

Solution 17.14

Linearization: Take the square root

$$\sqrt{y} = \frac{a + \sqrt{x}}{b\sqrt{x}}$$

or

$$\sqrt{y} = \frac{a}{b} \frac{1}{\sqrt{x}} + \frac{1}{b}$$

Therefore, a plot of \sqrt{y} versus $1/\sqrt{x}$ should yield a straight line with a slope of a/b and an intercept of $1/b$.

| x | y | $1/\sqrt{x}$ | \sqrt{y} | \sqrt{y} / \sqrt{x} | $1/x$ |
|-----|------|-----------------|-----------------|-----------------------|-----------------|
| 0.5 | 10.4 | 1.414214 | 3.224903 | 4.560702 | 2 |
| 1 | 5.8 | 1 | 2.408319 | 2.408319 | 1 |
| 2 | 3.3 | 0.707107 | 1.81659 | 1.284523 | 0.5 |
| 3 | 2.4 | 0.57735 | 1.549193 | 0.894427 | 0.333333 |
| 4 | 2 | 0.5 | 1.414214 | 0.707107 | 0.25 |
| | | 4.198671 | 10.41322 | 9.855078 | 4.083333 |

The slope and intercept can be computed as

$$a_1 = \frac{5(9.855078) - 4.198671(10.41322)}{5(4.083333) - 4.198671^2} = 1.992126$$

$$a_0 = \frac{10.41322}{5} - 1.992126 \frac{4.198671}{5} = 0.409788$$

The constants can then be computed as

$$b = \frac{1}{0.409788} = 2.440288$$

$$a = 1.992126(2.440288) = 4.861362$$

and the prediction calculated as

$$y(1.6) = \left(\frac{4.861362 + \sqrt{1.6}}{2.440288\sqrt{1.6}} \right)^2 = 3.93904$$