Solution 23.11

Here is a VBA program uses Eq. 23.9 to obtain first-derivative estimates for unequally spaced data.

```
Option Explicit
Sub TestDerivUnequal()
Dim n As Integer, i As Integer
Dim x(100) As Double, y(100) As Double, dy(100) As Double
Range("a5").Select
n = ActiveCell.Row
Selection.End(xlDown).Select
n = ActiveCell.Row - n
Range("a5").Select
For i = 0 To n
  x(i) = ActiveCell.Value
  ActiveCell.Offset(0, 1).Select
  y(i) = ActiveCell.Value
  ActiveCell.Offset(1, -1).Select
Next i
For i = 0 To n
  dy(i) = DerivUnequal(x, y, n, x(i))
Next i
Range("c5").Select
For i = 0 To n
  ActiveCell.Value = dy(i)
  ActiveCell.Offset(1, 0).Select
Next i
End Sub
Function DerivUnequal(x, y, n, xx)
Dim ii As Integer
If xx < x(0) Or xx > x(n) Then
  DerivUnequal = "out of range"
Else
  If xx < x(1) Then
    DerivUnequal = DyDx(xx, x(0), x(1), x(2), y(0), y(1), y(2))
  ElseIf xx > x(n - 1) Then
    DerivUnequal = _
        DyDx(xx, x(n - 2), x(n - 1), x(n), y(n - 2), y(n - 1), y(n))
  Else
```

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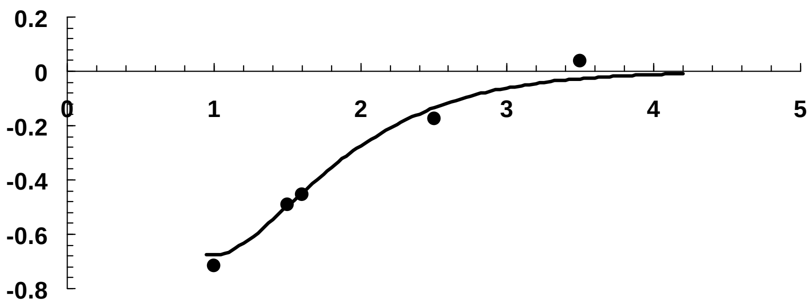
```
 For ii = 1 To n - 2
      If xx >= x(ii) And xx <= x(ii + 1) Then
        If xx - x(ii - 1) < x(ii) - xx Then
          'If the unknown is closer to the lower end of the range,
          'x(ii) will be chosen as the middle point
          DerivUnequal = _
          DyDx(xx, x(ii - 1), x(ii), x(ii + 1), y(ii - 1), y(ii), y(ii + 1))
        Else
          'Otherwise, if the unknown is closer to the upper end,
          'x(ii+1) will be chosen as the middle point
          DerivUnequal = _
          DyDx(xx, x(ii), x(ii + 1), x(ii + 2), y(ii), y(ii + 1), y(ii + 2))
        End If
        Exit For
      End If
    Next ii
  End If
End If
End Function
Function DyDx(x, x0, x1, x2, y0, y1, y2)
DyDx = y0 * (2 * x - x1 - x2) / (x0 - x1) / (x0 - x2) _
      + y1 * (2 * x - x0 - x2) / (x1 - x0) / (x1 - x2) _
       + y2 * (2 * x - x0 - x1) / (x2 - x0) / (x2 - x1)
End Function
```

When the program is run, the result is shown below:

	A	B	C
1	**Problem 23.11**		
2			
3	**Data:**		
4	**x**	**y**	**dy/dx**
5	**1**	**0.6767**	**-0.71793**
6	**1.5**	**0.3734**	**-0.49342**
7	**1.6**	**0.3261**	**-0.45258**
8	**2.5**	**0.08422**	**-0.17378**
9	**3.5**	**0.01596**	**0.037264**

Solution continued on the next page...

The results can be compared with the true derivatives which can be calculated with analytical solution, $f'(x) = 5e^{-2x} - 10xe^{-2x}$. The results can be displayed graphically below where the computed values are represented as points and the true values as the curve.



An even more elegant approach is to put cubic splines through the data (recall Sec. 20.2 and the program used for the solution to Prob. 20.10) to evaluate the derivatives. Here is the result of applying that program to this problem.