Problem 5.4

Which of the following, if any, is a valid way to prove P(n) → P(n+1).
(i) Let's see what happens if P(n+1) is T. → Valid derivations → Look! P(n) is True.
(ii) Let's see what happens if P(n+1) is F. → Valid derivations → Look! P(n) is False.

Only the second example is a valid way to show that P(n) → P(n+1)

The first is flawed because one cannot assume P(n+1) to be true when it is what you're trying to prove. As shown in class with the "7=4" example, one can do perfectly valid derivations and still get a result that doesn't make logical sense. Additionally, the first statement only attempts to show that P(n+1) → P(n) which is not the same as P(n) → P(n+1).

The second statement is a successful use of proof by contradiction. By showing that if P(n+1) is false then P(n) can never be true, you are also proving that if P(n) is true then P(n+1) can never be false.

Problem 5.3(c)

For which n is P(n) true? Explain by showing the chain of implications.
(c) P(2) is true and p(n) → (P(n^2) AND P(n-2) is True for n greater or equal to 2.

P(2) → P(4) AND P(0)
P(3) → P(9) AND P(1)
P(4) → P(16) AND P(2)

Complete Chain:

P(0) P(1) P(2) P(3) P(4) P(5) P(6) P(7) P(8) P(9) … P(16)

As demonstrated above, as n gets bigger, P(n-2) statements will get "filled in." P(2) being true also proves P(0) is true. P(3) being true also proves P(1) is true. Etc etc. Thus n is true for all numbers greater or equal to zero.

Problem 5.11(b)

The nth Harmonic number is Hn = 1 + ½ + 1/3 + … 1/n. Prove:
(b) 1 + 1/2 log sub e n less than H sub n less than 1 + log sub e n.

Problem 5.43a

(a) A robot moves one diagonal move at a time. Prove that no sequence of moves takes the robot to the shaded square.

Initial predicate: P(x, y) = A robot starting at position (x, y) cannot move to the point (x+1, y) when restricted to moving on diagonals.

Proof by induction:

We'll start with the base case P(1, 1).

The robot wants to move to the point (2, 1), so we can consider all possible moves to see if that point is reachable.

If the robot moves to the upper right diagonal, then it's coordinates will be (2, 2).
If the robot moves to the upper left diagonal, then it's coordinates will be (0, 2).
If the robot moves to the lower right diagonal, then it's coordinates will be (2, 0).
If the robot moves to the lower left diagonal, then it's coordinates will be (0, 0).

None of the possible moves result in moving to the shaded area. Reaching the shaded area will also be impossible because any move to the right (x+1) to get on the same x value of the shaded area must also be accompanied by a move in the y direction, putting the robot either above or below the shaded area.

Now to prove that P(x, y) → P(x ± 1, y ± 1). I'll proof by contraposition.

Assume P(x ± 1, y ± 1) is false.

Now, say that n = x + y.

P(x ± 1, y ± 1) = P(n) OR P(n+2) OR P(n – 2) and P(x, y) = P(n).

When P(x ± 1, y ± 1) = P(n), since P(x, y) = P(n), P(x ± 1, y ± 1) = P(x, y).
When P(x ± 1, y ± 1) = P(n+2), the robot can just move to the lower left diagonal and the sum of its new coordinates will be n.
When P(x ± 1, y ± 1) = P(n-2), the robot can just move to the upper right diagonal and the sum of its new coordinates will equal n.

In all instances, P(x ± 1, y ± 1) is either equivalent to or can be derived to be equal to P(x, y). Since we assumed P(x ± 1, y ± 1) to be false and showed that P(x, y) is equivalent to P(x ± 1, y ± 1), P(x, y) will also be false.

This is a proof by contraposition because if P(x ± 1, y ± 1) is false then P(x, y) can never be true, then if P(x, y) is true P(x ± 1, y ± 1) can never be false.

Thus, by induction, P(x, y) → P(x ± 1, y ± 1) which means P(x, y) is true for all coordinates.

Problem 5.43b

(b) One of the moves changed. Now prove that any square (m, n) can be reached by a finite sequence of moves.

Exercise 6.2

Use induction to prove the claim P(n): n^3 < 2^n, for n greater or equal to 10.

Exercise 6.4

Show that if the missing (blacked out) square is at position (n, n) in the 2^n by 2^n, you can still L-tile the patio. The patios for n = 1, 2, 3, 4 are illustrated.