Clarification on why every edge in the graph has at least one endpoint in the complement of the clique

In graph theory, an edge is defined by two endpoints (vertices). When you have a **clique**, it means that all the vertices within the **clique** are fully connected to each other. In other words, if you pick any pair of vertices within the **clique**, there is an edge connecting them. This is the definition of a **clique** - a set of vertices where every pair is connected by an edge.

Now, consider the vertices that are *not* in the clique, which we refer to as the *complement* of the clique. Since the vertices in the clique are all fully connected to each other, any vertex *outside* the clique will have at least one edge connecting it to a vertex inside the clique. Why is this the case?

- 1. If a vertex outside the clique is connected to no vertices inside the clique, it would mean the clique is not maximal. In other words, you could add that vertex to the clique, and it would still satisfy the definition of a clique, contradicting the assumption that you had a clique in the first place.
- 2. Since the complement vertices are not in the clique, there must be at least one edge from each of these vertices that connects them to the vertices inside the clique to satisfy the definition of a clique.

So, when you look at every edge in the graph, you can be certain that at least one of its endpoints is either inside the clique or in the complement of the clique. This is why it's said that every edge in the graph has at least one endpoint in the complement of the clique.







