Exercise 1

Homogeneous Coordinates.

a) 设 $\tilde{l}_1 = (a_1, b_1, c_1)^T$ $\rightarrow \begin{cases} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{cases}$ 解得 $\begin{cases} x = \dfrac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \\ y = \dfrac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1} \end{cases}$

$\tilde{l}_2 = (a_2, b_2, c_2)^T$

$$a_1a_2x+a_2b_1y+a_2c_1=0$$
$$a_1a_2x+a_1b_2y+a_1c_2=0$$
$$(a_1b_2y-a_2b_1y)+a_1c_2-a_2c_1=0$$
$$y=\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}$$

$$x=-\frac{c_1}{a_1}-\frac{b_1}{a_1}y$$
$$x=-\frac{c_1}{a_1}-\frac{b_1}{a_1}\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}$$
$$=-\frac{1}{a_1}\left(c_1+b_1\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}\right)$$
$$=-\frac{1}{a_1}\left(\frac{a_1b_2c_1-\cancel{a_2b_1c_1}+\cancel{a_2b_1c_1}-a_1b_1c_2}{a_1b_2-a_2b_1}\right)$$
$$=-\frac{b_2c_1-b_1c_2}{a_1b_2-a_2b_1}$$
$$=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$$

$$X = \tilde{l}_1 \times \tilde{l}_2$$
$$= [\tilde{l}_1]_\times \tilde{l}_2 = \begin{bmatrix} 0 & -c_1 & b_1 \\ c_1 & 0 & -a_1 \\ -b_1 & a_1 & 0 \end{bmatrix} \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} = \begin{pmatrix} b_1c_2-b_2c_1 \\ a_2c_1-a_1c_2 \\ a_1b_2-a_2b_1 \end{pmatrix}$$
$$= \begin{pmatrix} \dfrac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \\ \dfrac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1} \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$ 结果相同

b) $\tilde{x}_1 = (a_1, b_1, 1)$ 传统法: $y = \dfrac{b_2-b_1}{a_2-a_1}(x-a_1)+b_1$

$\tilde{x}_2 = (a_2, b_2, 1)$

$$\Downarrow \qquad a_2b_1-\cancel{a_1b_1}-a_1b_2+\cancel{a_1b_1}$$

$$(b_2-b_1)x-(a_2-a_1)y+b_1(a_2-a_1)-a_1(b_2-b_1)=0$$
$$\Rightarrow (b_2-b_1)x+(a_1-a_2)y+a_2b_1-a_1b_2=0$$
$$\therefore \tilde{l} = \left(\frac{b_2-b_1}{a_2b_1-a_1b_2}, \frac{a_1-a_2}{a_2b_1-a_1b_2}, 1\right)^T$$

$$\tilde{l} = \tilde{x}_1 \times \tilde{x}_2$$
$$= \begin{pmatrix} 0 & -1 & b_1 \\ 1 & 0 & -a_1 \\ -b_1 & a_1 & 0 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \\ 1 \end{pmatrix} = \begin{pmatrix} b_1-b_2 \\ a_2-a_1 \\ a_1b_2-a_2b_1 \end{pmatrix} = \begin{pmatrix} \dfrac{b_2-b_1}{a_2b_1-a_1b_2} \\ \dfrac{a_1-a_2}{a_2b_1-a_1b_2} \\ 1 \end{pmatrix}$$

结果相同

c) $\begin{cases} x+y+3=0 \\ -x-2y+7=0 \end{cases}$ 解得 $\begin{cases} x=-13 \\ y=10 \end{cases}$

$$X = \tilde{l}_1 \times \tilde{l}_2$$
$$= \begin{pmatrix} 0 & -3 & 1 \\ 3 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 7 \end{pmatrix} = \begin{pmatrix} -13 \\ 10 \\ 1 \end{pmatrix}$$

交点相同

d). $n_x = \frac{3}{5}$, $n_y = \frac{4}{5}$, $d=3$

$\therefore \tilde{l} = (\frac{3}{5}, \frac{4}{5}, 3)^T$

e) $\tilde{l} = (a,b,c)^T$ $\qquad \sqrt{2^2+5^2} = \sqrt{29}$.

$\downarrow$ normalized.

$$\tilde{l} = \left(\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}, \frac{c}{\sqrt{a^2+b^2}}\right)^T$$

$$\therefore \tilde{l} = \left(\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}, \frac{1}{5}\right)^T$$

$$n = \left(\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}\right)$$
$$d = \frac{1}{5}$$

Transformations.

a). $t = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$$[I \ \ t]_{2\times3} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

b)

$$T = [I \ \ t] = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \end{bmatrix} \qquad \bar{x}_i = \begin{pmatrix} x_1^i \\ x_2^i \\ 1 \end{pmatrix}$$

$$T \cdot \bar{x}_i = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \end{bmatrix} \begin{bmatrix} x_1^i \\ x_2^i \\ 1 \end{bmatrix} = (x_1^i + t_1 \ , \ x_2^i + t_2)^T$$

$$E(T) = E(t_1, t_2) = \sum_{i=1}^{N} \| T\bar{x}_i - y_i \|_2^2$$

$$= \sum_{i=1}^{N} \left[(x_i^1 + t_1 - y_i^1)^2 + (x_i^2 + t_2 - y_i^2)^2 \right] \qquad y_i = \begin{pmatrix} y_1^i \\ y_2^i \end{pmatrix}$$

$$\frac{\partial E(t_1, t_2)}{\partial t_1} = \frac{\partial \sum_{i=1}^{N} (x_i^1 + t_1 - y_i^1)^2}{\partial t_1}$$

$$= 2 \sum_{i=1}^{N} (x_i^1 + t_1 - y_i) = 2Nt_1 + 2\sum_{i=1}^{N} (x_1^i - y_1^i) = 0$$

$$\frac{\partial E(t_1, t_2)}{\partial t_2} = 2Nt_2 + 2\sum_{i=1}^{N} (x_2^i - y_2^i) = 0$$

$$\therefore \ t_1 = \frac{1}{N} \sum_{i=1}^{N} (y_1^i - x_1^i)$$

$$t_2 = \frac{1}{N} \sum_{i=1}^{N} (y_2^i - x_2^i)$$

c)

$$t_1 = \frac{1}{3} (3 + 2 + 1) = 2$$

$$t_2 = \frac{1}{3} (-6 - 1 - 5) = -4$$

$$\therefore \ T^* = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \end{bmatrix}$$

Camera Projection

a) $\tilde{P} = \begin{bmatrix} K & 0 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$ $R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$

$K = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$ $\varphi = 90^\circ$

$$\therefore \tilde{P} = \begin{bmatrix} 100 & 0 & 25 & 0 \\ 0 & 100 & 25 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 100 & 25 & 0 & 150 \\ 0 & 25 & -100 & 50 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $\tilde{x}_s = \tilde{P}\,\bar{x}_w$

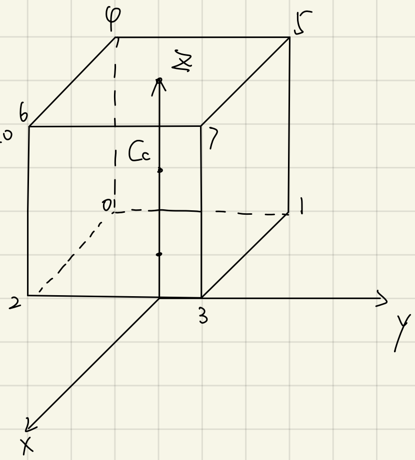
$\therefore \bar{x}_w = \tilde{P}^{-1}\tilde{x}_s$

$$= \begin{bmatrix} 0.01 & 0 & -0.25 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & -0.01 & 0.25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 25 \\ 50 \\ 1 \\ 0.25 \end{bmatrix} = \begin{bmatrix} -0.25 \\ 0.5 \\ -0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore X_w = [-1, 2, -1]^T$

c)

$C_0 \rightarrow C_c = (0, 0, 15)^T$ as center, $s = 20$



i) $K = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{bmatrix}$

中心坐标为 (0, 0, 15)

8个点坐标为

用 python 写了 "c)"