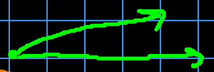
Section 5 dot products and cross products:
Part 63: Dot products

$\vec{a} \cdot \vec{b}$

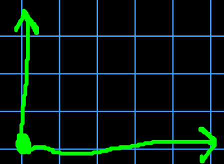
dot product

How much $\vec{a}$ and $\vec{b}$ point in the same direction



$\vec{a} \times \vec{b}$

Cross product
The length shows how much a and b point in different directions



2 dimensions

$\vec{a} = (a_1, a_2)$

$\vec{b} = (b_1, b_2)$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$

3 dimension

$\vec{a} = (a_1, a_2, a_3)$

$\vec{b} = (b_1, b_2, b_3)$

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 b_2 + a_3 b_3$

$\vec{b} = (-1, 2, 3)$

$\|\vec{b}\|^2 = \vec{b}\vec{b}$

$= (-1)(-1) + 2 \cdot 2 + (3)(3)$

$= 1 + 4 + 9 = 14$

$\sqrt{\|\vec{b}\|^2} = \sqrt{14} \Rightarrow \|\vec{b}\| = \sqrt{14}$

$\vec{a} = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

must always be row matrix times col matrix, so change: $\vec{a} \cdot \vec{b} = [a_1, a_2] \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

Dot product of itself below

$\|\vec{a}\| = \sqrt{a_1^2, a_2^2} \longrightarrow \|a\|^2 = \boxed{a_1^2 + a_2^2} = \vec{a} \cdot \vec{a} = a_1 \cdot a_1 + a_2 \cdot a_2 = a_1^2 + a_2^2$

(Set of rules for dot products)

1) Communicative $\rightarrow \vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 = b_1a_1 + b_2a_2$

2) Distributive

$\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

3) Associative

$(3\vec{r})\cdot\vec{s} = 3(\vec{r}\cdot\vec{s})$

Quiz 24: Dot Products

Q1: Find the dot product:

$\vec{x} = (5,-1)$

$\vec{y} = (3,2)$

$\vec{x}\cdot\vec{y} = 5\cdot3 + -1\cdot2$

$= 15 - 2 = 13$

Q2: Find the dot product:

$\vec{x} = (-4, 0, 12)$

$\vec{y} = (9, -12, 8)$

$\vec{x}\cdot\vec{y} = (-4\cdot9) + (0\cdot-12) + (12\cdot8)$

$-36 + 96 = \boxed{60}$

Q3: Use the dot prouct to find

$3\vec{x}\cdot(-2\vec{y} - \vec{z})$

$\vec{x} = (-4, -2, 7)$

$\vec{y} = (6, -1, -10)$

$\vec{z} = (3, -2, 0)$

$3\vec{x} = (-12, -6, 21)$

$-2\vec{y} = (-12, 2, 20)$

$-2\vec{y} - \vec{z} \Rightarrow \begin{bmatrix}-12\\2\\20\end{bmatrix} - \begin{bmatrix}3\\-2\\0\end{bmatrix} = \begin{bmatrix}-15\\4\\20\end{bmatrix}$

$\begin{bmatrix}-12 & -6 & 21\end{bmatrix}\begin{bmatrix}-15\\4\\20\end{bmatrix} = 180 - 24 + 420$

$= 576$

Section ~~4~~5.65: Cauchy-Schwarz Inequality

2 different vectors:

$$|\vec{a} \cdot \vec{b}| \leq \|\vec{a}\| \cdot \|\vec{b}\|$$

Only equal when $\vec{a}$ and $\vec{b}$ are some scalars of each other:

$=$

$\vec{a} = c \cdot \vec{b}$ $\quad c = 2$

$\vec{b} = c \cdot \vec{a}$ $\quad c = 1/2$

L.D.

$<$ L.I.

eg:

$\hat{i} = (1, 0)$

$\hat{j} = (0, 1)$

$\|i\| = \sqrt{1^2 + 0^2}$

$= \sqrt{1}$

$\|i\| = 1$ $\quad \|j\| = 1$

$|i \cdot j| = (1 \cdot 0) + (0 \cdot 1)$

$|i \cdot j| = 0$

$|i \cdot j| \leq \|i\| \cdot \|j\|$

$0 \leq 1$

$0 < 1$

Because we have < , we know that this relationship is linearly dependant

Another example, using known linearly dependant vectors:

$\vec{a} = (2, 1)$ $\quad \vec{b} = (-6, -3)$

$\|\vec{a}\| = \sqrt{2^2 + 1^2}$

$= \sqrt{4 + 1} = \sqrt{5}$

$\|\vec{b}\| = \sqrt{36 + 9} = \sqrt{45} = \sqrt{9} \cdot \sqrt{5}$

$\|\vec{a} \cdot \vec{b}\| \leq \|\vec{a}\| \|\vec{b}\|$

$|(2 \cdot -6) + (1 \cdot -3)| \leq \sqrt{45} \cdot \sqrt{5}$

$15 \leq \sqrt{9} \cdot 5$

$15 = 15$

Because these are equal , thus vectors $\vec{a}$ and $\vec{b}$ are linearly dipendant

Section ~~4~~ 5: Quiz 25: Cachy-Schwarz inequality

Recall: $|\vec{a}\cdot\vec{b}| \leq \|\vec{a}\|\cdot\|\vec{b}\|$

if : $|a\cdot b| < \|a\|\,\|b\|$; then L.I.

if : $|a\cdot b| = \|a\|\,\|b\|$ then L.D.

Q1

Use Cauchy-Schwarz inequality to say which vector set it Linearly Independant:

Try option D, as A, B, and C visually appear to be Linearly Dependant

$\vec{u} = (6,2) \quad \vec{v} = (-5,1)$

$\|-28\| \leq \|\vec{u}\|\,\|\vec{v}\|$

$\vec{a}\cdot\vec{b} = (6\cdot-5)+(2\cdot1)$

$= -30+2$

$= -28$

$\|\vec{u}\| = \sqrt{40} =$

$\|\vec{v}\| = \sqrt{-5^2+1^2}$

$= \sqrt{26}$

$28 \leq \sqrt{40}\cdot\sqrt{26} \implies 28 \leq \sqrt{1040}$

$28 < 32.2$

Since the absolute value of the dot product of $\vec{u}$ and $\vec{v}$ is less than the product of their lengths, we determine that the the set of vectors, $\vec{u}$, $\vec{v}$ are Linearly Dependar

Q2 Use the Cauchy-Schwarz inequality to say which vector set is Linearly Independent:

Try option D first, as vector sets A, B, and C appear visually to be Linearly Dependant

$\vec{u} = (-5, -5)$ $\quad \vec{v} = (10, -5)$

$|\vec{a} \cdot \vec{b}| = |(-5 \cdot 10) + (-5 \cdot -5)|$

$= |-50 + 25|$

$|\vec{a} \cdot \vec{b}| = 25$

$\|\vec{u}\| = \sqrt{50}$ $\quad \|\vec{v}\| = \sqrt{50}$

$\|u\| \cdot \|v\| = \sqrt{50} \cdot \sqrt{50}$

$= 50$

$25 \leq 50$

$25 < 50$

Thus since the absolute value of the dot productr of vectors $\vec{a}$ and $\vec{b}$ are less than the product of the length of $\vec{a}$ and $\vec{b}$, we can conclude that this set of vectors are Linearly Independent

Q3. Use the Cauchy-Schwarz inequality to say which vector set is Linearly Independant:

Try vector set B first, as A, C, & D all appear to be Linearly Dependent

B $\vec{u} = (6, 5)$ $\vec{v} = (4, 0)$

Set up the Cauchy-Scwarz Inequality:

$$|\vec{u}\cdot\vec{v}| \leq \|\vec{u}\| \cdot \|\vec{v}\|$$

$$|(6\cdot 4) + (5\cdot 0)| \leq \sqrt{36+25} \cdot \sqrt{16+0}$$

$$24 \leq \sqrt{61} \cdot 4$$

$$24 \leq 7.81 \cdot 4$$

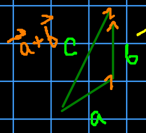
$$24 < \approx 31.24$$

As the absolute dot product of vectors $\vec{u}$ and $\vec{v}$ is less than the length of vectors $\vec{u}$ and $\vec{v}$, we can say that this set of vectors is Linearly Independent

B

5.67 vector triangle inequality

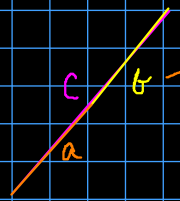
Can test for L.I. & L.D.



$\rightarrow c < a + b$ (L.I.)

$c \leq a + b$

$\leq$



$\rightarrow c = a + b$

(L.D.)

Triangle inequality : only works in $\mathbb{R}^2$ space

WE need vector triangle inequality

$$\|\vec{a}+\vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$$

Test for $\mathbb{R}^4$: $\vec{a} = (1, 1, 2, 2)$

$\vec{b} = (1, 0, -3, 1)$

$$\|\vec{a}\| = \sqrt{1^2+1^2+2^2+2^2} = \sqrt{10}$$

$$\|\vec{b}\| = \sqrt{1^2+3^2+1^2} = \sqrt{11}$$

$$\|a+b\| = (1+1, 1+0, 2+-3, 2+1)$$

$$= (2, 1, -1, 3)$$

$$= \sqrt{2^2+1^2+1^2+3^2} = \sqrt{15}$$

$\sqrt{15} \leq \sqrt{10} + \sqrt{11}$

$3.87 \leq 6.47$

$3.87 < 6.47$

Length of the sum of vectors < sum of length of vectors

Thus this set is Linearly Independent

Section 5 Quiz 26: Vector Triangle Inequality

Q1: Use vector triangle inequality to say which vector set is Linearly Independent:

Try vector set D first, as A, B, C seem to have vectors that are multiples of each one another (possibly Linearly D.)

$\vec{u} = (-9, -8) \quad \vec{v} = (-5, 6)$

$\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$

$\|\vec{u}\| = \sqrt{-9^2} = 9$

$\vec{u} + \vec{v} = ((-9 + -5), (-8 + 6))$

$= (-14, -2)$

$\|\vec{u} + \vec{v}\| = \sqrt{-14^2 + -2^2} = \sqrt{200}$

$\|\vec{v}\| = \sqrt{-5^2 + 6^2} = \sqrt{25 + 36}$

$\|\vec{v}\| = \sqrt{61}$

$\|\vec{u} + \vec{v}\| \cong 14.14$

$14.14 \leq \sqrt{61} + 9$

$14.14 \leq 16.81 \longrightarrow 14.14 < 16.81$

Choice D is linearly independent

Since the length of $(\vec{a} + \vec{b})$ is less than the sum of lengths of $\vec{a} + \vec{b}$, we know that this set of vectors is Linearly Independent

Q2: Use the vector triangle inequality to determine which vector set is linearly independent:

Try ~~A~~ B, as ~~B~~ A, C, D vector sets appear to be multiples of some constant c, infer they may be Linearly Dependent

vector set B: $\vec{u} = (-8, -3)$

$\vec{v} = (2, 8)$

find $\|\vec{u} + \vec{v}\|$

$(-8+2, -3+8)$

$(-6, 5) = \vec{u} + \vec{v}$

$\|\vec{u}+\vec{v}\| = \sqrt{-6^2 + 5^2}$

$= \sqrt{61}$

$\|\vec{u}+\vec{v}\| \cong 7.81$

find $\|\vec{u}\|$

$\|\vec{u}\| = \sqrt{-8^2 + -3^2} = \sqrt{73}$

find $\|\vec{v}\|$

$\|\vec{v}\| = \sqrt{2^2 + 8^2} = \sqrt{68}$

$7.81 \leq \sqrt{73} + \sqrt{68}$

$7.81 < \sqrt{73} + 68$

Since length of $(\vec{a} + \vec{b})$ is less than the sum of the lengths of $\vec{a}$ and $\vec{b}$, we know that this set of vectors is Linearly Independent

Q3: Find which vector set is linearly independent using vector triangle inequality

Try set C, as A, B, and D appear to be multiples of each other, infer probably these are Linearly Dependent sets

C = $\vec{u} = (5, -3)$

$\vec{v} = (0, 6)$

$\|u + v\| = \sqrt{25 + 9}$

$5.83 \cong \sqrt{34}$

$5.83 < 11.83$

$\|u\| = \sqrt{25+9}$

$= 5.83$

$\|v\| = \sqrt{36}$

$= 6$

Since length of $(\vec{a}+\vec{b})$ is less than the sum of length of $\vec{a}$ and length of $\vec{b}$,

We determined that this set of vectors is Linearly Independent

C

5.69 Angle Between Vectors

$\vec{a}\cdot\vec{b} = \|\vec{a}\|\cdot\|\vec{b}\|\cos\theta$ = formula always finds smallest angle

$\cos(90) = \emptyset$

$a\cdot b = \|a\|\|b\|\cdot\emptyset$

$a\cdot b = \emptyset$

Means vectors are perpendicular in 2d space or orthogonal in > 2d space

$\vec{a}\cdot\vec{b} = \|\vec{a}\|\cdot\|\vec{b}\|\cdot\cos\theta$

$a\cdot b = \emptyset$ always means $\theta = 90°$

$\mathbb{R}^4$ vectors:

$\vec{a} = (1, 0, -2, 0)$

$\vec{b} = (0, 3, 0, 1)$

$\vec{a}\cdot\vec{b} = ((1)(0) + (0)(3) + (-2)(0) + (0)(1))$

$= (0, 0, 0, 0)$ Orthogonal in $\mathbb{R}^4$

$0 = \|\vec{a}\|\|\vec{b}\|\cos\theta$

$0 = \cos\theta$

$\theta = 90°$

$\vec{a} = (0, 3)$ $\vec{b} = (-1, -4)$

$\vec{a}\cdot\vec{b} = (0\cdot -1 + 3\cdot -4)$

$\vec{a}\cdot\vec{b} = -12$

$\|\vec{a}\| = \sqrt{0^2+3^2} = 3$

$\|\vec{b}\| = \sqrt{-1^2+4^2} = \sqrt{17}$

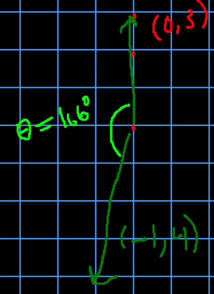
$-12 = 3\sqrt{17}\cos\theta$

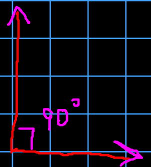
$\cos\theta = \frac{-4}{\sqrt{17}}$

$\arccos(\cos\theta) = \arccos(-4/\sqrt{17})$

$\theta = \arccos(-4/\sqrt{17})$

$\theta = 166°$





Quiz 27: Angle Between Vectors

Q1: Say whether or not the vectors are orthogonal:

$\vec{u} = i + j + 2k$

$\vec{v} = 2i + 2j + 4k$

Plug into equation: $\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos(\theta)$

$\vec{a} \cdot \vec{b} = (1 \cdot 2) + (1 \cdot 2) + (2)(4)$

$\vec{a} \cdot \vec{b} = 12$

since $\vec{a} * \vec{b} \neq 0$, these vectors are not orthogonal

$\vec{u} = (1, 1, 2)$

$\vec{v} = (2, 2, 4)$

$\|\vec{u}\| = \sqrt{1^2 + 1^2 + 2^2}$

$= \sqrt{6} \cong 2.45$

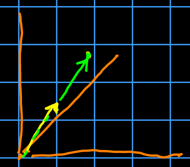
$\|\vec{v}\| = \sqrt{24} \cong 4.89$

$12 = (2.45)(4.89)\cos(\theta)$

$1 \cong \cos(\theta)$

$\cos^{-1}(\cos(\theta)) = \cos^{-1}(1)$

$\theta \cong (1)$



Q2) Find the angle between the vectors:

First consider the formula for the angle between two vectors:

$\vec{a}\cdot\vec{b} = \|\vec{a}\|\cdot\|\vec{b}\|\cdot\cos(\theta)$ $\quad \theta = ?$

$\vec{a} = (2,0,-1)$

$\vec{b} = (-1,4,2)$

$\vec{a}\cdot\vec{b} = (2)(-1) + (0)(4) + (-1)(2)$

$\vec{a}\cdot\vec{b} = -4$

$\|\vec{a}\| = \sqrt{2^2+0^2+1^2} = \sqrt{5} = \|\vec{a}\|$

$\|\vec{b}\| = \sqrt{-1^2+4^2+2^2} = \sqrt{21} = \|\vec{b}\|$

$-4 = \sqrt{5}\cdot\sqrt{21}\cdot\cos(\theta)$

$\dfrac{-4}{\sqrt{5}\cdot\sqrt{21}} = \cos(\theta)$

$\cos^{-1}(\cos(\theta)) = \left(\dfrac{-4}{\sqrt{5}\cdot\sqrt{21}}\right)\cos^{-1}$

$\theta \hat{=} \arcsin(-0.39036)$

$\theta \cong 113°$

Q2: Find the angle between the vectors:

$\vec{a} = (1, -3, 1)$

$\vec{b} = (0, 6, -2)$

$\vec{a} \cdot \vec{b} = 1 \cdot 0 + -3 \cdot 6 + 1 \cdot -2$

$= 0 - 18 - 2$

$\vec{a} \cdot \vec{b} = -20$

$\|\vec{a}\| = \sqrt{1^2 + -3^2 + 1^2}$

$\|\vec{a}\| = \sqrt{11}$

$\|\vec{b}\| = \sqrt{0^2 + 6^2 + -2^2}$

$\|\vec{b}\| = \sqrt{40}$

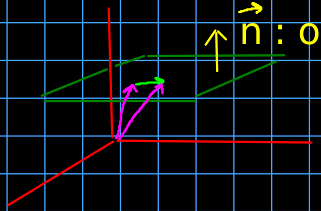
$-19 = \sqrt{11} \cdot \sqrt{40} \cdot \cos(\theta)$

$\cos(\theta) = -\dfrac{20}{\sqrt{11}\sqrt{40}} = \dfrac{-20}{\sqrt{440}} \cong -0.9535$

$\cos^{-1}(\cos\theta) \cong \cos^{-1}(-0.9535)$

$\theta \cong 162^\circ$

5.71 Equation of a plane, and normal vectors



$\vec{n}$: orthogonal to the plane

x_0 and x are the two magenta vectors

$\vec{x_0} = (x_0, y_0, z_0)$

$\vec{x} = (x, y, z)$

x - x_0 gives use the green vector

$$x - x_0 = \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix} \longrightarrow \vec{g}$$

$\vec{n} \cdot \vec{g} = 0$ orthogonal vectors' dot product is zero

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix} = 0$$

$$\left[a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \right]$$

: standard form of the equation of a plane

$\vec{n} = (2, 5, -3)$ — a, b, c

$(1, 0, -2)$ — x_0, y_0, z_0

$$2(x - x_0) + 5(y - 0) - 3(z + 2)$$

$$2x - 2 + 5y - 3z - 6$$

$$2x + 5y - 3z = 8 \longrightarrow \vec{n} = (2, 5, -3)$$

$$Ax + By + Cz = D$$

Quiz 28: Equation of a plane, and normal vectors

q1) What is the normal vector to the plane?

$3x + 5y + 9z = (-26)$ $\vec{n} = (3, 5, 9)$

Q2: Find the equation of the plane, given a point in the plane and the normal vector to the plane:

$(x, y, z) = (5, -8, -9)$

$\vec{n} = (8, 2, -1)$

plug in values given into standard form of equation for a plane:

$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ where: $\vec{n}\,(8, 2, -1)$
A B C

$8(x - 5) + 2(y + 8) - (z + 9) = 0$

$8x - 40 + 2y + 16 - z - 9 = 0$

$8x + 2y - z = 33$

Q3: Find the equation of the plane, given a point in the plane and the normal vector to the plane:

$$(x, y, z) = (-5, 3, -3)$$
$$\quad\quad\quad\quad\quad\quad x_0 \quad y_0 \quad z_0$$

$$\vec{n} = (-4, -3, 9)$$
$$\quad\quad\quad A \quad B \quad C$$

$$-4(x+5) - 3(y-3) + 9(z+3) = 0$$

$$-4x - 20 - 3y + 9 + 9z + 27 = 0$$

$$-4x - 3y + 9z = -16$$

Section 5.73: Cross Products

Determinant: Not a matrix

$$\vec{a} \cdot \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

(could be either direction)

$\vec{a} \cdot \vec{b}$ (orthogonal to both)

no matter how big determinant is, want to break it down to 2x2 size

$$= i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Cross multiply determinants:

$$= i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

Example:

$\vec{a} = (1, 0, 2)$

$\vec{b} = (-2, 1, 0)$

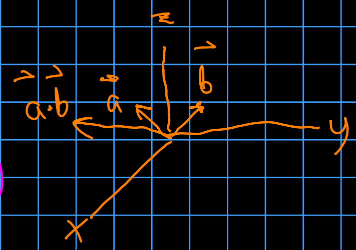
$$\begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ -2 & 1 & 0 \end{vmatrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$= i \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} - j \begin{vmatrix} 1 & 2 \\ -2 & 0 \end{vmatrix} + k \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix}$$

$$= i(0 - 2) - j(0 + 4) + k(1 - 0)$$

$$= -2i - 4j + k$$

$$\vec{a} \cdot \vec{b} = (-2, -4, +1)$$



① $\|a \times b\| = \|a\| \|b\| \sin\theta$

② $\|a \times b\| = \sqrt{-2^2 + -4^2 + 1^2}$

$= \sqrt{21}$

$\boxed{\|\vec{a}\vec{b}\| \cong 4.6}$

Q1: Find the cross product $\vec{a} \times \vec{b}$:

$\vec{a} = (1,-1,1) \quad \vec{b} = (-2,1,2)$

create a determinate:

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ -2 & 1 & 2 \end{vmatrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Break a 3x3 determinate into 2x2 determinates:

$$= i\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} + k\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix}$$

$$i(-2-1) - j(2+2) + k(1-2)$$

$$= -3i - 4j - k$$

$$\vec{a}\cdot\vec{b} = (-3, -4, -1)$$

Q2: Find the cross product $\vec{a} \times \vec{b}$

$\vec{a} = (4,2,0) \quad \vec{b} = (-1,-3,1)$

$$\begin{vmatrix} i & j & k \\ 4 & 2 & 0 \\ -1 & -3 & 1 \end{vmatrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$i\begin{vmatrix} 2 & 0 \\ -3 & 1 \end{vmatrix} - j\begin{vmatrix} 4 & 0 \\ -1 & 1 \end{vmatrix} + k\begin{vmatrix} 4 & 2 \\ -1 & -3 \end{vmatrix}$$

$$i(2-0) - j(4-0) + k(-12+2)$$

$$2i - 4j - 10k$$

$$\vec{a}\times\vec{b} = (2, -4, -10)$$

Q3: Find the cross product $\vec{a} \times \vec{b}$:

$\vec{a} = (6, 7, -5) \quad \vec{b} = (8, 7, -11)$

$$\begin{vmatrix} i & j & k \\ 6 & 7 & -5 \\ 8 & 7 & -11 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$i \begin{vmatrix} 7 & -5 \\ 7 & -11 \end{vmatrix} - j \begin{vmatrix} 6 & -5 \\ 8 & -11 \end{vmatrix} + k \begin{vmatrix} 6 & 7 \\ 8 & 7 \end{vmatrix}$$

$$i(-77+35) - j(-66+40) + k(42-56)$$

$$-42i + 26j - 14k$$

$$\vec{a} \times \vec{b} = (-42, +26, -14)$$

5.76 Dot and Cross products as opposite Ideas:

$\|\vec{a}\| = 5$

$\|\vec{b}\| = 2$

$0 \rightarrow \theta = 90^\circ$

$10 \rightarrow \theta = 0^\circ$

$-10 \rightarrow \theta = 180^\circ$

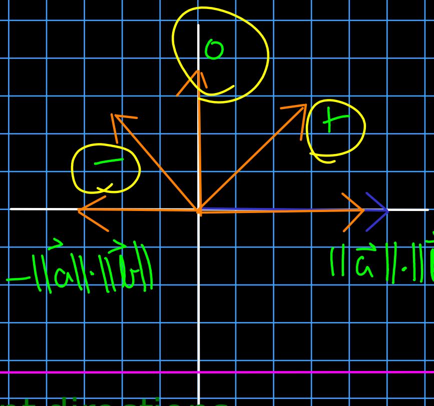
Dot product tells how much two vectors move in same direction

$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos\theta$

For $\theta = 90^\circ$: $\cos(90) = 0$: $\vec{a} \cdot \vec{b} = 0$

For $\theta = 0^\circ$: $\cos(0) = 1$: $\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\|$

For $\theta = 180^\circ$: $\cos(180) = -1$: $\vec{a} \cdot \vec{b} = -\|\vec{a}\| \cdot \|\vec{b}\|$



$-10 = -\|\vec{a}\| \cdot \|\vec{b}\|$

$\|\vec{a}\| \cdot \|\vec{b}\| = 10$

Cross Product: it measures how much vectors move in different directions

$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin\theta$

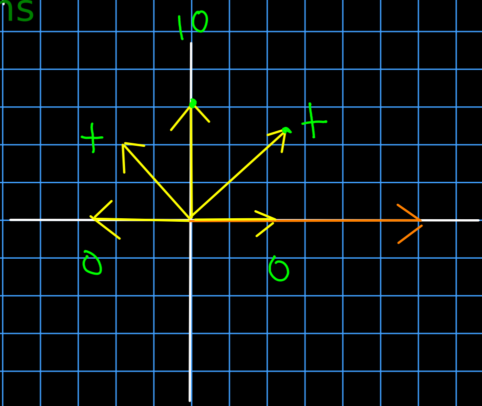
For $\theta = 90^\circ$: $\sin(90) = 1$: $\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \cdot \|\vec{b}\|$

For $\theta = 0^\circ$: $\sin(0) = 0$: $\|\vec{a} \times \vec{b}\| = 0$

For $\theta = 180$: $\sin(180) = 0$: $\|\vec{a} \times \vec{b}\| = 0$

$\|\vec{a}\| = 5$

$\|\vec{b}\| = 2$

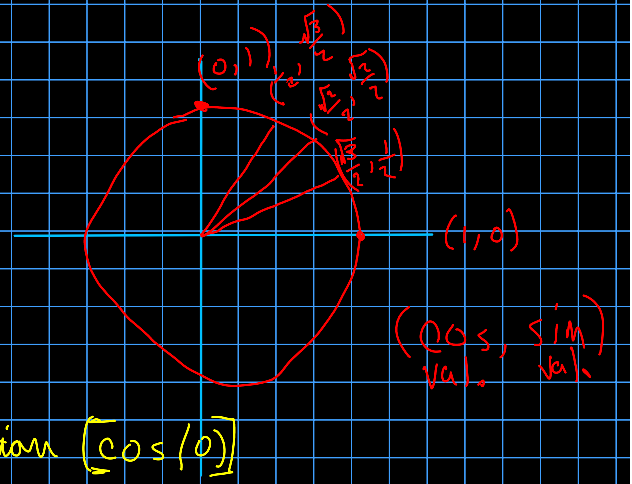


Quiz 30: Dot and cross product as opposite ideas

Q1) When is the dot product of two vectors maximized?

Let's suppose:

$$\vec{a}\cdot\vec{b} = \|\vec{a}\|\cdot\|\vec{b}\|\cos\theta$$



When $\cos\theta = 1$ is when we can maximize the dot product.

$$\cos(0) = 1$$

Thus, where $\vec{a}$ & $\vec{b}$ are in the exact same direction $[\cos(0)]$ can we maximize the value of $\vec{a}\cdot\vec{b}$

Q2: When two vectors point in exactly the same direction...

Length of cross products: $\|\vec{a} \times \vec{b}\|$

B The length of the cross products is $\emptyset$

Q3: Describe the dot product and the length of the cross product of the vector pair:

$\vec{v} = (3, 4) \quad \vec{w} = (6, 8)$

dot product:

$\vec{a}\cdot\vec{b} = \|\vec{a}\|\cdot\|\vec{b}\|\cos\theta \qquad \cos\theta = 1$

$(3\cdot 6) + (8\cdot 4)$

$\vec{a}\cdot\vec{b} = 50 = 50\cdot\cos\theta$

$\|\vec{a}\| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$\|\vec{b}\| = \sqrt{6^2 + 8^2} = \sqrt{36+64}$

$= 10$

$\|\vec{v} \times \vec{w}\| = \|v\|\|w\| \sin\theta$

$= 5 \cdot 10 \sin\theta$

$= 50 \sin\theta$

$= 50 \cdot \sin(0)$

$= 50 \cdot 0$

$\|\vec{v} \times \vec{w}\| = 0$

These 2 vectors point in the same direction