Solution 21.1

(a) Analytical solution:

$$\int_0^{\pi/2} (6+3\cos x)\; dx = \left[6x+3\sin x\right]_0^{\pi/2} = 6(\pi/2)+3\sin(\pi/2)-0 = 12.42478$$

(b) Trapezoidal rule (n = 1):

$$I = (1.570796-0)\frac{9+6}{2} = 11.78097 \qquad \varepsilon_t = \left|\frac{12.42478-11.78097}{12.42478}\right| \times 100\% = 5.182\%$$

(c) Trapezoidal rule (n = 2):

$$I = (1.570796-0)\frac{9+2(8.12132)+6}{4} = 12.26896 \qquad \varepsilon_t = 1.254\%$$

Trapezoidal rule (n = 4):

$$I = (1.570796-0)\frac{9+2(8.771639+8.12132+7.14805)+6}{8} = 12.38613 \qquad \varepsilon_t = 0.311\%$$

(d) Simpson's 1/3 rule:

$$I = (1.570796-0)\frac{9+4(8.12132)+6}{6} = 12.43162 \qquad \varepsilon_t = 0.055\%$$

(e) Simpson's rule (n = 4):

$$I = (1.570796-0)\frac{9+4(8.771639+7.14805)+2(8.12132)+6}{12} = 12.42518 \qquad \varepsilon_t = 0.0032\%$$

(f) Simpson's 3/8 rule:

$$I = (1.570796-0)\frac{9+3(8.598076+7.5)+6}{8} = 12.42779 \qquad \varepsilon_t = 0.024\%$$

(g) Simpson's rules (n = 5):

$$\begin{aligned} I &= (0.628319-0)\frac{9+4(8.85317)+8.427051}{6} \\ &\quad + (1.570796-0.628319)\frac{8.427051+3(7.763356+6.927051)+6}{8} \\ &= 5.533364+6.891665 = 12.42503 \qquad \varepsilon_t = 0.002\% \end{aligned}$$