# Exercise 2

## Epipolar Geometry

a) $K = R = I$

$\tilde{E}_1 = [t_1]_\times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad \tilde{E}_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ $\quad \tilde{E}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$t_1$时

设点为 $\bar{x}_1 = \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$ $\quad \bar{x}_2 = \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$

$\tilde{l}_2 = \tilde{E}\,\tilde{x}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ y_1 \end{pmatrix}$

$\tilde{l}_1 = \tilde{E}^T \tilde{x}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{pmatrix} 0 \\ 1 \\ -y_2 \end{pmatrix}$

两条线均为水平线

∴ 平行 ∴ 极点位于无穷远处

为 ideal point

$t_2$时 $\quad \tilde{l}_2 = \tilde{E}\,\tilde{x}_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -x_1 \end{pmatrix}$

$\tilde{l}_1 = \tilde{E}^T \tilde{x}_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{pmatrix} -1 \\ 0 \\ x_2 \end{pmatrix}$

为垂直线 → 平行 → 极点位于无穷远处

$t_3$时 $\quad \tilde{l}_2 = \tilde{E}\,\tilde{x}_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} -y_1 \\ x_1 \\ 0 \end{bmatrix}$

$\tilde{l}_1 = \tilde{E}^T \tilde{x}_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} y_2 \\ -x_1 \\ 0 \end{bmatrix}$

c)

$K_2^{-T}\tilde{E}K_1^{-1} = \tilde{F} = \tilde{E}$ 时

即要求 $\begin{cases} \bar{x}_2^T K_2^{-T} = \bar{x}_2^T \\ K_1^{-1}\bar{x}_1 = \bar{x}_1 \end{cases}$

$\Rightarrow \begin{matrix} K_2^{-T} = I & \therefore K_2 = I \\ K_1^{-1} = I & K_1 = I \end{matrix}$

$\tilde{x}_2^T \tilde{E}\tilde{x}_1 = 0$

$\tilde{x}_i = K_i^{-1}\bar{x}_i$

$\downarrow$

$\bar{x}_2^T K_2^{-T}\tilde{E}K_1^{-1}\bar{x}_1 = \bar{x}_2^T\tilde{F}\bar{x}_1 = 0$

$\therefore \begin{matrix} \tilde{x}_2 = \bar{x}_2 \\ \tilde{x}_1 = \bar{x}_1 \end{matrix} \quad \therefore \begin{matrix} K_2 = I \\ K_1 = I \end{matrix}$

Triangulation

a) $\tilde{x}_i^{(s)} = \tilde{P}_i\tilde{X}_w$

$\tilde{x}_1^{(s)} = \tilde{P}_1\tilde{X}_w = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{2} \\ 1 \end{pmatrix}$

$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\tilde{x}_2^{(s)} = \tilde{P}_2\tilde{X}_w = \begin{pmatrix} -\frac{1}{5} \\ \frac{1}{5} \\ 1 \end{pmatrix}$

$P_2 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 & -3 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$\underline{P = K[R|t]}$

由 $\begin{bmatrix} x_i^s\tilde{p}_{i3}^T - \tilde{p}_{i1}^T \\ y_i^s\tilde{p}_{i3}^T - \tilde{p}_{i2}^T \end{bmatrix}\tilde{X}_w = 0$ 得 $A = \begin{bmatrix} -1 & 0 & 0.25 & 0 \\ 0 & -1 & 0.5 & 0 \\ 2 & 0 & -1.2 & 2.8 \\ 0 & 2 & 0.8 & -0.8 \end{bmatrix}$

解得 $\tilde{X}_w = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \end{bmatrix}$

Stereo Vision

a)

$$z(d) = \frac{fb}{d}$$

$$z(d+\Delta d) = \frac{fb}{d+\Delta d}$$

$$\text{error} = z(d+\Delta d) - z(d)$$

对d导数 $-\frac{fb}{d^2} = -\frac{z^2}{fb}$

$$\frac{z(d+\Delta d) - z(d)}{\Delta d} = -\frac{z^2}{fb}$$

$$\therefore \quad \text{error} = -\Delta d \cdot \frac{z^2}{fb}$$

error 随 $z^2$ 增长

b)

增加 b 的大小

基线太长可能导致测量不准

Block Matching

a) ZNCC:

$$NCC(x,y,d) = \frac{(W_L(x,y) - \bar{W}_L(x,y))^T}{\| W_L(x,y) - \bar{W}_L(x,y) \|_2} \cdot \frac{(W_R(x-d,y) - \bar{W}_R(x-d,y))}{\| W_R(x-d,y) - \bar{W}_R(x-d,y) \|_2}$$

$W_i' = \alpha_i W_i + 1\beta_i$ 线性变化

$$\bar{W}_i' = \alpha_i \bar{W}_i + 1\beta_i$$

$$W_i' = \alpha_i W_i + 1\beta_i$$

$$\therefore \quad NCC'(x,y,d) = \frac{(W_L'(x,y) - \bar{W}_L'(x,y))^T}{\| W_L'(x,y) - \bar{W}_L'(x,y) \|_2} \cdot \frac{(W_R'(x-d,y) - \bar{W}_R'(x-d,y))}{\| W_R'(x-d,y) - \bar{W}_R'(x-d,y) \|_2}$$

$$= \frac{\alpha_1 (W_L(x,y) - \bar{W}_L(x,y))^T}{\alpha_1 \| W_L(x,y) - \bar{W}_L(x,y) \|_2} \cdot \frac{\alpha_2 (W_R(x-d,y) - \bar{W}_R(x-d,y))}{\alpha_2 \| W_R(x-d,y) - \bar{W}_R(x-d,y) \|_2}$$

$$= NCC(x,y,d)$$

b)

3×3 窗口

$$SSD(x,y,d) = \| W_L(x,y) - W_R(x-d,y) \|_2^2$$

d=0

$$SSD(x,y,0) = 3\times(10-5)^2 + 3\times(5-6)^2 + 3\times(6-7)^2$$
$$= 3 \times (25+1+1) = 3\times 27 = 81$$

d=1

$$SSD(x,y,1) = 3\times\left((10-4)^2 + (5-5)^2 + (6-6)^2\right)$$
$$= 3\times 36 = 108$$

d=2

$$SSD(x,y,2) = 3\times\left((10-10)^2 + (5-4)^2 + (6-5)^2\right)$$
$$= 3\times 2 = 6$$

∴ d=2 为 best match

c)

Cyan left 坐标为 (2,6). d=1 ∴ right ⇒ (2,6-1) = (2,5),

right (2,5), d=1 ∴ left → (2,5+1) = (2,6) 一致

Green. left (3,4). d=2. → right (3, 4-2) = (3,2)

right (3,2) d=2 ∴ left → (3, 2+2) = (3,4) 一致.

Red. left (5,3) d=2. ∴ right (5, 3-2) = (5,1)

right (5,1) d=2. ∴ left (5, 1+2) = (5,3) 一致.

Test 没有成功确定错误估计

right 最左边一列出错了

Learned Stereo and End-to-End Models

a) $$N = \frac{(W-F+2P)}{S} + 1$$

W：输入大小
F：kernel 大小
P：补丁
S：步长.

$$\text{params} = \text{channels}_{out} \times (\text{kernel}_w \times \text{kernel}_h \times \text{channels}_{in}) + \text{channels}_{out}$$

$$\text{param\_Memory} = \text{params} \times 4 / 1024 / 1024$$

$$\text{activate\_Memory} = \text{out}_w * \text{out}_h * \text{out}_{channel} * 4$$

| Layer | Input shape. | Output shape | # Trainable Parameters | Memory |
|---|---|---|---|---|
| Conv2D (32.64.3) | (32,128,128) | (64, 128, 128) | $64\times(3\times3\times32)+64$ | |
| Conv2D (64,128,3) | (64,128,128) | (128, 128, 128) | $128\times(3\times3\times64)+128$ | 12.352 MB |
| Conv3D (1, 64, 3) | (1.32,128,128) | (64, 32,128,128) | $64\times3\times3\times3\times1+64$ | |
| Conv3D (64,128,3) | (64.32,128,128) | (128, 32,128,128) | $128\times3\times3\times3\times64+128$ | 391.10 MB |

2D. $$(64\times128\times128 + 128\times128\times128)\times4 + \text{param\_Memory} = 12952320 \text{ Byte} = 12.3522 \text{ MB}$$

3D. $$(64\times32\times128\times128 + 128\times32\times128\times128)\times4 + (64\times3^3+64+128\times3^3\times64+128)\times4 = 410099200 \text{ Byte} = 391.10 \text{ MB}$$

c)

对于 $P_1$,

$$\sigma(-C_d) = [0.44, 0.06, 0.00, 0.06, 0.44]$$

$$E(d) = 0\times0.44 + 1\times0.06 + 2\times0 + 3\times0.06 + 4\times0.44 = 2$$

对于 $P_2$

$$\sigma(-C_d) = [0, 0.21, 0.58, 0.21, 0]$$

$$E(d) = 0\times0 + 1\times0.21 + 2\times0.58 + 3\times0.21 + 4\times0 = 2$$