Solution 21.8

Analytical solution:

$$\int_0^3 \left(5+3\cos x\right) dx = \left[5x+3\sin x\right]_0^3 = 15.42336$$

(a) Trapezoidal rule (n = 1):

$$I = (3-0)\frac{8+2.030023}{2} = 15.04503 \qquad \varepsilon_t = \left|\frac{15.42336-15.04503}{15.42336}\right| \times 100\% = 2.453\%$$

(b) Simpson's 1/3 rule (n = 2):

$$I = (3-0)\frac{8+4(5.212212)+2.030023}{6} = 15.43943 \quad \varepsilon_t = 0.104\%$$

(c) Simpson's 3/8 rule:

$$I = (3-0)\frac{8+3(6.620907+3.751559)+2.030023}{8} = 15.43028 \quad \varepsilon_t = 0.045\%$$

(d) Simpson's rules (n = 5):

$$I = (1.2-0)\frac{8+4(7.476007)+6.087073}{6} + (3-1.2)\frac{6.087073+3(4.318394+2.787819)+2.030023}{8}$$
$$= 8.79822 + 6.62304 = 15.42126 \qquad \varepsilon_t = 0.014\%$$

(e) Boole's rule:

$$I = (3-0)\frac{7(8)+32(7.195067)+12(5.212212)+32(3.115479)+7(2.030023)}{90} = 15.42314$$

$$\varepsilon_t = 0.0014\%$$

(f) Midpoint method:

$$I = (3-0)5.212212 = 15.63663 \qquad \varepsilon_t = 1.383\%$$

(g) 3-segment-2-point open integration formula:

$$I = (3-0)\frac{6.620907+3.751559}{2} = 15.5587 \qquad \varepsilon_t = 0.877\%$$

(h) 4-segment-3-point open integration formula:

$$I = (3-0)\frac{2(7.195067)-5.212212+2(3.115479)}{3} = 15.40888 \qquad \varepsilon_t = 0.094\%$$