



42104

# Technical University of Denmark

## Introduction to Financial Engineering

### Written Examination 2019

**Date:** December 19, 2019

**Duration:** 4 hours

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#### INSTRUCTIONS

1. This exam consists of three sections
2. Section A consists of five questions
3. Section B consists of nine questions
4. Section C consists of six Multiple Choice questions. Any ambiguity about the chosen answer will be awarded 0 marks.
5. All aids (excluding Internet use) are allowed
6. This final examination counts for 100% of the course grade
7. Please submit one pdf-file and/or a handwritten set of pages for marking

## SECTION A: Numerical questions

There are five questions in this section. Attempt all questions.

1. Your colleague has used a bond's duration and convexity to calculate that the effect of an interest rate increase of 1% will lead to a percentage reduction in the bond price of 17%.
  - a. Using only the information above, can you conclude if the bond has a duration of more than 17, less than 17 or exactly 17? Justify your answer.
  - b. The convexity of the bond is 200. Use this information together with the above information to calculate the effect of an interest rate decrease of 0.5%.
2. Compute the following (using geometric averages for calculation of average returns):
  - a. If the average weekly return of a stock over the past six months is 0.01 (1%), what is the average return in annual terms?
  - b. A stock has returned 2% the first month, -1% the second month and 5% the third month. Calculate the monthly average of returns and convert this to annual terms.
  - c. If the standard deviation of daily returns is 0.015, what is the variance in annual terms?
  - d. If the standard deviation of daily returns is 0.02, what is the standard deviation in annual terms?
3. Calculate the following:
  - a. Suppose that Stock A trades at \$35 and that Stock B trades at \$50. A portfolio has 500 shares of Stock A and 25 shares of Stock B. What are the weights of Stocks A and B in this portfolio? What is the initial cost of setting up this portfolio?
  - b. A Zero-Coupon-Bond with a nominal of \$100 expires in 27 years. Assume that the yield of the bond is 0.01 in annual terms. What is the value of the bond today?

4. You are analysing different stocks to construct a portfolio. You love air planes, so you have already decided to invest in Boeing and you are deciding which of your other two interests – Internet surfing or frozen French fries – is best suited for your portfolio. You have therefore gathered the following historical information about the stocks of Boeing, Google and Lamb Weston Holdings<sup>1</sup>

Company	Annual expected return	Annual standard deviation of returns	Correlation with Boeing
BO	0.18	0.29	–
GOOGL	0.25	0.24	0.26
LW	0.16	0.24	0.05

Table 1: Stock data

You believe that these numbers are an accurate description of the future stock returns and will make your decisions based on them along with the assumption that the risk free rate is 0.02 per year.

- Which of the two stocks, Google or Lamb Weston, will be the best choice to obtain the smallest risk (measured as variance or standard deviation) possible? Answer this question without using information about the expected returns given in the above table.
- Which of the two stocks, Google or Lamb Weston, will be the preferred stock to use if solely looking at the Sharpe Ratio of the stocks?
- Pick one of the two stocks, Google or Lamb Weston. Find the Global Minimum Variance portfolio and the tangent portfolio using your chosen stock and Boeing. Report the weights, the expected return and the standard deviation of the portfolios.
- Compute the Sharpe Ratio of the tangent portfolio. How should the value of the Sharpe Ratio of the tangent portfolio compare to the Sharpe Ratio of an arbitrarily chosen portfolio of the same two stocks?

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<sup>1</sup>Lamb Weston Holdings, Inc. is an American food processing company that is one of the world's largest producers and processors of frozen french fries and other frozen potato products.

5. Assume that the Capital Asset Pricing Model (CAPM) holds. Suppose that the risk-free interest rate is  $\mu_f = 0.03\%$ , that the expected return on the market portfolio is  $\mu_M = 0.12$  and the volatility (standard deviation) of the market return is  $\sigma_M = 0.15$ . Answer the following (when relevant, notation is as seen in Ruppert-Matteson):
- What is the expected return on an efficient portfolio with  $\sigma_R = 0.06$ ?
  - Stock A returns have a covariance of 0.005 with market returns. What is the beta of stock A?
  - Stock B has a beta of 1.5 and  $\sigma_\epsilon^B = 0.08$ . Stock C has a beta of 1.2 and  $\sigma_\epsilon^C = 0.15$ . What is the expected return on a portfolio with 40% of the wealth invested in Stock B and 60% of the wealth invested in Stock C?
  - What is the volatility (standard deviation) of a portfolio with 50% of the wealth invested in Stock B and 50% of the wealth invested in Stock C? Assume that the  $\epsilon$ s are independent.

## SECTION B: Verbal questions

There are nine questions in this section. Attempt all questions.

6. Figure 1 shows how financial returns data usually compare with the normal distribution. Explain what the plot tells you about the distribution of this set of financial returns. What are potential problems when financial data often exhibits characteristics as seen in the figure?

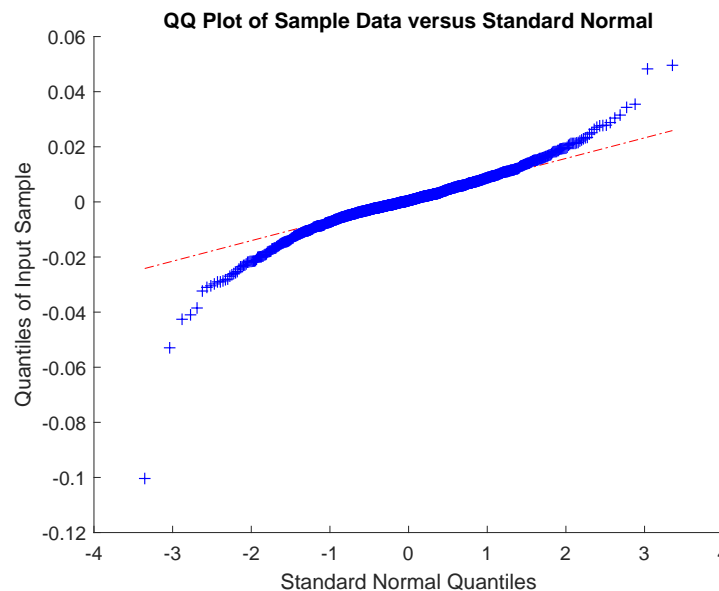


Figure 1: QQ plot of the normal distribution vs. financial returns data

7. Answer the following:
- What does the Capital Asset Pricing Model (CAPM) say about excess return of an asset?
  - Write up an expression for excess returns using a factor model and explain why both CAPM and the Fama-French three-factor model fit this framework.
  - What is the main assumption about the residuals in a factor models? What consequence does it have, if this assumption is not fulfilled?
  - Why is it useful to use factor models for estimating correlations (or equivalent covariance) between asset returns instead of using sample correlation (or sample covariance)?

8. The risk free rate is 0.01 (1%) and the excess return on the market is 0.09 (9%).
- a. You have computed expected returns and betas for numerous stocks in order to empirically test the Capital Asset Pricing Model (CAPM). Consider Figure 2. Are the empirical findings for this dataset consistent with CAPM? Justify your answer.

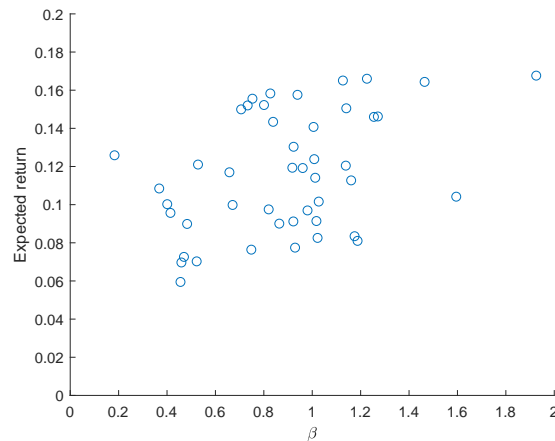
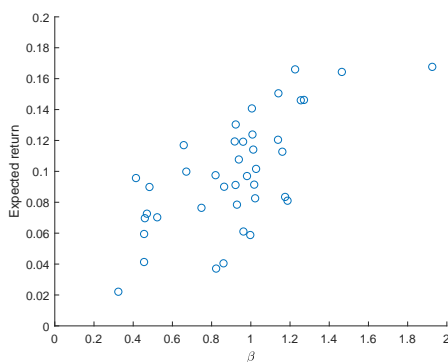
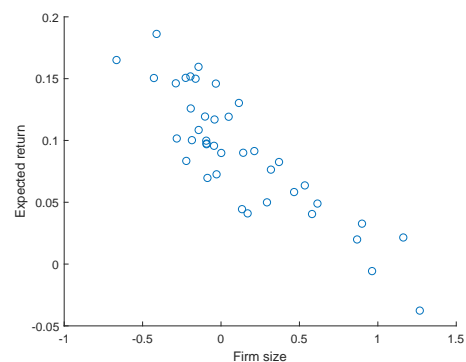


Figure 2: Betas of stocks vs returns

- b. You have computed expected returns and betas for numerous stocks as well as regressed on characteristics of the company such as firm size. Consider Figure 3. Are the empirical findings for this dataset consistent with CAPM? Justify your answer.



(a) Betas of stocks vs expected returns



(b) Firm size vs expected returns

Figure 3: Results of joint regression of returns on betas and firm size.

- c. Give reasons for why empirical testing of CAPM often leads to a rejection of the CAPM.

9. Define the yield of a bond (using a formula) and explain what it represents.
10. Empirical investigations as for instance in Frazzini & Pedersen: Betting Against Beta (Journal of Financial Economics, 2015) as well as references therein suggest that the Security Market Line is different from the one dictated by the CAPM. How is it different and what is the main explanation offered for this difference?
11. Is the following statement true or false? Justify your answer.  
*Any portfolio (with portfolio weights adding up to 1) constructed from two or more efficient portfolios is also efficient*
12. Consider the figure in Figure 4, where standard deviation is on the x-axis and return is on the y-axis. The hyperbola shows the risky-assets only Efficient Frontier, and the dots show the seven individual assets making up the Efficient Frontier when there is no restrictions on short-selling. Carefully illustrate or explain in words, where the Efficient Frontier would be located, if a restriction is added that minimum 10% of the wealth must be allocated to each asset.

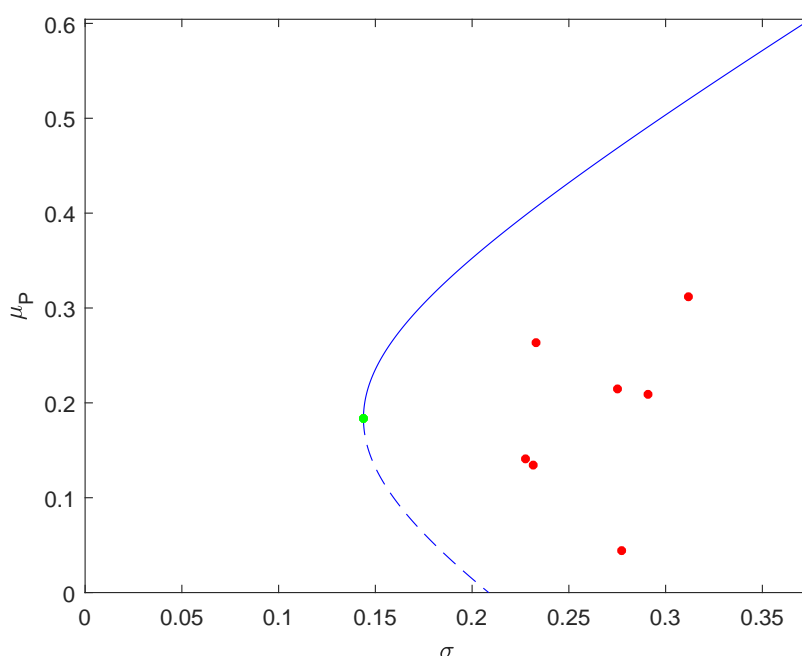


Figure 4: Efficient Frontier and assets

13. Discuss the pros and cons of using the Modified duration as opposed to using the Fisher-Weil duration.
  
14. You have done portfolio optimisation for  $N$  stocks using historical stock data. Based on your advice, a client considered whether to invest in the Global Minimum Variance portfolio with an expected return of 10% and a volatility (standard deviation) of 13% or in the tangent portfolio with an expected return of 25% and a volatility (standard deviation) of 35%. He ended up investing in the tangent portfolio because he would like to earn a lot of money. After one year he has earned 10% on his investment and he is furious and accuses you of having mixed up the two portfolios. Offer him at least two explanations for why his portfolio performance is not inconsistent with your calculations.



## SECTION C: Multiple Choice questions

There are six questions in this section. Attempt all questions and clearly state only one answer per question.

15. Consider 100 assets. All of them can be described by the characteristic line (17.13) and it is assumed that residuals are uncorrelated. Asset 1 has a beta of 1 and the average of all 100 betas is also 1. When you change your investment from solely asset 1 to an equal weighted portfolio of all 100 assets, how does the total risk change? The market risk  $\sigma_M$  is calculated to be 0.15 and all residuals have  $\sigma_{\epsilon,j} = 0.25$ .
- a. Total risk changes from 29.15% to 15.21%
  - b. Total risk changes from 29.15% to 2.92%
  - c. Total risk changes from 29.15% to 20.92%
  - d. Total risk changes from 29.15% to 16.96%
  - e. None of the above
16. Consider a coupon bearing bond with a nominal of \$100 that matures in 23 years and has an annual coupon of 4 per cent. If the price is \$98, what is the yield to maturity?
- a. 4.00%
  - b. 3.86%
  - c. 4.14%
  - d. Not possible to say based on the above information
17. Researchers in factor models talk about the "factor zoo" in relation to factor models. How many factors explaining excess returns have been proposed in the finance literature in total?
- a. Several hundred different factors
  - b. Three factors – the ones originally proposed by Fama & French
  - c. Between ten and fifty different factors
  - d. Somewhere around a hundred different factors

18. Which of the following is a false statement about stocks:
- a. Owning a stock of a company is like owning a part of the company
  - b. Owning a stock of a company limits the owners loss to the initial investment
  - c. Stocks can pay negative dividends
  - d. None of the above statements are false
19. Consider the payments illustrated in Figure 5. Which type of bond has this cash flow?

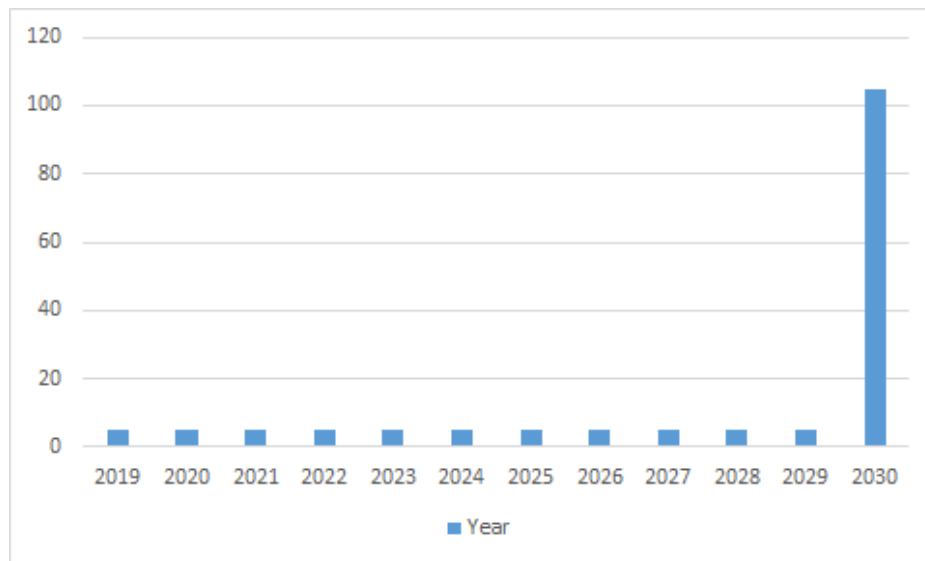


Figure 5: Cash flow for bond

- a. Serial bond
  - b. Zero coupon bond
  - c. Annuity
  - d. Bullet bond
20. Is it possible to see banks offering negative interest rates?
- a. Interest rates must always be positive, otherwise it is better to store money "in the mattress"
  - b. Yes, negative interests can be seen
  - c. Interest rates can be zero, but never negative

**END OF PAPER**