SI214C: Homework #2

Qing Wu - 202031070

wuqing@shanghaitech.edu.cn

November 5, 2020

Contents

1 Problem 1

1.1 The Statement of Problem 1

Given $f \in L_2(0,1)$ find $u \in H_0^1(0,1)$ such that

$$\int_0^1 u'v' = \int_0^1 fv \quad \forall v \in H_0^1(0,1) \tag{1}$$

About the bases, ϕ_1 is written in the equation below.

$$\phi_1(x) = \begin{cases} \frac{1}{\Delta x}x & if \quad 0 < x < \Delta x \\ 2 - \frac{1}{\Delta x}x & if \quad \Delta x < x < 2 \times \Delta x \\ 0 & if \quad 2 \times \Delta x < x < 1 \end{cases} \tag{2}$$

We can freely choose the number k of the bases.

1.1.1 Input & Output

Input:

- the number of the bases k.
- the parameters $[\xi_1, \xi_2]$ of $f(x)$.

$$f(x) = \begin{cases} \xi_1 & if \quad 0 < x < 0.5 \\ \xi_2 & if \quad 0.5 \leq x < 1 \end{cases} \tag{3}$$

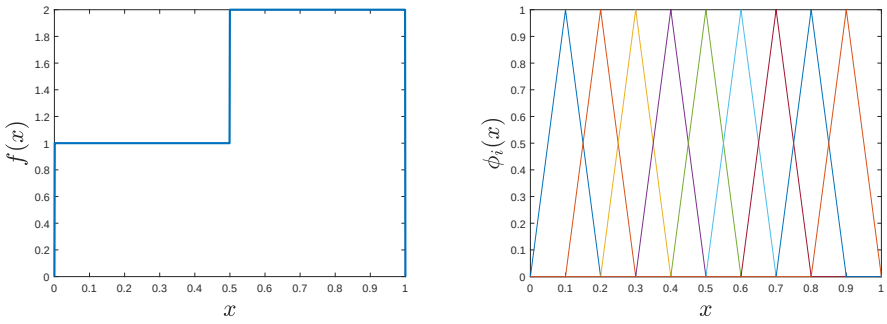
Output:

- the numerical solution U_k of Equation 1.

1.2 Implementation Details

1.2.1 Parameter Setting

In our experiment, the number of the bases k was 9, and ξ_1, ξ_2 for $f(x)$ were 1 and 2, respectively. the function $f(x)$ is shown in Figure 1.2.1 (a) and the bases $\phi_i(x)$ are shown in Figure 1.2.1 (b).



(a) the function $f(x)$ (b) the bases $\phi_i(x)$

Figure 1: $f(x)$ and $\phi_i(x)$ over x between 0 and 1, $\Delta x = 0.01$.

1.3 Result

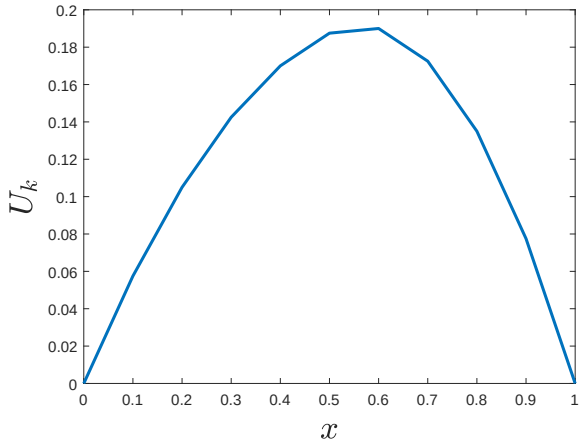


Figure 2: the numerical solution U_k for this problem.