4:58

Basis:

Subpace:
- Vector set that is
    1) Closed under multiplication
    2) Closed under addition

Basis
    1) Spans the subspace
    2) Linearly independent

e $\mathbb{R}^2$: 2 vectors, L.I., in $\mathbb{R}^2$

$V = \left\{ \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\3\end{bmatrix}, \begin{bmatrix}4\\6\end{bmatrix} \right\}$

Basis for $\mathbb{R}^2$?

$V = \left\{ \begin{bmatrix}2\\-3\end{bmatrix}, \begin{bmatrix}5\\1\end{bmatrix} \right\}$

$c_1 \begin{bmatrix}2\\-3\end{bmatrix} + c_2 \begin{bmatrix}5\\1\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$

$\left[\begin{array}{cc|c} 2 & 5 & x \\ -3 & 1 & y \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 5/2 & x/2 \\ 0 & 17/2 & y + \frac{3x}{2} \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 5/2 & x/2 \\ 0 & 1 & \frac{2y}{17} + \frac{3x}{17} \end{array}\right]$

$R_1 - \frac{5}{2} \cdot R_2 \rightarrow R_1$

$\left[\begin{array}{cc|c} 1 & 0 & \frac{1}{17}x - \frac{5}{17}y \\ 0 & 1 & \frac{2y}{17} + \frac{3x}{17} \end{array}\right]$

$V = \left\{ \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\3\end{bmatrix} \right\}$

$\frac{5}{2}\left(\frac{2y}{17} + \frac{3x}{17}\right)$

$\frac{x}{2} - \left(\frac{10y}{34} + \frac{15x}{34}\right)$

$\frac{17x}{34} - \frac{10y}{34} - \frac{15x}{34}$

$c_1 = \frac{1}{17}x - \frac{5}{17}y$

$c_2 = \frac{2}{17}x + \frac{3x}{17}$

$= \frac{1}{17}x - \frac{5}{17}y$

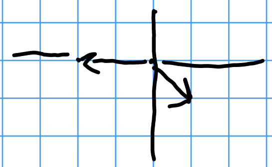
We can choose any x and y,
and x, y will tell us what to apply into
c1 and c23

Plug in 0, 0 for x, y to determine if V set is L.I.

$c_1 = 0$; $c_2 = 0$

Quiz 23: Basis

Q1: Does the span pf V form a basis for $\mathbb{R}^2$?



$$V = \left\{ \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

' set up as a augmented matrix and set augment = x, y:

$$\left[\begin{array}{cc|c} -2 & 1 & x \\ 0 & -1 & y \end{array}\right]$$

$-\frac{1}{2}R_1 \to R_1$

$$\left[\begin{array}{cc|c} 1 & 1/2 & -x/2 \\ 0 & -1 & y \end{array}\right]$$

$-R_2 \to R_2$

$$\left[\begin{array}{cc|c} 1 & 1/2 & -x/2 \\ 0 & 1 & -y \end{array}\right]$$

$R_1 - \frac{1}{2}R_2 \to R_1$

$$\left[\begin{array}{cc|c} 1 & 0 & y/2 - x/2 \\ 0 & 1 & -y \end{array}\right]$$

$-x/2 - \frac{1}{2}(-y)$

$\frac{y}{2} - x/2$

build a system of equations:

$$c_1 \begin{bmatrix} -2 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

plug in 0, 0 for x, y:

$c_1 = \frac{0}{2} - \frac{0}{2}$

$c_2 = -0$

$c_1 = 0$; $c_2 = 0$ $\to$ This L.I.

Q23 Question 2:
Does the span of V form the basis of $\mathbb{R}^2$? short answer no due to graphing it, lets see with math:

$$V = \left\{ \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -6 \\ 2 \end{bmatrix} \right\}$$

$$\left[\begin{array}{cc|c} 3 & -6 & x \\ -1 & 2 & y \end{array}\right]$$

$$-R_2 \longleftrightarrow R$$

$$\left[\begin{array}{cc|c} 1 & 2 & y \\ 3 & -6 & x \end{array}\right]$$

$$R_2 - 3 \cdot R_1 \longrightarrow R_2$$

$$\left[\begin{array}{cc|c} 1 & 2 & y \\ 0 & -12 & x-3y \end{array}\right] \qquad x-3y$$

$$-\frac{1}{12} R_2 \longrightarrow R_2$$

$$\left[\begin{array}{cc|c} 1 & 2 & y \\ 0 & 1 & -\frac{x}{12} + \frac{3}{12} y \end{array}\right]$$

$$R_1 - 2 \cdot R_2 \longrightarrow R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & \frac{3}{4} y + \frac{x}{6} \\ 0 & 1 & -\frac{x}{12} + \frac{3}{12} y \end{array}\right]$$

$$y - 2\left(-\frac{x}{12} + \frac{3}{12} y\right)$$

$$y + \frac{x}{6} - \frac{1}{4} y$$

$$\frac{3}{4} y + \frac{x}{6}$$

$$c_1 = \frac{3}{4} y + \frac{x}{6}$$

$$c_2 = \frac{1}{4} y - \frac{x}{12}$$

determine if Linear independent:

Set up a system of equations

$$c_1 \begin{bmatrix} 3 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} -6 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$3c_1 - 6c_2 = x$$

$$-c_1 + 2c_2 = y \longrightarrow -3 \cdot c_1 + 6 c_2 = 3y$$

equation 2 (mult. by 3)

add the 2 equations:

$$\begin{aligned} 3c_1 - 6c_2 &= x \\ + \; (-3c_1 + 6c_2) &= 3y \\ \hline 0 &= 3y + x \end{aligned}$$

From this result, we can't get to any vector in $\vec{V} = (x, y)$ in $\mathbb{R}^2$

Question 4: Does the span of V form a basis for $\mathbb{R}^3$?

$$V = \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix} \right\}$$

in order for V to form the basis of $\mathbb{R}^3$
we need to meet two conditions
1) the vectors in V need to span $\mathbb{R}^3$
2) the vectors in V need to be linearly independent

to span R^3 we need to be able to get to any vector in $\mathbb{R}^3$
by using any combination of the vectors in the set

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 0 & 1 & -3 & y \\ -2 & 0 & 1 & z \end{array}\right] \xrightarrow{2R_1 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 0 & 1 & -3 & y \\ 0 & -2 & 1 & 2x+z \end{array}\right] \xrightarrow{R_2 + R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -3 & x+y \\ 0 & 1 & -3 & y \\ 0 & -2 & 1 & 2x+z \end{array}\right]$$

$$\xrightarrow{2R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & x+y \\ 0 & 1 & -3 & y \\ 0 & 0 & -5 & 2x+2y+z \end{array}\right] \xrightarrow{-\frac{1}{5} R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & x+y \\ 0 & 1 & -3 & y \\ 0 & 0 & 1 & -\frac{2}{5}x - \frac{2}{5}y - \frac{1}{5}z \end{array}\right]$$

$$\xrightarrow{3 \cdot R_3 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & x+y \\ 0 & 1 & 0 & -\frac{6}{5}x - \frac{1}{5}y - \frac{3}{5}z \\ 0 & 0 & 1 & -\frac{2}{5}x - \frac{2}{5}y - \frac{1}{5}z \end{array}\right] \xrightarrow{3R_3 + R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{5}x - \frac{1}{5}y - \frac{3}{5}z \\ 0 & 1 & 0 & -\frac{6}{5}x - \frac{1}{5}y - \frac{3}{5}z \\ 0 & 0 & 1 & -\frac{2}{5}x - \frac{2}{5}y - \frac{1}{5}z \end{array}\right]$$

$$3\left(-\frac{2}{5}x - \frac{2}{5}y - \frac{1}{5}z\right)$$

$$-\frac{6}{5}x - \frac{6}{5}y - \frac{3}{5}z$$

Thus we can get to any location in $\mathbb{R}^3$

$$c_1 = \frac{1}{5}x - \frac{1}{5}y - \frac{3}{5}z$$

$$c_2 = -\frac{6}{5}x - \frac{1}{5}y - \frac{3}{5}z$$

$$c_3 = -\frac{2}{5}x - \frac{2}{5}y - \frac{1}{5}z$$

With any combination of x, y, z, we can arrive at c_1 c_2 c_3
We can say that this set of vectors spans $\mathbb{R}^3$

Now let's demonstrate the set of equations are linearly independent by constructing an augmented matrix and obtaining reduced row echelon form:

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ -2 & 0 & 1 & 0 \end{array}\right] \xrightarrow{2 \cdot R_1 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right] \xrightarrow{R_2 + R_1 \to R_1 \ \& \ 2R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & -5 & 0 \end{array}\right]$$

$-\frac{1}{5}R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

rref

$c_1 = 0$

$c_2 = 0$

$c_3 = 0$

c_1 c_2 c_3 are all zero and this is the only solution for the zero vector. Thus we say that this set of 3 vectors are linearly independent. Since we determined earlier that this set also spans R^3

We conclude then that set V forms the basis of $\mathbb{R}^3$