Exercises from Chapter 9

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Problem 1.1. Let $p(x, y, z) = 2x^2y - 3xy^3z + 4y^2z^5$ and $q(x, y, z) = 7x^2 + 5x^2y^3z^4 - 3x^2z^3$ be polynomials in $\mathbb{Z}[x, y, z]$.

(a) Write each p and q as a polynomial in x with coefficients in $\mathbb{Z}[y, z]$.

(b) Find the degree of each of p and q.

(c) Find the degree of p and q in each of the three variables x, y, and z.

(d) Compute pq and find the degree of pq in each of the three variables x, y, and z.

(e) Write pq as a polynomial in the variable z with coefficients in $\mathbb{Z}[x, y]$

Proof. For part a, $p = (2y)x^2 - (3y^3z)x + (4y^2z^5)x^0$ and $q = (7 + 5y^3z^4 - 3z^3)x^2$. For part b, the degree of p is the degree of the last term $2 + 5 = 7$ and the degree of q is the degree of the center second term $2 + 3 + 4 = 9$. For part c, x, y, z degrees of p are $2, 3$, and 5 respectively and for q they are $2, 3$, and 4 respectively. For part d,

$$pq = (2x^2y - 3xy^3z + 4y^2z^5)(7x^2 + 5x^2y^3z^4 - 3x^2z^3)$$

$$= 14x^4y - 21x^3y^3z - 6x^4yz^3 + 9x^3y^3z^4 + 10x^4y^4z^4 + 28x^2y^2z^5 - 15x^3y^6z^5 - 12x^2y^2z^8 + 20x^2y^5z^9$$

The degrees of x, y, and z are $4, 6$, and 9 respectively. Lastly, for part e, we have

$$(20x^2y^5)z^9 - (12x^2y^2)z^8 + (28x^2y^2 - 15x^3y^6)z^5 + (9x^3y^3 + 10x^4y^4)z^4 - (6x^4y)z^3 - (21x^3y^3)z + (14x^4y)z^0$$

$\square$

Problem 1.2. Repeat the preceding exercise under the assumption that the coefficients are of p and q are in $\mathbb{Z}/3\mathbb{Z}$.

Proof. content... $\square$