DMC Problem 13.3

The choices for breakfast (B), lunch (L), and dinner (D) are:
B ∈ {hot sausages, hot eggs, cold cereal, cold fruit}
L ∈ {hot pasta, hot burger, cold sandwich}
D ∈ {hot steak, hot pizza, cold salad, cold beer}
You can't have two hot or cold meals in a row. How many different daily menus can you create?

Let BH1 = the first hot meal you can have for breakfast (sausages), BH2 the second hot meal you can have (eggs), BC1 equal the first cold meal you have for breakfast (cereal), and BC2 equal fruit.

Similarly let LH1 be pasta, LH2 be burgers, and LC1 be sandwiches. Let the same convention apply to dinner meals as well.

Now, we just need to consider two cases: one, where you have a hot meal for breakfast and two, where you have a cold meal for breakfast.

Let's list the possible breakfast, lunch, and dinner choices when you have a hot meal for breakfast using the notation from above.

BH1, LC1, DH1
BH1, LC1, DH2

Because of temperature rule, there are only 2 sequences of meals possible if you start with a hot meal. Since there are two hot meals, that means there are 4 total combinations possible.

Now let us consider the cold meals case.

BC1, LH1, DC1
BC1, LH1, DC2
BC1, LH2, DC1
BC1, LH2, DC2

There are 4 sequences of meals per cold meal and two possible cold meals to choose from at breakfast. This gives a total of 8 combinations. Thus, since 4 + 8 = 12, there are 12 total daily menus to create.

DMC Problem 13.4

Every day, you wake up and decide to go back to sleep or walk a mile. After 20 days, you have walked 12 miles. In how many different ways could you have done this?

I know from the book that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ = the number of subsets of size k from n possible choices.

In the context of this problem k = 12 = the number of days we choose to run and n = 20 = the number of total days passed.

$$\binom{20}{12} = \frac{20!}{12!\,(20-12)!} = \frac{20 * 19 * 18 * 17 * 16 * 15 * 14 * 13}{8 * 7 * 6 * 5 * 4 * 3 * 2 * 1}$$

$$= \frac{(\cancel{5} * \cancel{4}) * 19 * (\cancel{6} * \cancel{3}) * 17 * (\cancel{8} * \cancel{2}) * (5 * 3) * (\cancel{7} * 2) * 13}{\cancel{8} * \cancel{7} * \cancel{6} * \cancel{5} * \cancel{4} * \cancel{3} * \cancel{2} * 1}$$

$$= \frac{19 * 17 * 5 * 3 * 2 * 13}{1} = 26 * 19 * 17 * 15 = 125970$$

DMC Problem 13.20(c)

Solve with build up counting: tinker, invent notation; go from small to large.
(c) How many 6 digit numbers 000000 through 999999 have digits which sum to 27?

Let s = the sum of the digits in a number and n = the total number of 6 digit numbers that have digits that sum to s.

S = 0, then n= 0
S = 1, then n = 6 because there are 6 possible locations for a 1.
S = 2, then…

000002
000011
000020
000101
000110
000200
001001
001010
001100
002000
010001
010010
010100
011000
020000
100001
100010
100100
101000
110000
200000

N=21

S	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
N	1	6	21	56	126	252	462	792	1287	1996	2967	4242	5852	7812	10116	12732	15597

From the table I extrapolated and a got a formula Ax = -15x^4 - 10x^3 -105x^2 + 34x - 24 / (4(n-31))

Plugging in 27 I got 29492.

DMC Problem 13.40

A sequence is non decreasing if $0 \le z_1 \le z_2 \le \ldots \le z_k \le n$. Count non-decreasing sequences using a bijection to non-negative solutions of $x_1 + x_2 + \ldots x_k \le n$.

DMC Problem 14.32a

(a) Use build up counting. Let P(n,m) be the number of different paths from (0,0) to (n,m). Show that P(n,m) = P(n – 1,m) + P(n,m-1).

I used a recursive C++ function below to fill out the table.

```
int UghRecursion(int x, int y, int end_x, int end_y) {
	if ((x==end_x)||(y==end_y)) {
		return 1;
	} else {
		return UghRecursion(x+1,y,end_x,end_y) + UghRecursion(x,y+1,end_x,end_y);
	}
}
```

P(n, m)		n				
		0	1	2	3	4
m	0	1	1	1	1	1
	1	1	2	3	4	5
	2	1	3	6	10	15
	3	1	4	10	20	35
	4	1	5	15	35	70

As we can see the rule P(n, m) = P(n-1,m) + P(n,m+1) holds true for all the values in area we’re working with.

DMC Problem 14.32b

(b) What are P(0 ,m) and P(n, 0)?

P(0, m) and P(n, 0) are both always 1 for all values of m and n. P(0, m) represents the vertical line from the origin moving up. P(n ,0) represents the horizontal line moving right from the origin. Since we can only move in two directions (up and right), there is only one way to travel in a straight line.

DMC Problem 14.32c

(c) Using (a), (b) compute P(4,4), the number of paths from 0,0 to (4,4).

Using the results of the table computed by the program as well as P(3,4) = 35 and P(4,3) = 35, I know that P(4,4) = 70.

DMC Problem 14.32d

(d) Explain why P(n,m) = $\binom{\boldsymbol{n+m}}{\boldsymbol{n}}$

Traditionally when we use the form $\binom{n}{k}$, n represents the total number of choices and k represents the number of elements we want to include in a set.

In the context of the problem, n represents how far right one has to travel and m represents how far up one has to travel, thus n + m = the total distance one has to travel. This is analogous to the total number of choices to make.

DMC Problem 12.19

Prove or disprove: There exists $n \geq 3$ for which there are preferences of n boys and n girls such that every matching is stable. (To prove, give the preferences and show that every matching is stable. To disprove, you must show that for any $n \geq 3$ and any set of preferences, at least one matching is unstable.