Solution 4.7

True value:

$$f'(x) = 75x^{2} - 12x + 7$$
$$f'(2) = 75(2)^{2} - 12(2) + 7 = 283$$

function values:

$$x_{i-1} = 1.8$$
 $f(x_{i-1}) = 50.96$
 $x_i = 2$ $f(x_i) = 102$
 $x_{i+1} = 2.2$ $f(x_{i+1}) = 164.56$

forward:

$$f'(2) = \frac{164.56 - 102}{0.2} = 312.8$$
 $\varepsilon_t = \left| \frac{283 - 312.8}{283} \right| \times 100\% = 10.53\%$

backward:

$$f'(2) = \frac{102 - 50.96}{0.2} = 255.2$$
 $\varepsilon_t = \left| \frac{283 - 255.2}{283} \right| \times 100\% = 9.823\%$

centered:

$$f'(2) = \frac{164.56 - 50.96}{2(0.2)} = 284$$
 $\varepsilon_t = \left| \frac{283 - 284}{283} \right| \times 100\% = 0.353\%$

Both the forward and backward have errors that can be approximated by (recall Eq. 4.15),

$$|E_t| \approx \frac{f''(x_i)}{2}h$$

 $f''(2) = 150x - 12 = 150(2) - 12 = 288$
 $|E_t| \approx \frac{288}{2}0.2 = 28.8$

This is very close to the actual error that occurred in the approximations

forward:
$$|E_t| \approx |283 - 312.8| = 29.8$$

backward: $|E_t| \approx |283 - 255.2| = 27.8$

Solution continued on the next page...

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The centered approximation has an error that can be approximated by,

$$E_t \approx -\frac{f^{(3)}(x_i)}{6}h^2 = -\frac{150}{6}0.2^2 = -1$$

which is exact: $E_t = 283 - 284 = -1$. This result occurs because the original function is a cubic equation which has zero fourth and higher derivatives.

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