POINT CLOUD REGISTRATION IN MULTIPLE COORDINATE SYSTEMS

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1 PRELIMINARY IDEAS

Consider N points in R^d and M local coordinate systems that are related through unknown rigid transforms. For each point, we are given (possibly noisy) measurements of its local coordinates in some of the coordinate systems. Alternatively, for each coordinate system, we observe the coordinates of a subset of the points. The problem of estimating the global coordinates of the N points (up to a rigid transform) from such measurements comes up in distributed approaches to molecular conformation and sensor network localization, and also in computer vision and graphics.

In the investigation of 3D reconstruction technology, we found that point cloud registration is closely related to matrix calculation.Therefore, our team is going to study this field and dig ICP algorithm deeply. In addition to learning the traditional ICP algorithm, We aim to learn and implement the most advanced algorithms. The essence of point cloud registration is actually a least squares problem. If the point cloud is divided into multiple small blocks, and each small block is cross-registered separately, it seems that the spatial registration under multiple coordinate systems may also be solved.

2 DESCRIPTION OF THE PROBLEM

The problem of point cloud registration comes up in computer vision Sharp et al. (2002) and graphics and in distributed approaches to molecular conformation and sensor network localization. The registration problem in question is one of determining the coordinates of a point cloud P from the knowledge of (possibly noisy) coordinates of smaller point cloud subsets (called patches) $P_1, ..., P_M$ that are derived from P through some general transformationWilliams & Bennamoun (2000). In certain applications,Mitra et al. (2004) one is often interested in finding the optimal transforms (one for each patch) that consistently align $P_1, ..., P_M$. This can be seen as a subproblem in the determination of the coordinates of P Cucuringu et al. (2012a).

In this project, we consider the problem of rigid registration in which the points within a given P_i are (ideally) obtained from P through an unknown rigid transform. Moreover, we assume that the correspondence between the local patches and the original point cloud is known; that is, we know beforehand which points from P are contained in a given P_i. In fact, one has a control on the correspondence in distributed approaches to molecular conformation Cucuringu et al. (2012b) and sensor network localization. While this correspondence is not directly available for certain graphics and vision problems, such as multiview registrationPottmann et al. (2006), it is in principle possible to estimate the correspondence by aligning pairs of patches Huang & Guibas (2013), e.g., using the ICP (iterative closest point) algorithm.

Iterative closest point (ICP) Besl & McKay (1992) is an algorithm employed to minimize the difference between two clouds of points. ICP is often used to reconstruct 2D or 3D surfaces from different scans, to localize robots and achieve optimal path planning (especially when wheel odometry is unreliable due to slippery terrain), to co-register bone models, etc.

3 DESCRIPTION OF THE ICP

Point Cloud Registration refers to the input of two point clouds P_s (source) and P_t (target) and the output of a transformation T such that T(P_s) and P_t coincide as much as possible. The transform

T may or may not be rigid, but in this paper only rigid transforms are considered, i.e. transforms include only rotation and translation.

The point cloud alignment can be divided into two steps, the Coarse Registration and the Fine Registration. Coarse registration refers to the rough registration when the transformation between two point clouds is completely unknown, the purpose is mainly to provide a good initial value of the transformation for the fine registration; the fine registration criterion is given an initial transformation, and further optimization to obtain a more accurate transformation.

At present, the most widely used point cloud fine alignment algorithm is the Iterative Closest Point (ICP) and various variants of ICP algorithm.

4 DESCRIPTION OF THE ALGORITHM

For the case where T is a rigid transformation, the point cloud alignment problem can be described as:

$$R^*, t^* = \arg\min_{R,t} \frac{1}{|P_s|} \sum_{i=1}^{|P_s|} \left\| p_t^i - \left(R \cdot p_s^i + t\right) \right\|^2 \tag{1}$$

Here p_s and p_t are the one-to-one correspondence points in the source and target point clouds.

The intuitive idea of the ICP algorithm is as follows:

1. If we know the correspondence between points on two point clouds, then we can use Least Squares to solve for the R, t parameter.
2. How do you know the correspondence of the points? If we already know a roughly reliable R, t parameter, then we can find the correspondence between points on two point clouds by being greedy (directly looking for the closest point as a correspondence).

4.1 FIND CLOSET POINT

Use the initial R_0, t_0 or R_{k-1}, t_{k-1} obtained from the previous iteration to transform the initial point cloud to obtain a temporary transformed point cloud, and then compare this point cloud with the target point cloud to find the source point cloud The nearest neighbor of each point in the target point cloud.

If you directly compare to find the nearest neighbor, you need to perform a double loop, and the computational complexity is $O(|Ps||Pt|)$. This step will be time-consuming. Common acceleration methods are:

Set the distance threshold. When the distance between the point and the point is less than a certain threshold, the corresponding point is considered to be found, without traversing the entire point set; Use ANN (Approximate Nearest Neighbor) to speed up the search, commonly used KD-tree; the computational complexity of KD-tree building is $O(Nlog(N))$, and the search usually has a complexity of $O(log(N))$ (the worst Case $O(N)$).

4.2 FIND BEST TRANSFORM

For the point-to-point ICP problem, there is a closed-form solution to find the optimal transformation, which can be calculated with the help of SVD decomposition.

The conclusion is given first. In the case where the correspondence of points is known, let $\hat{p}_s^i = p_s^i - \bar{p}_s, \hat{p}_t^i = p_t^i - \bar{p}_t$ represent the center of mass of the source and target point clouds respectively, let $H = \sum_{i=1}^{P_{|s|}} \hat{p}_s^i \hat{p}_t^{iT}$, which is a 3x3 matrix, SVD decomposition of H gives $H = U\Sigma V^T$, then the optimal rotation of the point-to-point ICP problem is.

$$R^* = VU^T$$

The optimal translation is:

$$t* = \hat{p}_t - R^* \hat{p}_s$$

4.3 Iteration

At each iteration, we will get the current optimal transform parameter R_k, t_k, and then apply this transform to the current source point cloud; the steps of "find the nearest corresponding point" and "solve the optimal transform" are iterated until the iteration termination condition is satisfied. Commonly used termination conditions are:

1. The variation of R_k,t_k is less than a certain value.
2. loss less than a certain value
3. Maximum number of iterations reached

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