42104 – Introduction to Financial Engineering – Exam Date: 12/19/19
Weston Jones – Study Number: 191380.

Question 1a

According to the definition of duration, for every 1% change in interest rates (increase or decrease), a bond's price will change approximately 1% in the opposite direction, for every year of duration.
Thus if a 1% increase causes a reduction of 17% in price, then duration must be 17.

Question 1b

Change in price = Duration effect + Convexity effect
≈ (−AnnModDuration × ΔYield) + (.5 × AnnConvexity × (ΔYield)^2)

Plugging in numbers gets:
(-17 * -.0005) + (.5 * 75 * -.0005^2) = .0084

The interest decrease would lead to a .84% increase in price.

Question 2a.

Average Annual Return = (1+WeeklyReturn)^52 – 1
(1+.01)^52-1=.677688

Average Annual Return is 67.77%

Question 2b.

{[(1+.02)(1-.01)(1+.05)]^(1/3)}-1 = .0197

The average monthly return is 1.97%

We can annualize this with:

(1+.0197)^12 - 1 = .263859

The annualized average return is then 26.39%

Question 2c.

Daily Std to Annual Std → .015 * sqrt(252) = .23811

Annual Std to Annual Variance → sqrt(.23811) = .4879

Annual Variance = 48.79%

Question 2d.

Daily Std to Annual Std → .02 * sqrt(252) = .31749

Annual Std = 31.75%

Question 3a.

To calculate weights, calculate total wealth invested. Divide the wealth invested in each stock by total wealth invested.

Total wealth = 500*$35 + 25*$50 = $18,750. << This is also the initial cost for setting up the portfolio.

Weights on Stock A = (500*35)/18750 = .93
Weights on Stock B = (25*50)/18750 = .0666666

Question 3b

A Zero-Coupon-Bond with a nominal of $100 expires in 27 years. Assume that the yield of the bond is 0.01 in annual terms. What is the value of the bond today?

Question 4a

Both Google and Lamb Weston have the same Annual Standard deviation of returns / risk. However, when considering the stock we are already committed to investing in, Boeing, Lamb Weston has a much lower correlation. Therefore investing in Lamb Weston instead of google will diversify the portfolio more and thus lower risk.

Question 4b

Sharpe Ratio = Excess Return / Risk. If only looking at the Sharpe Ratio of the stocks, Google would be better than Lamb Weston to invest in as its annual expected return is much higher while its standard deviation / risk is the same.

Question 4c

I will invest in Boeing and Lamb Weston.

I did the following calculations in Matlab:

```
mu = [.18; .16];
mu_e = mu - .02;

sigma = [.29^2, .05 * .29 * .24; .05 * .29 * .24, .24^2];
```

```
A = [mu, ones(2,1)]' * inv(sigma) * [mu, ones(2,1)];

a = mu' * inv(sigma) * mu;
b = mu' * inv(sigma) * ones(2,1);
c = ones(2,1)' * inv(sigma) * ones(2,1);

GMVu = b/c;
GMVstd = sqrt(1/c);
GMVw = (1/c) * inv(sigma) * ones(2,1);

TANu_e = (mu_e' * inv(sigma) * mu_e) / (ones(2,1)' * inv(sigma) * mu_e);
TANu = TANu_e + .02;
TANstd = sqrt(mu_e' * inv(sigma) * mu_e) / (ones(2,1)' * inv(sigma) * mu_e);
TANw = inv(sigma) * mu_e * (1/(ones(2,1)' * inv(sigma) * mu_e));
```

Global Minimum Variance Portfolio:

- -GMVstd: 0.189372721395474
- -GMVu: 0.168033249220721,
- -GMVw: [0.401662461036070;0.598337538963931], (First is Boeing, Second is Lamb Weston)

Tangent Portfolio:

- -TANstd: 0.189832173929203,
- -TANu: 0.168752431565226,
- -TANw: [0.437621578261306;0.562378421738694] (First is Boeing, Second is Lamb Weston)

Question 4d

Sharpe Ratio = Excess Return / Risk

Building off the Matlab code above:
TANsharpe = TANu_e / TANstd = 0.783599684322755

The Tangent portfolio is calculated specifically to be the portfolio with the highest Sharpe Ratio. Therefore the sharpe ratio calculated above should always be higher than any sharpe ratio calculated from any other portfolio given the problem data and constraints.

Question 7a

$\mu R - \mu f = (\sigma R / \sigma M) * (\mu M - \mu f)$

(.06 / .15) * (.12 - .0003) = .04788. The question asks for expected return not excess return, so this is the final answer. About 4.7% return

Question 7b

Beta for an asset is equal to the covariance with the market divided by the variance of the market.

.005 / .15^2 = .22 = Beta for stock A.

Question 7c

$$\mu_{P,t} = \sum_{j=1}^{N} w_j(\mu_f + \beta_j(\mu_M - \mu_f)) = \mu_f + \beta_P(\mu_M - \mu_f)$$

Return on portfolio = weighted sum of asset returns
Return on asset = risk free rate + Beta(market return – risk free rate)

(.4 * (.0003 + 1.5(.12 - .00003))) + (.6 * (.0003 +1.2(.12 - .00003))) = .158304
The specified portfolio ahs a return of about 15%.

Question 7d

$$\sigma_P^2 = (w^T\beta)^2\sigma_M^2 + \sum_{j=1}^{N} w_j^2\sigma_{\epsilon,j}^2$$

The beta in the above equation can just be the average of the two asset betas as they are weighted the same.
([.5; .5] * (1.2+1.5 / 2))^2 * .15^2) + (.5 * .08^2) + (.5 * .15^2) = .0350

This gives the variance of the portfolio. Take the square root of that to get the standard deviation which is .1870

Question 8a

Yes. If the assumptions of CAPM held and the model is correct, there should be a linear relationship with slope = to the market risk premium (The market return – risk free rate part of the model) between beta and expected return. Since beta is a proxy for risk, you should be rewarded for taking more risk. While the data is a bit scattered, possibly because the CAPM in an imperfect model, one could conceivably draw a trend line starting at .01, the risk free rate and continuing up with a slope of .08.

Question 8c

CAPM relies on lots of faulty assumptions about markets and investor behavior. It assumes that all investors will hold some proportion of their wealth in the market portfolio and the rest in the risk free asset. Most investors don't use this high level theory when making decisions and thus invest their wealth differently.

Question 9

Question 10

Question 11

No. The assets that make up the portfolios could be correlated in a way that means they are no longer efficient.

Question 12

Question 13

Question 14

1. Any calculation done using historical data cannot predict the future with certainty.
2. The calculated return of the tangent portfolio is an expected return / average. Given that the standard deviation is high (The annual returns are expected to be spread out around 35%), it seems like this could just be an unlucky year. If he were to hold the tangent portfolio for say 100 years and the annual return stayed around 10%, then his claim might have more validity. If the calculations are correct we'd expect the annual return to average around 35% given a large enough sample size to account for randomness.

Question 15

A

Question 16

B

Question 17

A

Question 18

C

Question 19

C

Question 20

B