Foo Kai Yan

33085625

G in this question represents a graph where n represents the number of vertices of G and m represents the number of edges of G. φG is the Boolean expression in CNF.

From the question it is known that the vertices present in the graph will be one of the 3 colours which are Red, Black and White. It is also known that vertices connected by an edge could never be of the same colour like for example there is a graph F and have 3 vertices, if vertex1 (v1) is connected to vertex2 (v2) by an edge and v1 is of Red colour then v2 could not be Red colour and can only be of either Black or White unless v2 is connected to other coloured vertices like vertex3 (v3) of White colour then v2 could only be of Black colour for graph F to be of a valid coloured graph. Hence from here, we can say that for a vertex, there could only be 3 options of colour it could be so there is a possibility of 3n for the vertices to have a colour which in this case is Red, Black and White and also this possibility will be applied to each vertex present in the graph so there are n number of clauses of this type where each vertex will be assigned with a colour.

From the example above one can see that if 2 vertices are connected by an edge then both vertices must be of different colours. Graph G stated in the question has 4 vertices with 5 edges present to connect the said vertices where some vertex is connected to 2 edges and some connected to 3 edges. For example, v1 is connected to v2 and v3 but not to v4 so there would be 2 clauses of this type to state that v1 could not be of the same colour as v2 and v3. Hence there would be 3m number of clauses of this type.

From the example above one can see that if a vertex is assigned a colour, let's say Red, then the said vertex could never be of White or Black colour. With this said, there would be n number of clauses of this type as it has to be applied for each vertex present within the graph as each vertex can only be of one colour.

In conclusion, after adding all the clauses to be an expression to be the number of clauses of φG would be $4n + 3m$.