ANSWERS TO END OF CHAPTER QUESTIONS

QUESTIONS

1. The price value of a basis point will be the same regardless if the yield is increased or decreased by 1 basis point. However, the price value of 100 basis points (i.e., the change in price for a 100-basis-point change in interest rates) will not be the same if the yield is increased or decreased by 100 basis points. Why?

The convex relationship explains why the price value of a basis point (i.e., the change in price for a 1-basis-point change in interest rates) will be roughly the same regardless if the yield is increased or decreased by 1 basis point, while the price value of 100 basis points will not be the same if the yield is increased or decreased by 100 basis points. More details are below.

When the price-yield relationship for any option-free bond is graphed, it exhibits a convex shape. When the price of the option-free bond declines, we can observe that the required yield rises. However, this relationship is not linear. The convex shape of the price–yield relationship generates four properties concerning the price volatility of an option-free bond. First, although the prices of all option-free bonds move in the opposite direction from the change in yield required, the percentage price change is not the same for all bonds. Second, for very small changes in the yield required (like 1 basis point), the percentage price change for a given bond is roughly the same, whether the yield required increases or decreases. Third, for large changes in the required yield (like 100 basis points), the percentage price change is not the same for an increase in the required yield as it is for a decrease in the required yield. Fourth, for a given large change in basis points, the percentage price increase is greater than the percentage price decrease.

2. Calculate the requested measures in parts (a) through (f) for bonds A and B (assume that each bond pays interest semiannually). Needed bond details are below.

	Bond A	*Bond B*
Coupon	8%	9%
Yield to maturity	8%	8%
Maturity (years)	2	5
Par	$100.00	$100.00
Price	$100.00	$104.055

(a) What is the price value of a basis point for bonds A and B?

For **Bond A**, we get a bond quote of $100.00 for our initial price if we have a 2-year maturity, an 8% coupon rate and an 8% yield. If we change the yield one basis point so the yield is 8.01%, then we have the following variables and values: C = $40, y = 0.0801 / 2 = 0.04005 and n = 2(2) = 4. Inserting these values into the present value of the coupon payments formula, we get:

$$P = C\left[\frac{1-\left[\frac{1}{(1+r)^{n}}\right]}{r}\right] = \$40\left[\frac{1-\left[\frac{1}{(1.04005)^{4}}\right]}{0.04005}\right] = \$145.179$$

Computing the present value of the par or maturity value of $1,000 gives:

$$\frac{M}{(1+r)^n} = \frac{\$1{,}000}{(1.04005)^4} = \$854.640.$$

If we add a basis point to the yield, we get the value of Bond A as: P = $145.179 + $854.640 = $999.819 with a bond quote of $99.9819. For **bond A the** price value of a basis point is about $100 – $99.9819 = **$0.0181** per $100.

Using the bond valuation formulas as just completed above, the value of **bond B** with a yield of 8%, a coupon rate of 9%, and a maturity of 5 years is: P = $364.990 + $675.564 = $1,040.554 with a bond quote of $104.0554. If we add a basis point to the yield, we get the value of Bond B as: P = $364.899 + $675.239 = $1,040.139 with a bond quote of $104.0139. For **bond B**, the price value of a basis point is $104.0554 – $104.0139 = **$0.0416** per $100.

(b) Compute the Macaulay durations for the two bonds..

For **bond A** with C = $40, n = 4, y = 0.04, P = $1,000 and M = $1,000, we have:

$$\text{Macaulay duration (half years)} = \frac{\frac{1C}{1+y} + \frac{2C}{(1+y)^2} + \ldots + \frac{nC}{(1+y)^n} + \frac{nM}{(1+y)^n}}{P} =$$

$$\frac{\frac{1(\$40)}{1.04} + \frac{2(\$40)}{(1.04)^2} + \ldots + \frac{4(\$40)}{(1.04)^4} + \frac{4(\$1{,}000)}{(1.04)^4}}{\$1{,}000} = \frac{\$3{,}775.09}{\$1{,}000} = 3.77509.$$

Macaulay duration (years) = Macaulay duration (half years) / 2 = 3.77509 / 2 = **1.8875.**

For **bond B** with C = $45, n = 10, y = 0.04, P= $1,040.55 and M = $1,000, we have:

$$\text{Macaulay duration (half years)} = \frac{\frac{1C}{1+y} + \frac{2C}{(1+y)^2} + \ldots + \frac{nC}{(1+y)^n} + \frac{nM}{(1+y)^n}}{P} =$$

$$\frac{\frac{1(\$45)}{1.04} + \frac{2(\$45)}{(1.04)^2} + \ldots + \frac{10(\$45)}{(1.04)^4} + \frac{10(\$1{,}000)}{(1.04)^4}}{\$1{,}040.55} = \frac{\$8{,}645.2929}{\$1{,}040.55} = 8.3084.$$

Macaulay duration (years) = Macaulay duration (half years) / 2 = 8.3084 / 2 = **4.1542.**

(c) Compute the modified duration.

Taking our answer for the Macaulay duration in years in part (b), we can compute the modified for **bond A** by dividing by 1.04. We have: modified duration = 1.8875 / (1.04) = **1.814948.**

Taking our answer for the Macaulay duration in years in part (b), we can compute the modified duration for **bond B** by dividing by 1.04. We have: modified duration = 4.1542 / (1.04) = **3.994417**.

[NOTE. We could get the same answers for both bonds A and B by computing the modified duration using an alternative formula that does not require the extensive calculations required by the procedure in parts (a) and (b). This shortcut formula is:

$$\text{modified duration} = \frac{\dfrac{C}{y^2}\left[1-\dfrac{1}{(1+y)^n}\right]+\dfrac{n(100-[C/y])}{(1+y)^{n+1}}}{P}$$

where C is the semiannual coupon payment, y is the semiannual yield, n is the number of semiannual periods, and P is the bond quote in 100's.

For **bond A** (expressing numbers in terms of a \$100 bond quote), we have: C = \$4, y = 0.04, n = 4, and P = \$100. Inserting these values in our modified duration formula, we can solve as follows:

$$\frac{\dfrac{C}{y^2}\left[1-\dfrac{1}{(1+y)^n}\right]+\dfrac{n(100-[C/y])}{(1+y)^{n+1}}}{P} = \frac{\dfrac{\$4}{0.04^2}\left[1-\dfrac{1}{(1.04)^4}\right]+\dfrac{4(\$100-[\$4/0.04])}{(1.04)^5}}{\$100}$$

= (\$362.98952 + \$0) / \$100 = 3.6299. Converting to annual number by dividing by two gives a modified duration for **bond A** of **1.8149** which is the same answer shown above.

For **bond B**, we have C = \$4.5, n = 20, y = 0.04, and P = \$104.055. Inserting these values in our modified duration formula, we can solve as follows:

$$\frac{\dfrac{C}{y^2}\left[1-\dfrac{1}{(1+y)^n}\right]+\dfrac{n(100-[C/y])}{(1+y)^{n+1}}}{P} = \frac{\dfrac{\$4.5}{0.04^2}\left[1-\dfrac{1}{(1.04)^{10}}\right]+\dfrac{4(\$100-[\$4.5/0.04])}{(1.04)^{11}}}{\$104.055}$$

= (\$912.47578 – \$811.9762) / \$104.055 = 7.988834 or about 7.99. Converting to annual number by dividing by two gives a modified duration for **bond B** of **3.9944** which is the same answer shown above.]

(d) Compute the approximate duration using the shortcut formula by changing yields by 20 basis points and compare your answer with that calculated in part (c).

To compute the approximate measure for **bond A**, which is a 2-year 8% coupon bond trading at 8% with an initial price (P_0) of \$1,000 (thus, it trades at its par value of \$1,000), we proceed as follows.

First, we increase the yield on the bond by 20 basis points from 8% to 8.20%. Thus, Δy is 0.002. The new price (P+) can be computed using our bond valuation formula. Doing this we get \$996.379 with a bond price quote of \$99.6379.

Second, we decrease the yield on the bond by 20 basis points from 8% to 7.8%. The new price (P_-) can be computed using our bond valuation formula. Doing this we get \$1,003.638 with a bond price quote of \$100.3638.

Third, with the initial price, P_0, equal to \$100 (when expressed as a bond quote), the duration can be approximated as follows:

$$\text{approximate duration (years)} = \frac{P_- - P_+}{2(P_0)(\Delta y)}$$

where Δy is the change in yield used to calculate the new prices (in decimal form). What the formula is measuring is the average percentage price change (relative to the initial price) per 20-basis-point change in yield. Inserting in our values, we have:

$$\text{approximate duration (years)} = \frac{\$100.3638 - \$99.6379}{2(\$100)(0.02)} = \mathbf{1.814948}.$$

This compares with 1.814948 computed in part (c). Thus, the approximate duration measure is virtually the same as the modified duration computed in part (c).

To compute the approximate measure for **bond B**, which is a 5-year 9% coupon bond trading at 8% with an initial price (P_0) of \$104.0554, we proceed as follows.

First, we increase the yield on the bond by 20 basis points from 8% to 8.20%. Thus, Δy is 0.002. The new price ($P+$) can be computed using our bond valuation formula. Doing this we get \$1,032.283 with a bond price quote of \$103.2283.

Second, we decrease the yield on the bond by 20 basis points from 8% to 7.8%. The new price (P_-) can be computed using our bond valuation formula. Doing this we get \$1,048.909 with a bond price quote of \$104.8909.

Third, with the initial price, P_0, equal to \$104.0554 (when expressed as a bond quote), the duration can be approximated as follows:

$$\text{approximate duration} = \frac{P_- - P_+}{2(P_0)(\Delta y)}$$

where Δy is the change in yield used to calculate the new prices (in decimal form). What the formula is measuring is the average percentage price change (relative to the initial price) per 20-basis-point change in yield. Inserting in our values, we have:

$$\text{approximate duration} = \frac{\$104.8909 - \$103.2283}{2(\$104.0554)(0.02)} = \mathbf{3.994400}.$$

This compares with 3.994417 computed in part (c). Thus, the approximate duration measure is virtually the same as the modified duration computed in part (c).

Besides the above approximate duration approach, there is another approach that is shorter than the Macaulay duration and modified duration approach. With this approach, we proceed as follows. For **bond A**, we add 20 basis points and get a yield of 8.20%. We now have C = \$40, y = 4.10%, n = 4, and M = \$1,000. Before we use this shortcut approach, we first compute P. As given above, we can use our bond valuation formula to get **\$996.379**.

Now we can compute the modified duration for **bond A** using the below formula:

$$\text{modified duration} = \frac{\frac{C}{y^2}\left[1-\frac{1}{(1+y)^n}\right]+\frac{n(100-[C/y])}{(1+y)^{n+1}}}{P}.$$

Putting in all applicable variables in terms of \$100, we have: C = \$4, n = 4, y = 0.041, and P = \$99.6379. Inserting these values in our formula gives:

$$\text{modified duration} = \frac{\frac{\$4}{0.041^2}\left[1-\frac{1}{(1.041)^4}\right]+\frac{4(\$100-[\$4/0.041])}{(1.041)^5}}{\$99.6379} =$$

(\$353.30310 + \$79.8036) / \$99.6379 = 3.6260 or about 3.63. Converting to annual number by dividing by two gives a modified duration for **bond A** of 1.8130. This compares with 1.8149 computed in part (c). Since both round off to **1.81**, the 20 point change in basis does not exercise any noticeable effect on our computation as the difference is only 0.0019.

For **bond B**, we add 20 basis points and get a yield of 8.20%. We now have C = \$45, y = 4.10%, n = 10, and M = \$1,000. Before we use the modified duration formula, we first compute P. Using our bond valuation formula to get **\$1,032.283**.

Now we can compute the modified duration for bond B as above for Bond B. Given C = \$4.5, n = 10, y = 0.041, and P =\$103.2237, and inserting these values into the above formula for modified duration gives:

modified duration = (\$885.80511 – \$627.0730) / \$103.2283 = 7.97357 or about 7.97.

Converting to annual number by dividing by two gives a modified duration for bond B of 3.9868. This compares with 3.9944 computed in part (c). Since both round off to **3.99**, the 20-point change in basis does not exercise any noticeable effect on our computation. However, for the two year bond, we only had a difference of 0.0019, while for the five year bond, we have a difference of 0.0076. Thus, this shortcut approach gives a wider disagreement for the longer-term bond (bond B).

(e) Compute the convexity measure for both bond A and B.

In half years, the convexity measure $= \frac{d^2P}{dy^2}\frac{1}{P}$. Noting that

$$\frac{d^2P}{dy^2} = \left[\frac{2C}{y^3}\left[1-\frac{1}{(1+y)^n}\right]-\frac{2Cn}{y^2(1+y)^{n+1}}+\frac{n(n+1)(100-[c/y])}{(1+y)^{n+2}}\right],$$

we can insert this quantity into our convexity measure (half years) formula to get:

$$\text{convexity measure} = \left[\frac{2C}{y^3}\left[1-\frac{1}{(1+y)^n}\right]-\frac{2Cn}{y^2(1+y)^{n+1}}+\frac{n(n+1)(100-[c/y])}{(1+y)^{n+2}}\right]\left[\frac{1}{P}\right].$$

For **bond A**, we have a 2-year 8% coupon bond trading at 8% with an initial price (P_0) of \$1,000 with a bond quote of \$100. Expressing numbers in terms of a \$100 bond quote, we have: C = \$4, y = 0.04, n = 4 and P = \$100. Inserting these numbers into our convexity measure formula gives:

convexity measure (half years) =

$$\left[\frac{2(\$4)}{(0.04)^3}\left[1-\frac{1}{(1.04)^4}\right]-\frac{2(\$4)4}{(0.04)^2(1.04)^5}+\frac{4(5)(100-[\$4/0.04])}{(1.04)^6}\right]\left[\frac{1}{\$100}\right]=$$

$$[\$125{,}000[0.14519581]-\$18{,}149.4761+\$0]\left[\frac{1}{\$100}\right]=1{,}710.93[0.01]=\mathbf{17.109354}.$$

Convexity measure (years) = convexity measure (half years) / 4 = 17.1093 / 4 = **4.2773350.**
Dollar convexity measure = convexity measure (years) times P = 4.2773350(\$100) equals about **\$427.73.**

[NOTE. We can get the same convexity by proceeding as follows. First, we increase the yield on the bond by 10 basis points from 8% to 8.1%. Thus, Δy is 0.001. The new price (P+) can be computed using our bond valuation formula. Doing this we get \$998.187 with a bond quote of \$99.8187. Second, we decrease the yield on the bond by 10 basis points from 8% to 7.9%. The new price (P_-) can be computed using our bond valuation formula. Doing this we get \$100.1817. Third, with the initial price, P_0, equal to \$100, the convexity measure of any bond can be approximated using the following formula:

$$\text{approximate convexity measure} = \frac{P_+ + P_- - 2P_0}{P_0(\Delta y)^2}.$$

Inserting in our values, the approximate convexity measure for **bond A** is

$$\text{approximate convexity measure} = \frac{\$100.1817+\$99.8187-2(\$100)}{\$100(0.001)^2}=4.2773384.$$

The approximate convexity measure of 4.2773384 is virtually identical to the convexity measure of 4.2773350 computed above.]

For **bond B**, we have a 5-year 9% coupon bond trading at 8% with an initial price (P_0) of

\$104.055. Expressing numbers in terms of a \$100 bond quote, we have: C = \$4.5, y = 0.04, n = 10, and P = \$104.0554. Inserting these numbers into our convexity measure formula gives:

convexity measure (half years) =

$$\left[\frac{2(\$4.5)}{(0.04)^3}\left[1-\frac{1}{(1.04)^{10}}\right]-\frac{2(\$4.5)10}{y^2(1.04)^{11}}+\frac{10(11)(100-[\$4.5/0.045])}{(1.04)^{12}}\right]\left[\frac{1}{\$104.0554}\right]=$$

$$[\$140{,}625[0.32443583]-\$45{,}623.7888+-\$858.8209]\left[\frac{1}{\$104.0554}\right]=$$

8,226.04[0.0096103] = **79.0544**.

Convexity measure (years) = convexity measure (half years) / 4 = 79.0544 / 4 = **19.7636077**. Dollar convexity measure = convexity measure (years) times P = 19.7636077(\$1,040.55) equals about **\$2,056.51**.

[NOTE. We can get the same convexity measure by proceeding as follows. First, we increase the yield on the bond by 10 basis points from 8% to 8.1%. Thus, Δy is 0.001. The new price (*P+*) can be computed using our bond valuation formula. Doing this we get \$1,036.408 with a bond quote of \$103.6408. Second, we decrease the yield on the bond by 10 basis points from 8% to 7.9%. The new price (P_-) can be computed using our bond valuation formula. Doing this we get \$104.4721. Third, with the initial price, P_0, equal to \$104.0554, the convexity measure of any bond can be approximated using the following formula:

$$\text{approximate convexity measure} = \frac{P_+ + P_- - 2P_0}{P_0(\Delta y)^2}.$$

Inserting in our values, the approximate convexity measure for **bond B** is

$$\text{approximate convexity measure} = \frac{\$104.4721+\$103.6408-2(\$104.0554)}{\$104.0554(0.001)^2} =$$

19.7636548. The approximate convexity measure of 19.7636548 is virtually identical to the convexity measure of 19.763608 computed above.]

(f) Compute the approximate convexity measure using the shortcut formula by changing yields by 20 basis points and compare your answer to the convexity measure calculated in part (e).

To compute the approximate convexity measure for **bond A**, which is a 2-year 8% coupon bond trading at 8% with an initial price (P_0) of \$100, we proceed as follows.

First, we increase the yield on the bond by 20 basis points from 8% to 8.2%. Thus, Δy is 0.002. The new price (*P+*) can be computed using our bond valuation formula. Doing this we get \$99.6379.

Second, we decrease the yield on the bond by 20 basis points from 8% to 7.8%. The new price

(P_-) can be computed using our bond valuation formula. Doing this we get \$1,003.638 with a bond price quote of \$100.3638.

Third, with the initial price, P_0, equal to \$100, the convexity measure of any bond can be approximated using the following formula:

$$\text{approximate convexity measure} = \frac{P_+ + P_- - 2P_0}{P_0(\Delta y)^2} .$$

Inserting in our values, the approximate convexity measure is

$$\text{approximate convexity measure} = \frac{\$100.3638 + \$99.6379 - 2(\$100)}{\$100(0.002)^2} = 4.2773486.$$

This answer of 4.2773486 for the approximate convexity measure is very similar to that computed in part (e) using the convexity measure where we got 4.2773350. [NOTE. The 4.2773486 for a change of 20 basis points is virtually identical to the 4.2773384 that we can compute for a change of 10 basis points.]

To compute the approximate convexity measure for **bond B** which is a 5-year 9% coupon bond trading at 8% with an initial price (P_0) of \$104.0554 (worth \$1,040.554 and with a par value = M = \$1,000), we proceed as follows.

First, we increase the yield on the bond by 20 basis points from 8% to 8.2%. Thus, Δy is 0.001. The new price ($P+$) can be computed using our bond valuation formula. Doing this we get \$103.2283.

Second, we decrease the yield on the bond by 20 basis points from 8% to 7.8%. The new price (P_-) can be computed using our bond valuation formula. Doing this we get \$1,048.909 with a bond price quote of \$104.8909.

Third, with the initial price, P_0, equal to \$104.0554, the convexity measure of any bond can be approximated using the following formula:

$$\text{approximate convexity measure} = \frac{P_+ + P_- - 2P_0}{P_0(\Delta y)^2} .$$

Inserting in our values, the approximate convexity measure is

$$\text{approximate convexity measure} = \frac{\$104.8909 + \$103.2283 - 2(\$104.0554)}{\$104.0554(0.002)^2} = 19.763824$$

This answer of 4.2773486 for the approximate convexity measure is very similar to that computed in part (e) using the convexity measure where we got 19.7636077 [NOTE. The 19.7636077 for a change of 20 basis points is virtually identical to the 19.7636548 computed for a change of 10 basis points.]

3. Can you tell from the following information which of the following three bonds will have the

on dollar duration and is our approximation of the price change based on duration. In equation (2), the first term on the right-hand side is the approximate percentage change in price based on modified duration. The second term in equations (1) and (2) includes the second derivative of the price function for computing the value of a bond. It is the second derivative that is used as a proxy measure to correct for the convexity of the price–yield relationship. Market participants refer to the second derivative of bond price function as the dollar convexity measure of the bond. The second derivative divided by price is a measure of the percentage change in the price of the bond due to convexity and is referred to simply as the convexity measure.

(e) Without working through calculations, indicate whether the duration of the two bonds would be higher or lower if the yield to maturity is 10% rather than 8%.

Like term to maturity and coupon rate, the yield to maturity is a factor that influences price volatility. Ceteris paribus, the higher the yield level, the lower the price volatility. The same property holds for modified duration. Thus, a 10% yield to maturity will have both less volatility that a 8% yield to maturity and also a smaller duration.

There is consistency between the properties of bond price volatility and the properties of modified duration. When all other factors are constant, a bond with a longer maturity will have greater price volatility. A property of modified duration is that when all other factors are constant, a bond with a longer maturity will have a greater modified duration. Also, all other factors being constant, a bond with a lower coupon rate will have greater bond price volatility. Also, generally, a bond with a lower coupon rate will have a greater modified duration. Thus, bonds with greater durations will greater price volatilities.

5. State why you would agree or disagree with the following statement: As the duration of a zero-coupon bond is equal to its maturity, the price responsiveness of a zero-coupon bond to yield changes is the same regardless of the level of interest rates.

As seen in Exhibit 4-3, the price responsiveness of a zero-coupon bond is different as yields change. Like other bonds, zero-coupon bonds have greater price responsiveness for changes at higher levels of maturity as interest rates change. Like other bonds, zero-coupon bonds have greater price responsiveness for changes at lower levels of interest rates compared to higher levels of interest rates.

Except for long-maturity deep-discount bonds, bonds with lower coupon rates will have greater modified and Macaulay durations. Also, for a given yield and maturity, zero-coupon bonds have higher convexity and thus greater price responsiveness to changes in yields.

6. State why you would agree or disagree with the following statement: When interest rates are low, there will be little difference between the Macaulay duration and modified duration measures.

The Macaulay duration is equal to the modified duration times one plus the yield. Rearranging this expression gives:

$$\text{modified duration} = \frac{\textit{Macaulay duration}}{1+y}$$

Consider a 6% 25-year bond selling at \$70.3570 to yield 9%. The dollar duration is 747.2009. For a 1-basis-point (0.0001) increase in the required yield, the estimated price change per \$100 of face value is

dP = –(dollar duration)(dy) = –(\$747.2009)(0.0001) = **–\$0.07472.**

If we change the yield one basis point so the yield is 9.01%, then the value of the bond is: P = \$592.378 + \$110.445 = \$702.824 with a bond quote of \$70.2824. The price value of a basis point is about \$70.2824 – \$70.3570 = **–\$0.07464**. Notice that the dollar duration for a 1-basis point change gives about the same value as the price value of a basis point as both round off to **–\$0.0747**.

If we add a basis point to the yield, we get the value of Bond A as: P = \$145.179 + \$854.640 = \$999.819 with a bond quote of \$99.9819. For **bond** A the price value of a basis point is about \$100 – \$99.9819 = **\$0.0181** per \$100.

9. The November 26, 1990, issue of *BondWeek* includes an article, "Van Kampen Merritt Shortens." The article begins as follows: "Peter Hegel, first v.p. at Van Kampen Merritt Investment Advisory, is shortening his \$3 billion portfolio from 110% of his normal duration of 6½ years to 103–105% because he thinks that in the short run the bond rally is near an end." Explain Hegel's strategy and the use of the duration measure in this context.

If Hegel thinks the bond rally is over it implies that he thinks bond prices will not go up. This implies the belief that Hegel thinks interest rates will stop falling. If interest rates begin going up then one does not want to lock in longer-term bonds at lower rates. This implies you want your portfolio of bonds to focus more on shorter-term bonds. Thus, you want a portfolio with a shorter duration. A shorter duration will mean not only less sensitivity to interest rates but if interest rates go up then Hegel will later capitalize on this because as bonds in his portfolio mature quicker (than would be achieved with a portfolio with a higher duration) he will be able to buy new bonds and lock in higher rates. In brief, Hegel uses the duration measure to optimize the value of his portfolio based upon his belief about how interest rates change.

10. Consider the following two Treasury securities:

Bond	Price	Modified duration (years)
A	\$100	6
B	\$80	7

Which bond will have the greater dollar price volatility for a 25-basis-point change in interest rates?

The estimated dollar price change can be obtained by using the below equation:

dP = –(modified duration)P(dy)

Inserting in our values for **bond A,** we have:

dP = –(modified duration)P(dy) = –(6)\$100(0.0025) = **–\$1.50** which is the estimated dollar price change or volatility for a 25-basis-point change. The percentage change in price is:

$$\frac{dP}{P} = \frac{\$1.50}{\$100} = -0.0150 \text{ or } -1.50\%.$$

Inserting in our values for **bond B**, we have:

$dP = -(\text{modified duration})P(dy) = -(7)\$80(0.0025) = \mathbf{-\$1.40}$ which is the estimated dollar price change or volatility for a 25-basis-point change. The percentage change in price is:

$$\frac{dP}{P} = \frac{\$1.40}{\$80} = -0.0175 \text{ or } -1.750\%.$$

Thus, we see that while bond A has a greater estimated dollar price volatility compared to bond B, it has a lower percentage change in price. From an investor's point of view, every dollar invested in bond B has greater volatility.

11. What are the limitations of using duration as a measure of a bond's price sensitivity to interest-rate changes?

Below we discuss three limitations of using duration.

First, duration measures are only approximations for small changes in yield. They do not capture the effect of the convexity of a bond on its price performance when yields change by more than a small amount. To get improved accuracy, the duration measure should be supplemented with an additional measure to capture the curvature or convexity of a bond. It is important to note that investors can be mislead if they rely on duration as the sole measure of the price volatility of a bond.

Second, in the derivation of the relationship between modified duration and bond price volatility, we started with the bond price equation that assumes that all cash flows for the bond are discounted at the same discount rate. In essence we are assuming that the yield curve is flat and all shifts are parallel. This assumption does not always hold. This is very important when we try to use a portfolio's duration to quantify the responsiveness of a portfolio's value to a change in interest rates. If a portfolio has bonds with different maturities, the duration measure may not provide a good estimate for unequal changes in interest rates of different maturities.

Third, we must be careful when applying our duration equations to bonds that are not option-free bonds. When changes in yields result in a change in the expected cash flow for a bond, which is the case for bonds with embedded options, the duration and convexity measures are appropriate only in certain circumstances.

12. The following excerpt is taken from an article titled "Denver Investment to Make \$800 Million Treasury Move," which appeared in the December 9, 1991, issue of *BondWeek*, p. 1:"Denver Investment Advisors will swap \$800 million of long zero-coupon Treasuries for intermediate Treasuries. . . . The move would shorten the duration of its \$2.5 billion fixed-income portfolio. . . ." Why would the swap described here shorten the duration of the portfolio?

Duration captures the price sensitivity of a fixed-income investment to changes in yields. Thus, lowering the duration should lower the sensitivity. This is desired if one feels interest rates are going to increase in which case the value of your fixed-income investment would decline.

Denver Investment Advisors are swapping $800 million long zero-coupon Treasuries for intermediate Treasuries. As a percentage of its portfolio, the proposed swap involves $800 million ÷ $2.5 billion = 0.32 or 32%. Because the portfolio duration is the weighted average of its individual investments, the swap of $800 million long zero-coupon Treasuries for intermediate Treasuries will lower its duration if the $800 million being swapped actually has a lower duration or price sensitivity.

As seen in Exhibit 4-3, the price responsiveness of a zero-coupon bond is different as yields change. Like other bonds, zero-coupon bonds have greater price responsiveness for changes at higher levels of maturity as interest rates change. Furthermore, like other bonds, zero-coupon bonds have greater price responsiveness for changes at lower levels of interest rates. However, the exhibit also shows that zero-coupon bonds have greater percentage price changes especially for longer-term securities. This indicates that swapping its long zero-coupon Treasuries for intermediate Treasuries could have an important impact on lowering Denver Investment's duration.

13. You are a portfolio manager who has presented a report to a client. The report indicates the duration of each security in the portfolio. One of the securities has a maturity of 15 years but a duration of 25. The client believes that there is an error in the report because he believes that the duration cannot be greater than the security's maturity. What would be your response to this client?

Unfortunately, market participants often confuse the main purpose of duration by constantly referring to it as some measure of the weighted average life of a bond. This is because of the original use of duration by Macaulay where the cash flow for each period divided by the market value formed a weight with the weights adding up to one. If you rely on this interpretation of duration, it will be difficult for you to understand why a security with a maturity of 15 years can have a duration greater than 25 years. For example, consider collateralized mortgage obligation (CMO) bond classes. Certain CMO bond classes have a greater duration than the underlying mortgage loans (because CMO bond classes are leveraged instruments whose price sensitivity or duration are a multiple of the underlying mortgage loans from which they were created). That is, a CMO bond class can have a duration of 25 although the underlying mortgage loans from which the CMO is created can have a maturity of 15 years.

The answer to the puzzle (about duration being greater than maturity) is that duration is the approximate percentage change in price for a small change in interest rates. Thus, a CMO bond class with a duration of 25 does not mean that it has some type of weighted average life of 15 years. Instead, it means that for a 100-basis-point change in yield, that bond's price will change by roughly 40%. Similarly, we interpret the duration of an option in the same way. A call option can have a duration of 25 when the time to expiration of the option is much less than 25 years. This is confusing to someone who interprets duration as some measure of the life of an option.

14. Explain why the duration of an inverse floater is a multiple of the duration of the collateral from which the inverse floater is created.

In general, an inverse floater is created from a fixed-rate security. The security from which the inverse floater is created is called the collateral. From the collateral two bonds are created: a floater and an inverse floater. The two bonds are created such that (i) the total coupon interest paid to the two bonds in each period is less than or equal to the collateral's coupon interest in each period, and (ii) the total par value of the two bonds is less than or equal to the collateral's total par value. Equivalently, the floater and inverse floater are structured so that the cash flow