Solution 17.30

The sum of the squares of the residuals for this case can be written as

$$S_r = \sum_{i=1}^{n}\left(y_i - a_1 x_i - a_2 x_i^2\right)^2$$

The partial derivatives of this function with respect to the unknown parameters can be determined as

$$\frac{\partial S_r}{\partial a_1} = -2\sum\left[(y_i - a_1 x_i - a_2 x_i^2)x_i\right]$$

$$\frac{\partial S_r}{\partial a_2} = -2\sum\left[(y_i - a_1 x_i - a_2 x_i^2)x_i^2\right]$$

Setting the derivative equal to zero and evaluating the summations gives

$$\left(\sum x_i^2\right)a_1 + \left(\sum x_i^3\right)a_2 = \sum x_i y_i$$

$$\left(\sum x_i^3\right)a_1 + \left(\sum x_i^4\right)a_2 = \sum x_i^2 y_i$$

which can be solved for

$$a_1 = \frac{\sum x_i y_i \sum x_i^4 - \sum x_i^2 y_i \sum x_i^3}{\sum x_i^2 \sum x_i^4 - \left(\sum x_i^3\right)^2}$$

$$a_2 = \frac{\sum x_i^2 \sum x_i^2 y_i - \sum x_i y_i \sum x_i^3}{\sum x_i^2 \sum x_i^4 - \left(\sum x_i^3\right)^2}$$

Solution continued on the next page...

The model can be tested for the data from Table 17.28.

x	y	x^2	x^3	x^4	xy	x^2y
10	25	100	1000	10000	250	2500
20	70	400	8000	160000	1400	28000
30	380	900	27000	810000	11400	342000
40	550	1600	64000	2560000	22000	880000
50	610	2500	125000	6250000	30500	1525000
60	1220	3600	216000	12960000	73200	4392000
70	830	4900	343000	24010000	58100	4067000
80	1450	6400	512000	40960000	116000	9280000
Σ		20400	1296000	87720000	312850	20516500

$$a_1 = \frac{312850(87720000) - 20516500(1296000)}{20400(87720000) - (1296000)^2} = 7.771024$$

$$a_2 = \frac{20400(20516500) - 312850(1296000)}{20400(87720000) - (1296000)} = 0.119075$$

Therefore, the best-fit model is

$$y = 7.771024x + 0.119075x^2$$

The fit, along with the original data can be plotted as