Solution 21.19

The required quantities can be computed at the sample points and tabulated as

y, m	*H*, m	*U*, m/s	*HU* (m²/s)
0	0	0	0
1	1	0.1	0.1
3	1.5	0.12	0.18
5	3	0.2	0.6
7	3.5	0.25	0.875
8	3.2	0.3	0.96
9	2	0.15	0.3
10	0	0	0

The cross-sectional area can be computed with a combination of the trapezoidal rule and two applications of Simpson's 3/8 rule,

$$A_c = \int_0^y H(y)\,dy = (1-0)\frac{0+1}{2} + (7-1)\frac{1+3(1.5+3)+3.5}{8} + (10-7)\frac{3.5+3(3.2+2)+0}{8}$$
$$= 0.5 + 13.5 + 7.1625 = 21.1625 \text{ m}^2$$

The average depth is

$$H = \frac{A_c}{y} = \frac{21.1625 \text{ m}^2}{10 \text{ m}} = 2.11625 \text{ m}$$

The flow is

$$Q = \int_0^y H(y)U(y)\,dy = (1-0)\frac{0+0.1}{2} + (7-1)\frac{0.1+3(0.18+0.6)+0.875}{8} + (10-7)\frac{0.875+3(0.96+0.3)+0}{8}$$
$$= 0.05 + 2.48625 + 1.745625 = 4.281875 \ \frac{\text{m}^2}{\text{s}}$$

The average velocity is

$$U = \frac{Q}{A_c} = \frac{4.281875 \text{ m}^3/\text{s}}{21.1625 \text{ m}^2} = 0.202333 \ \frac{\text{m}}{\text{s}}$$