Solution 17.5

The sum of the squares of the residuals for this case can be written as

$$S_r = \sum_{i=1}^{n} \left(y_i - a_1 x_i \right)^2$$

The partial derivative of this function with respect to the single parameter a_1 can be determined as

$$\frac{\partial S_r}{\partial a_1} = -2\sum \left[(y_i - a_1 x_i) x_i \right]$$

Setting the derivative equal to zero and evaluating the summations gives

$$0 = \sum y_i x_i - a_1 \sum x_i^2$$

which can be solved for

$$a_1 = \frac{\sum y_i x_i}{\sum x_i^2}$$

So the slope that minimizes the sum of the squares of the residuals for a straight line with a zero intercept is merely the ratio of the sum of the dependent variables (y) times the sum of the independent variables (x) over the sum of the independent variables squared (x^2). Application to the data gives

x	y	xy	x^2
2	1	2	4
4	2	8	16
6	5	30	36
7	2	14	49
10	8	80	100
11	7	77	121
14	6	84	196
17	9	153	289
20	12	240	400
		688	1211

Solution continued on the next page...

Therefore, the slope can be computed as 688/1211 = 0.5681. The fit along with the data can be displayed as