Solution 22.18

$$S = \frac{h}{6}\left(f(a) + 4f(c) + f(b) \right) \qquad S_2 = \frac{h}{12}\left(f(a) + 4f(d) + 2f(c) + 4f(e) + f(b) \right)$$

$$Q = S_2 + \frac{S_2 - S}{15}$$

$$= \frac{h}{12}\left(f(a) + 4f(d) + 2f(c) + 4f(e) + f(b) \right) + \frac{\frac{h}{12}\left(f(a) + 4f(d) + 2f(c) + 4f(e) + f(b) \right) - \frac{h}{6}\left(f(a) + 4f(c) + f(b) \right)}{15}$$

Collecting terms

$$Q = \frac{4h}{45}\left(f(a) + 4f(d) + 2f(c) + 4f(e) + f(b) \right) - \frac{h}{90}\left(f(a) + 4f(c) + f(b) \right)$$

$$Q = \frac{7}{90}hf(a) + \frac{16}{45}hf(d) + \frac{2}{15}hf(c) + \frac{16}{45}hf(e) + \frac{7}{90}hf(b)$$

$$Q = \frac{h}{90}\left[7f(a) + 32f(d) + 12f(c) + 32f(e) + 7f(b) \right]$$

which is Boole's Rule.