Solution 23.9

The first forward difference formula of $O(h^2)$ from Fig. 23.1 can be used to estimate the velocity for the first point at $t = 0$,

$$f'(0) = \frac{-58 + 4(32) - 3(0)}{2(25)} = 1.4 \frac{\text{km}}{\text{s}}$$

The acceleration can be estimated with the second forward difference formula of $O(h^2)$ from Fig. 23.1

$$f''(0) = \frac{-78 + 4(58) - 5(32) + 2(0)}{(25)^2} = -0.0096 \frac{\text{km}}{\text{s}^2}$$

For the interior points, centered difference formulas of $O(h^2)$ from Fig. 23.3 can be used to estimate the velocities and accelerations. For example, at the second point at $t = 25$,

$$f'(25) = \frac{58 - 0}{2(25)} = 1.16 \frac{\text{km}}{\text{s}}$$

$$f''(25) = \frac{58 - 2(32) + 0}{(25)^2} = -0.0096 \frac{\text{km}}{\text{s}^2}$$

For the final point, backward difference formulas of $O(h^2)$ from Fig. 23.2 can be used to estimate the velocities and accelerations. The results for all values are summarized in the following table.

t	*y*	*v*	*a*
0	0	1.40	-0.0096
25	32	1.16	-0.0096
50	58	0.92	-0.0096
75	78	0.68	-0.0096
100	92	0.44	-0.0096
125	100	0.20	-0.0096