Solution 21.2

(a) Analytical solution:

$$\int_0^3 \left(1-e^{-2x}\right) dx = \left[x+0.5e^{-2x}\right]_0^3 = 3+0.5e^{-2(3)} - 0 - 0.5e^{-2(0)} = 2.501239$$

(b) Trapezoidal rule (n = 1):

$$I = (3-0)\frac{0+0.997521}{2} = 1.496282$$

$$\varepsilon_t = \left|\frac{2.501239-1.496282}{2.501239}\right| \times 100\% = 40.18\%$$

(c) Trapezoidal rule (n = 2):

$$I = (3-0)\frac{0+2(0.950213)+0.997521}{4} = 2.17346 \quad \varepsilon_t = 13.10\%$$

Trapezoidal rule (n = 4):

$$I = (3-0)\frac{0+2(0.77687+0.950213+0.988891)+0.997521}{8} = 2.411051 \qquad \varepsilon_t = 3.61\%$$

(d) Simpson's 1/3 rule:

$$I = (3-0)\frac{0+4(0.950213)+0.997521}{6} = 2.399186 \quad \varepsilon_t = 4.08\%$$

(e) Simpson's rule (n = 4):

$$I = (3-0)\frac{0+4(0.77687+0.988891)+2(0.950213)+0.997521}{12} = 2.490248 \qquad \varepsilon_t = 0.44\%$$

(f) Simpson's 3/8 rule:

$$I = (3-0)\frac{0+3(0.864665+0.981684)+0.997521}{8} = 2.451213 \quad \varepsilon_t = 2.00\%$$

(g) Simpson's rules (n = 5):

$$I = (1.2-0)\frac{0+4(0.698806)+0.909282}{6} + (3-1.2)\frac{0.909282+3(0.972676+0.99177)+0.997521}{8}$$

$$= 0.740901 + 1.755032 = 2.495933 \qquad \varepsilon_t = 0.21\%$$