PERMUTATIONS AND COMBINATIONS

MULTIPLICATION RULE

- If one event can occur in x ways, a second event in y ways, and a third event in z, then the three events can occur in xyz ways
- Example:

Erza has 5 tops, 6 skirts and 4 hats to choose an outfit. In how many ways can she choose her outfit?

$5 \times 6 \times 4 = 120$ ways

REPETITION OF AN EVENT

- If one event with n outcomes occurs r times with repetition allowed, then the number of ordered arrangement is n^r
- Example:

A car plate has 3 letters and 3 numbers. How many car plates are there:

a) If there's no restrictions?

letters (a-z) = 26

Numbers (0-9) = 10

$26^3 \times 10^3 = 17576000$

b) That begin with ABC?

$1^3 \times 10^3 = 1000$

FACTORIAL REPRESENTATION

- $n! = n \times (n-1) \times (n-2) \times \ldots\ldots$
  - ↳ Example : $5! = 5 \times 4 \times 3 \times 2 \times 1$
- $0! = 1$
- Example :

  In how many ways can 6 people arrange in a row?

  $6! = 720$

PERMUTATIONS

- ARRANGEMENT = Orders matter
- $${}_nP_r = \frac{n!}{(n-r)!}$$ n = number of objects, r = number of positions
- Example :

  i) How many ways can 5 boys and 4 girls arrange on a bench if:

  a) There's no restrictions ?

  ${}^9P_9 = 362880$

  b) Boys and girls alternate ?

  ${}^5P_5 \times {}^4P_4 = 2880$

  c) Boys and girls sit in 2 groups ?

  ${}^5P_5 \times {}^4P_4 \times 2! = 5760$

d) A couple wants to sit together?

$^{7}P_{2} \times 8! = 80640$

2) There are 7 horses in a race. How many ways can the horses finish, and how many ways can the placement be?

$7! = 5040$

$^{7}P_{3} = 210$

PERMUTATIONS WITH REPETITIONS

- If there's n element, with x of one kind, y of another kind and z of another another kind, then:

$$\frac{n!}{x!\,y!\,z!}$$

- Example:

How many different arrangement of PARRAMATTA are possible?

4 A's, 2 R's, 2 T's

$$\frac{10!}{4!\,2!\,2!} = 37800$$

CIRCULAR PERMUTATIONS

- Objects are arranged in a circle
- To calculate the number of ways n objects can be arranged in a circle, we fix the position of one object
  ↳ Number of arrangement = $(n-1)!$
- Example:
  There are 12 people at a dinner party. In how many ways can they sit if:

a) There's no restrictions?

$(12-1)! = 11! = 39916800$

b) A couple wants to sit together?

$2! \times (11-1)! = 2! \times 10! = 7257600$

c) Neither A or B wants to sit with C?

The 2 seats beside C

${}^{9}P_{2} = 72$

The remaining seats

$(10-1)! = 9! = 362880$

$72 \times 362880 = 26127360$

COMBINATIONS

- **UNORDERED** = Orders don't matter
- $^{n}C_{r} = \frac{^{n}P_{r}}{r!} = \frac{n!}{r!(n-r)!}$
- Example :

A committee of 5 people is chosen from 5 men and 4 women. How many ways can the committee be chosen if :

a) There's no restrictions ?

$^{9}C_{5} = 126$

b) There's majority of women ?

4 women 1 man

$^{4}C_{4} \times {}^{5}C_{1} = 5$

3 women 2 man

$^{4}C_{3} \times {}^{5}C_{2} = 40$

$5 + 40 = 45$

c) A particular person must be included ?

$1 \times {}^{8}C_{4} = 70$

d) Two people are excluded ?

$^{7}C_{5} = 21$

FURTHER PERMUTATIONS AND COMBINATIONS

- Example :

i) There are 4 Maths books selected from 6 Maths books, and 3 English books from 5 English books, how many ways can the 7 books be arranged on a shelf if :

a) Maths and English books alternate ?

$${}^6P_4 \times {}^5P_3 = 21600$$

b) The 4 Maths book remain together ?

$${}^6P_4 \times {}^5C_3 \times 4! = 86400$$

c) A Maths book at the beginning and an English book in middle ?

$${}^6P_1 \times {}^5P_1 \times {}^5C_3 \times {}^4C_2 \times 5! = 216000$$

2) How many different 8 letter words can be made from SYLLABUS :

a) There's no restriction ?

2 A's , 2 L's

$$\frac{8!}{2!2!} = 10080$$

b) Begin and end with L ?

2 A's

$$\frac{6!}{2!} = 360$$