Introduction to Financial Engineering

Week 37

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Bonds

- Owning a bond is lending to the issuer of the bond
- Bond holders get their money before stock owners
- Government bonds are generally viewed as safe assets, where the bond owner gets a fixed stream of coupons and principal back
- Corporate bonds are generally not completely safe assets, as the company can go bankrupt
- The current market interest rate (and the potential risk) is reflected the bond price

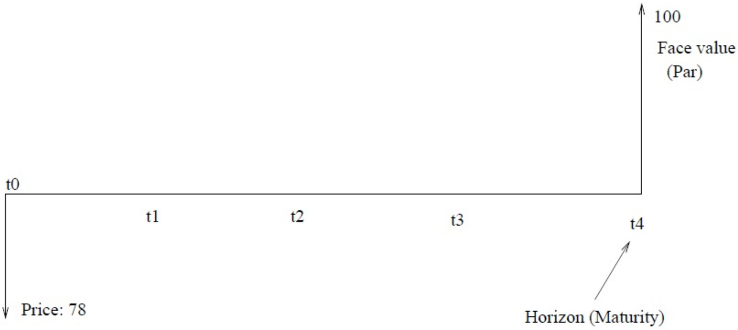
Types of bonds

The basic bond types are

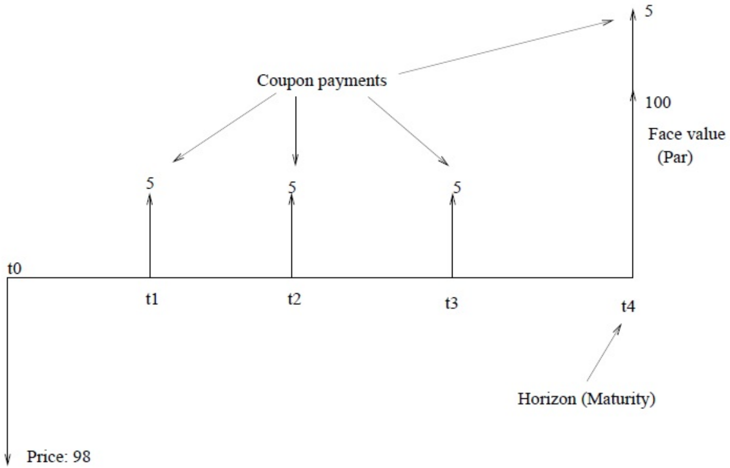
- Zero Coupon Bond
- Bond with coupons
- Annuity
- (Other types, e.g., serial loans)

The bond has a cash flow depending on type, price, coupons, maturity.

The Zero Coupon Bond is the most simple



A bond with coupon has payments before maturity



An annuity pays the same every time

Insert picture from blackboard yourself

For a serial loan repayments are constant

Insert picture from blackboard yourself

Market price of bonds

- A zero coupon bond pays back the par value at maturity
- It usually sells below par
- A coupon bond pays an annual coupon of C plus par value at maturity. In case of coupons being paid semi-annually, $C/2$ is paid twice a year etc.
- Assume that the market interest rate is r with annual compounding
- The market value or present value or market price of a zero coupon bond with maturity T is the par value discounted back at the rate r: $PAR(1+r)^{-T}$
- If the market interest rate change, so does the value of the bond
- Bond prices and interest rates move in the same/opposite direction?

Finding yields from ZCB

- Often, we consider the opposite problem. From prices of bonds, we infer market interest rates (or yields as they are then called)
- From the cash flow, it is possible to extract the *yield to maturity* implied by that bond
- The yield to maturity $y_t(T)$ at time t for a zero coupon bond maturing at time T solves the equation

$$P_t(T) = \frac{PAR}{(1 + y_t(T))^{(T-t)}},$$

  where $P_t(T)$ is the price at time t for a bond maturing at time T.
- Note: The book uses a slightly different notation and assume a semi-annual yield
- We'll stick to annual yields, because the bonds we'll look at pays annual coupons

Finding yields from bonds with coupons

- The principle for extracting yields from coupon-bearing bonds is the same
- The question is "which rate should I use for discounting the cash flows to obtain the quoted price of the bond?"
- Denote the payment at time t_i from the bond by C_{t_i}
- The yield to maturity solves the equation:

$$P_t(T) = \sum_{i=t_1}^{t_N} \frac{C_i}{(1 + y_t(T))^{i-t}}$$

- The yield $y_t(T)$ denotes the yield to maturity at time t for a bond maturing at time T. If it's clear whan t or T is, we might leave it out and just write y

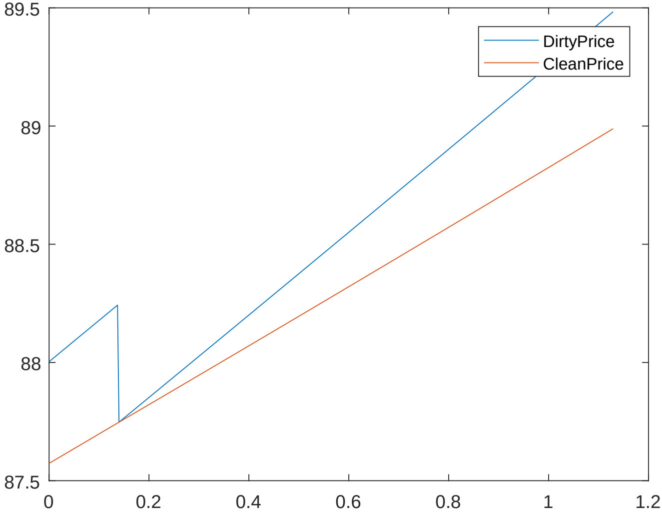
Some additional comments

- For this course, the dependence of T is more relevant than the dynamic dependence t
- The function $T \to y_t(T)$ is called the term structure of yields/interest rates on day t
- As the formulation on the previous slide reveal, yield (and other) calculations can be made on any day regardless of time of payments being an integer. If for instance, the next payment is in a month, then $t_1 = 1/12$ etc.
- There are numerous conventions for how to calculate days when working with bonds. For the purpose of this course, we'll use actual time. If you are curious about this, then NASDAQ Copenhagen has a 24 page long document with guidelines.

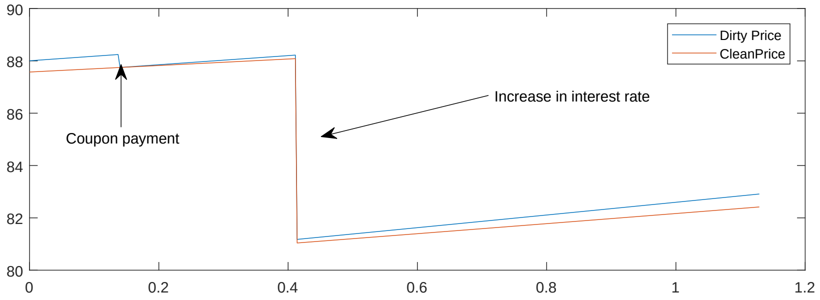
Quoted prices vs present value of cash flows

- There two types of bond prices:
  - Clean price: The price the exchange quotes
  - Dirty price: The price you actually have to pay – it is the present value of all future payments
  - Clean price = Dirty price - accrued interest
  - The accrued interest is the amount of coupon that the previous owner has to be compensated for. The accrued interest is the coupon times time passed since last coupon date.
- When setting up the cash flow and calculating yields, the dirty price needs to be calculated from the clean price and the coupons
- Clean prices are more "clean". When they change, it's because of changes in the market
- The dirty price can change just because of time passing and coupons being paid

Clean vs. dirty prices at constant interest rate



Prices when interest rates increase



When clean prices move/jump, interest rates have changed. When dirty prices change, there is either a coupon payment or something happening with the interest rates.

Topics for next week

- Duration and convexity
- Nelson-Siegel term structure model
- Analysis of interest rate risk
- Reading 1: RM Chapter 11.3. Mainly pay attention to the parametrisation of the yield curve
- Reading 2: Note on duration and convexity in discrete time

For todays exercises

- Finish the exercise from last week if you haven't
- Calculate annual returns/standard deviations
- Extract prices from NASDAQ
- Convert the clean prices into dirty prices
- Set up the cash flow (dates, coupons, principals) as a vector or matrix
- Calculate the yield for each bonds
- Plot the yields as a function of time to maturity – does any of them stand out?