MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #6 and Additional Practice Questions

Tutorial Questions

1. Let $X = \{a, b, c, d, e, f\}$.

   (a) Make up a relation on X which is reflexive and transitive but is not symmetric and is not antisymmetric. Draw its arrow diagram.

   (b) Make up a relation on X which is symmetric but is not reflexive and is not transitive. Draw its arrow diagram.

   (c) Make up an equivalence relation on X. Draw its arrow diagram.

2. Let S and T be binary relations defined as follows.

   - S is defined on $\mathcal{P}(\{1, 2, 3, 4\}) - \{\emptyset\}$ by ASB if and only if $\min(A) = \min(B)$ (where $\min(A)$ means the smallest element of the nonempty set A).
   - T is defined on the set of finite binary strings by cTd if and only if $c = d$ or c can be obtained from d by deleting some bits (for example 0,1,01,10 and 11 can all be obtained from 101).

   For each relation S and T, state whether the relation is reflexive, symmetric, antisymmetric and transitive, and explain why in each case.

3. State which (if any) of S and T are equivalence relations and which (if any) are partial order relations. For those that are equivalence relations, describe the equivalence classes. For those that are partial order relations, state whether they are total order relations and whether they are well-order relations, and explain why.

4. Let Q, R, S and T be relations on $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   (a) If $3Q4$ and $4 \not{Q} 3$, can Q be reflexive? symmetric? antisymmetric? transitive?

   (b) If $1R2$, $2R1$ and $1 \not{R} 1$, can R be reflexive? symmetric? antisymmetric? transitive?

   (c) Suppose S is antisymmetric and transitive. If $5S6$, $6S7$ and $6S8$, then what else can we definitely say about S?

   (d) If $7T8$, $8T4$ and $4T7$ can T be both transitive and antisymmetric? Can it be an equivalence relation?

(See over for practice questions.)

Practice Questions

1. For the following (informally defined) relations on the set of all people in Australia, decide whether they are reflexive, symmetric, antisymmetric and transitive. If they are equivalence relations find their equivalence classes and if they are partial order relations find whether they are total order relations and well-order relations.

   (a) x is related to y if and only if x has shaken hands with y.

   (b) x is related to y if and only if x is a (biological) ancestor of y.

   (c) x is related to y if and only if x and y were born in the same month.

   (d) x is related to y if and only if x has ever beaten y at chess.

2. Let $X = \{a, b, c, d\}$.

   (a) How many possible relations are there on X?

   (b) How many of these are reflexive?

   (c) How many of these are reflexive and symmetric?

   (d) How many of these are equivalence relations?

   (e) What would the answers to (a), (b) and (c) be if $|X| = n$ instead of $|X| = 4$?

3. Let A and B be sets such that $A \cap B = \emptyset$. Suppose R is a well-order relation on A and S is a well-order relation on B.

   (a) Can you find a well-order relation on the set $A \cup B$?

   (b) Can you find a well-order relation on the set $A \times B$?

R S A T → or is it ~~relation~~... reflective QAQ

· reflective

· symmatry

· anti-reflective

· transition

→ if have all properties... order

aRa for all a

↳ reflection for all a elements (or relation...)

symmatry

eg. sym: $aWb \xrightarrow{?} bWa$

transitive

eg. $aWb \wedge bWc \xrightarrow{?} aWc$

"a same colour shirt as b" "b same colour shirt as c" HENCE, OFC a wear same colour shirt as c

AS : Antisymmary

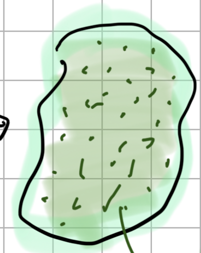
$aWb \wedge bWa \xrightarrow{?} aWb$

"a same colour shirt as b" "b same colour shirt as c" It doesn't mean that a & b is the same guy, they just wear the same colour shirt only

W: xWy

x related to y if and only if x and y ... (condition)

W ⇒

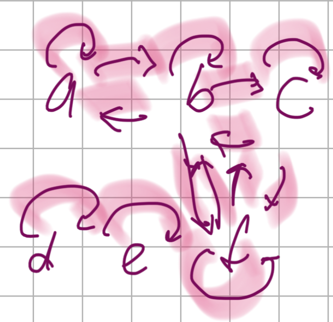


set

elements in set

RST ⇒ Equivalence 1)(c)

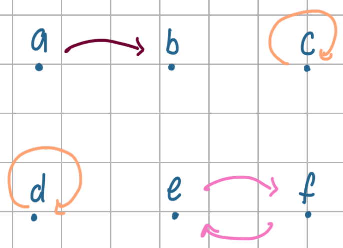
RAT ⇒ partial



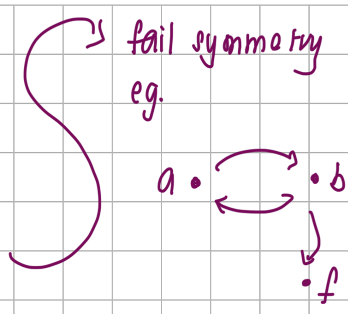
QAQ y a nver

1) Let $X = \{a, b, c, d, e, f\}$

(a) R ✓ relation } ✓ want
T ✓ transitive } leave till last! most complicated T^T
AS ✗ antisymmetrical } ✗ want
S ✗ symmetric

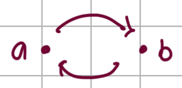


fail symmetry
eg.



cus no arrow f R b

* symmetrical need both forward & backward where ...

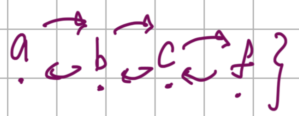


if only a • → • b
or a • → • b

then it's not symmetrical
↳ have aRb / bRa
but no bRa / aRb

1 fail = fail!

* Transitive
aRb but bRa
aRb ∧ bRb → aRb ✓
↳ don't fail



} fail. no aRa, bRf

* antisymmetry need
only a • → • b or a • → • b
but NOT a • ⇄ • b } back & forth

If you want AS, you'll never have S

AS & S is mutually exclusive!

BUT! can don't have both AS & S

* Reflective
means all have loop like

if one don't have like

the still fail!

[a] ⇒ write down equivalent class where a is a member of

eg.

$[a] = \{\{a, b\}, \{a\}, \{a, b, c\}\}$

2) $S = P(\{1,2,3,4\}) - \{\emptyset\}$ by ASB if and only if $\min(A) = \min(B)$

T = finite binary strings by cTd if and only if $c = d$ or deleting bits from d

- Reflexive ?
- Symmetric ?
- Transitive ?
- Antisymmetric ?

$ASB \rightarrow BSA$

$\min(A) = \min(B)$ is the same as $\min(A) = \min(B)$

so it's symmetric

$\underset{①}{ASB} \wedge \underset{②}{BSC} \xrightarrow{?} \underset{③}{ASC}$

① $\min(A) = \min(B)$

② $\min(B) = \min(C)$

③ so... $\min(A) = \min(C)$

ASA for all A?

min of A is the min of A

so it's Reflexive

AS: $ASB \wedge BSA \xrightarrow{?} A = B$

NOPE, so it's not AS

Reason ① = AS conditions is opposite of S

since this have S, AS would not be here

♥ HENCE... S have equivalence relation!

cTc for all c

- Reflexive ?
- Symmetric ?
- Transitive ?
- Antisymmetric ?

$cTd \wedge dTe \rightarrow cTe$

01101

001 00100

11011 10001

101 10001

10010101 0110

10010011

1110 1101110

0110110 0110

11010

sets of finite binary string

$\leftarrow = T$

$a = 101$

$b = 11011$

a is substring of 11011

but b is NOT a substring of a

∴ b longer than a so yea...

① 101 T 11011

cus can get 101 from 11011

~~1~~ 1 0 1 ~~1~~

101

101 is substring of 11011

② 001 T 10001

cus can get 001 from 10001

~~1~~ ~~0~~ 0 0 1

001

001 is substring of 10001

AS: $cTd \wedge dTc \rightarrow c = c$ ✓

♥ HENCE... T have partial order

ORDER:

PARTIAL	TOTAL	WELL
RAT		
$a \rightarrow b$	$a \rightarrow b$	$a \quad b$
$a \leftarrow b$	$a \leftarrow b$	$a \quad b$
$a \quad b$	* any pair of members can only have $a \rightarrow b$ or $a \leftarrow b$ only! " partial order $\left\{ xRy \text{ OR } yRx \right\}$ "	not related $x \not R y \wedge y \not R x$