Introduction to Financial Engineering

Week 36

Nina Lange

Management Science, DTU

Week 36

Lectures Wednesdays 8-12

Classes will have the following (rough) structure:

- $\sim 3 \times 30$ minutes of lecturing and sometimes with small classroom discussion/exercises
- $\sim$ two hours of supervised group work doing exercises in R or Matlab

Expected preparation:

- Required readings will be listed $\sim$ one week before class
- Slides and exercises be uploaded $\sim$ one day before class
- Solutions (not always full) for exercises are published with a one-two week delay
- Therefore, make sure to have solved the previous week's exercises before class

Textboook

- Ruppert, D., and Matteson, D.S. (2015). Statistics and Data Analysis for Financial Engineering, Springer.
- We will use parts of the book (chapters 1, 2, 3, 4, 5, 16, 17, 18)
- There are lots data examples and exercises in the book (using R), but the focus is analysis of financial data
- Therefore, the textbook is supplemented with additional readings focusing on a more overall understanding of financial markets

Additional readings

- Elton, E.J., Gruber, M.J., Brown, S.J., and Goetzmann, W.N. (2014). Modern Portfolio Theory and Investment Analysis, Wiley. Chapter 2 and 7 (p. 130-142)
- Grinblatt, M., and Titman, S. (2011). Financial Markets and Corporate Strategy, McGraw-Hill. Chapter 5 (particular 5.8).
- Lando, D., Nielsen, S.E., and Poulsen, R. (2015). Lecture Notes for Finance 1 (and More): Chapter 9, University of Copenhagen http://web.math.ku.dk/ rolf/ifnotes.pdf
- Sections of books, web references and papers uploaded or linked to on Campusnet

Recommended videos

You might find it relevant or enlightening to watch the following videos

- Khan Academy: Short videos explaining different concepts
- The Ascent of Money: A bit on the history of finance and assets – good for watching while on the bus, ironing or being tired after a long week of studying (available on YouTube)

Assessment of course

- Group projects (groups of three students): Handed out in week 38, 43, 47. Due Monday in week 40, 45, 49 at 8AM. The projects needs to be approved in order to take the exam. If a project is not approved, there will be an opportunity to fix the mistakes and hand-in again by a set deadline.
- A written exam on which your course grade is based. The exam takes place on the course allocated exam day.
- Hint: The exam will be a lot easier, if you do the projects well.

Calculus

- Derivatives and partial derivatives
- Finding minima/maxima of functions
- Lagrange multipliers
- Taylor expansions

Probability

- Random variables/probability distributions
- The normal distribution and its' limitations
- Expected values and variance/covariance

Linear algebra

- Solving systems of linear equations
- Add, multiply, transpose and invert matrices, and compute determinants
- And yes, I do expect you to be able to invert a 2-by-2 matrix without the use of a computer

Programming

- In the course and in the project, most calculations will be done in R/Matlab and very few by hand
- You will be required to make small scripts and function to calculate various quantities, import and process financial data, illustrate results in plots etc.
- I don't care if you use R or Matlab. I do most of my work in Matlab. The TA is very good (and probably a better source than me) with R. The textbook and the R book linked to under links on Campusnet is a very good source for doing financial calculations in R.

Course topics

- What are stocks and bonds
- Working with financial data
- Key concepts in bond analysis
- Portfolio choice – risk vs. return (Markowitz-framework)
- Capital Asset Pricing Model
- Analysis/evaluation of portfolios

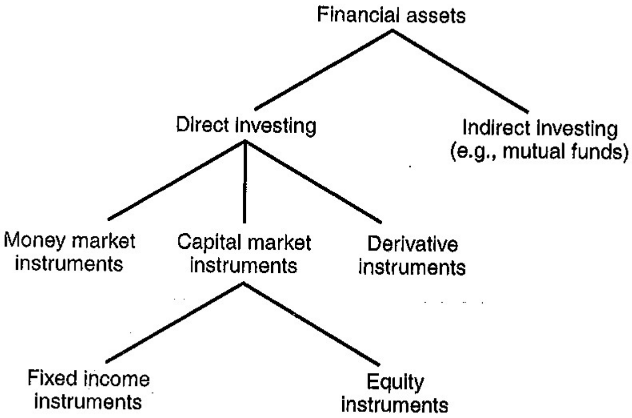
Recordings

- You may not record the lectures and share them with others
- The same goes for photographing examples on the board
- And; it is MUCH better to draw your own notes/drawings step by step while the examples are explained rather than having the final picture

Working with financial data

- Always look at the data to get a feeling for the data. For instance various plots and summary statistics.
- All models are false – but some are useful!
- There is a tradeoff between bias and variance
- Financial markets data are not normally distributed
- Variances (or standard deviation or volatility) are not constant over time

Financial assets



Stocks

What is a stock?

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https://www.khanacademy.org/economics-finance-domain/core-finance/stock-and-bonds/modal/v/what-it-means-to-buy-a-company-s-stock

Bonds

What is a bond?

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https://www.khanacademy.org/economics-finance-domain/core-finance/stock-and-bonds/modal/v/introduction-to-bonds

What affects the prices of stocks?

Spend five minutes researching and/or discussing with the student next to you `https://e.ggtimer.com/5%20minutes`

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Other financial assets

Derivatives

- Financial assets whose value is *derived* from something else
- Examples are options, forwards, futures
- The underlying actually doesn't have to be a financial asset, it could be rainfall or the price of a physical good

Funds

- Mutual funds: Pool of money from many investors. Investments in many different assets and professionally managed
- Exchange traded funds: Similar to a mutual fund, but trades on exchanges

The rate of return or the net return

- The rate of return (or just the **net return**) over a time-period is defined as the difference in asset price at the two points in time measured as a fraction of initial value:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

- Often in finance, log returns are used instead:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = p_t - p_{t-1}$$

where $p_t = \log P_t$ is called the log-price. Note: $\log$ means the natural logarithm, sometimes denoted as $\ln$ in some computer programs or calculators

When/why are these two expressions (almost) the same?

Unit and scale

- Returns are scale-free – the unit of prices does not matter
- Returns are *not* unit-free – the their unit is time, i.e., stating a return does not make sense unless the time-frame is specified
- We write the return over the most recent k periods as $R_t(k)$, where

$$
\begin{aligned}
R_t(k) &= \frac{P_t - P_{t-k}}{P_{t-k}} = \frac{P_t}{P_{t-k}} - 1 \\
&= \frac{P_t}{P_{t-1}} \frac{P_{t-1}}{P_{t-2}} \cdots \frac{P_{t-k+1}}{P_{t-k}} - 1 \\
&= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}) - 1
\end{aligned}
$$

Calculating average returns

- Often average returns per unit of time is more relevant that individual returns
- Example:
  - Stock 1 increase by 10% every year for three years
  - Stock 2 increase by 60% the first year, decrease by 50% the next year and increase by 20% the third year
  - Which stock do you prefer? Why?

Geometric average

- The arithmetic average of the two stocks are both 10%
- But the questions is really; at which rate $\bar{R}$ does my stock grow per time period (here a year) on average?

$$(1+\bar{R})^3 = (1+R_1)(1+R_2)(1+R_3)$$

- Or in more general terms: The average rate of return per time period is:

$$\bar{R} = \left(\prod_{i=1}^{N}(1+R_i)\right)^{(1/N)} - 1$$

- What is $\bar{R}$ for the two stocks on the previous slide?

Standard deviations and variance/covariance

- In many applications, we are more interested in the variation of returns than the average rate of return
- For this we use the normal definition of standard deviation and use the build-in functions in the data tool we are using
- The variation of returns gives a (simple) measure for the riskiness of the financial asset
- In today's exercise you will be asked to compute the standard deviation of daily returns
- You will also be asked to compute the covariance matrix and correlation matrix for a group of stocks
- The covariance/correlation matrix says something about how different stocks co-move

Annualising

- In the example before, we used three different annual returns to compute the average *annual* rate of return over those three years
- If we use N daily returns and compute the average, we get a *daily* average rate of return for the period that our daily returns cover
- In practice, we almost always work with daily or weekly returns
- If we have calculated an average daily or weekly return for a period of time (regardless of this time period being half a year or ten years), a relevant question is: "If we get a this return on average per day/week, what would we get in one year?"
- The answer is $(1+\bar{R})\cdots(1+\bar{R})-1$ where we multiply as many times as there are days/weeks in one year:

$$\bar{R}^A = (1+\bar{R}^W)^{52} - 1 = (1+\bar{R}^D)^{252} - 1$$

Annualising log returns

- If we use log returns, then we can just use the simple arithmetic average:

$$\bar{r} = \frac{1}{k} \sum_{i=1}^{N} r_{t-i+1}$$

- Annualising also becomes simpler:

$$\bar{r}^{A} = 52 \times \bar{r}^{W} = 252 \times \bar{r}^{D}$$

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Annualising standard deviations

- It can be seen that the standard deviation of annual returns can be extracted from the standard deviation of daily or weekly returns
- At least if we assume that returns are independent, have constant standard deviation and we are using log returns (see argument on blackboard)
- In conclusion, the standard deviation of daily returns are converted into an annual number by

$$\sigma^A = \sqrt{52}\sigma^W = \sqrt{252}\sigma^D$$

- The annualised standard deviation of returns is often also denoted the (annual) volatility

Next week we will talk about

- Cash flow of bonds
- Yield to maturity
- Duration and convexity (mostly covered in next week's slides/lecture)
- Readings: Chapter 3