Solution 4.7

True value:

$$f'(x) = 75x^2 - 12x + 7$$
$$f'(2) = 75(2)^2 - 12(2) + 7 = 283$$

function values:

$x_{i-1} = 1.8$ $\quad f(x_{i-1}) = 50.96$

$x_i = 2$ $\quad f(x_i) = 102$

$x_{i+1} = 2.2$ $\quad f(x_{i+1}) = 164.56$

<u>forward:</u>

$$f'(2) = \frac{164.56 - 102}{0.2} = 312.8 \qquad \varepsilon_t = \left|\frac{283 - 312.8}{283}\right| \times 100\% = 10.53\%$$

<u>backward:</u>

$$f'(2) = \frac{102 - 50.96}{0.2} = 255.2 \qquad \varepsilon_t = \left|\frac{283 - 255.2}{283}\right| \times 100\% = 9.823\%$$

<u>centered:</u>

$$f'(2) = \frac{164.56 - 50.96}{2(0.2)} = 284 \qquad \varepsilon_t = \left|\frac{283 - 284}{283}\right| \times 100\% = 0.353\%$$

Both the forward and backward have errors that can be approximated by (recall Eq. 4.15),

$$|E_t| \approx \frac{f''(x_i)}{2} h$$
$$f''(2) = 150x - 12 = 150(2) - 12 = 288$$
$$|E_t| \approx \frac{288}{2} 0.2 = 28.8$$

This is very close to the actual error that occurred in the approximations

forward: $|E_t| \approx |283 - 312.8| = 29.8$

backward: $|E_t| \approx |283 - 255.2| = 27.8$

Solution continued on the next page...

The centered approximation has an error that can be approximated by,

$$E_t \approx -\frac{f^{(3)}(x_i)}{6}h^2 = -\frac{150}{6}0.2^2 = -1$$

which is exact: E_t = 283 – 284 = –1. This result occurs because the original function is a cubic equation which has zero fourth and higher derivatives.