Leap-Frogging Vortices

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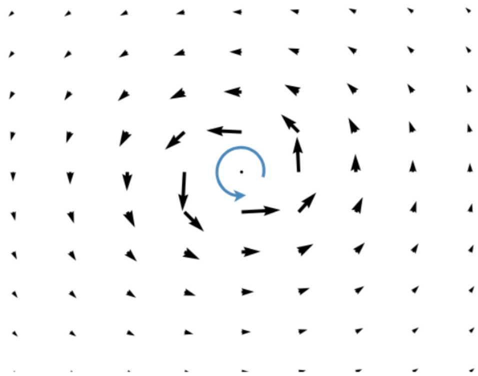
1 Introduction

A moving fluid can be idealized as a spinning vortex: being created by air passing through a boundary that creates friction with the fluid or by air passing by an airfoil. These are just two examples of vortices being formed, but the important thing about vortices is that they induce a velocity around them. For the case of this report, the focus is on vortex rings, which are just a circular formation of vortices. These vortices only induce a velocity on the other vortices that are directly below and directly behind the vortex source with respect to their orientation. In other words, a vortex moving in the x-y plane with induced velocity that rotate above the z-axis will only affect the other vortices in thex-y plane. This effect stays constant as different vortices around the ring are viewed since the axes described can be shift with respect to new orientation of the vortex. A vortex can't induce a velocity on itself, but it can on other vortices. This leads to the leap-frogging phenomenon of two vortex rings as their vortices induce velocities on one another.

The angular induced velocity can be calculated with the following equation.

$$V_\theta = \frac{\Gamma}{2\pi r} \tag{1}$$

This equation leads to a model that can be visualized as such:



2 Methods

For this assignment, the top and bottom of each vortex were the points of interest being used to model the movement of the vortices. the leap-frogging motion of the vortex rings can be simplified by only evaluating these four vortice since the rings are symmetrical. What ever relationship in motion these four share will also be shared by another four vortices in the same plane. The high and low point of an individual vortex were intially spaced one meter apart, and the distance between the vortices was the same value.

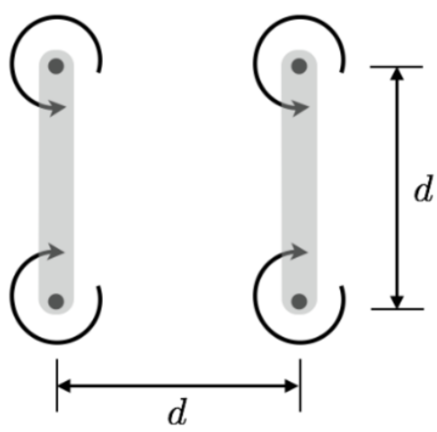


Figure 1: This visual represents the initial formation of the vortices

Although the movement of these vortices is difficult to track due to the constantly changing velocities, they are possible to mimic through the use of a program. For the model created in this report, Julia was the chosen language. By creating loops that calculated the velocities at each interval and multiplied these velocities by $\Delta t = 0.01$ seconds, a matrix was able to be created containing the position of the top and bottom of each vortex.

3 Results

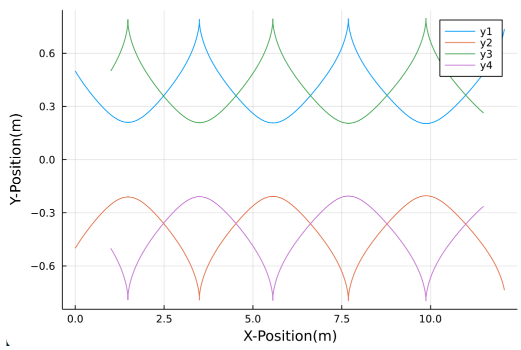


Figure 2: The parametric lines created from each point's matrix

Lines $y1$ and $y2$ represent the vortices that started in the back while lines $y3$ and $y4$ show the other. When graphed, they follow a dependent path of one another. As shown below, the motion is very symmetrical, and the vorticies switch back and forth between two different patterns of movement (seen below).

By switching certain parameters of the original simulation, such as circulation strength and initial distances, new and interesting results can be obtained. Some of these variations are shown below:

As seen in figure 3, the two vortices have quick and somewhat abrupt 'leap-frogging' motions. However, as the simulation continues to run, their spacing increases, and their movements begin to resemble the original simulation.

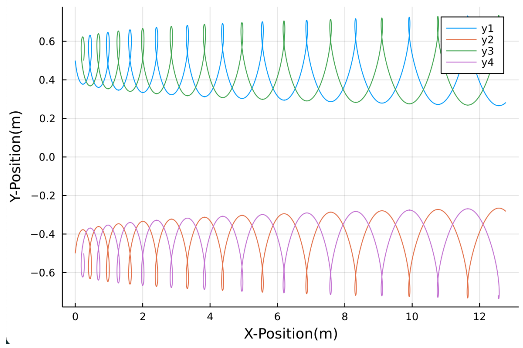


Figure 3: Vortex movement when the initial x-axis spacing between the vortices was 0.25 meters

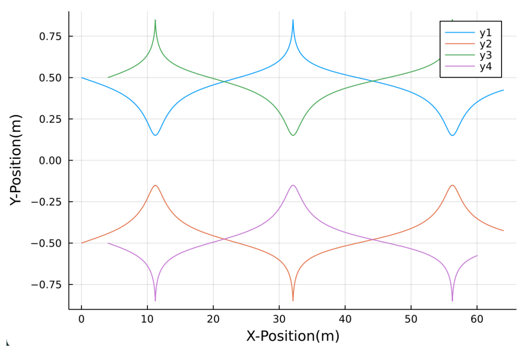


Figure 4: Vortex movement when the initial x-axis spacing between the vortices was 4 meters

In the case of figure 4, the movement of the vortices is much slower and obtuse, and the time it takes for a single pass through to occur is much longer.

For figure 5 (below), the simulation resembles the motion of the rings when the x-axis spacing was increased to 4 meters. The leap-frogging effect is still present, but it is much more obtuse and slow than the original.

Changing the initial distance of the vortices inevitably affects the induced velocity that they have on each other. The equation $V_\theta = \frac{\Gamma}{2\pi r}$ shows that when the initial distance is smaller, the induced velocity on other vortices is

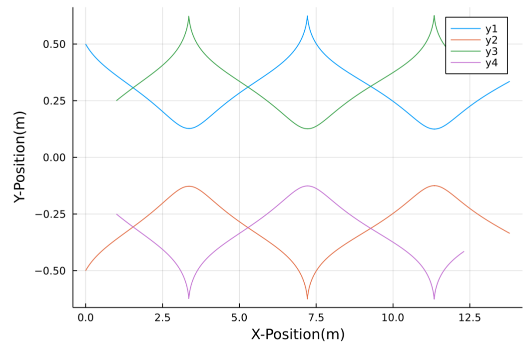


Figure 5: Vortex movement when the diameter of the front vortex was decreased by 0.5 meters

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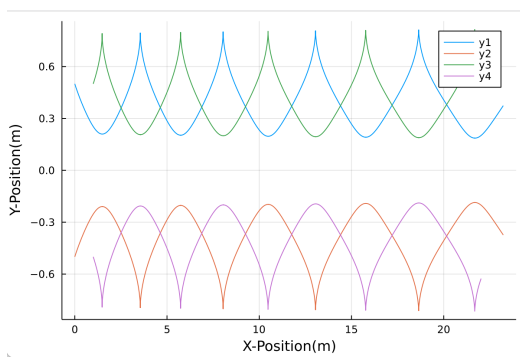

Figure 6: Vortex movement when the circulation strength is equal to 2

much larger: leading to them passing through each other much faster. On the contrary, when the induced velocity is initially lower, the rings take a longer time to pass each other.

Another parameter that can be changed is the circulation strength of the vortices.

This changed caused the frequency of pass throughs to increase while still preserving the orderly movement of the rings that is found with all of the initial conditions remaining unchanged.

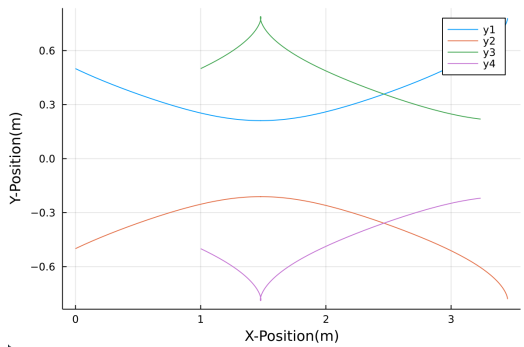

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Figure 7: Vortex movement when the circulation strength is equal to 0.25

On the other hand, when the circulation strength is reduced to 0.25, the rings' frequency of pass throughs decreased. Although the overall movement of the rings is limited to a single pass through do to their slow velocities, it can be seen that they act similaraly to the $\Gamma = 2$ example: in which they keep orderly motion.

What is learned from these three different cases is that the initial parameters of the simulation play a huge role in how the 'leap-frogging' effect plays out; slight adjustments ultimately bring drastically different results. A large change will bring more obvious changes. In terms of circulation strength, it seems that consistent motion can be kept no matter how the constant is change; however, the rate at which the rings pass through each other seems to change proportionally to the change in the strength.

4 Conclusion

Working through this problem was extremely beneficial for me. Not only did it introduce me to an interesting phenomenon but it taught me how to use Julia and how to work through complex problems. On the surface, this project seemed pretty straight forward; however, as I continued to dive deeper into it, I realized that I completely underestimated the work required.

Initially, I only had a two equations to model the movement of these vortices, but by the end, my code required nine in order to represent the simulation correctly. It was a battle of trial and error, and I was constantly learning new things as problems arose. If I could go back and do this project again, I would approach the induced velocities as vectors instead of scalars. This would have saved me a lot of time and headache when trying to compute the motion of the vortices. Although this project was only an onboarding assignment, I still feel extremely proud of myself for accomplishing it since the coding and problem solving were harder than anything that I've ever done before. I hope that the skills learned through this assignment will be a foundation for the rest of my time in the lab, and I can't wait to keep obtaining more information through projects like this.