Duration and Convexity to Measure Bond Risk

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What Are Duration and Convexity?

Duration and convexity are two tools used to manage the risk exposure of fixed-income investments. Duration measures the bond's sensitivity to interest rate changes. Convexity relates to the interaction between a bond's price and its yield as it experiences changes in interest rates.

With coupon bonds, investors rely on a metric known as duration to measure a bond's price sensitivity to changes in interest rates. Because a coupon bond makes a series of payments over its lifetime, fixed-income investors need ways to measure the average maturity of a bond's promised cash flow, to serve as a summary statistic of the bond's effective maturity. The duration accomplishes this, letting fixed-income investors more effectively gauge uncertainty when managing their portfolios.

KEY TAKEAWAYS

- With coupon bonds, investors rely on a metric known as “duration” to measure a bond's price sensitivity to changes in interest rates.
- Using a gap management tool, banks can equate the durations of assets and liabilities, effectively immunizing their overall position from interest rate movements.

Duration of a Bond

In 1938, Canadian economist Frederick Robertson Macaulay dubbed the effective-maturity concept the “duration” of the bond. In doing so, he suggested that this duration be computed as the weighted average of the times to maturity of each coupon, or principal payment, made by the bond. Macaulay's duration formula is as follows:

$$\sum_{i=1} \frac{C}{(1+r)^t} + \frac{F}{(1+r)^t}$$

where:

$D = \text{The bond's MacAulay duration}$

$T = \text{the number of periods until maturity}$

$i = \text{the } i^{th} \text{ time period}$

$C = \text{the periodic coupon payment}$

$r = \text{the periodic yield to maturity}$

$F = \text{the face value at maturity}$

Duration in Fixed Income Management

Duration is critical to managing fixed-income portfolios, for the following reasons:

1. It's a simple summary statistic of the effective average maturity of a portfolio.
2. It's an essential tool in immunizing portfolios from interest rate risk.
3. It estimates the interest rate sensitivity of a portfolio.

The duration metric carries the following properties:

- The duration of a zero-coupon bond equals time to maturity.
- Holding maturity constant, a bond's duration is lower when the coupon rate is higher, because of the impact of early higher coupon payments.
- Holding the coupon rate constant, a bond's duration generally increases with time to maturity. But there are exceptions, as with instruments such as deep-discount bonds, where the duration may fall with increases in maturity timetables.
- Holding other factors constant, the duration of coupon bonds is higher when the bonds' yields to maturity are lower. However, for zero-coupon bonds, duration equals time to maturity, regardless of the yield to maturity.
- The duration of level perpetuity is (1 + y) / y. For example, at a 10% yield, the duration of perpetuity that pays $100 annually will equal 1.10 / .10 = 11 years. However, at an 8% yield, it will equal 1.08 / .08 = 13.5 years. This principle makes it obvious that maturity and duration may differ widely. Case in point: the maturity of the perpetuity is infinite, while the duration of the instrument at a 10% yield is only 11 years. The present-value-weighted cash flow early on in the life of the perpetuity dominates the duration computation.

Duration for Gap Management

Many banks exhibit mismatches between asset and liability maturities. Bank liabilities, which are primarily the deposits owed to customers, are generally short-term in nature, with low duration statistics. By contrast, a bank's assets mainly comprise outstanding commercial and consumer loans or mortgages. These assets tend to be of longer duration, and their values are more sensitive to interest rate fluctuations. In periods when interest rates spike unexpectedly, banks may suffer drastic decreases in net worth, if their assets drop further in value than their liabilities.

A technique called gap management, developed in the late 1970s and early 1980s, is a widely used risk management tool, where banks attempt to limit the "gap" between asset and liability

On the other side of the balance sheet, the introduction of longer-term bank certificates of deposit (CDs) with fixed terms to maturity, serve to lengthen the duration of bank liabilities, likewise contributing to the reduction of the duration gap.

Understanding Gap Management

Banks employ gap management to equate the durations of assets and liabilities, effectively immunizing their overall position from interest rate movements. In theory, a bank's assets and liabilities are roughly equal in size. Therefore, if their durations are also equal, any change in interest rates will affect the value of assets and liabilities to the same degree, and interest rate changes would consequently have little or no final effect on net worth. Therefore, net worth immunization requires a portfolio duration, or gap, of zero.

Institutions with future fixed obligations, such as pension funds and insurance companies, differ from banks in that they operate with an eye towards future commitments. For example, pension funds are obligated to maintain sufficient funds to provide workers with a flow of income upon retirement. As interest rates fluctuate, so do the value of the assets held by the fund and the rate at which those assets generate income. Therefore, portfolio managers may wish to protect (immunize) the future accumulated value of the fund at some target date, against interest rate movements. In other words, immunization safeguards duration-matched assets and liabilities, so a bank can meet its obligations, regardless of interest rate movements.

Convexity in Fixed Income Management

Unfortunately, duration has limitations when used as a measure of interest rate sensitivity. While the statistic calculates a linear relationship between price and yield changes in bonds, in reality, the relationship between the changes in price and yield is convex.

In the image below, the curved line represents the change in prices, given a change in yields. The straight line, tangent to the curve, represents the estimated change in price, via the duration statistic. The shaded area reveals the difference between the duration estimate and the actual price movement. As indicated, the larger the change in interest rates, the larger the error in estimating the price change of the bond.

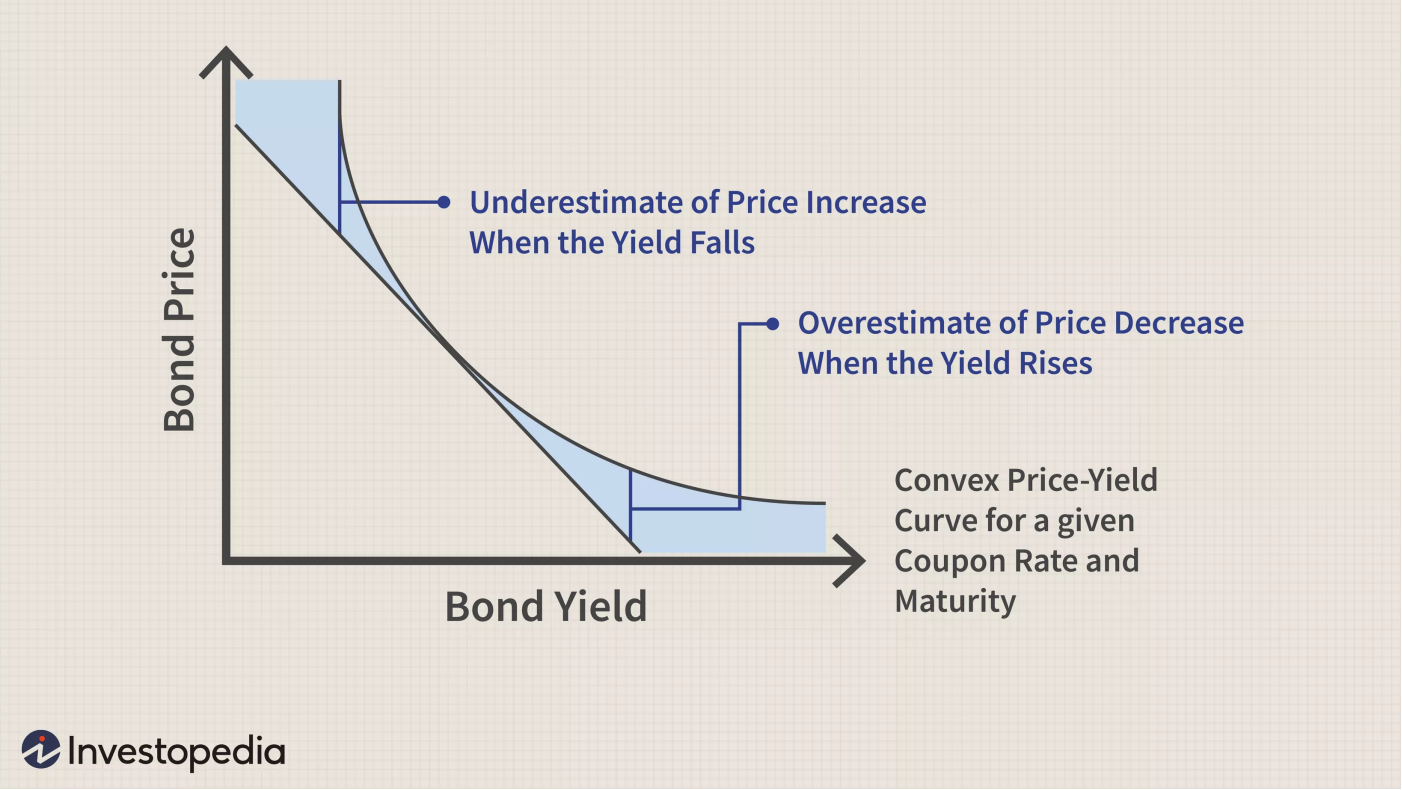

Image by Julie Bang © Investopedia 2019

Convexity, a measure of the curvature of the changes in the price of a bond, in relation to changes in interest rates, addresses this error, by measuring the change in duration, as interest rates fluctuate. The formula is as follows:

$$C = \frac{d^2\left(B\left(r\right)\right)}{B * d * r^2}$$

where:

$C = \text{convexity}$

$B = \text{the bond price}$

$r = \text{the interest rate}$

$d = \text{duration}$

In general, the higher the coupon, the lower the convexity, because a 5% bond is more sensitive

larger fluctuations in price when interest rates move.

The opposite is true of low convexity bonds, whose prices don't fluctuate as much when interest rates change. When graphed on a two-dimensional plot, this relationship should generate a long-sloping U shape (hence, the term "convex").

Low-coupon and zero-coupon bonds, which tend to have lower yields, show the highest interest rate volatility. In technical terms, this means that the modified duration of the bond requires a larger adjustment to keep pace with the higher change in price after interest rate moves. Lower coupon rates lead to lower yields, and lower yields lead to higher degrees of convexity.

The Bottom Line

Ever-changing interest rates introduce uncertainty in fixed-income investing. Duration and convexity let investors quantify this uncertainty, helping them manage their fixed-income portfolios.

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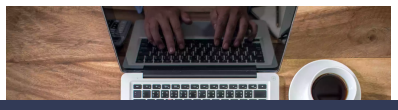

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Duration Definition

Duration indicates the years it takes to receive a bond's true cost, weighing in the present value of all future coupon and principal payments. more

Understanding Convexity Adjustments

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