Exercises from Chapter 1

Wesley Basener

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Problem 1.1. Beginning with the formula for the sum of a geometric series, use differentiation to obtain the identity.

$$\sum_{n=0}^{\infty} ne^{-An} = \frac{e^{-A}}{(1-e^{-A})^2}$$

Solution. First, we integrate the left side to get the expression into geometric series form.

$$\int \sum_{n=0}^{\infty} ne^{-An} dA = \sum_{n=0}^{\infty} e^{-An}$$

Next, recall that the sum of a geometric series is $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$. Using this fact, the previous result can be rewritten,

$$\frac{-1}{1-e^{A}}$$

Finally, we take the derivative with respect to A, to undo our previous integration.

$$\frac{d}{dA}\frac{-1}{1-e^{A}} = \frac{e^{-A}}{(1-e^{-A})^2}$$

$\square$