# Project 3 - Stock portfolio CAPM/FF

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## 1 Introduction

This paper builds off of a previous report - further analyzing the historical data of eight stocks across different sectors of industry in order to fit the two models CAPM and Fama-French and give elaborate on the meaning of $\beta$-values.

## 2 Model fitting

The chosen stocks for this paper are as follows: Cisco Systems, Inc. (CSX), FedEx Corporation (FDX), Alphabet Inc. Class C (GOOG), JPMorgan Chase & Co. (JPM), Coca-Cola Co (KO), Newmont Goldcorp Corp (NME), Pfizer Inc. (PFE), Phillips 66 (PSX). The stocks were chosen for their spread across the Global Industry Classification Standard (GICS) groups and their relatively stable and positive returns.

Historical price and returns data for the eight stocks was collected from January 1, 2014 until January 1, 2019. A summary of the stock's change in value overtime is shown below.

### 2.1 Summary of General Statistics

| Ticker | Name | Industry | Annualized Return | Annualized Risk |
|---|---|---|---|---|
| CSCO | Cisco Systems | Information Technology | .1813 | .2111 |
| FDX | FedEx Corporation | Industrials | .0372 | .2234 |
| GOOG | Alphabet Inc. Class C | Communication services | .1334 | .2345 |
| JPM | JPMorgan Chase & Co. | Financials | .1378 | .2084 |
| KO | Coca-Cola Company | Consumer Staples | .0649 | .1368 |
| NEM | Newmont Goldcorp | Materials | .0867 | .3770 |
| PFE | Pfizer Inc. | Health Care | .1139 | .1746 |
| PSX | Phillips 66 | Energy | .0542 | .2426 |

Of the chosen stocks, all have positive annualized returns and risk less than 0.4. The tech and financial stocks - Cisco, Google, and JPMorgan - seem to offer the highest average annual rates of return, though they are relatively more risky. The Coca-Cola company offers the lowest returns but least amount of risk.

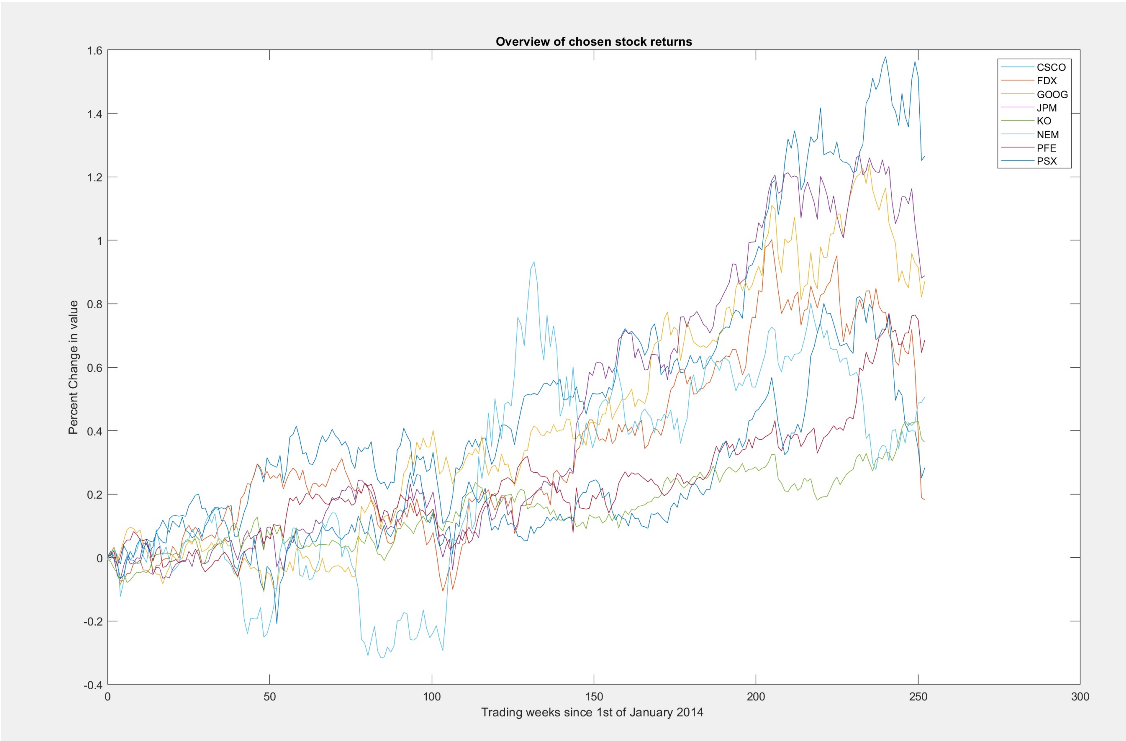


Figure 1: Historical performance of the 8 stocks.

### 2.2 CAPM-model

The chosen stocks are used to fit the factor model $CAPM$ which is fitted by:

$$R_{j,t} = \beta_{0,j} + \beta_{1,j} F_{1,t} + \epsilon_{j,t} \tag{1}$$

where $R_{i,t}$ is the excess return on the $j$th asset at time $t$ (see p527, Statics and Data Analysis for financial engineering, D. Ruppert & D. Matteson, Springer). In this fit, the only risk besides the individual asset's risk, is the market risk factor. This is obviously a generalization, since the price of an asset is affected by numerous factors. However the simplicity eases the use of the model. Each of the chosen stocks is fitted using the LinearModel.fit() function in Matlab and the residuals and the correlation between each stocks return is plotted on the next page:

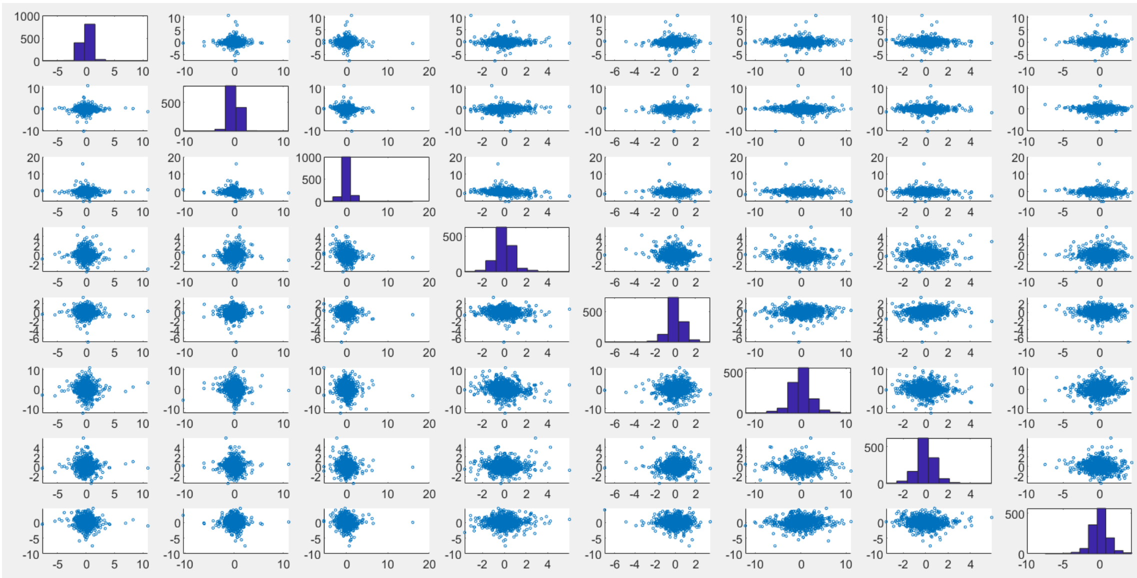

Figure 2: Residuals of CAPM-fit

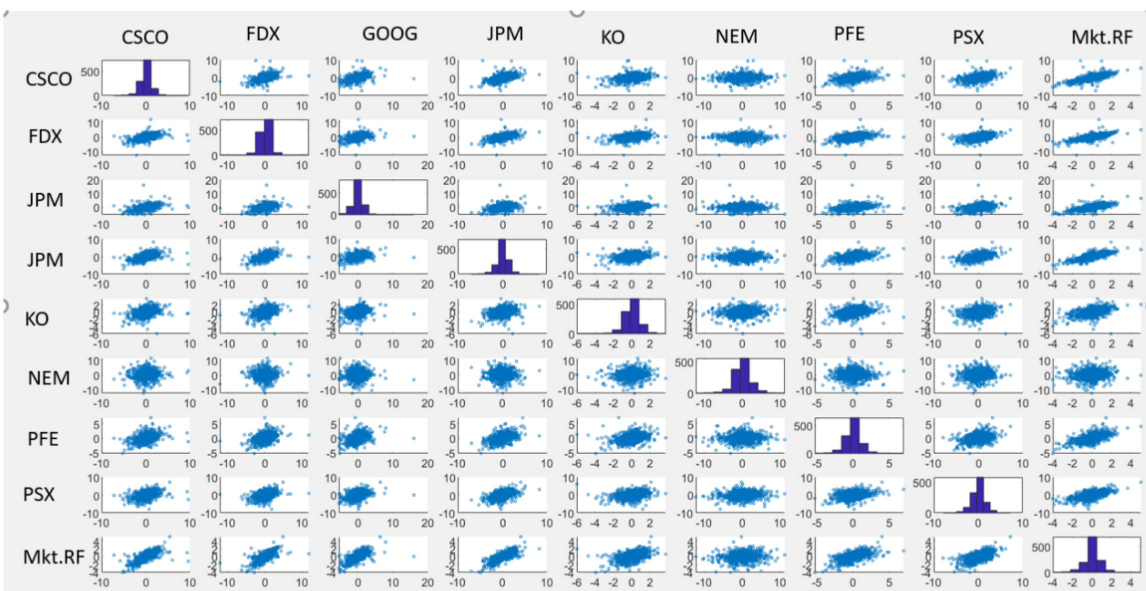


Figure 3: Correlation plot of the returns of stocks

## 2.3 Visual test of CAPM-model

In order to test the fitting of the CAPM-model, the expected return of each of the assets is plotted as a function of the stock's $\beta$. The $\beta$s are used along with the expected returns to fit a linear model of the return. The function is plotted below as the orange line. This line is more commonly known as the Securities Market Line (SML). Most of the chosen assets are placed well around the SML however the stocks FDX and PSX are positioned far below and seem to have too high volatility compared to the expected return given our chosen stocks. However the model isn't adequate to model the whole market since it only consists of 8 stocks. CSCO on the other hand is likely too perform very well.

| "" | "CSCO" | "FDX" | "GOOG" | "JPM" | "KO" | "NEM" | "PFE" | "PSX" |
|---|---|---|---|---|---|---|---|---|
| "Intercept" | "0.036721" | "-0.014693" | "0.02005" | "0.018935" | "0.010741" | "0.048342" | "0.020808" | "-0.0064404" |
| "Mkt.RF" | "1.0887" | "1.116" | "1.1567" | "1.1696" | "0.47517" | "0.32273" | "0.78051" | "1.1156" |

Figure 4: Efficient frontiers of the models and significant portfolios:

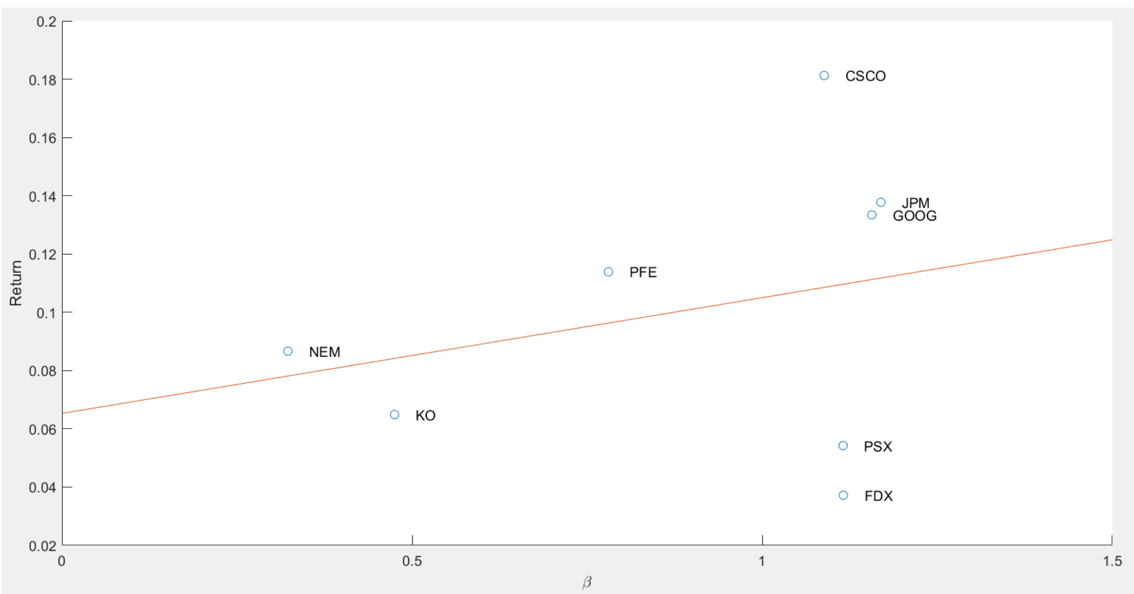


Figure 5: Asset return as a function of $\beta$

## 2.4 Fama-French-model

The chosen stocks are also used to fit another factor model The Fama-French, which is fitted by:

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}(R_{M,t} - \mu_{f,t}) + \beta_{2,j}SMB_t + \beta_{2,j}HML_t + \epsilon_{j,t} \tag{2}$$

This fit include the two additional factors of HML and SMB, which is the outperformance of the market for growth stocks and size effect of the stock respectively.

## 2.5 Interpretation of FF-coefficients:

Notice the negative relation between GOOG and HML-factor (last plot in third column in the scatter plot below). This indicates that the stocks is a growth stock. The opposite is the case with JPM (fourth in last row), which seems to have a "strong" positive relation with the High Minus Low (HML), this indicates that the stock is a value stock. Both indications seem fair, since GOOGLE doesn't pay any dividends in order to reinvest in growth and JPM does pay and is placed in a less explosive growth sector than the tech industry. In order to investigate the stocks further to coefficients of the model fit is shown in the table below. Along with an indication whether the stock excess returns behaves like a small/large company or growth/wealth stock. Two labels haven't been placed (TBD), that's whether KO is a growth or wealth stock. This can't be said from this data since the relation to HML is close to zero. The same is the case with PSX concerning its behaviour as a Large or small company since the relation to SMB is even closer to zero.

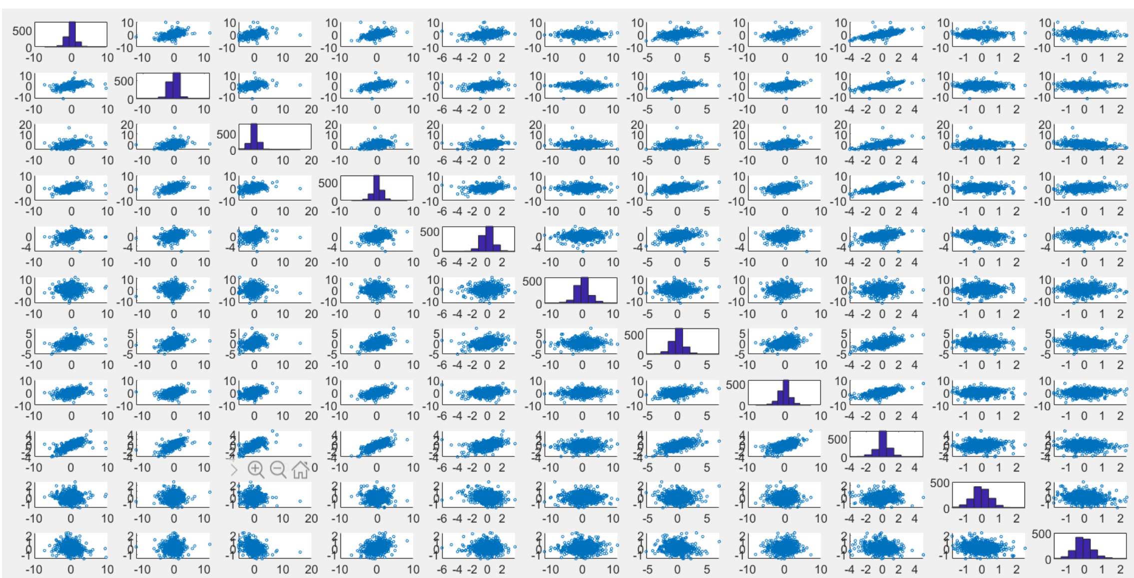

Figure 6: Scatter plots of the stocks returns and the three FF-factors

| "" | "CSCO" | "FDX" | "GOOG" | "JPM" | "KO" | "NEM" | "PFE" | "PSX" |
|---|---|---|---|---|---|---|---|---|
| "Intercept" | "0.032264" | "-0.012364" | "0.0092773" | "0.026784" | "0.0056032" | "0.051318" | "0.016935" | "-0.0023156" |
| "Mkt.RF" | "1.1015" | "1.1321" | "1.1373" | "1.2443" | "0.51436" | "0.33129" | "0.79213" | "1.1474" |
| "SMB" | "-0.24993" | "0.0079337" | "-0.33248" | "-0.083874" | "-0.41994" | "0.074701" | "-0.21989" | "-0.0042629" |
| "HML" | "-0.14843" | "0.24538" | "-0.73039" | "0.97815" | "0.0091175" | "0.22527" | "-0.12529" | "0.45962" |
| "Growth/Wealth" | "Growth" | "Value" | "Growth" | "Value" | "TBD" | "Value" | "Growth" | "Value" |
| "Large/Small" | "Large" | "Small" | "Large" | "Large" | "Large" | "Small" | "Large" | "TBD" |

Figure 7: Table of FF-model coefficients

## 2.6 Efficient frontiers:

In order to compare the different models (the CAPM and the Farma-French) the efficient frontier of each of the models is calculated. The efficient frontier of stocks only using the covariance matrix of the excess returns of the stocks without any fitting. Besides the frontiers the global minimum variance (GMV) and the Tangency portfolio (TAN) is plotted along with the stocks. The risk reduction of the portfolios are immediately seen compared to a cruel approach of taking the stock with the highest expected return.

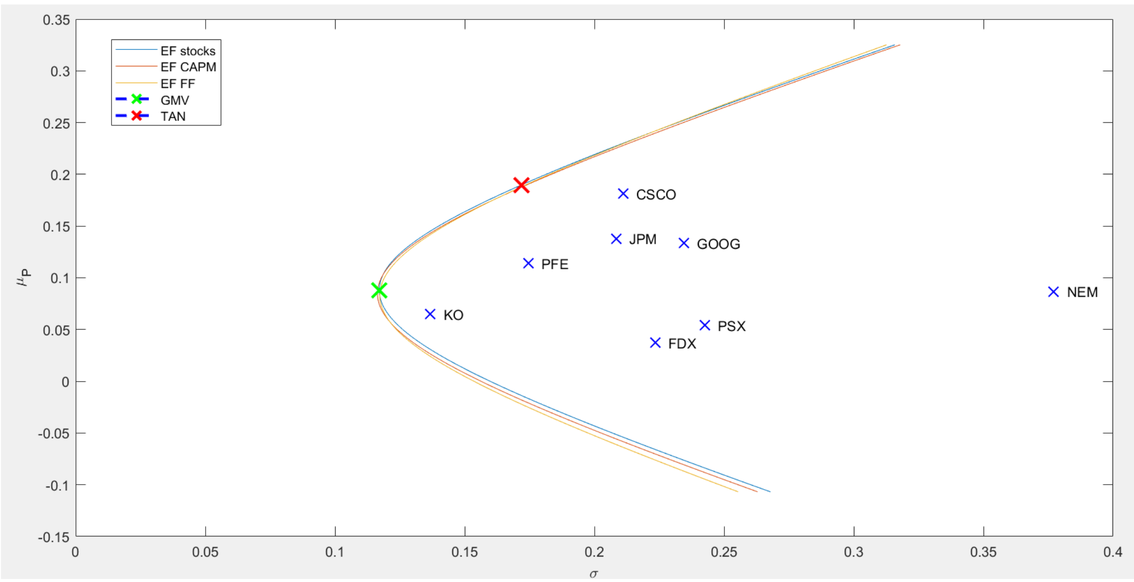


Figure 8: Efficient frontiers of the models and significant portfolios:

## 2.7 Diversification:

Next, we can observe the effect diversification has on non-systemic risk using an equal weights portfolio.

First, to obtain non-systemic risk, the variance of each stock was broken down into its component parts. The variance of each stock's daily returns was used to represent total risk. The square of each stock's beta times the variance of the market was used to represent systemic risk. Subtracting each stock's systemic risk from its total risk yielded each stock's unique risk.

Next, because weights in the considered portfolio are all equal, the unique risk of the portfolio is the average of the stock's unique risks divided by 8 - the number of stocks in the portfolio. Taking the square root of both sides gives the relationship:

$$\sigma_{\epsilon,P} = \sigma_\epsilon / \sqrt{N} \tag{3}$$

Plugging in the numbers computed from above yields.

$$\sigma_{\epsilon,P} = .0044 and \sigma_\epsilon = .0124 \tag{4}$$

The ratio between them is about .356

## 2.8 Rolling Beta Estimates

Finally, yearly rolling beta estimates were considered to better get a sense of something. The data was analyzed one 252-trading-day frame at a time - starting from January 2, 2014 to January 2, 2015 and moving the start and end bounds of the frame forward a day.

Beta values were calculating similarly to the methods described above. The stock returns were fitted to the CAPM model using stock returns data and market data from within the current frame using linearModel.fit function, then beta values for each of the stocks was estimated using the linearModel.Coefficients.estimate function. Plotting the results of this process over time yields the following figure:


Figure 9: Rolling yearly beta estimates for the eight stocks. Note that CSCO is represented by the blue line that starts around .8 and ends at 1.3

Beta is an indirect measure of nondiversifiable or systemic risk and is strongly related to the correlation between the stock and the market. With this information, we can draw some conclusions from the above plot.

The most standout feature is the fact that NEM drops into the negative betas at some point. This indicates that NEM is inversely related to the market at some points in time. This seems odd, but when compared to the historical price chart at the beginning of this paper, one can see there are instances in which NEM moves in the opposite direction from the rest of stocks. Given that the other stocks are always somewhat positively correlated to the market, we can roughly say that NEM does indeed move in the opposite direction of the market at some points, which rationalizes this chart.

Aside from NEM, the rest of the results seem fairly normal. Higher betas mean more return and higher non market risk. CSCO, GOOG, and JPM are the three highest returning stocks and also have the highest beta values.

Lower betas mean less return and lower non-market risk (i.e. the stock is less volatile than the market). Ignoring NEM and focusing on KO, the stock with the lowest risk, the chart again makes sense. While lower betas don't necessarily mean less risk for a particular stock, one can see from the returns data that KO is consistently less volatile than the rest of stocks. While they rapidly increase or decrease in value, KO stays relatively stable.

These observations of the Betas and Historical returns data seem to corroborate one another.