Exercises – Week 40

Introduction to Financial Engineering

Note: You may choose to work in R or Matlab. Sometimes solutions will be available in one language, sometimes in both.

1. (returns, portfolios, diversification) Collect 10 years of weekly data for McDonalds, Coca Cola and Microsoft for the period January 1, 1991 to January 1, 2001.

   (a) Calculate the weekly returns for each of the stocks

   (b) Calculate the annualised mean and covariance matrix for the returns of the stocks

   (c) Consider all portfolios weights $(0, 0, 1), (0, 0.1, 0.9), (0.1, 0.1, 0.8) \ldots, (0.9, 0.1, 0), (1, 0, 0)$. For each of the portfolios calculate the annualised mean and annualised standard deviation. Plot mean against standard deviation for all these portfolios in one single plot.

   (d) Which of the portfolios has the maximal mean? (Could you have answered this question without doing c)?)

   (e) Which of the portfolios has the lowest standard deviation? (Could you have answered this question without doing c)?)

   (f) Which of the portfolios has the highest ratio of mean to standard deviation? Why is this portfolio interesting?

2. (portfolios, diversification, efficient frontier) Assume that

   - Asset 1 has an annual expected return of 10% and the standard deviation is 10%
   - Asset 1 has an annual expected return of 20% and the standard deviation is 20%

   (a) Find the Efficient Frontier (combinations of $(\mu; \sigma)$) when short selling is allowed and with no riskless lending/borrowing for each of the following correlation coefficient between the returns of assets 1 and 2

   - $\rho = 0$
   - $\rho = 0.5$
   - $\rho = -0.5$

   Use different colours for different values of the correlation coefficient. Comment on your findings. What happens to the efficient frontier when $\rho \to \pm 1$? It's a good idea to program this function for an arbitrary number of assets.

3. (portfolios, diversification, efficient frontier) If time permits, add the efficient frontier to the plot made in question 1. You have most likely programmed a function that illustrates the efficient frontier, when you did question 2.