**Exercício 1**

Variables A B C : Prop.

Theorem frist\_theorem: A -> B -> A.

Proof.

intro Ha.

intro Hb.

assumption.

Qed.

### Exercício 2

Variables A B C : Prop.

Theorem frist\_theorem: (A -> B) -> (B -> C) -> A -> C.

Proof.

intro Hab.

intro Hbc.

intro Ha.

apply Hbc.

apply Hab.

assumption.

Qed.

### Exercício 3

Variables A B C : Prop.

Theorem frist\_theorem: (A /\ B) /\ C -> A /\ (B /\ C).

Proof.

intro Habc.

destruct Habc as [Hab Hc].

destruct Hab as [Ha Hb].

split.

+

assumption.

+

split.

\*

assumption.

\*

assumption.

Qed.

### Exercício 4

Variables A B C : Prop.

Theorem frist\_theorem: ((A /\ B) -> C) -> (A -> B -> C).

Proof.

intro Habc.

intro Ha.

intro Hb.

apply Habc.

split.

+

assumption.

+

assumption.

Qed.

### Exercício 5

Variables A B C : Prop.

Theorem frist\_theorem: (A -> B -> C) -> ((A /\ B) -> C).

Proof.

intro Habc.

intro Hab.

destruct Hab as [Ha Hb].

apply Habc.

+

assumption.

+

assumption.

Qed.

### Exercício 6

Theorem frist\_theorem: ((A -> B) /\ (A -> C)) -> A -> (B /\ C).

Proof.

intro Habc.

intro Ha.

destruct Habc as [Hab Hac].

split.

+

apply Hab.

assumption.

+

apply Hac.

assumption.

Qed.

### Exercício 7

Theorem frist\_theorem: ((A \/ B) /\ ~ A) -> B.

Proof.

intro Habna.

destruct Habna as [[Ha|Hb] Hna].

+

contradiction.

+

assumption.

Qed.

### Exercício 8

Theorem frist\_theorem: (A \/ (B /\ C)) -> (A \/ B) /\ (A \/ C).

Proof.

intro Habc.

destruct Habc as [Ha | [Hb Hc]].

split.

+

left.

assumption.

+

left.

assumption.

+

split.

\*

right.

assumption.

\*

right.

assumption.

Qed.

### Exercício 9

Lemma ex9 : forall x : U, P x -> exists y : U, P y.

Proof.

intro x.

intro Hp.

exists x.

assumption.

Qed.

### Exercício 10

Hypothesis U : Set.

Hypothesis u : U.

Hypothesis P : U -> Prop.

Hypothesis Q : U -> Prop.

Hypothesis R : U -> Prop.

Lemma ex10 : (forall x : U, P x -> ~ Q x) -> ~ exists y : U, P y /\ Q y.

Proof.

intro Hpq.

unfold not.

intro Hpq2.

destruct Hpq2 as [x [Hpx Hqx] ].

destruct (Hpq x).

assumption.

assumption.

Qed.

### Exercício 11

Hypothesis U : Set.

Hypothesis u : U.

Hypothesis P : U -> Prop.

Hypothesis Q : U -> Prop.

Hypothesis R : U -> Prop.

Lemma ex11 : (forall x : U, P x -> Q x) ->

(forall x : U, ~ Q x) -> (forall x : U, ~ P x).

Proof.

intro Hpq.

intro Hq.

intro x.

unfold not.

intro Hp.

destruct (Hq x).

apply Hpq.

assumption.

Qed.