

Solution 22.2

The integral can be evaluated analytically as,

$$I = \int_1^2 \left(x + \frac{1}{x} \right)^2 dx = \int_1^2 x^2 + 2 + x^{-2} dx$$

$$I = \left[\frac{x^3}{3} + 2x - \frac{1}{x} \right]_1^2 = \frac{2^3}{3} + 2(2) - \frac{1}{2} - \frac{1^3}{3} - 2(1) + \frac{1}{1} = 4.8333$$

The tableau depicting the implementation of Romberg integration to $\varepsilon_s = 0.5\%$ is

| iteration→ | 1 | 2 | 3 |
|-----------------------------|------------|------------|------------|
| $\varepsilon_t \rightarrow$ | 6.0345% | 0.0958% | 0.0028% |
| $\varepsilon_a \rightarrow$ | | 1.4833% | 0.0058% |
| 1 | 5.12500000 | 4.83796296 | 4.83347014 |
| 2 | 4.90972222 | 4.83375094 | |
| 4 | 4.85274376 | | |

Thus, the result is 4.83347014.