dot product : $xy = \sum_{i=p}^{n} x_i y_i$

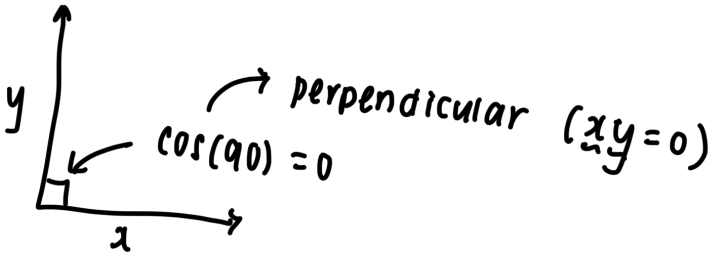
$\begin{pmatrix}1\\2\\3\end{pmatrix} \times \begin{pmatrix}4\\5\\6\end{pmatrix} = 1(4) + 2(5) + 3(6)$
$= 4 + 10 + 18$
$= 32$ #

↳ length/magnitude/normal : $|\underset{\sim}{v}|$

↳ $|\underset{\sim}{x}| = \sqrt{\underset{\sim}{x} \times \underset{\sim}{x}} = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

↳ angle

↳ $\underset{\sim}{x} \times \underset{\sim}{y} = |x||y| \cos\theta$



unit vector : $\hat{\underset{\sim}{u}} = \frac{\underset{\sim}{u}}{|u|}$ ($\underset{\sim}{u}$ → vector, $|u|$ → magnitute)

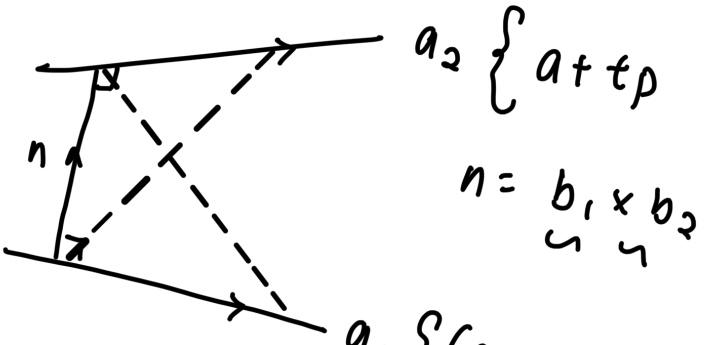
magnitute of $\begin{pmatrix}1\\2\\3\end{pmatrix} = \sqrt{1^2+2^2+3^2}$
$= \sqrt{14}$

$\hat{\underset{\sim}{u}} = \frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix}$ #

scalar projection :

* Finding distance

$v_w = \underset{\sim}{v} \times \hat{\underset{\sim}{w}}$ ← unit vector

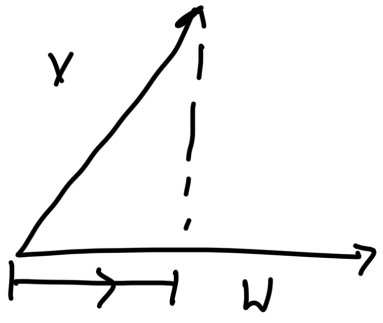


$a_2 \{ a + tp$

$n = \underset{\sim}{b_1} \times \underset{\sim}{b_2}$

$a_1 \{ (a_1 - a_2) \times \hat{\underset{\sim}{n}}$

vector projections :



v_w → vector projection "shadow"

$$\underset{\sim}{v}_w = v \times \hat{\underset{\sim}{w}}$$

$$\underset{\sim}{v}_w = v \times \hat{\underset{\sim}{w}} \times \hat{\underset{\sim}{w}} = v \times \frac{\underset{\sim}{w}}{|w|} \times \frac{\underset{\sim}{w}}{|w|}$$

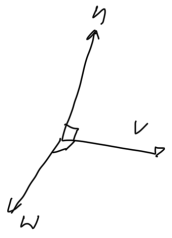
($v \times \hat{w}$ is a scalar)

$$= \frac{v \times w}{|\underset{\sim}{w}|^2} \times \underset{\sim}{w}$$

cross product :

$$\underset{\sim}{v} \times \underset{\sim}{w} = \begin{pmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

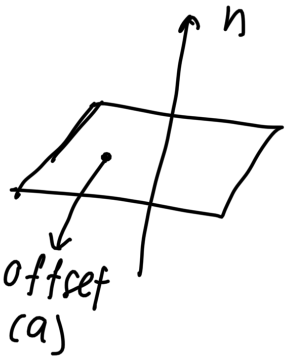
$$\underset{\sim}{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \underset{\sim}{w} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$



$$= \begin{pmatrix} 2(6) - 3(5) \\ 3(4) - 1(6) \\ 1(5) - 2(4) \end{pmatrix} - [1(6) - 3(4)]$$

area :

$rn = an$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$= 2 + 4 + 6$$

$$= 12$$

plane equation → $x + 2y + 3z = 12$

QUESTIONS

1.16 Find eq joining 2 lines

A B

$d = A - B$

$a = A$

$l = a + \lambda d$

1.19 check intersection

$l_1 = l_2$

$\begin{pmatrix} 1+2\lambda \\ 1+3\lambda \\ 1+4\lambda \end{pmatrix} = \begin{pmatrix} 2+4t \\ 2+5t \\ 2+6t \end{pmatrix}$

$\lambda = t$

1.33 check point in line

↳ substitute point in line

1.34 check line intersection

↳ make $l_1 = l_2$

1.37 check line & plane intersection

↳ substitute line into plane

$l_1 = \begin{pmatrix} 1+3t \\ 2+2t \\ 3+1t \end{pmatrix}$; $3x + 4y + 8z = 1$

$3(1+3t) + 4(2+2t) + 8(3+1t) = 1$

get t

1.38 plane & plane intersection

$n = n_1 \times n_2$

- normal line to both of their directions

$3x + 4y + 5z = 0$

$6x + y + z = 1$

let $z = 0$

$3x + 4y = 0 \quad 6x + y = 1$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = a \qquad l = a + tn$

↳ find a

1.39 3-plane intersection

- simultaneous equation

* continue next page

★ distance between 2 point

↳ $\sqrt{(x-x)^2+(y-y)^2+(z-z)^2}$

★ distance between point & line

↳ $\left((\ell - A)-(\ell)=0\right)$

★ distance between line & line

↳ $\left(\frac{(a_1-a_2)\times(b_1\times b_2)}{|b_1\times b_2|}\right)$

★ distance between plane & line & point

↳ $\left(\frac{ax+by+(z-d)}{\sqrt{a^2+b^2+c^2}}\right)$

plane : $ax+by+cz=d$

$3x+4y+5z=6$

$P=\begin{pmatrix}x\\y\\z\end{pmatrix}\begin{pmatrix}2\\2\\4\end{pmatrix}$

$d=\frac{3(2)+4(2)+5(4)-6}{\sqrt{3^2+4^2+5^2}}$

$r(n)=a(n)$

$\begin{pmatrix}x\\y\\z\end{pmatrix}\hat{n}=Q\hat{n}$

$d=(P-Q)\times\hat{n}$

$=\frac{(P\hat{n})-(Q\hat{n})}{|n|}\rightarrow=\frac{P\hat{n}-d}{|n|}$

nan?