DMC Problem 22.1

A barber shaves all people who do not shave themselves and only people who do not shave themselves. Who shaves the barber?

The question is a paradox and has no well defined answer. As per the terms stated in the question, the barber must be shaved but cannot be shaved without violating those same terms.

If the barber is not shaved, then he falls under the category of people who do not shave themselves and thus must be shaved by the barber.

If the barber shaves himself then he no longer is the barber because the barber only shaves people who do not shave themselves.

DMC Problem 27.1

Answer Yes or No and explain your reasoning: "Is the correct answer to Problem 27.1 No?"

This question is another paradox and has no well defined answer.

If one answers yes, then they're saying that the correct answer to the problem is no, which contradicts the initial answer of yes.

If one answers no, then they're saying the statement "the correct answer to the problem is no" is false when it should be true (the correct answer should be yes) because of the initial response of no.

DMC Problem 22.18(a)

Prove or disprove

Z^2 is the set of pairs $\{(z_1, z_2)|z_1, z_2 \in Z\}$. Z^2 is countable.

First we prove a bijection between Z and N following the example we did in class. One possible equation is:

$$Z = f(n) = \begin{cases} n/2 & n \text{ is even} \\ -\dfrac{n+1}{2} & n \text{ is odd} \end{cases}$$

From there, while studying this problem I learned about the Cantor Pairing function which maps (N, N) to distinct values of N. The function goes like so:

$$f(n_1, n_2) = \frac{1}{2}(n_1 + n_2)(n_1 + n_2 + 1) + n_2$$

We can prove that there are no values of (a1, a2) in N and (b1, b2) in N that produce the same values of f(n1, n2) by setting the equation equal to itself and plugging in a1, a2 on one side and b1, b2 on the other side. One an see that with some simple algebra, the two sides will cancel.

Thus we've proven that Z is in N and that (N, N) is in N which proves (Z, Z) is in N and is thus countable.

DMC Problem 22.18(b)

Q is the set of rational numbers, Q = {r|r = a/b, a in Z and b in N}. Q is countable.

As per the example in the book, we can prove that the rational numbers, Q, can be listed which proves they are countable.

Q		Z								
		0	+1	-1	+2	-2	+3	-3	+4	-4
	1	0/1	1/1	-1/1	2/1	-2/1	3/1	-3/1	4/1	-4/1
	2	0/2	½	-1/2	2/2	-2/2	3/2	-3/2	4/2	-4/2
N	3	0/3	1/3	-1/3	2/3	-3/2	3/3	-3/3	4/3	-4/3
	4	0/4	¼	-1/4	2/4	-4/2	3/4	-3/4	4/4	-4/4
	5	0/5	1/5	-1/5	2/5	-5/2	3/5	-3/5	4/5	-4/5

One can traverse the rationals in a snake like manner starting at 0/1, then going to 1/1, ½, 0/2, 0/3, 1/3, -1/3… etc.

The fact that we can list the rationals in this way proves they are countable.

DMC Problem 22.18(c)

DMC Problem 21.7

DMC Problem 21.37

DMC Problem 21.38