Solution 21.2

(a) Analytical solution:

$$\int_{0}^{3} \left(1 - e^{-2x} \right) dx = \left[x + 0.5e^{-2x} \right]_{0}^{3} = 3 + 0.5e^{-2(3)} - 0 - 0.5e^{-2(0)} = 2.501239$$

(b) Trapezoidal rule (n = 1):

$$I = (3-0)\frac{0+0.997521}{2} = 1.496282$$

$$\varepsilon_t = \left| \frac{2.501239 - 1.496282}{2.501239} \right| \times 100\% = 40.18\%$$

(c) Trapezoidal rule (n = 2):

$$I = (3-0)\frac{0+2(0.950213)+0.997521}{4} = 2.17346 \quad \varepsilon_t = 13.10\%$$

Trapezoidal rule (n = 4):

$$I = (3-0)\frac{0 + 2(0.77687 + 0.950213 + 0.988891) + 0.997521}{8} = 2.411051 \qquad \varepsilon_t = 3.61\%$$

(d) Simpson's 1/3 rule:

$$I = (3-0)\frac{0+4(0.950213)+0.997521}{6} = 2.399186 \quad \varepsilon_t = 4.08\%$$

(e) Simpson's rule (n = 4):

$$I = (3-0)\frac{0 + 4(0.77687 + 0.988891) + 2(0.950213) + 0.997521}{12} = 2.490248 \qquad \varepsilon_t = 0.44\%$$

(f) Simpson's 3/8 rule:

$$I = (3-0)\frac{0+3(0.864665+0.981684)+0.997521}{8} = 2.451213 \quad \varepsilon_t = 2.00\%$$

(g) Simpson's rules (n = 5):

$$\begin{split} I &= (1.2-0)\frac{0+4(0.698806)+0.909282}{6} \\ &+ (3-1.2)\frac{0.909282+3(0.972676+0.99177)+0.997521}{8} \\ &= 0.740901+1.755032 = 2.495933 \qquad \varepsilon_t = 0.21\% \end{split}$$

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