Airframe Analysis Write Up

Weston Pace

October 2022

1 Dictionary

Wingspan/Span (b) = A planar projection of the lateral extent of the wing. A vertical winglet will change the length of the wing, but it won't change the wingspan.

Wing Area/Reference Area (S) = The flat projected area of the total wing. Usually, the reference area is idealized as a trapezoidal shape even if the wing itself is not trapezoidal in shape.

Sweep (Λ) = This is the shear angle about the quarter chord line of a wing or the leading edge of the wing.

Dihedral (ϕ) = This is a rotative angle that shows the angle that the front of the wing makes with the horizontal ground.

Root Chord (c_r)= Chord length of the middle segment of a wing

Tip Chord (c_t)= Chord length of the end of the wing.

Twist (θ)= The rotation of the wing with respect to the horizontal flat surface of the ground. It creates a change in the angle of attack of the wing.

Aspect Ratio (AR) = Shows the ratio between the span and reference area $\frac{b^2}{S}$

Taper Ratio (λ) = Shows the ratio between the tip chord and root chord $\frac{c_t}{c_r}$

Mean Geometric Chord ($\tilde{c}$) = $\frac{S}{b}$

Mean Aerodynamic Chord (c_{mac}) = This is used in stability and control calculations, to normalize pitching moment, to compute static margin, and as the length scale in Reynold's number calculations.

Vortex Filament = A line of vortices that induce a velocity on the things around it. A vortex filament cannot abruptly end; it either needs to be a closed loop or go on for infinity. This is required to satisfy the fact that the circulation strength around a filament is constant.

Winglet = An angle created at the tip of the wing with the horizontal surface of the ground. It is like a dihedral angle but jsut at the tip.

Inviscid Span Efficiency (e_{inv}) = A coefficient that is made up of all the coefficients that represent induced drag. It gives a good representation of how much drag will be produced with respect to the lift being generated. $\frac{CL^2}{\pi * AR * CD}$

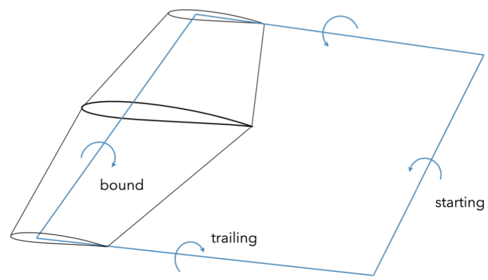

Figure 1: Downwash Model from Vortex Filaments

Side Slip Angle = The angle between the relative wind and the direction that an aircraft is traveling. It is like angle of attack, but it is with respect to the axis of yaw.

Tail Volume Ratio = Tail Volume ratio gives a relationship between how large and how far away a component of an airplane's tail is with respect to the wing. This can be calculated for both the horizontal and vertical stabilizers. An example for the Tail Volume Ratio of the horizontal stabilizer is: $\frac{S_t * x_t}{S_w * x_w}$

Airframe Stability Derivatives: These derivatives are extremely important for analyzing how an aircraft will travel through a space. Aircrafts have natural oscillating motions that are caused by the stabilizers implemented into the them. Stability derivatives tell us whether these implementations will cause the plane to naturally self correct itself and have consistent motion or if the plane will become unstable and lose control.

Coefficient of Drag (3D) CD = The drag coefficient in lifting line theory is induced by the generation of lift. The downwash that is creating lift is also responsible for changing the effective angle of attack of the incoming freestream. This rotates the lift vector a little bit, which inevitably creates drag from this newly oriented lift. In short, though, this is the drag of a 3D body in motion.

Coefficient of Lift (3D) CL = The lift coefficient in lifting line theory is created in a very different method than panel method in a program like Xfoil.jl. Lifting line theory uses horseshoe vortex filaments to describe the fluid flow around and behind the wing and tail. The down that is created by these vortex filaments are crucial for the generation of lift for the whole body of the plane. In short, though, it is the non-dimensionalized value of how much lift is being generated for the aircraft.

Coefficient of Moment (3D) CM = The coefficient of moment in 3D describes a similar phenomenon as its 2D counterpart does. It describes how the flow of the fluid around the body of the plane affects the rotational motion of the plane. By using the horseshoe vortex filaments created in lifting line theory, we can see what moment is created for the plane's body. Although moments are not desirable for an aircraft, the airplane has stabilizers that keep these effects from getting out of control.

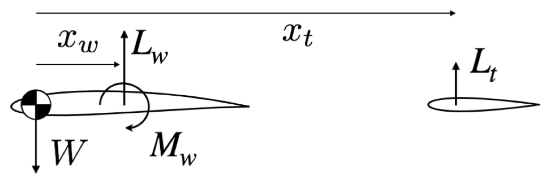

Figure 2: Tail Volume Ratio Parameters

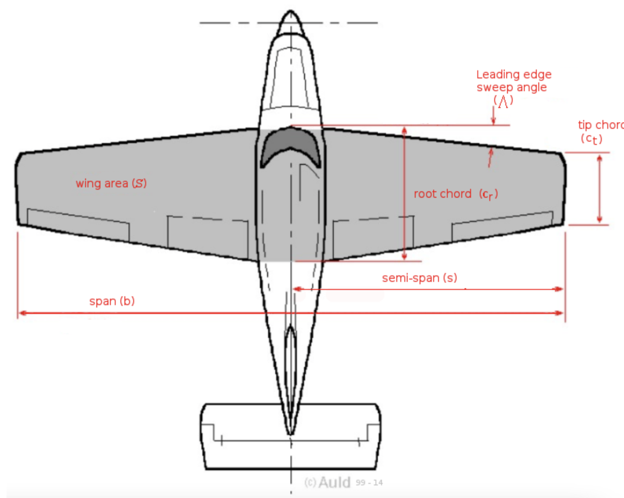

Figure 3: Wing Dimension Terms

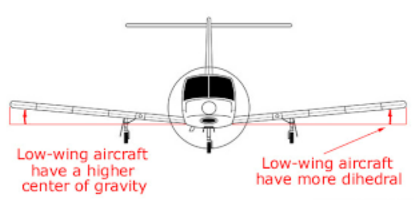

Figure 4: Wing Dihedral

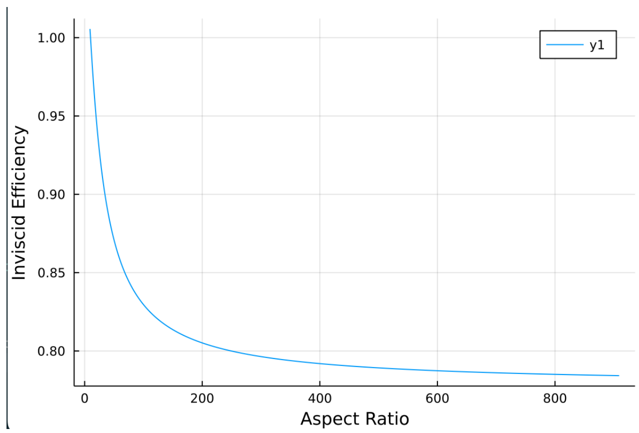

Figure 5: Hershey Bar Wing Efficiencies

2 Lifting Line Theory

- Where Thin Airfoil Theory focuses on the flow of a fluid around an airfoil, Lifting Line Theory takes into account the whole body of the aircraft and, instead of flow around the wing, the induced velocities produced by vortices that span across the wing and stabilizers of the aircraft.

- These vortices are in groups that are referred to as horseshoe vortex filaments, and a major effect that they produce is called downwash. Downwash is responsible for the fluid pushing down on the wing, but it is also the major contributor for lift.

3 VLM

- Vortex Lattice Method (VLM) is a package in Julia that uses Lifting Line Theory to find the aerodynamic coefficients and stability derivatives of an aircraft.

- A user just has to input wing and tail geometry, specify some free stream parameters, and identify what values need to be outputted after running the solver.

4 Wing Aspect Ratio vs. Wing Efficiency

- As shown in the dictionary, wing aspect ratio gives the non-dimensionalized ratio between the span of a wing squared and the reference area of said wing.

- Wing Efficiency pops up in an equation that relates both the coefficient of lift and the coefficient of drag: $\frac{CL^2}{\pi * AR * CD}$. A high efficiency means that the coefficient of drag is reasonably lower than the coefficient of lift produced after evaluating using lifting line theory.

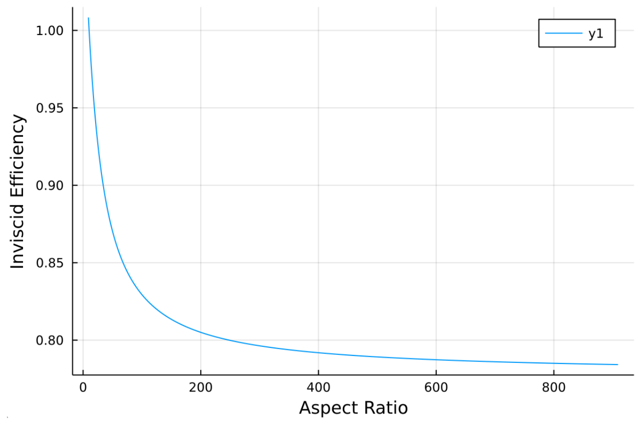

Figure 6: Dihedral Hershey Bar Efficiencies

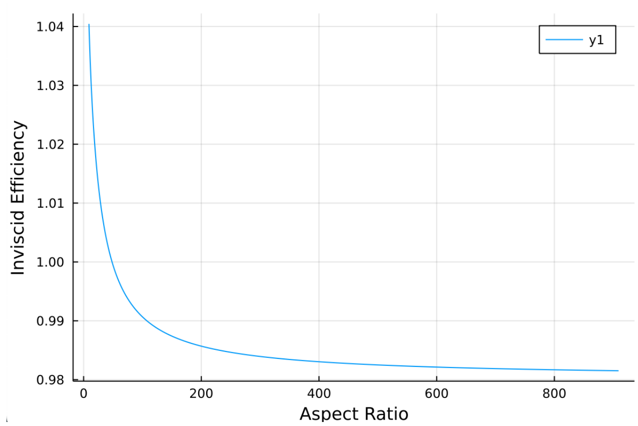

Figure 7: Non Hershey Bar Dihedral Efficiencies

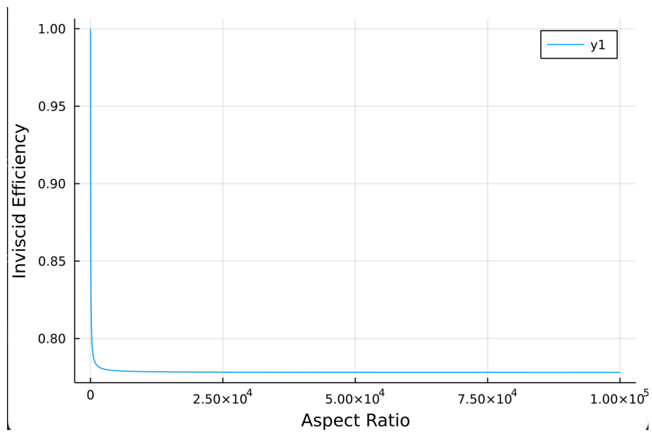

Figure 8: Constant Area Efficiencies

- Figures 5,6, and 8 all used hershey bar wings in them. With hershey bar wings, the efficiency can never be higher than one; however, for a non-hershey bar wing like figure 7, it is shown that the efficiency can actually be higher than 1: which explains why most wings are not designed like a rectangle. These non-hershey bar wings can produce a noticeably less amount of drag with respect to lift, which is extremely desirable in aircraft design.

- Another takeaway from these graphs is that as aspect ratio increases, which was created by increasing the span of the wing, the efficiency decreases in a rational manner.

5 Stability Derivatives

- Stability derivatives give essential information for an aircraft. They describe whether a mode of motion for an aircraft is stable, meaning the aircraft will natural correct its motion, or if it is unstable, meaning the aircraft will lose control. An example of a mode of motion is the naturally occurring oscillation that moves the aircraft in the "up" and "down" direction (Figure 9).

- The ideal sign for a stability derivative is negative. In an ODE, a characteristic equation with a negative sign corresponds to a solution that converges to a stable value. This is desirable in different modes of flight since an aircraft that stabilizes itself is much better than one who goes out of control.

- In these simulations, the tail volume ratio, $\frac{S_t * x_t}{S_w * x_w}$, was changed in order to see the effects on the stability derivatives. This change was created in three ways. One, the chord length of the tail was changed. Second, the span of the tail was altered. And finally, the distance of the tail from the aircraft's center of gravity was changed. Each of these effect the tail volume ratio, but their influences on the stability derivatives are different.

- In figure 10, the derivative Cmq, which is the derivative for pitching motion,

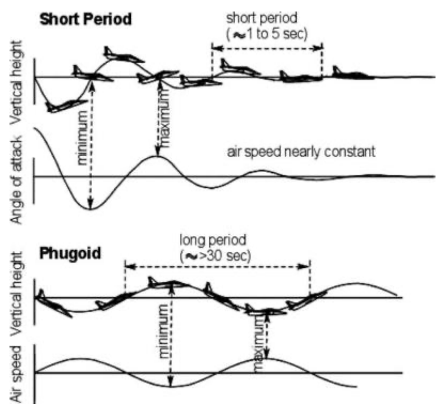

Figure 9: Phugoid Mode (Oscillating "Up" and "Down" Motion)

decreases to a more negative value as the tail volume ratio increases. This means that this mode gets more stable, and it is not a surprise that changing the tail volume ratio would create more stable "up" and "down" motion since the whole point of the tail is stabilize the plane as it naturally pitches up and down.

- Figure 11 shows a similar effect as tail volume ratio increases. The derivative Cma refers to pitch stiffness, which is similar to what Cmq relates to. As tail volume ratio increases, the pitch stiffness derivative becomes more stable, which isn't a surprise since this is what the horizontal stabilizer is for.

- For the other derivatives, their changes were so insignificant that changing tail volume ratio should not be linked to them. Also, some derivatives were so close to zero that they also can be somewhat overlooked when considering tail volume ratio.

6 Angle of Attack vs. Lift

- In order to model how angle of attack affected the coefficient of lift in VLM.jl, I just changed the freestream angle of attack. I could have changed the twist of the wing, as well, but just focusing on the freestream was more than enough.

- Figure 12 shows that as the angle of attack changes, the coefficient of lift changes in a linear manner. More specifically, the coefficient of lift changes with a slope of 2π.

- This is a limitation of Lifting Line Theory, and it arises from the fact that this theory is an inviscid method. This means that Lifting Line Theory doesn't account for viscosity in a fluid: which means that fluids won't separate at higher angle of attacks and lead to stall. This is why the linear relationship of lift to alpha exists for such high angle of attacks. In Xfoil.jl, viscosity is accounted for, which is why a decrease in lift and a change from a linear relationship can be seen in the lift polars when using that package. However, VLM.jl doesn't account for this, so this linear relationship goes on for any value of alpha. All

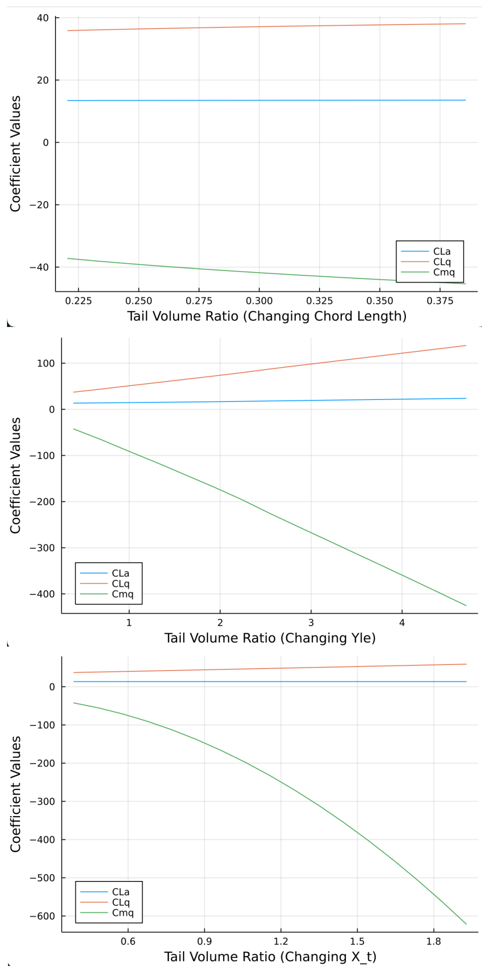

Figure 10: Larger Magnitude Stability Derivatives

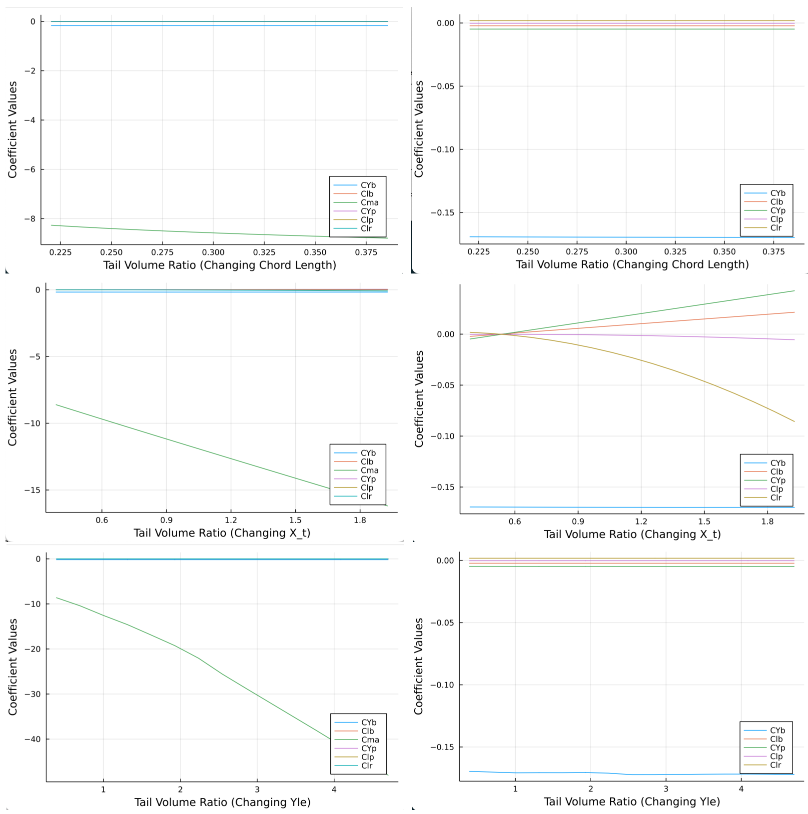

Figure 11: Smaller Magnitude Stability Derivatives

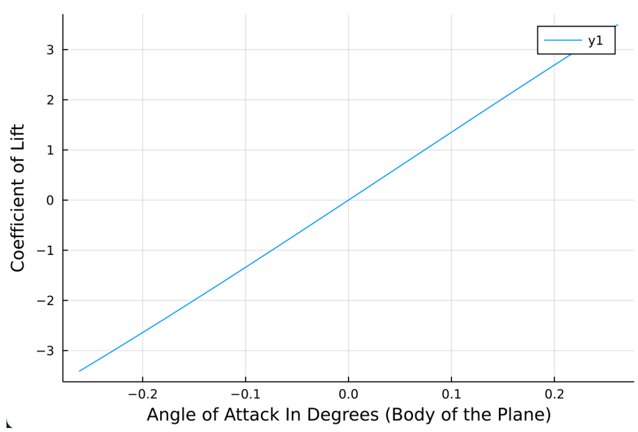

Figure 12: Lift vs. Angle of Attack (Radians)

of this stems from the assumptions made in Lifting Line Theory.