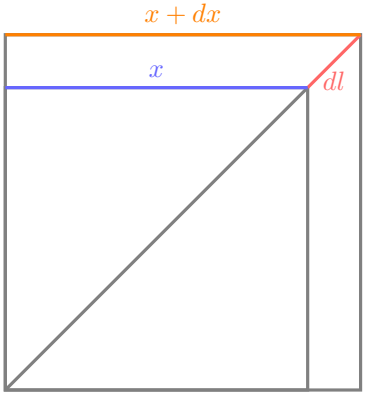


$$S = a^2, \quad dx = \frac{dl}{\sqrt{2}}$$

$$S_a = x^2, \quad S_b = (x + dx)^2$$

$$dS = S_b - S_a$$

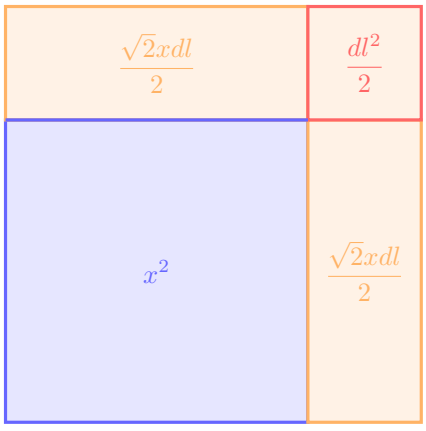
$$dS = (x+dx)^2 - x^2 = x^2 + 2xdx + dx^2 - x^2 =$$

$$= 2xdx + dx^2 = 2x\frac{dl}{\sqrt{2}} + \left(\frac{dl}{\sqrt{2}}\right)^2 =$$

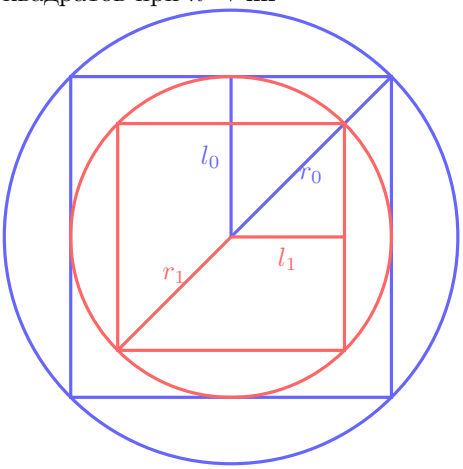
$$= \sqrt{2}xdl + \frac{dl^2}{2}$$

$$\frac{dS}{dl} = \frac{\sqrt{2}xdl + \frac{dl^2}{2}}{dl} = \sqrt{2}x + \frac{dl}{2} \sim \sqrt{2}x$$

Ответ: $\sqrt{2}xdl + \frac{dl^2}{2}$



В круг радиуса r вписан квадрат, в квадрат вписан круг и так n раз. Найдите предел суммы площадей всех кругов и предел суммы площадей всех квадратов при $n \to \inf$



$$S(r_i) = \pi {r_i}^2,\ \frac{S(l_i)}{4} = {l_i}^2,\ c = \cos(\pi/4) = \frac{\sqrt{2}}{2}$$

$$r_0 = r,\ l_0 = r_0 c,\ r_1 = l_0 c,\ \dots\ r_i = rc^{2i},\ l_i = rc^{2i+1}$$

$$S_r = \sum_{i=0}^{\infty} S(r_i) = \sum_{i=0}^{\infty} \pi {r_i}^2 = \pi \sum_{i=0}^{\infty} \left(rc^{2i}\right)^2 =$$

$$= \pi r^2 \sum_{i=0}^{\infty} c^{4i} = \frac{\pi r^2}{1-c^4} = \frac{\pi r^2}{1-1/4} = \frac{\pi r^2}{3/4} = \frac{4\pi r^2}{3}$$

$$\frac{S_l}{4} = \sum_{i=0}^{\infty} S(l_i) = \sum_{i=0}^{\infty} {l_i}^2 = \sum_{i=0}^{\infty} \left(rc^{2i+1}\right)^2 = \tag{1}$$

$$= (cr)^2 \sum_{i=0}^{\infty} c^{4i} = \frac{(cr)^2}{1-c^4} = \frac{(cr)^2}{1-1/4} = \frac{(cr)^2}{3/4} = \frac{4(cr)^2}{3}$$

$$S_l = 4\frac{4(cr)^2}{3} = \frac{(cr)^2}{3} = \frac{\frac{1}{2}r^2}{3} = \frac{r^2}{6}$$