		
7	2		

Proof:

$$(a^m)(a^n) = \underbrace{a \cdot a \cdot a \cdots a}_m \cdot \underbrace{a \cdot a \cdot a \cdots a}_n = a^{m+n}$$

Practice

Simplify Each Of The Following:

i. x^5x^7

iii. $\kappa^{41}\kappa^{23}$

v. $\varphi^4\varphi^{234}$

vii. $\heartsuit^{12}\heartsuit^3$

ii. m^3m^{13}

iv. $\alpha^3\alpha^7\alpha^{20}$

vi. $\tau^{16}\tau^{12}$

viii. $\alpha^5\alpha^{15}\alpha^2$

2. **Prove That** $a^m \div a^n = a^{m-n}$:

m	n	$\frac{a^m}{a^n}$	a^{m-n}
8	2	$\frac{a^8}{a^2} = \frac{\cancel{a} \cdot \cancel{a} \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a}{\cancel{a} \cdot \cancel{a}}$	a^6
5	4		
3	1		
4	3		
3	2		

Proof:

$$\frac{a^m}{a^n} = \underline{\hspace{2cm}} = a^{m-n}$$

Practice

Simplify Each Of The Following:

i. $\frac{x^{15}}{x^7}$

iii. $\frac{\phi^{10}}{\phi^3}$

v. $\frac{\pi^{300}}{\pi^{100}}$

vii. $\frac{\omega^{435}}{\omega^{17}\omega^{12}}$

ii. $\frac{\gamma^{17}}{\gamma^7}$

iv. $\frac{2^8}{2^4}$

vi. $\frac{y^{20}}{y^{10}y^7}$

viii. $\frac{r^{1234}}{r^{456}}$

3. **Prove That** $(a^m)^n = a^{mn}$:

m	n	$(a^m)^n$	a^{mn}
8	2	$(a^8)^2 = (a^8) \cdot (a^8) = (a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a) \cdot (a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a)$	a^{16}
5	4		
3	1		
4	3		
3	2		
5	3		
7	3		
4	2		
10	3		

Proof:

$$(a^m)^n = \quad \quad \quad = a^{mn}$$

Practice

Simplify Each Of The Following:

i. $(x^{10})^2$

iii. $(\heartsuit^{17})^3$

v. $(z^2 z^2)^3$

vii. $(\gamma^5)^a$

ii. $(\alpha^9)^2$

iv. $(m^{15})^3$

vi. $(\kappa^4)^2$

viii. $(\phi^5 \phi^3)^4$

4. **Prove That** $(ab)^m = a^m b^m$:

m	$(ab)^m$	$a^m b^m$
2	$(ab)^2 = (ab) \cdot (ab) = a \cdot b \cdot a \cdot b = a \cdot a \cdot b \cdot b$	$a^2 b^2$
3		
4		
5		
6		
7		
8		
9		
10		

Proof:

$$(ab)^m = \qquad \qquad \qquad = a^m b^m$$

Practice

Simplify Each Of The Following:

- | | | | |
|-----------------|---------------------------|-----------------------------|-----------------------|
| i. $(xy)^5$ | iii. $(\alpha\beta)^{10}$ | v. $(\gamma^2 x)^2$ | vii. $(x^3 y^2)^{10}$ |
| ii. $(xy)^{17}$ | iv. $(ab)^7$ | vi. $(\omega^3 \alpha^2)^5$ | viii. $(xyz)^7$ |

5. **Prove That** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$:

n	$\left(\frac{a}{b}\right)^n$	$\left(\frac{a^n}{b^n}\right)$
2	$\left(\frac{a}{b}\right)^2 = \left(\frac{a}{b}\right) \cdot \left(\frac{a}{b}\right) = \frac{a \cdot a}{b \cdot b}$	$\left(\frac{a^2}{b^2}\right)$
3		
4		
5		
6		
7		

Proof:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Practice

i. $\left(\frac{a}{b}\right)^{12}$

iii. $\left(\frac{x}{z}\right)^5$

v. $\left(\frac{m}{n}\right)^{54}$

vii. $\left(\frac{\gamma^4 \alpha}{\beta}\right)^3$

ii. $\left(\frac{\alpha}{\beta}\right)^{25}$

iv. $\left(\frac{x}{z}\right)^5$

vi. $\left(\frac{\alpha \beta}{x}\right)^{13}$

viii. $\left(\frac{\gamma^3}{\beta^2 \alpha}\right)^3$