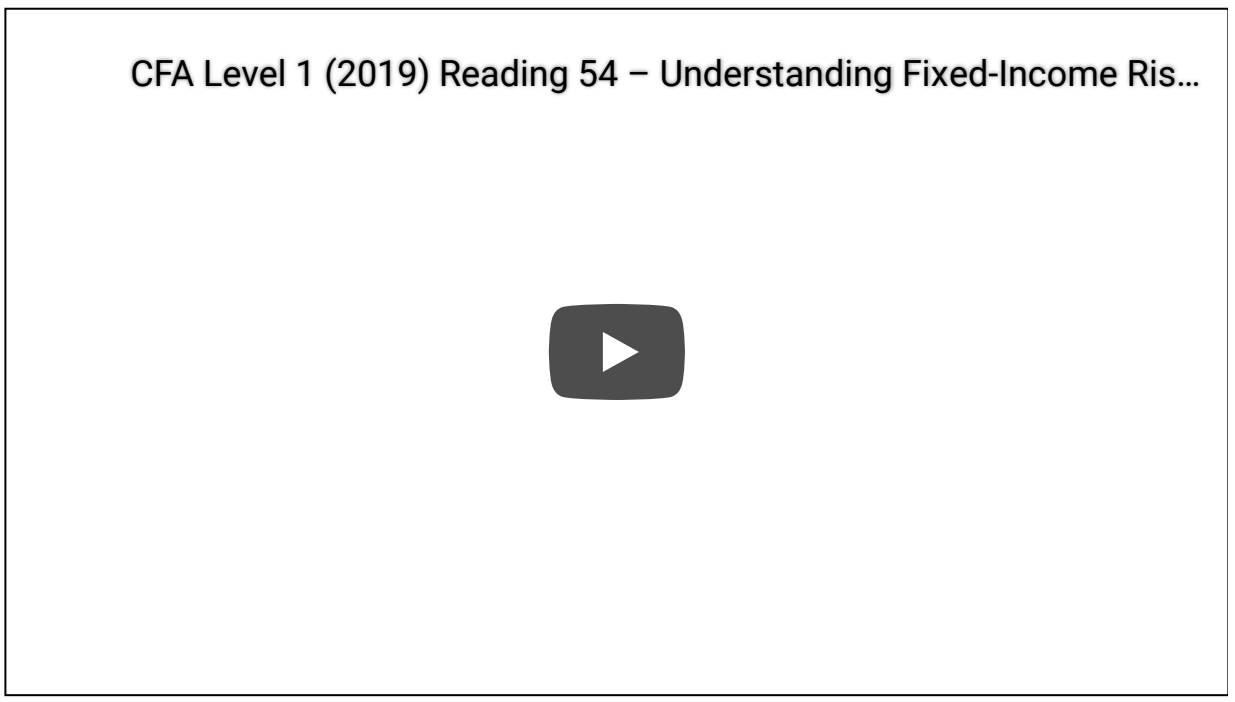
September 6, 2019 in Fixed Income

# Duration and Convexity Effect on the Price Change of a Bond




The change in the price of a bond can be summarized as follow:

$$Change \quad in \quad price = Duration \quad effect + Convexity \quad effect$$

$$\approx (-AnnModDur \times \Delta Yield) + (\frac{1}{2} \times AnnConvexity \times (\Delta Yield)^2)$$

## Example 1

Suppose the yield-to-maturity is expected to fall by 10 bps tomorrow, from 2.95% to 2.85%. A bond has an annual (modified) duration of 24.500 and annual convexity of 775.0. What is the percentage price gain from this fall in interest rate?

$$\%\Delta PV^{FULL} \approx (-24.500 \times -0.0010) + (\frac{1}{2} \times 775.0 \times (-0.0010)^2)$$

$$\approx 0.0245 + 0.0004 \cong 0.0249$$

The modified duration alone underestimates the gain to be 2.45%. The convexity adjustment adds 4 basis points.

## Example 2

A pension scheme holds a large position in a 6.5% annual coupon payment government bond that matures on 10th March 2034. The bond's yield-to-maturity is 6.75% for settlement on 15th May 2019, stated as an effective annual rate. That settlement date is 65 days into the 360-day year using the 30/360 day count convention.

(a) Calculate the full price of the bond per 100 of par value.

The full price of the bond is 98.845543 per 100 per value.

$$PV_0 = [\frac{6.5}{1.0675^1} + \frac{6.5}{1.0675^2} + \cdots + \frac{100 + 6.5}{1.0675^15}] \times 1.0675^{65/360} = 98.845543$$

(b) Calculate the approximate modified duration and approximate convexity using a 1 bp increase and decrease in the yield-to-maturity.

$$PV_+ = [\frac{6.5}{1.0676^1} + \frac{6.5}{1.0676^2} + \cdots + \frac{100 + 6.5}{1.0676^15}] \times 1.0676^{65/360} = 98.755130$$

$$PV_- = [\frac{6.5}{1.0674^1} + \frac{6.5}{1.0674^2} + \cdots + \frac{100 + 6.5}{1.0674^15}] \times 1.0674^{65/360} = 98.936070$$

The approximate modified duration is 9.1527.

$$Approx. \quad ModDur = \frac{98.936070 - 98.755130}{2 \times 0.0001 \times 98.845543} = 9.1527$$

The approximate convexity is 115.3315.

$$Approx. \quad conv = \frac{98.936070 + 98.755130 - (2 \times 98.845543)}{0.0001^2 \times 98.845543} = 115.3315$$

(c) Calculate the estimated convexity-adjusted percentage price change resulting from a 100 bp increase in the yield-to-maturity

The convexity-adjusted percentage price drop resulting from a 100 bp increase in the yield-to-maturity is estimated to be 8.576%. Notably, modified duration alone estimates the percentage drop to be 9.1527%. The convexity adjustment adds 57.67 bps.

(100 bps = 1% = 0.0100)

$$\%\Delta PV^{FULL} \approx (-9.1527 \times 0.0100) + (\frac{1}{2} \times 115.3315 \times (-0.0100)^2)$$
$$\approx -0.091527 + 0.005767 = -0.08576$$

(d) How does the estimated percentage price change compare with the actual change, assuming the yield-to-maturity jumps to 7.75% on that settlement date?

The new full price if the yield-to-maturity goes from 6.75% to 7.75% on 15$^{th}$ May 2019 is 90.344807.

$$PV^{FULL} = [\frac{6.5}{1.0775^1} + \frac{6.5}{1.0775^2} + \cdots + \frac{100 + 6.5}{1.0775^15}] \times 1.0775^{65/360} = 90.34$$

$$\%\Delta PV^{FULL} \approx \frac{90.344807 - 98.845543}{98.845543} = -0.086000$$

As these calculations show, the actual percentage change in the bond price is –8.6%. The convexity-adjusted estimate is –8.576%, whereas the estimated change using modified duration alone is –9.1527%. As such, it is evident that convexity adjustment is paramount.

# Question

*An investment bank holds a considerable position in a 7% annual coupon paying bond. The bond's yield-to-maturity is 8%. The settlement date is 83 days into the 360-year. The approximate modified duration is 9 years and approximate convexity is 105. What is the estimated convexity-adjusted percentage price change resulting from a 100 bps increase in the yield-to-maturity?*

*A. 0.0953*

*B. 0.0875*

*C. 0.0925*

*Solution*

*The correct answer is A.*

$$\%\Delta PV^{FULL} \approx (-9.00 \times -0.0100) + (\frac{1}{2} \times 105.00 \times (-0.01)^2)$$

$$\approx 0.09 + 0.0053 \approx 0.0953$$

*The convexity-adjusted percentage price drop resulting from a 100 bps increase in the yield-to-maturity is estimated to be 9.53%. The modified duration alone underestimates the gain to be 9.00%, and the convexity adjustment adds 53.0 bps.*

*Reading 54 LOS 54i:*

*Estimate the percentage price change of a bond for a specified change in yield, given the bond's approximate duration and convexity*

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