Solution 23.8

(a)

	x	f(x)
x_{i-2}	-0.5	-17.125
x_{i-1}	-0.25	-16.0156
x_i	0	-15
x_{i+1}	0.25	-13.9844
x_{i+2}	0.5	-12.875

$$f'(x) = \frac{-(-12.875) + 8(-13.9844) - 8(-16.0156) - 17.125}{12(0.25)} = 4$$

Analytical: $f'(x) = 3x^2 + 4 = 3(0) + 4 = 4$

$$f''(x) = \frac{-(-12.875) + 16(-13.9844) - 30(-15) + 16(-16.0156) - (-17.125)}{12(0.25)^2} = 0$$

Analytical: $f''(x) = 6x = 6(0) = 0$

Therefore, for this case, the results are exact.

(b)

	x	f(x)
x_{i-2}	0.2	0.039203
x_{i-1}	0.3	0.08598
x_i	0.4	0.14737
x_{i+1}	0.5	0.219396
x_{i+2}	0.6	0.297121

$$f'(x) = \frac{-(0.297121) + 8(0.219396) - 8(0.08598) + 0.039203}{12(0.1)} = 0.674504$$

Analytical: $f'(0.4) = 2x \cos x - x^2 \sin x = 2(0.4) \cos(0.4) - 0.4^2 \sin(0.4) = 0.674542$

$$\varepsilon_t = \left| \frac{0.674542 - 0.674504}{0.674542} \right| \times 100\% = 0.0056\%$$

$$f''(x) = \frac{-(0.297121) + 16(0.219396) - 30(0.14737) + 16(0.08598) - (0.039203)}{12(0.1)^2} = 1.071654$$

Solution continued on the next page...

Analytical:

$$f'(0.4) = 2\cos x - x^2 \cos x - 4x \sin x = 2\cos(0.4) - 0.4^2 \cos(0.4) - 4(0.4)\sin(0.4) = 1.071683$$

$$\varepsilon_t = \left|\frac{1.071683 - 1.071654}{1.071683}\right| \times 100\% = 0.0027\%$$

(c)

	x	f(x)
x_{i-2}	2	0.786843
x_{i-1}	2.5	1.100778
x_i	3	1.557408
x_{i+1}	3.5	2.338254
x_{i+2}	4	4.131729

$$f'(x) = \frac{-(4.131729) + 8(2.338254) - 8(1.100778) + 0.786843}{12(0.5)} = 1.092486$$

Analytical: $f'(3) = \dfrac{\tan(x/3)^2 + 1}{3} = \dfrac{\tan(1)^2 + 1}{3} = 1.14184$

$$\varepsilon_t = \left|\frac{1.14184 - 1.092486}{1.14184}\right| \times 100\% = 4.3223\%$$

$$f''(x) = \frac{-(4.131729) + 16(2.338254) - 30(1.557408) + 16(1.100778) - (0.786843)}{12(0.5)^2} = 1.127902$$

Analytical: $f''(3) = \dfrac{2\tan(x/3)\left(\tan(x/3)^2 + 1\right)}{9} = \dfrac{2\tan(1)\left(\tan(1)^2 + 1\right)}{9} = 1.18554$

$$\varepsilon_t = \left|\frac{1.18554 - 1.127902}{1.18554}\right| \times 100\% = 4.8617\%$$

(d)

	x	f(x)
x_{i-2}	0.6	0.62948
x_{i-1}	0.8	0.540569
x_i	1	0.479426
x_{i+1}	1.2	0.433954
x_{i+2}	1.4	0.398355

Solution continued on the next page...

$$f'(x) = \frac{-(0.398355) + 8(0.433954) - 8(0.540569) + 0.62948}{12(0.2)} = -0.25908$$

Analytical: $f'(1) = \dfrac{\cos\left(\sqrt{x}/2\right)}{4x^{3/2}} - \dfrac{\sin\left(\sqrt{x}/2\right)}{2x^2} = \dfrac{\cos(1/2)}{4} - \dfrac{\sin(1/2)}{2} = -0.26003$

$$\varepsilon_t = \left|\frac{-0.26003 - (-0.25903)}{-0.26003}\right| \times 100\% = 0.365\%$$

$$f''(x) = \frac{-(0.398355) + 16(0.433954) - 30(0.479426) + 16(0.540569) - (0.62948)}{12(0.2)^2} = 0.378652$$

Analytical:

$$f'(1) = -\frac{x\sin\left(\sqrt{x}/2\right) - 32\sin\left(\sqrt{x}/2\right) + 10\sqrt{x}\cos\left(\sqrt{x}/2\right)}{16x^3} = -\frac{\sin(0.5) - 32\sin(0.5) + 10\cos(0.5)}{16} = 0.380398$$

$$\varepsilon_t = \left|\frac{0.380398 - 0.378652}{0.380398}\right| \times 100\% = 0.459\%$$

(e)

	x	f(x)
x_{i-2}	1.6	6.553032
x_{i-1}	1.8	7.849647
x_i	2	9.389056
x_{i+1}	2.2	11.22501
x_{i+2}	2.4	13.42318

$$f'(x) = \frac{-(13.42318) + 8(11.22501) - 8(7.849647) + 6.553032}{12(0.2)} = 8.38866$$

Analytical: $f'(2) = 1 + e^x = 1 + e^2 = 8.389056$

$$\varepsilon_t = \left|\frac{8.389056 - 8.38866}{8.389056}\right| \times 100\% = 0.0047\%$$

$$f''(x) = \frac{-(13.42318) + 16(11.22501) - 30(9.389056) + 16(7.849647) - (6.553032)}{12(0.2)^2} = 7.388924$$

Solution continued on the next page...

Analytical: $f'(2) = e^x = e^2 = 7.389056$

$$\varepsilon_t = \left| \frac{7.389056 - 7.388924}{7.389056} \right| \times 100\% = 0.0018\%$$