

**Solution 23.16****(a)**

$$\frac{\partial f}{\partial x} = 3y + 3 - 3x^2 = 3(1) + 3 - 3(1)^2 = 3$$

$$\frac{\partial f}{\partial y} = 3x - 9y^2 = 3(1) - 9(1)^2 = -6$$

$$\frac{\partial f}{\partial x \partial y} = 3$$

**(b)**

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{f(x + \Delta x, y) - f(x - \Delta x, y)}{2\Delta x} = \frac{f(1.0001, 1) - f(0.9999, 1)}{2\Delta x} \\ &= \frac{[3(1.0001)1 + 3(1.0001) - (1.0001)^3 - 3(1)^3] - [3(0.9999)1 + 3(0.9999) - (0.9999)^3 - 3(1)^3]}{0.0002} \\ &= \frac{2.00029997 - 1.99969997}{0.0002} = 2.99999999 \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial y} &= \frac{f(x, y + \Delta y) - f(x, y - \Delta y)}{2\Delta y} = \frac{f(1, 1.0001) - f(1, 0.9999)}{2\Delta y} \\ &= \frac{[3(1)1.0001 + 3(1) - (1)^3 - 3(1.0001)^3] - [3(1)0.9999 + 3(1) - (1)^3 - 3(0.9999)^3]}{0.0002} \\ &= \frac{1.99939991 - 2.00059991}{0.0002} = -6.00000003 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 f}{\partial x \partial y} &= \frac{f(x + \Delta x, y + \Delta y) - f(x + \Delta x, y - \Delta y) - f(x - \Delta x, y + \Delta y) + f(x - \Delta x, y - \Delta y)}{4\Delta x \Delta y} \\ &= \frac{f(1.0001, 1.0001) - f(1.0001, 0.9999) - f(0.9999, 1.0001) + f(0.9999, 0.9999)}{4\Delta x \Delta y} \\ &= \frac{1.99969991 - 2.00089985 - 1.99909985 + 2.00029991}{4(0.0001)(0.0001)} = 2.999999982 \end{aligned}$$