

Solution 21.3

(a) Analytical solution:

$$\int_{-2}^4 (1-x-4x^3+2x^5) dx = \left[x - \frac{x^2}{2} - x^4 + \frac{x^6}{3} \right]_{-2}^4 = 1104$$

(b) Trapezoidal rule ($n = 1$):

$$I = (4 - (-2)) \frac{-29 + 1789}{2} = 5280 \quad \varepsilon_t = \left| \frac{1104 - 5280}{1104} \right| \times 100\% = 378.26\%$$

(c) Trapezoidal rule ($n = 2$):

$$I = (4 - (-2)) \frac{-29 + 2(-2) + 1789}{4} = 2634 \quad \varepsilon_t = 138.59\%$$

Trapezoidal rule ($n = 4$):

$$I = (4 - (-2)) \frac{-29 + 2(1.9375 - 2 + 131.3125) + 1789}{8} = 1516.875 \quad \varepsilon_t = 37.398\%$$

(d) Simpson's 1/3 rule ($n = 2$):

$$I = (4 - (-2)) \frac{-29 + 4(-2) + 1789}{6} = 1752 \quad \varepsilon_t = 58.7\%$$

(e) Simpson's 3/8 rule:

$$I = (4 - (-2)) \frac{-29 + 3(1 + 31) + 1789}{8} = 1392 \quad \varepsilon_t = 26.087\%$$

(f) Boole's rule ($n = 5$):

$$I = (4 - (-2)) \frac{7(-29) + 32(1.9375) + 12(-2) + 32(131.3125) + 7(1789)}{90} = 1104 \quad \varepsilon_t = 0\%$$