Investments
FIN-405

Project

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Contents

1 Introduction

Please find all results on the following github repository : https://github.com/Wfiles/global-tactical-fund-simulator

2 Data

All data were downloaded from the WRDS Monthly World Indices database. The dataset includes the CRSP value-weighted return index for the U.S., the 1-month Treasury Bill rate (T-Bill, crsp.tfz mth rf on WRDS) , monthly exchange rates (CHF/USD, JPY/USD, USD/AUD, USD/EUR, USD/GBP), monthly stock market indices for Australia, Japan, France, Germany, Switzerland, and the United Kingdom, as well as the corresponding 3-month interbank rates for each of these countries. The sample period spans from April 2002 to December 2024, since the interbank rates for Japan were only available from April 2002 onwards.

- **Exchange Rate Computation:**
  For all currencies in our dataset, we ensured that exchange rates were available in both directions, as USD per unit of foreign currency (XXX/USD) and foreign currency per USD (USD/XXX). For example, when only USD/AUD was available, we computed AUD/USD as:

  $$\text{AUD/USD} = \frac{1}{\text{USD/AUD}}$$

  This procedure was systematically applied across all currency to allow for flexible conversions in either direction.

- The 3-month interbank rate since the date was indexed on the beginning of the month, were adjusted by shifting each observation from $01\text{-Month}_i - \text{Year}$ to $31\text{-Month}_{i-1} - \text{Year}$. The rates were divided by 100 to express them in percentage terms. A missing data point in the U.S. 3-month interbank rate for March 2020 was identified and addressed through linear interpolation.

- The 1-month T-Bill rate was provided as an annualized rate and in percentage. To convert it to a monthly rate, we divided the values by 12 and then by 100 to express them in decimal.

For consistency, all dates were aligned to the last day of each respective month. (e.g. 20/03/2003 would be mapped to 31/03/2003)

3 The international diversification strategy (DIV)

3.1 Returns computation of each stock market indexes in USD

The unhedged dollar return R_{noh} of \$1 invested in the foreign stock market is:

$$R_{\text{noh}} = \frac{P_{t+1} \cdot X_{t+1}}{P_t \cdot X_t} - 1$$

Where :

- X_t: Exchange rate (USD per unit of foreign currency)
- P_t: Foreign stock index (including cumulative dividends)

Using the returns we have for each indices and the fact that a return can be expressed as :

$$Return_{t+1} = \frac{P_{t+1} - P_t}{P_t} \quad (1)$$

$$= \frac{P_{t+1}}{P_t} - 1 \quad (2)$$

We rewrited the above formula to convert returns into USD returns as :

$$R_{\text{noh}} = \frac{X_{t+1}}{X_t} \cdot (Return_{t+1} + 1) - 1$$

Building on this formula, we converted each foreign index returns into USD returns except for the US, were we directly used the CRSP return index since it's already in USD and no exchange rate adjustment is needed.

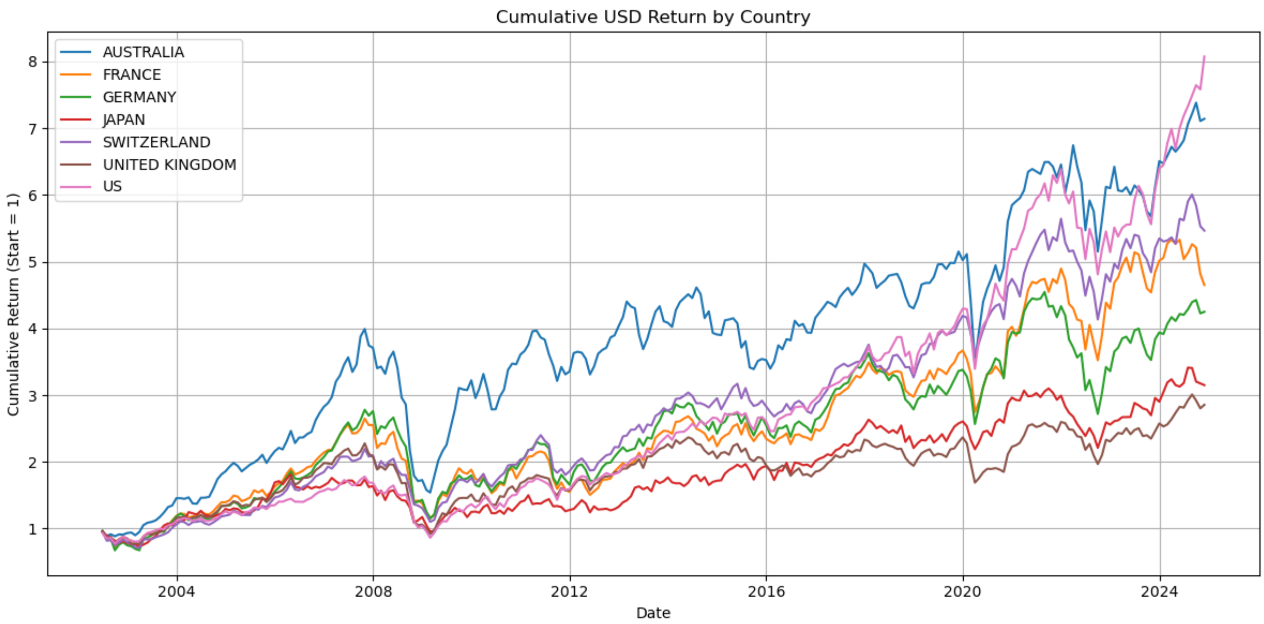

Figure 1: Cumulative performance of \$1 invested in country indexes, expressed in USD returns.

We found this result quite surprising: indeed, the Australian index outperformed from 2002 to 2024. One possible explanation is the commodity supercycle triggered by China's industrial expansion in the early 2000s, which significantly boosted Australia's resource-driven economy and equity market. However, more recent underperformance of the Australian dollar in 2024 may have allowed the U.S. to catch up in cumulative USD returns.(Source: Blackwell Global)

3.2 Currency-Hedged Excess Return in USD

The excess return in USD to a \$1 investment in the foreign currency is :

$$X_{t+1}^{foreign} = \frac{S_{t+1}}{S_t}(1 + r^{foreign}) - (1 + r^{US}) \tag{3}$$

where :

- S_t : Spot exchange rate at time t, in USD per foreign currency reported at monthly frequency
- $r^{foreign}$: The foreign 3-month interbank rate, reported at a monthly frequency
- r^{US}: The U.S. 3-month interbank rate, reported at a monthly frequency

The idea behind the currency-hedged return is to decompose the return of a USD investment in a foreign stock index into two components: the portion attributable to changes in the foreign currency, and the additional return arising from exposure to the foreign equity market. Using the computed excess return for each currency (AUD, EUR, JPY, CHF, GBP), we converted the local currency index returns into currency-hedged index returns, expressed in USD, as follows:

$$F_t^{foreign,US} = R_t^{foreign} - X_t^{foreign} \tag{4}$$

where :

- $F_t^{foreign,US}$: Currency hedged return on the foreign Index
- $R_t^{foreign}$: Return in USD of the foreign index.
- $X_t^{foreign}$ Excess return in USD from the currency-hedged investment in the foreign currency (defined in 3)

We applied the following formula for each country (Australia, France, Germany, Japan, Switzerland, United Kingdom), obtaining for each index the currency hedged return. Again, for the US, we used directly the CRSP return without currency hedging as it is already in USD and the investor is not exposed to currency risks.

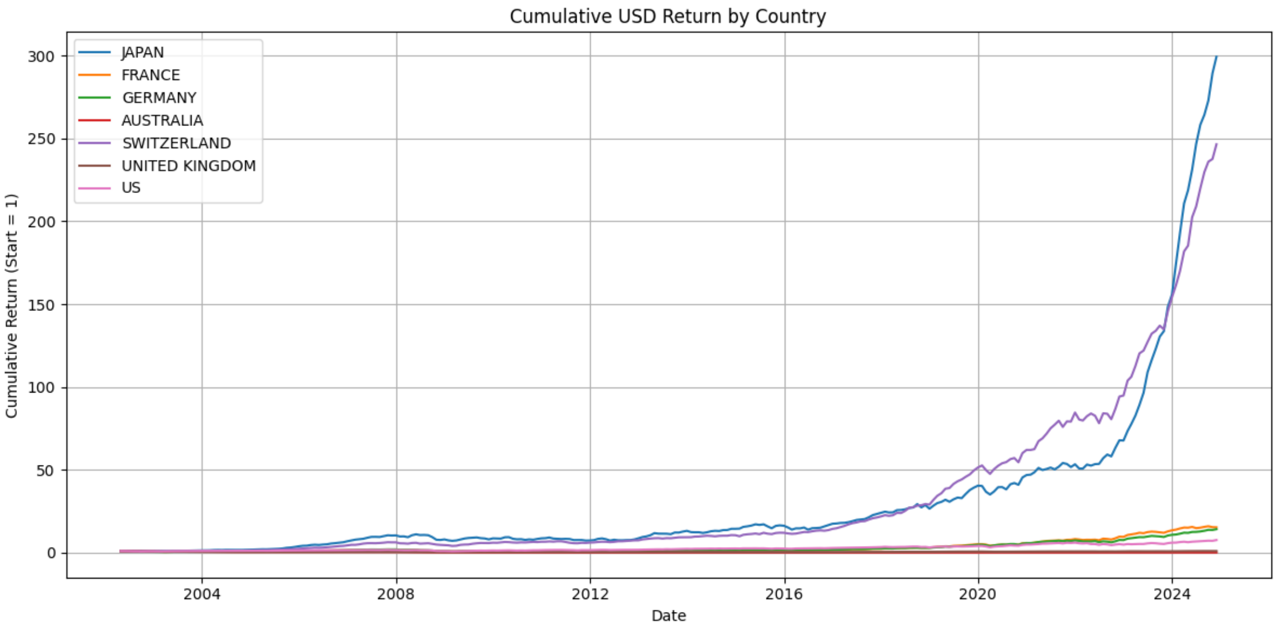

Figure 2: Cumulative performance of $1 invested in currency hedged country indexes, expressed in USD returns.

3.3 Evaluating International Diversification Strategies for a U.S. Investor

For all following exercises, we computed the following metrics as:

Mean Annual Return	**Annual Standard Deviation**	**Annualized Sharpe Ratios**
$\mu_p = 12 \cdot \mathbb{E}[R_p]$	$\sigma_p = \sqrt{12 \cdot \text{Var}(R_p)}$	Non zero cost: $Sharpe_p = \frac{\mu_p - \mathbb{E}[R_f]}{\sigma_p}$ Zero cost: $Sharpe_p = \frac{\mu_p}{\sigma_p}$

where :

- R_f: US T-bill return over the same period
- R_p : Realized portfolio return series

3.3.1 Equal Weight Strategy

- The portfolio return for an equal weight strategy are simply the average of asset returns at each time point

$$R_t^{\text{EW}} = \frac{1}{N} \sum_{i=1}^{N} R_{i,t}$$

Where for the non hedged returns we took the returns converted to USD derived in 3.1 and for the hedged we used 4.

- This strategy yielded the following performance :

  - **Unhedged Returns :**
    Mean Annual Excess Return : 0.07048
    Annual Standard Deviation : 0.15243
    Annualized Sharpe Ratio : 0.462
  - **Currency Hedged Returns:**
    Mean Annual Excess Return : 0.09830
    Annual Standard Deviation : 0.13911
    Annualized Sharpe Ratio : 0.707

3.3.2 Risk-Parity based on a 60 months rolling window estimate of the strategy returns volatilities

- In a risk parity strategy, portfolio weights are assigned inversely proportional to asset volatility in our case, based on the rolling 60-month standard deviations and then normalized to ensure they sum to one:

  $$w_{i,t} = \frac{1/\hat{\sigma}_{i,t}}{\sum_j 1/\hat{\sigma}_{j,t}}$$

  with $\hat{\sigma}_{i,t}$ the rolling 60-month standard deviation of asset i. We implemented this strategy using the same unhedged and currency hedged returns as in the equal weighted portfolio.

- We found the following performance for our portfolios :

  - **Unhedged Returns :**
    Mean Annual Excess Return : 0.04940
    Annual Standard Deviation : 0.15479
    Annualized Sharpe Ratio : 0.319
  - **Currency Hedged Returns:**
    Mean Annual Excess Return : 0.09712
    Annual Standard Deviation : 0.13981
    Annualized Sharpe Ratio : 0.695

3.3.3 Mean-variance optimal combination based on the rolling window mean and covariance matrix

- A mean-variance investor aims to maximize the utility function:

  $$U(w) = \mathbb{E}[R_p] - \frac{\gamma}{2}\mathrm{Var}(R_p)$$

  where $R_p = w^\top R$ denotes the portfolio return, w is the vector of portfolio weights, $\mathbb{E}[R_p]$ is the vector of expected excess returns, $\mathrm{Var}(R_p) = w^\top \Sigma w$ is the portfolio variance, Σ is the covariance matrix of asset returns, and $\gamma > 0$ represents the investor's risk aversion.

  Maximizing this utility function with respect to w leads to the first-order condition:

  $$\frac{\partial U}{\partial w} = \mathbb{E}[R] - \gamma \Sigma w = 0$$

  Solving for w yields the mean-variance optimal portfolio weights:

  $$w_t = \frac{\Sigma_t^{-1}(\mu_t - r_{f,t})}{\gamma}$$

  where :

  - Σ_t : Rolling 60 month covariance matrix
  - μ_t : Rolling 60-month mean returns
  - $r_{f,t}$: Rolling 60-month T-bills mean
  - γ : Investor's risk aversion

  To simplify our calculation, we set the risk aversion to $\gamma = 1$, we obtained the following performance :

- Performance Summary :

  - **Unhedged Returns :**
    Mean Annual Excess Return : 0.59717
    Annual Standard Deviation : 2.00254
    Annualized Sharpe Ratio : 0.298

  - **Currency Hedged Returns:**
    Mean Annual Excess Return : 22.30633
    Annual Standard Deviation : 7.41078
    Annualized Sharpe Ratio : 3.010

Note that for Risk-Parity and Mean-variance strategy, weights are shifted one period forward to ensure realism. At time t, investors can only use information available up to time t to decide the weights for their investment in period $t+1$.

4 Equity Index Momentum Strategy (MOM)

4.1 Return to a long-short momentum strategy portfolio

The MOMENTUM strategy exploits the cross-section of past 11-month returns, skipping the most recent month, across currency-hedged equity indices. For each country index i, we define the momentum signal as the cumulative return from month $t-12$ to $t-1$:

$$\mu_{i,t} = \prod_{s=t-12}^{t-1} (1 + r_{i,s}) - 1 \tag{5}$$

Let $\mathrm{Rank}(\mu_{i,t}) \in \{1, \ldots, N\}$ be its cross-sectional rank (1 = lowest momentum, N = highest). We then form zero-cost weights:

$$w_{i,t} = Z \cdot position_{i,t}$$

where $position_{i,t} = \left(\mathrm{Rank}(\mu_{i,t}) - \frac{N+1}{2}\right)$ and Z is a scaling factor such that:

$$\sum_{i:w_{i,t}>0} w_{i,t} = +1, \qquad \sum_{i:w_{i,t}<0} w_{i,t} = -1$$

To compute Z:

- we split the positions into long and short components:

$$long_{i,t} = \max(position_{i,t}, 0), short_{i,t} = \min(position_{i,t}, 0)$$

- we define

$$Z_t = \frac{1}{\sum_{i=1}^{N} long_{i,t}}$$

  Because the positions are centered around zero, the sum of the short positions will be the mirror image of the long side in magnitude :

$$\sum_i \max(\mathrm{position}_{i,t}, 0) = -\sum_i \min(\mathrm{position}_{i,t}, 0)$$

  Thus, normalizing the long leg to sum to +1 automatically ensures the short leg sums to –1. This results in a zero-cost portfolio exactly as intended.

The one period momentum return is given by:

$$R_{t+1}^{\mathrm{MOM}} = \sum_{i=1}^{N} w_{i,t} F_{t+1}^{i}$$

with F_{t+1}^{i} denoting the currency-hedged index return of index i in month $t+1$. We write $w_{i,t}$ because the portfolio weights are formed at time t using information available up to t, and are then applied to returns realized at $t+1$.

4.2 Mean, standard deviation, and Sharpe ratios of the long, short legs of the strategy and the strategy itself

We computed the MOM performance, and found :

Strategy	Mean Annual Excess Return	Annual Std. Dev.	Annualized Sharpe Ratio
MOM Strategy	0.27178	0.08321	3.266
MOM Strategy: only the Long leg	0.28634	0.13691	2.091
MOM Strategy: only the Short leg	0.02458	0.14369	0.171

Table 1: Performance summary for the MOM strategy and its long/short legs.

The Sharpe ratio of the MOM strategy was computed using zero as the baseline return, since it is a zero cost portfolio. In contrast, for the long and short legs taken individually, the Sharpe ratios were computed using the risk free rate (T-bills) as the baseline, since these legs involve net capital investment.

By running a t-test we obtained a p value extremely close to **0**. As $p < 0.05$, we can reject the null hypothesis at significance level of $\alpha = \mathbf{0.05}$ that the average return of the strategy is zero. The results of the different strategies are shown in Table 1.

4.3 Regress the MOM strategy return on the DIV return

Regressing the MOM strategy against the DIV strategy, estimatinh the following regression model:

$$R_{t+1}^{MOM} = \alpha + \beta R_{t+1}^{DIV} + \varepsilon_{t+1}$$

We obtained :

$$\alpha = 0.0194 \quad (t = 12.13,\ p = 0), \quad \beta = -0.0304 \quad (t = -0.786,\ p = 0.433)$$

The MOM strategy delivers a statistically significant positive alpha of 1.94%, indicating it offers returns not explained by the DIV factor. Its statistically insignificant beta implies low correlation with DIV, which translates into strong diversification potential. This suggests that MOM captures an orthogonal source of return, and combining it with DIV may improve the efficiency of the investor's portfolio by increasing the maximum attainable Sharpe ratio.

5 Equity Index Long Term Reversal strategy (REV)

5.1 Return to a long-short reversal strategy portfolio

The Equity Index Long-Term Reversal (REV) strategy is a zero cost, long-short portfolio constructed monthly. For each month t, we rank currency-hedged stock indexes based on their cumulative return from $t-60$ to $t-12$, i.e., a 5-year return lagged by 12 months. Therefore, similarly to 5:

$$\mu_{i,t} = \prod_{s=t-60}^{t-12} (1 + r_{i,s}) - 1$$

Denoting $\text{Rank}_{i,t}$ as the cross-sectional rank of index i at time t (with 1 = highest past return), the portfolio weight assigned to index i is:

$$w_{i,t} = Z \left(\frac{N+1}{2} - \text{Rank}_{i,t} \right)$$

where N is the number of available indexes, and Z is a scaling factor ensuring the long leg sums to $+1$ and the short leg to -1. (Note that the formula has a reversed sign compared to the MOM's one) The scaling factor was computed exactly in the same way as in the MOM's case.

The weights are shifted by one month before applying to returns in order to avoid look-ahead bias. Thus, $w_{i,t}$ is computed using information up to time t and applied to returns at time $t+1$: The one period reversal return is given by:

$$R_{t+1}^{REV} = \sum_{i=1}^{N} w_{i,t} F_{t+1}^{i}$$

5.2 Mean, standard deviation, and Sharpe ratios of the long, short legs of the strategy and the strategy itself

This strategy yielded the following performance :

Strategy	Mean Annual Excess Return	Annual Std. Dev.	Annualized Sharpe Ratio
REV Strategy	-0.20293	0.07309	-2.776
REV Strategy: only the Long leg	-0.02420	0.14339	-0.169
REV Strategy: only the Short leg	-0.20390	0.14011	-1.455

Table 2: Performance summary for the REV strategy and its long/short legs.

Similar to 4.2, the Sharpe ratio used zero as the baseline return for the REV strategy and the risk-free rate (T-bills) for the long and short legs taken individually.

A one-sample t-test was conducted to test whether its average return is statistically different from zero. The results indicate that the strategy's average return is significant at $\alpha = \mathbf{0.05}$, with a p-value < 0.05 ($\approx \mathbf{0}$). The results of the different strategies are shown in Table 2.

5.3 Regress the REV strategy return on the DIV return

To understand whether the REV strategy offers incremental value beyond the DIV factor, we regress REV returns on the DIV strategy's returns:

$$R_{t+1}^{REV} = \alpha + \beta \cdot R_{t+1}^{DIV} + \varepsilon_{t+1}$$

$$\alpha = -0.0172 \quad (t = -11.540,\ p = 0), \quad \beta = 0.0303 \quad (t = 0.840,\ p = 0.402)$$

The regression of the REV strategy's returns on DIV yields a statistically significant negative alpha, indicating underperformance, and an insignificant beta, suggesting low correlation between the two. Despite its negative alpha, including REV as a short position with DIV may improve the overall Sharpe ratio. If we decide to long REV it can still impact positevely the portfolio. This is due to diversification: the Sharpe ratio can increase if the drop in volatility σ_p from combining uncorrelated strategies outweighs the loss in expected return. When $\rho \approx 0$, the portfolio volatility simplifies to:

$$\sigma_p = \sqrt{w^2 \sigma_{\text{DIV}}^2 + (1-w)^2 \sigma_{\text{REV}}^2}.$$

Thus, longing REV can be beneficial for a DIV investor, provided the volatility reduction offsets REV's underperformance.

6 Currency Carry Strategy (CARRY)

6.1 Return to a long-short currency carry strategy portfolio

The CARRY strategy exploits the cross-section of 3-month interest-rate differentials. Denote by r_t^i the 3-month risk-free rate in currency i and by r_t^{US} the 3-month US T-bill rate. Define the excess carry signal

$$\kappa_{i,t} = r_t^i - r_t^{US}. \tag{6}$$

Let $\text{rank}(\kappa_{i,t}) \in \{1, \ldots, N\}$ be its cross-sectional rank (1 = lowest carry, N=highest). We then form zero cost weights

$$w_{i,t} = Z\Big(\text{rank}(\kappa_{i,t}) - \tfrac{N+1}{2}\Big) \tag{7}$$

where Z scales so that $\sum_{i:w_{i,t}>0} w_{i,t} = +1$ and $\sum_{i:w_{i,t}<0} w_{i,t} = -1$ (Same as in previous cases). The one-period carry return is

$$R_{t+1}^{\text{CARRY}} = \sum_{i=1}^{N} w_{i,t} \, X_{t+1}^{i} \tag{8}$$

with X_{t+1}^{i} the currency-hedged excess return as in (3).

6.2 Mean, standard deviation, and Sharpe ratios of the long, short legs of the strategy and the strategy itself

Note that since this is a zero-cost portfolio, we take as the baseline 0 to compute the Sharpe ration and not the risk-free rate r_f. This strategy gave the following performance :

Strategy	Mean Annual Excess Return	Annual Std. Dev.	Annualized Sharpe Ratio
Carry Strategy	0.36204	0.09236	3.920
Carry Strategy: only the Long leg	0.15060	0.09942	1.515
Carry Strategy: only the Short leg	0.18058	0.08848	2.041

Table 3: Performance summary for the Carry strategy and its long/short legs.

By running a t-test, we obtained a p-value ≈ 0 and therefore, we can reject the null hypothesis that the average return of the strategy is zero at significance of $\alpha = 0.05$. The results of the different strategies are shown in Table 3

6.3 Regress the CARRY strategy return on the DIV return

We estimated the model

$$R_{t+1}^{\text{CARRY}} = \alpha + \beta \, R_{t+1}^{\text{DIV}} + \varepsilon_{t+1}, \tag{9}$$

and obtained

$$\alpha = 0.0230 \quad (\text{t} = 12.883,\ p < 0.001), \qquad \beta = 0.1088 \quad (\text{t} = 2.512,\ p = 0.013),$$

Since $\beta > 0$, CARRY and DIV returns move in the same direction, CARRY is not a hedge for DIV. Moreover, the large positive α suggests CARRY offers a significant independent return source, making it a diversifying (but not hedging) complement to DIV. Hence, a DIV investor should take a long position in the CARRY strategy.

7 Currency Dollar Strategy (DOLLAR)

7.1 Return of the DOLLAR Strategy

The DOLLAR strategy goes long USD and shorts an equally-weighted basket of all N foreign currencies. Its one-period excess return is simply

$$R_{t+1}^{\text{DOLLAR}} = -\frac{1}{N} \sum_{i=1}^{N} X_{t+1}^{i}, \tag{10}$$

where X_{t+1}^{i} is the hedged excess return on currency i as defined in (3).

7.2 Mean, standard deviation, and Sharpe ratios of the strategy

The strategy yielded the following performance :

Metric (Annually)	DOLLAR
Mean Excess Return	0.02899
Annual Deviation	0.07761
Sharpe Ratio	0.374

Table 4: Performance for the DOLLAR strategy

A one-sample t-test yields $p = 0.0765 > 0.05$, so we cannot reject the null hypothesis that the mean return is zero at significance $\alpha = 0.05$

7.3 Regression on DIV

We estimated

$$R_{t+1}^{\text{DOLLAR}} \;=\; \alpha + \beta\, R_{t+1}^{\text{DIV}} + \varepsilon_{t+1}, \tag{11}$$

and obtained

$$\alpha = 0.0046\ (t = 3.124,\ p = 0.002), \quad \beta = -0.0212\ (t = -0.588,\ p = 0.557),$$

The regression shows that the DOLLAR strategy has a statistically insignificant negative exposure to the DIV strategy, with a beta of –0.0212, and a statistically significant alpha of 0.46% (p = 0.002). A DIV investor would be better off going long (or allocating a small weight) in DOLLAR to capture that independent 0.46%. return, rather than shorting it.

8 Optimal Fund Portfolio Return (STRAT)

8.1 Finding a for a fund return of 15%

Assuming we are running a fund that is currently investing in the **1-month T-Bill** and the **diversified stock-index strategy (DIV)** (see 3). Which means we have a convex combination of the form

$$R_{\text{FUND}} = (1-a)R_{\text{TBill}} + aR_{\text{DIV}} \tag{12}$$

where $\boldsymbol{a}$ is chosen so that the annualized volatility of our fund return is **15%**. Which means we want :

$$\text{Var}(R_{\text{FUND}}^{annual}) = 0.15^2 = 0.0225$$

Now, from Equation 12, by assuming that t-bills' volatility is equal to 0. We have that

$$\text{Var}(R_{\text{FUND}}) = a^2 \cdot \text{Var}(R_{\text{DIV}})$$

which means

$$a = \sqrt{\frac{\text{Var}(R_{\text{FUND}}^{\text{annual}})}{\text{Var}(R_{\text{DIV}}^{\text{annual}})}} = \sqrt{\frac{\text{Var}(R_{\text{FUND}}^{\text{annual}})}{\text{Var}(R_{\text{DIV}}^{\text{monthly}}) \times 12}} \approx 1.0755$$

8.2 Generation of the Optimal Fund Portfolio Return (STRAT)

To construct the strategy return R_{STRAT} we combine the four strategies MOM[4], REV[5], $CARRY$[6], $DOLLAR$[7] using a risk-parity approach based on rolling volatility estimates. First, we aligned the time series of all strategies to ensure a common date range. Then, we calculated the rolling 60-month standard deviation of each strategy to estimate strategy returns volatilities. The weight of each strategy was set inversely proportional to its volatility, and normalized so that the total weights summed to one at each point in time (We followed the same demarch as in 3.3.2). Finally, we computed R_{STRAT} as the weighted sum of the individual strategy returns using these dynamic weights. Note that since R_{strat} is made of zero cost strategies, it is also a zero cost strategy.

This risk-parity based strategy yielded the following performance :

Metric (Annually)	Value
Mean Excess Return	0.09784
Annual Deviation	0.03326
Zero Cost Sharpe Ratio	2.942

Table 5: Performance metrics of the risk parity strategy with currency hedge.

8.3 Solving for b and c

First, note that from all the previous questions, we derived that DOLLAR, CARRY, REV, and MOM are zerocost portfolios. Therefore, the resulting risk parity portfolio will also be a zero cost portfolio.

We seek weights b, c for the portfolio:

$$R_p = R_{\text{Tbills}} + b \cdot R^e_{\text{DIV}} + c \cdot R_{\text{strat}}.$$

A mean-variance investor will maximize:

$$\max_{b,c} \mathbb{E}[R_p] - \frac{a}{2}\mathbb{V}[R_p]$$

Note that R_{strat} is a pure alpha bet, as it is a zero cost portfolio. So we write:

$$R_{\text{strat}} = \alpha + \epsilon$$

which implies $\alpha = \mathbb{E}[R_{\text{strat}}]$ and $\text{Var}(R_{\text{strat}}) = \sigma^2_\epsilon$.

Using the first-order condition for the above utility function, the optimal weights for an investor are:

$$c = \frac{\alpha}{a\sigma^2_\epsilon}, \quad b = \frac{\mu_{\text{DIV}} - R_{\text{Tbills}}}{a\sigma^2_{\text{DIV}}} \tag{13}$$

With a denoting the investor's risk aversion parameter. Note that a is unknown. Instead of estimating a time varying b and c each month, we aim to find a single constant value of a such that the resulting portfolio exhibits an annualized volatility of 15%.

To achieve this, we construct a loss function that captures the squared deviation between the portfolio's annualized volatility and the 15% target. For each candidate value of a, we derive the corresponding exposures b and c using the expressions in Equation 13, and compute the resulting portfolio return series.

The portfolio's volatility is then measured and scaled to an annual level. The optimal risk aversion parameter a^* is obtained by minimizing this loss function, ensuring that the constructed portfolio satisfies the 15% volatility constraint.

We hence find that the optimal value of the risk aversion parameter is:

$$a^* \approx 28.00$$

By substituting this value into the expressions for b and c from Equation 13, we obtain the following average exposures:

$$\bar{b} \approx 0.443, \quad \bar{c} \approx 4.499$$

Consequently, the average resulting weights that the fund would invest in each substrategy are reported in Table 6. Where we computed the resulting weights in each strategy composing R_{strat} by multiplying by the risk parity weights. The weights in T-bills as (1-b) and in DIV as b.

Strategy	T-BILL	DIV	CARRY	DOLLAR	MOM	REV
Mean	0.557	0.443	1.24	1.31	0.95	1.00

Table 6: Resulting weights into T-BILL, DIV, and each sub-strategy (CARRY, DOLLAR, MOM, REV) over time.

8.4 Strategy (A) vs Strategy (C)

In this section, we refer to **Strategy (A)**, as defined in subsection 8.1, as the strategy that involves investing exclusively in T-Bills and the diversified stock-index strategy (DIV). We also define **Strategy (C)**, described in subsection 8.3, as the strategy that allocates across STRAT, T-Bills, and DIV.

Consequently, when comparing the cumulative performance of \$1 invested in Strategy (A) versus Strategy (C) both adjusted to maintain an annual volatility target of 15%, we observe a clear divergence in outcomes. As illustrated in Figure 3, Strategy C outperforms Strategy A over the period. While Strategy A exhibits steady growth, Strategy C follows a trajectory of exponential growth.

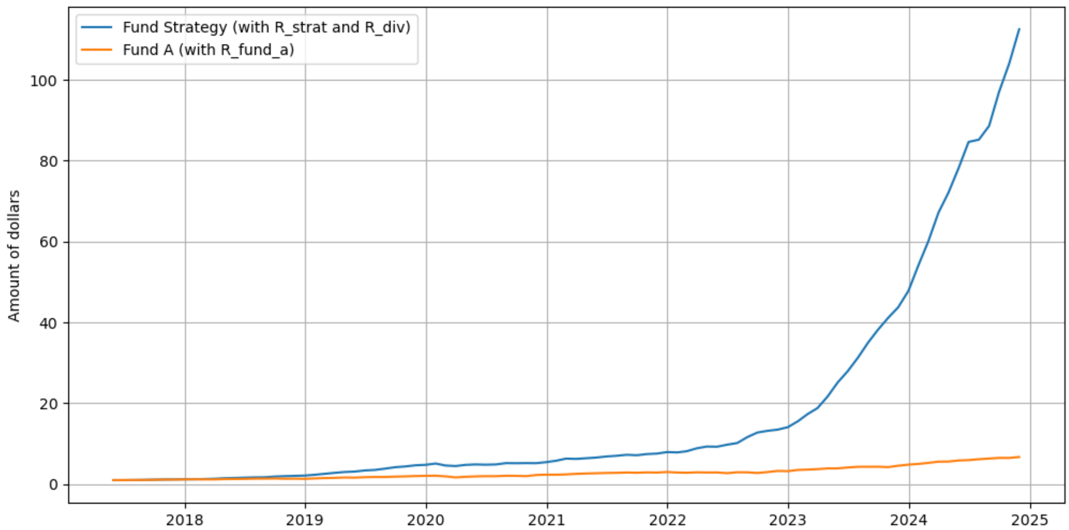

Figure 3: Cumulative performance of \$1 invested in Strategy (A) and Strategy (C), both calibrated to a 15% annual volatility.

Moreover, this difference is also supported by the performance metrics reported in Table 7 : Strategy (C) delivers a mean annual excess return of 0.6286 with a Sharpe ratio of 4.132, significantly higher than the 0.1045 return and 0.695 Sharpe ratio of Strategy (A). These results confirm that including the STRAT component built from risk-parity exposure to MOM, REV, CARRY, and DOLLAR, improves the portfolio's risk-adjusted performance, while still respecting the target volatility constraint of 15%. Note that the standard deviation in both cases is not exactly 15%, this is because we assumed that the T-bills had zero Variance/Covariance but in practice it is not the case.

Metric (Annually)	Strategy A	Strategy C
Mean Excess Return	0.1045	0.6286
Annual Deviation	0.1503	0.15215
Sharpe Ratio	0.695	4.132

Table 7: Comparison of the performance metrics between Strategy (A) and Strategy (B)

9 Performance and risk analysis for the Fund strategy

9.1 Regression of our Strategy on Fama-French 5 research factors

To assess whether the performance of our fund strategy can be explained by traditional US equity risk premia, we regress the fund's monthly excess returns on the five Fama-French factors (Mkt-RF, SMB, HML, RMW, CMA). Since the returns were expressed in percentage points, we divided by 100 to obtain decimal values. The regression follows this formula:

$$R^e_{\text{Fund},t} = \alpha + \beta_{\text{MKT}}(R_{\text{MKT},t} - R_{f,t}) + \beta_{\text{SMB}} \cdot \text{SMB}_t + \beta_{\text{HML}} \cdot \text{HML}_t + \beta_{\text{RMW}} \cdot \text{RMW}_t + \beta_{\text{CMA}} \cdot \text{CMA}_t + \varepsilon_t$$

Factor	**coef**	**std err**	**t**	**P>** $\lvert t \rvert$	**[0.025**	**0.975]**
alpha	0.0517	0.005	10.982	0.000	0.042	0.064
Mkt-RF	0.1323	0.103	1.283	0.203	-0.073	0.337
SMB	-0.1954	0.196	-1.000	0.320	-0.584	0.193
HML	0.0785	0.173	0.454	0.651	-0.265	0.422
RMW	-0.2356	0.237	-0.993	0.323	-0.707	0.236
CMA	-0.3205	0.251	-1.277	0.205	-0.819	0.178
R-squared			0.064			

Table 8: OLS regression results of the fund's excess returns on the five Fama-French factors

The results of the OLS regression, presented in Table 8 reveal that none of the factor loadings (betas) are statistically significant at confidence levels. The only statistically significant term is the intercept (alpha), estimated at 0.0517 with a t-statistic of 10.982, indicating that the strategy delivers a large and statistically significant excess return that is unexplained by the Fama-French model. Moreover, to support this idea, we note that the R-squared value is 0.064, suggesting that only 6.4% of the variation in the fund's excess returns is explained by the model. This low R-squared indicates that the five Fama-French factors do not capture the key drivers of the strategy's performance.

9.2 Explanation of the performance of our strategy

EMH predicts that once you adjust for all known risk factors, no strategy should earn a **persistent, statistically significant** alpha. Under the EMH, any "abnormal" return can reflect either true mispricing or a flawed risk-adjustment model. Our large, robust alpha combined with near-zero betas suggests either omitted risk premia beyond the five-factor model or genuine persistent mispricings. By the joint-test critique, we cannot reject EMH without also validating the completeness of our factor specification.

CAPM predicts that the expected excess returns are proportional to the market's beta. This means that the market should be the only source of systematic risk, hence we should not have other systematic factors other than the market and we should also have an alpha of zero to have a strategy consistent with CAPM which isn't the case here. Therefore this strategy is not consistent with CAPM.

APT allows multiple systematic factors, and an alpha equal to zero for most of the assets under equilibrium. This is an asymptotic arbitrage. In our case, even though we have an alpha different from zero, this doesn't imply that the strategy is inconsistent with APT. The non zero alpha may still be consistent with APT if important factors are omitted (e.g. momentum, liquidity).