**MAT1830 - Discrete Mathematics for Computer Science**
**Assignment #5**
Submit by uploading a pdf to moodle by 11:55pm Wednesday in week 12

Assessment questions/solutions for this unit must not be posted on any website.

Explanations required for Questions 1(iii) and (iv) and perhaps for 2(iv) and 3(i), (ii), (iii) and (iv).

**Note:** This assignment includes material from lecture 31.

(1) A matrix $M$ and its 7th power are given below.

$$M = \begin{array}{c} \\ \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \qquad M^7 = \begin{array}{c} \\ \\ \\ 3 \\ 4 \\ \\ \end{array}\begin{array}{c} \begin{array}{ccccc} & 2 & & & \end{array} \\ \begin{pmatrix} 0 & 82 & 105 & 0 & 0 \\ 82 & 0 & 0 & 46 & 82 \\ 105 & 0 & 0 & 59 & 105 \\ 0 & 46 & 59 & 0 & 0 \\ 0 & 82 & 105 & 0 & 0 \end{pmatrix} \end{array}$$

Let $G$ be the graph with vertex set $\{V_1, V_2, V_3, V_4, V_5\}$ whose adjacency matrix is $M$, where vertex $V_i$ corresponds to the $i$th row and column of $M$ for each $i \in \{1, 2, 3, 4, 5\}$.

(i) Draw a picture of $G$ (with the vertices labelled). **[1]**

(ii) Is $G$ bipartite? If so, divide the vertices of $G$ into sets $A$ and $B$ such that each edge of $G$ joins a vertex in $A$ to a vertex in $B$. If not, give an example of an odd length cycle in $G$. **[1]**

(iii) How may walks of length 7 that start at $V_3$ are there in $G$? Explain your answer. (v) **[2]**

(iv) How many walks of length 84 from $V_4$ to $V_2$ are there in $G$? Explain your answer. **[2]**

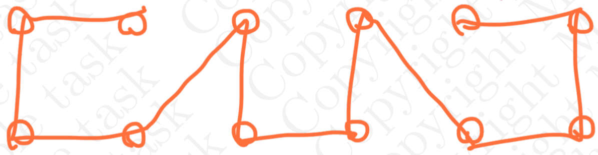
13 14 12 11

$12 \times 7 = 84$

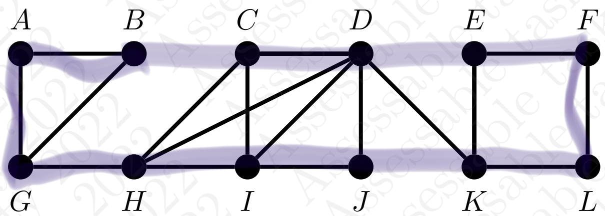
$105 + 59 + 105$
$= 210 + 59$
$= 269$

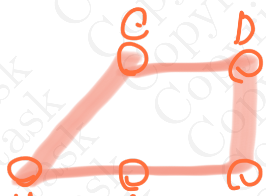
**[Explanations required only for (iii) and (iv).]**

(iv) euler:



(2) All parts of this question refer to the graph below.




(i) How many cycles of length 5 does the graph contain? One HIJDCH **[1]**

(ii) How many cycles of length 3 does the graph contain? 7 ABGA CHIC HIDH **[1]**

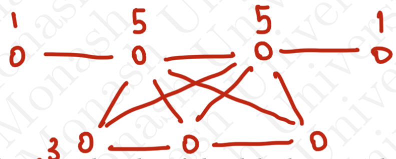
(iii) How many paths of length 12 from $B$ to $E$ are there in the graph? Write down each path as a sequence of vertices (if there are any such paths). 0 (1) **[2]**

(iv) Does the graph have an Euler trail? If so, give one. If not, explain why it does not. **[1]**

(v) Suppose we want to add edges to the graph until we reach a graph that has a **closed** Euler trail, but we wish to add the fewest edges possible to achieve this objective. Which edge(s) should be added?
(Note: we must add edges in such as way that the graph remains simple.) **[2]**

**[Answers only required, except perhaps in (iv).]**

**[See next page for question 3.]**




(3) For each of the graphs described below, either draw an example of such a graph or explain why such a graph does not exist.

should be 4 but can't connect

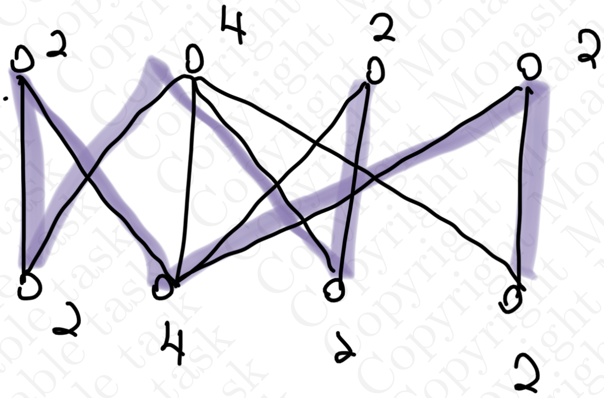
(i) A connected graph with 7 vertices with degrees 5, 5, 4, 4, 3, 1, 1. do not exist **[1]**

(ii) A connected graph with 7 vertices and 7 edges that contains a cycle of length 5 but does not contain a path of length 6. do not exist **[2]**

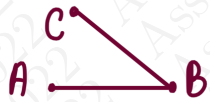
(iii) A graph with 8 vertices with degrees 4, 4, 2, 2, 2, 2, 2, 2 that does not have a closed Euler trail. do not exist **[2]**

(iv) A graph with 7 vertices with degrees 5, 3, 3, 2, 2, 2, 1 that is bipartite. do not exist **[2]**

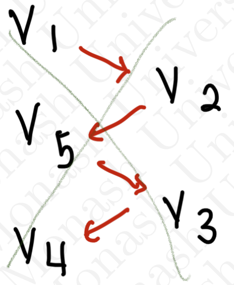
**[An explanation or a picture required for each part.]**



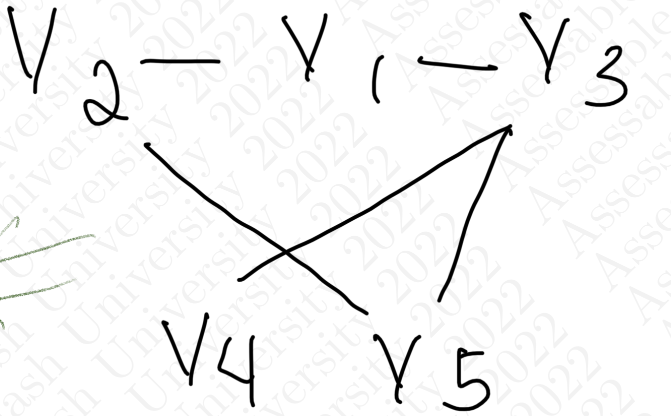

* The degree of a vertex A in a graph G is the number of edges of G that include A

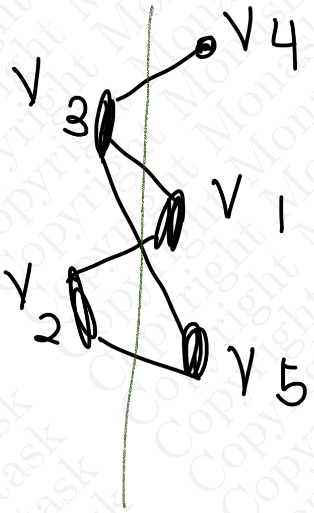



degree : 2 → B
1 → C, A




1)




~~NOT~~ BIPARTITE

1) G vertex set $\{v_1, v_2, v_3, v_4, v_5\}$ ; adjacency matrix $m$
& $v_i$ : ith row & column $m$ for $i \in \{1,2,3,4,5\}$

tips :
* bipartite no odd cycles (no 3, 5, 7, 9, 11, 13)
(ii) tells bipartite
(iii) 7 odd
(iv) $7 \times 12 = 84$
if barpartite = $v_4$ & $v_2$ at opposite sides

$$m = \begin{array}{c c} & \begin{array}{ccccc} & v_2 & & & \\ & \downarrow & & & \end{array} \\ \begin{array}{c} \\ \\ \\ v_4 \rightarrow \\ \\ \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

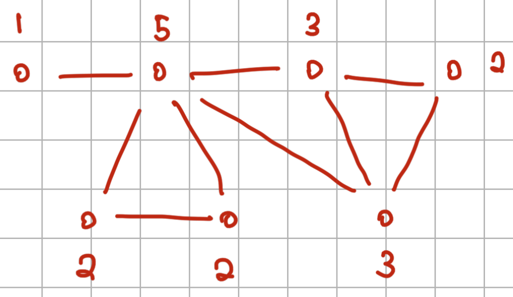
1. bipartite ?
2. $v_2$ opposite $v_2$ ?

* every 7 moves leaves you on opposite or same side?

| L | R |
|---|---|
| | x |
| x | |
| | x |
| x | |
| | x |
| x | |
| | x |

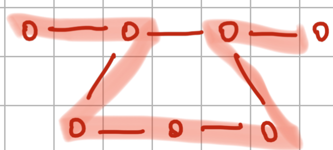
| L | R |
|---|---|
| x | |
| | x |
| x | |
| | x |
| x | |
| x | |
| x | |

3)(iv) bipartite, 7 vertices, degrees 5 3 3 2 2 2 1




NOT BIPARTITE SO DOES NOT EXIST

3) (ii) connected graph, 7 vertices, 7 edges, cycle of length 5
NOT path of length 6



DOES NOT EXIST, THERE IS PATH OF 6

1) (iii) when looking at start from a vertices to another vertices, we usually see the row so to look at start from V3 we look at row 3 and to look at the sum of lengths starting from V3 we sum all the length present in V3 so if summing 105, 59 and 105

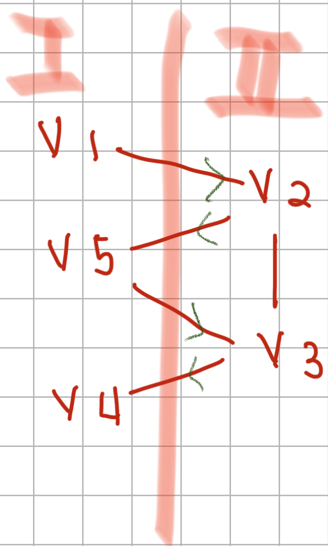
(iv) $12 \times 7 = 84$

$m^7 \times m^{12} = m^{84}$

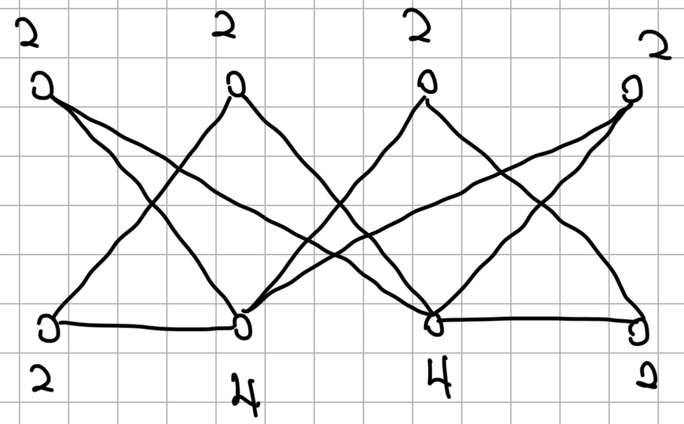
$$\left( \begin{pmatrix} 0 & 49 & 59 & 0 & 0 \end{pmatrix} \begin{pmatrix} 82 \\ 0 \\ 0 \\ 46 \\ 82 \end{pmatrix} \right) \times 12$$

$(0 \times 82) + (49 \times 0) + (59 \times 0) + (46 \times 0) + (82 \times 0) = 0$

zero walk 84 any x zero = zero




8 vertices 4 4 2 2 2 2 2 2 closed euler




num of walks

1 $V_4 \longrightarrow V_2$

2 $V_4 \rightleftarrows V_2$

3 $V_4 \rightleftarrows V_2$

4 $V_4 \rightleftarrows V_2$

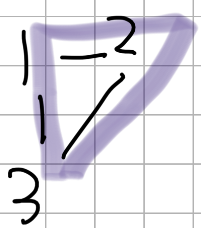
you see even will eventually go back to itself

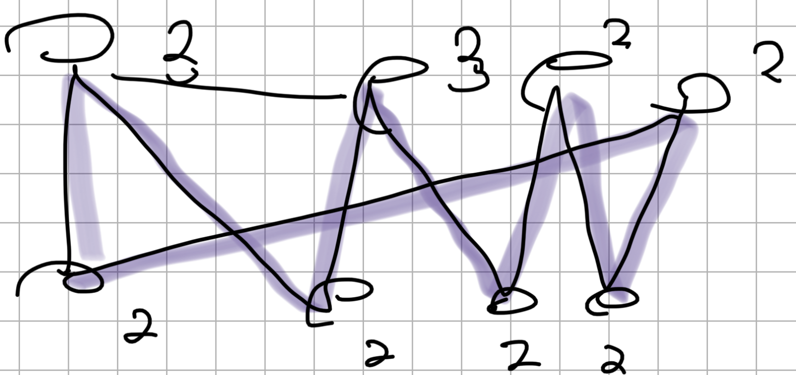
$84/2 = 42$

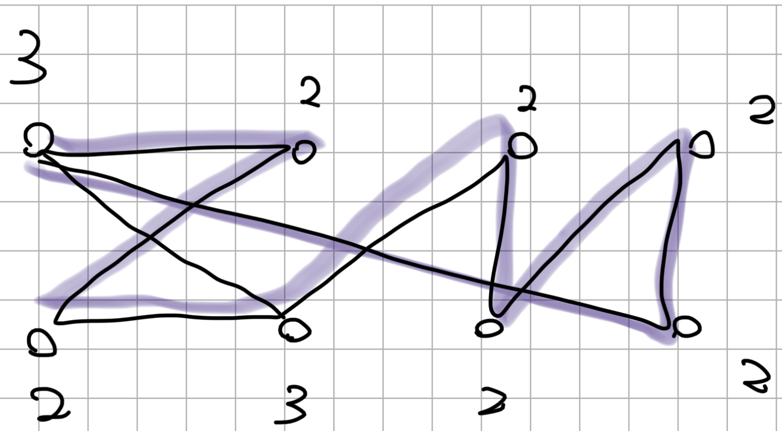
not odd

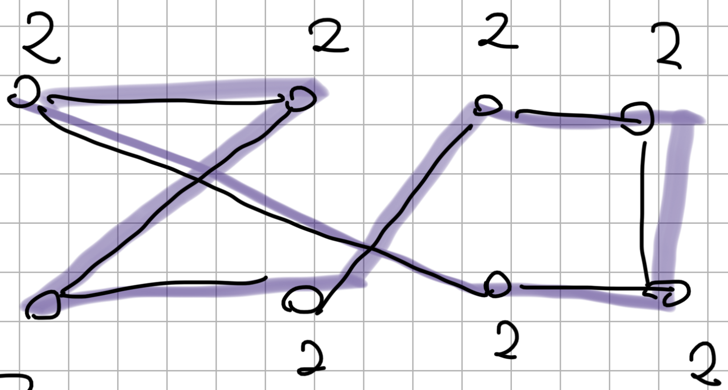
so $V_4$ cannot go $V_2$

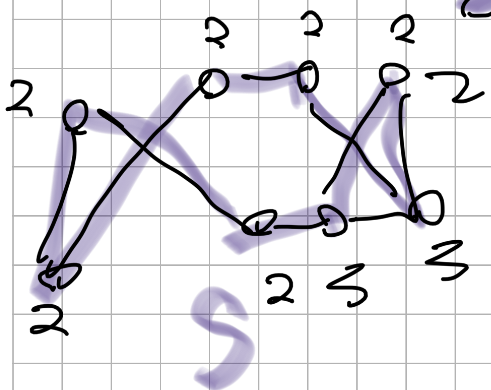
euler → trail
→ edge used only once
→ start & end diff point

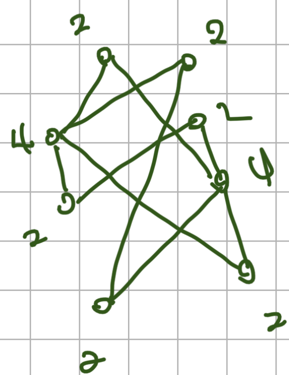


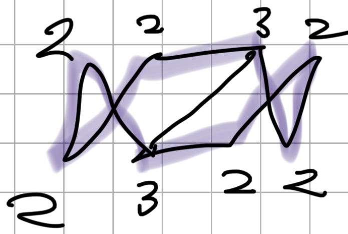



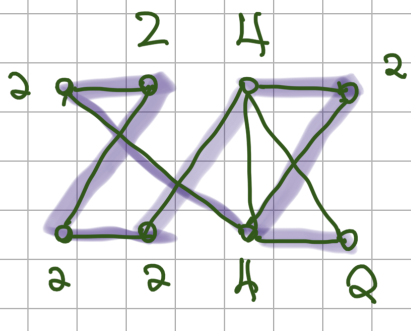

3 2 2 2
2 3 2 2

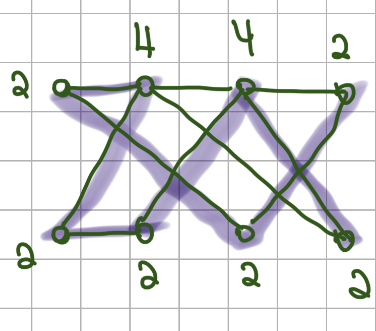

2 2 2 2
2 2 2

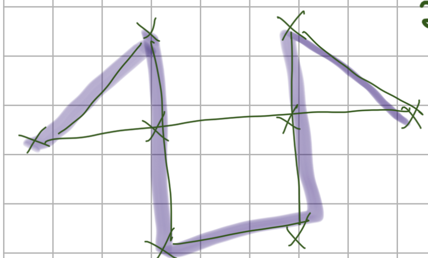

2 2 2 2 2
2 2 3 3

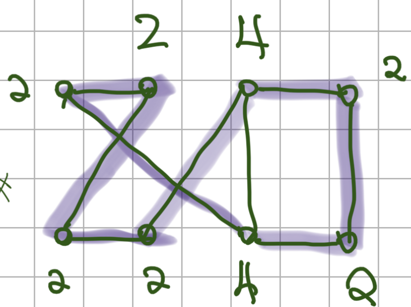

2 2 4 2 4 2 2 2

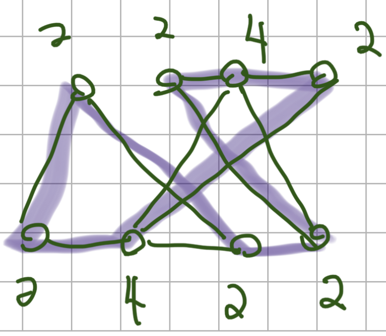

2 2 3 2
2 3 2 2

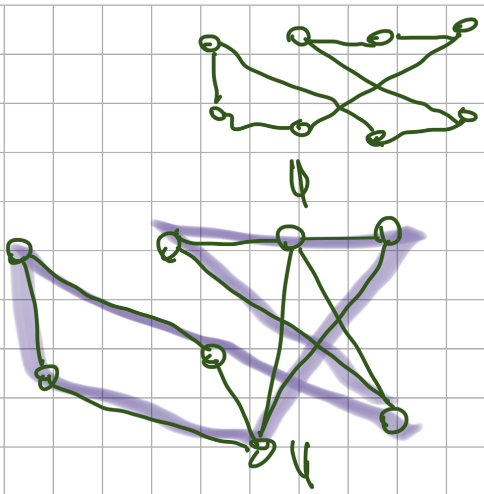

2 2 4 2
2 2 4 Q

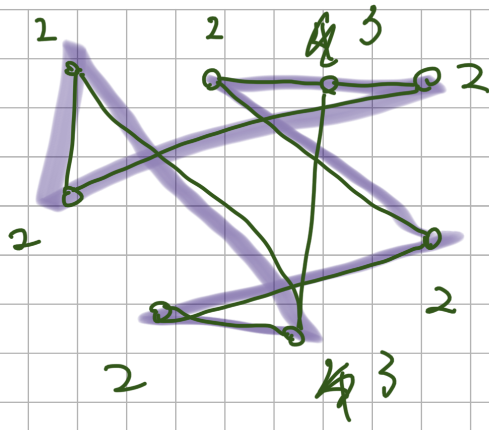

2 4 4 2
2 2 2 2




2 2 4 2
2 2 4 Q


2 2 4 2
2 4 2 2




euler
↳ edge only used once
↳ see ah, I got confused with edge and vertices