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Report

Analysis of Bond Investing

Name	Student nr.
Ben Rosoff	**s191379**
Jakob Laursen Vork	**s180225**
Weston Jones	**s191380**

Technical University
of Denmark

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1 Introduction

This report will analyse an investment of DKK 100.000 in Danish Government bonds on behalf of a client. The analysis will be based on current issued bonds with prices taken from the NASDAQ[1]. The prices used are from 2019-09-19. All information used can be found in the appendix and the calculations used can be found in the MATLAB file attached.

2 Yield

Since the NASDAQ[1] listed prices (See appendix table 2) are considered to be "clean", they must first be converted into the "dirty" price by taking into account the time since the last coupon payment and then compensating the previous owner for the bonds. Since the last coupon payment for each bond occurred on 2018-11-15, the previous owner gets compensation for that time in the form of an increase in price, which was calculated with the following equation (2.1).

$$\text{Dirty Price} = \text{Clean Price} + \text{Coupon} * (\frac{\text{Days Between 15/11/18 and 19/9/19}}{365\ Days}) \tag{2.1}$$

A different representation of the ratio calculation is found in the included MATLAB file. The value is the fraction of a year between the last coupon payment and the current day, in this case it is 0.8438. This means that the current seller held the bond for 84.38% of a year from the last coupon payment and will be given 84.38% of the next coupon amount in addition to the clean price. The dirty prices are listed on Table 1, showing the price increase in order to purchase each bond available on 2019-09-19.

Table 1: Danish government bonds listed on NASDAQ clean prices converted to dirty prices

Name	**Coupon**	**Clean price**	**Dirty prices**
DGBI 2023 GB	0.10	106.950	107.034
1,5ST.I.23 GB	1.50	109.600	110.866
3ST.I. 21 GB	3.00	108.250	110.782
4,5 St.I 39 GB	4.50	199.540	203.337
4St.I. 19 GB	4.00	100.833	104.208
DANSKE STAT 2022	0.25	103.860	104.071
DANSKE STAT 2029	0.50	110.350	110.772
DGBi	0.10	122.150	122.234
Danske Stat 2020	0.25	101.304	101.515
Danske Stat 2025	1.75	115.270	116.747
Danske Stat 2027	0.50	109.738	110.160
7 St.I 24 GB	7.00	140.480	146.387

Using the dirty price for each bond, the yield to maturity can be calculated with the following equation (2.2). $P_t(T)$, C_i, and $y_t(T)$ represent the dirty price, coupon, and yield to maturity respectively.

$$P_t(T) = \sum_{i=t_1}^{t_N} \frac{C_i}{(1 + y_t(T))^{i-t}} \tag{2.2}$$

For each bond, the last coupon is the coupon amount plus the base value of the bond (DKK 100). These yields are indicative of the percent return on investment if the bond were to be held until maturity and cashed out. The yields and their respective maturity year can be seen on the graph in Figure 1.

[1] http://www.nasdaqomxnordic.com/bonds/denmark

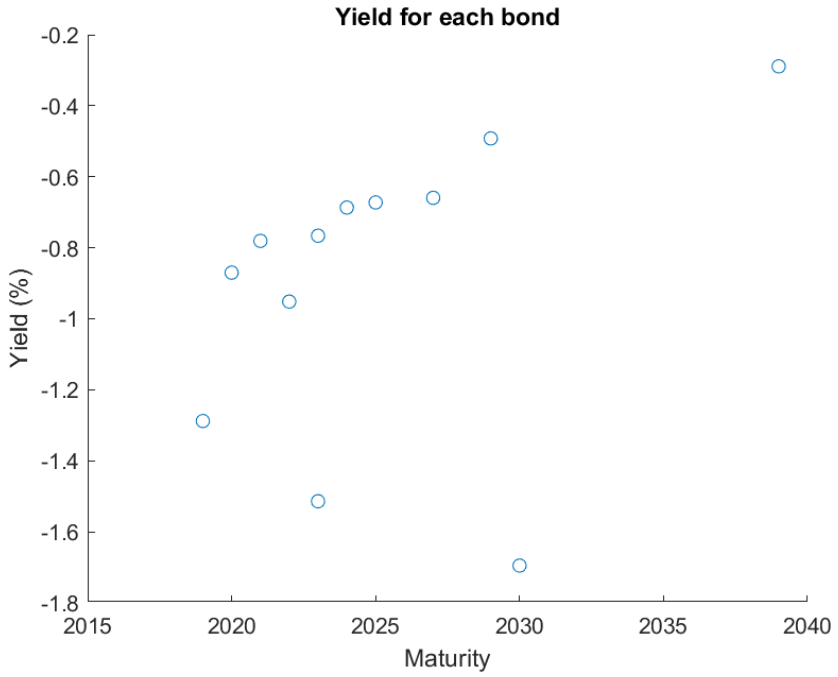

Figure 1: Yield for each bond

Three of the bonds in Figure 1: DGBI 2023 GB, 4St.I. 19 GB and DGBi are significant outliers from the rest of the data. The DGBI bonds are index bonds[2] and thus will mature very soon and be more sensitive to changes in time. This is also true of the 4St.I. 19 GB bond. These three bonds will therefore be omitted in future calculations and modeling in order to have the most accurate results for the client.

In order to calculate the yield for the Danish government bond market as a whole, the Nelson-Siegel model will be used:

$$y(T) = \beta_0 + \beta_1 \frac{1 - e^{-T/\tau}}{T/\tau} + \beta_2 (\frac{1 - e^{-T/\tau}}{T/\tau} - e^{-T/\tau}) \tag{2.3}$$

Calculating these values based on the Danish Government bonds (excluding the three bonds mentioned above) gives the following values:

$$\beta_0 = -0.02, \beta_1 = -0.77, \beta_2 = -1.28, \tau = 2.63 \tag{2.4}$$

This yield vs time function for the entire Danish government bond market can be seen alongside the nine remaining bonds in Figure 2.

[2] http://www.nationalbanken.dk/da/statsgaeld/IR/Sider/Indeksobligation.aspx

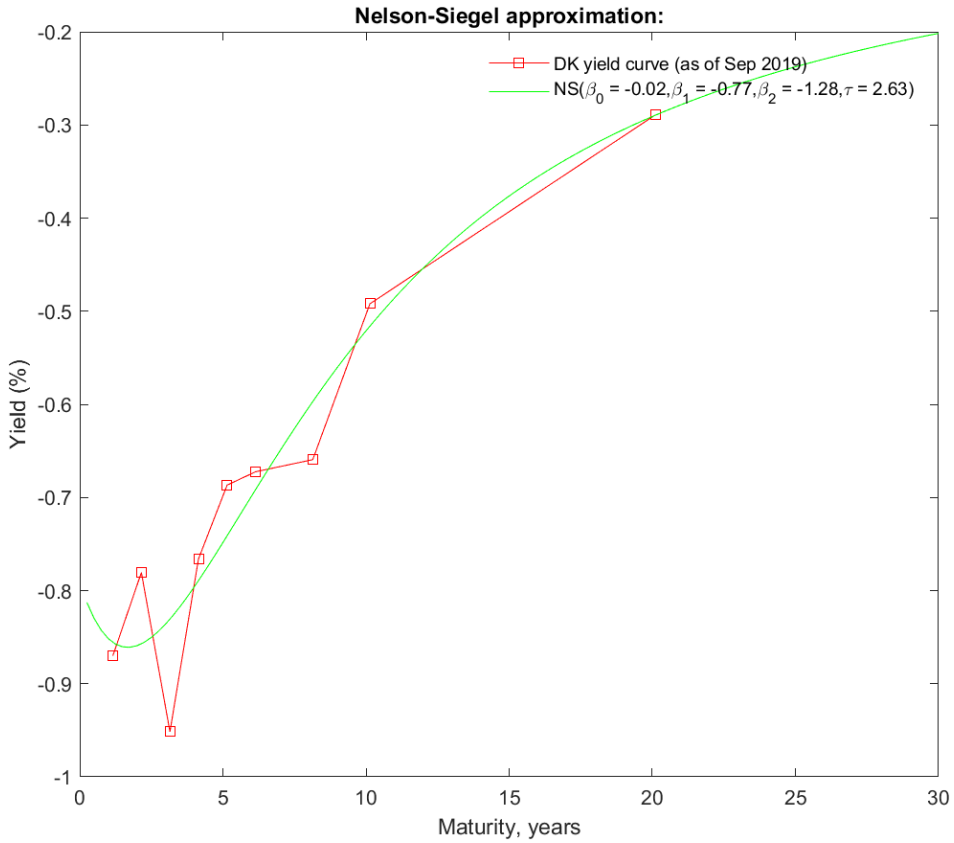

Figure 2: Each bond's yield as well a a yield fit using the Nelson-Siegel model.

3 Bond portfolio

In this section three bonds will be selected for the portfolio for the customer to invest DKK 100.000. The customer has requested a portfolio of three bonds with a duration of 8.5 years[3]. Assuming that the interest is not flat, the duration of each bond can be calculated using the Fisher-Weil duration formula:

$$D = \frac{1}{P}\sum_{t_i=t_1}^{t_N}\frac{t_i C_{t_i}}{(1+r_{t_i})^{t_i+1}} \tag{3.1}$$

Assmuing a portfolio, F, of three bonds A, B and C, the coupon payment of the portfolio at a given time t_i would be the coupon payment of each bond at time t_i giving:

$$C_{Ft_i} = C_{At_i} + C_{Bt_i} + C_{Ct_i} \tag{3.2}$$

The duration of the portfolio must be the duration of each weight of the bond in the portfolio, meaning that if one bond consists of 70% of the portfolio this bond also effect the duration with 70% of the duration, giving:

$$D_F = w_A D_A + w_B D_B + w_C D_C \tag{3.3}$$

The overall duration of the portfolio can be derived:

$$D_F = w_A\frac{1}{P_A}\sum_{t_i=t_1}^{t_N}\frac{t_i C_{At_i}}{(1+r_{t_i})^{t_i+1}} + w_B\frac{1}{P_B}\sum_{t_i=t_1}^{t_N}\frac{t_i C_{Bt_i}}{(1+r_{t_i})^{t_i+1}} + w_C\frac{1}{P_C}\sum_{t_i=t_1}^{t_N}\frac{t_i C_{Ct_i}}{(1+r_{t_i})^{t_i+1}} \tag{3.4}$$

[3]This is derived from the three student's student numbers

Note that the interest rate is the same for all three terms. Since the interest rate needs to be calculated at multiple times t_i the Nelson-Siegel fit is used instead of the yield of each bond as the Nelson-Siegel model gives yield as a continuous function of time t_i.

The three bonds selected for this client's portfolio are: 1,5ST.I.23 GB, 4,5 St.I 39 GB and DANSKE STAT 2029. In order to get a duration for the entire portfolio of 8.5 years the customer would have to invest his DKK 100,000 in the following manner:

1. 55.5% in 1,5ST.I.23 GB - DKK 55,500
2. 33.3% in 4,5 St.I 39 GB - DKK 33,300
3. 11.2% in DANSKE STAT 2029 - DKK 11,200

In order to properly estimate the impact of interest rate changes on the value of the portfolio, the convexity is calculated.

$$C = \frac{1}{P} \sum_{t_i=t_1}^{t_N} \frac{(t_i + 1)t_i C_{t_i}}{(1 + r_{t_i})^{t_i+2}} \tag{3.5}$$

Using both the duration and the convexity, the change in value of the entire portfolio, and ultimately better knowledge of the risk of the assets, can be found for an change of 1% for the interest rate (or yield) to be -0.0791 or -7.91%; meaning if a 1% increase in interest occurred the portfolio would lose 7.91% of it's value and vice versa for a decrease in interest rates. For a portfolio of DKK 100,000 a change of 1% in interest rates would change the value of the portfolio by DKK 7,910.

$$P_{Change} = -D^{FW} r_{change} + \frac{1}{2} C r_{change}^2 \tag{3.6}$$

Given that the only formal requirement from the client was that the duration of the portfolio equal 8.5 and that the individual bond weights be positive, the above bond selection is satisfactory.

4 Appendix

4.1 Bond prices

The bonds and prices used for the analysis in this report are based on the Danish Government bonds listed on the NASDAQ on 2019-09-19. The bonds from the Faroe Islands are excluded by the clients request; he requests the capital is invested in Danish bonds and not in self-governing regions of the Danish Realm. The used prices and bonds are shown in table 2. Although the expiration date for the '7 St.I 24 GB'-bond is 2024-11-10 it will be assumed that the expiration date for this bond is 2024-11-15. Additionally, for some bonds, there were no recent trades and so the latest prices available were used instead and taken to be as of 2019-09-19.

Table 2: Danish government bonds listed on NASDAQ. Prices are from 2019-09-19 and the bonds for the Faroe Islands are excluded in this analysis.

Name	**ISIN**	**Coupon**	**Price**	**(Used) Exp.**	**Currency**
DGBI 2023 GB	DK0009922916	0.10	106.950	2023-11-15	DKK
1,5ST.I.23 GB	DK0009923054	1.50	109.600	2023-11-15	DKK
3ST.I. 21 GB	DK0009922676	3.00	108.250	2021-11-15	DKK
4,5 St.I 39 GB	DK0009922320	4.50	199.540	2039-11-15	DKK
4St.I. 19 GB	DK0009922403	4.00	100.833	2019-11-15	DKK
DANSKE STAT 2022	DK0009923997	0.25	103.860	2022-11-15	DKK
DANSKE STAT 2029	DK0009923807	0.50	110.350	2029-11-15	DKK
DGBi	DK0009923724	0.10	122.150	2030-11-15	DKK
Danske Stat 2020	DK0009923641	0.25	101.304	2020-11-15	DKK
Danske Stat 2025	DK0009923138	1.75	115.270	2025-11-15	DKK
Danske Stat 2027	DK0009923567	0.50	109.738	2027-11-15	DKK
7 St.I 24 GB	DK0009918138	7.00	140.480	2024-11-15	DKK