Exercises from Chapter 7

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Problem 1.1. Show that $-1^2 = 1$ in R.

Proof. $-1 + (-1)^2 = -1(1 + -1) = -1(0) = 0$ $\square$

Problem 1.2. Prove that if u is a unit, then so is $-u$.

Proof. Let w be the multiplicative inverse of u and consider $-u \cdot w \cdot w$, by problem 1, when we multiply this by $-u$, we get $(-u)^2 \cdot w^2 = u \cdot w = 1$. Thus, $-u$ is a unit. $\square$

Problem 1.3. Let R be a ring with identity and S be a subring of R containing the identity. Prove that if u is a unit in S, then it is a unit in R. Show by example that the converse is false.

Proof. If u is a unit in S, then there is some v in S, such that $uv = 1$. Since S is a subset of R, v is also in R. So, u is a unit in R. For the second part, consider the ring of rational numbers $\mathbb{Q}$, which has the integers $\mathbb{Z}$ as a subring. Although 2 is a unit in $\mathbb{Q}$, with inverse $1/2$, it is not a unit in $\mathbb{Z}$. $\square$

Problem P. rove that the intersection of any nonempty set of subrings is also a subring.

Proof. Let $S_1, S_2, ...$ be a set of subrings in R and let $S = S_1 \cap S_2 \cap ...$ be the intersection of all the subrings. $\square$