Boolean Logic Logic gates Logisim MARIE (basic)

Boolean Logic, Logic gates, Logisim

- Give a boolean expression
 - Reduce using boolean identities
 - o Reduce using K-maps
 - Construct using Logisim

Identity Name	AND Form	OR Form
Identity Law	1x = x	0+x=x
Null (or Dominance) Law	0x = 0	1+ <i>x</i> = 1
Idempotent Law	xx = x	X+X=X
Inverse Law	$x\overline{x} = 0$	$x+\overline{x}=1$
Commutative Law	xy = yx	x+y=y+x
Associative Law	(xy)z = x(yz)	(x+y)+z=x+(y+z)
Distributive Law	x+yz = (x+y)(x+z)	x(y+z) = xy+xz
Absorption Law	X(X+Y)=X	X+XY=X
DeMorgan's Law	$(\overline{xy}) = \overline{x} + \overline{y}$	$(\overline{X+Y}) = \overline{X}\overline{Y}$
Double Complement Law	$\overline{\overline{X}} = X$	

Using Boolean identities, simplify C + (BC)'. Does it give True or False?

Expression	Rule used
C + (BC)' → C + (B' + C')	DeMorgan's (AND) (AB)' = A' + B'
$C + (B' + C') \rightarrow C + B' + C'$	Associative (OR) (A + B) + C = A + (B + C)
C + B' + C' → C + C' + B'	Commutative (OR) A + B = B + A
C + C' + B' → 1 + B'	Complement (OR) A + A' = 1
1 + B' → 1 (True)	Null (OR) 1 + A = 1

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Using boolean identities, simplify X = ABC + A'B + ABC'.

Expression	Rule used
ABC + A'B + ABC' → ABC + ABC' + A'B	Commutative (OR) A + B = B + A
ABC + ABC' + A'B → AB(C + C') + A'B	Distributive (OR) A(B + C) = AB + AC
AB(C + C') + A'B → AB + A'B	Complement (OR) A + A' = 1
AB + A'B → (A + A')B	Distributive (OR) A(B + C) = AB + AC
(A + A')B → 1B	Complement (OR) A + A' = 1
1B = B	Identity (AND) 1A = A

Using K-maps:

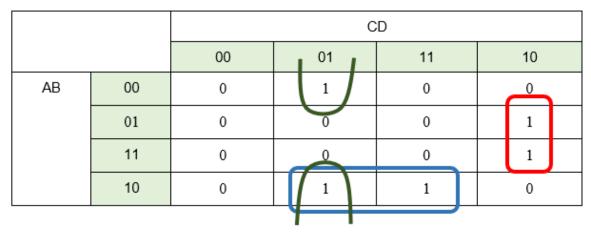
- No group contains 0
- Groups cannot be diagonal
- Grouping in 2^k (1, 2, 4, 8...)
- Each group should be as large as possible
- Groups can overlap
- Each "1" must be part of a group
- Groups can warp around map (left & right)
- There should be as few groups as possible

Simplify F(A,B,C) = ABC + A'BC + ABC' using K-maps:

		BC				
		00	01		11	10
А	0	0	0		1	0
	1	0	0		1	1

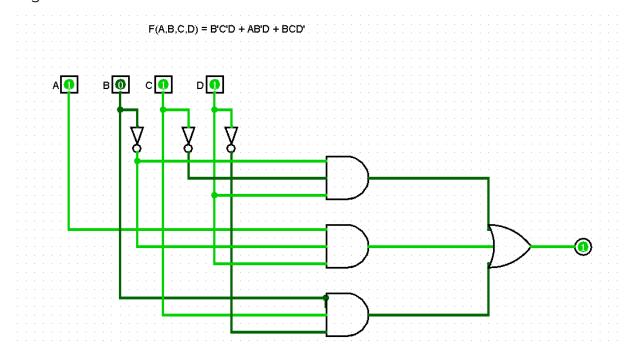
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Simplify function F(A,B,C,D) = A'B'C'D + AB'C'D + AB'CD' + ABCD' using SOP', then use Logisim to build the simplified circuit.



F(A,B,C,D) = B'C'D + AB'D + BCD'

Logisim circuit:



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MARIE Instructions

Туре	Instruction	Summary
Arithmetic	Add X	Adds value in AC at address X into AC, AC ← AC + X
	Subt X	Subtracts value in AC at address X into AC, AC ← AC - X
	Addl X	Add Indirect: Use the value at X as the actual address of the data operand to add to AC
	Clear	AC ← 0
Data Transfer	Load X	Loads Contents of Address X into AC
	Store X	Stores Contents of AC into Address X
1/0	Input	Request user to input a value
	Output	Prints value from AC
Branch	Jump X	Jumps to Address X
	Skipcond (C)	Skips the next instruction based on C: if (C) = - 000: Skips if AC < 0 - 400: Skips if AC = 0 - 800: Skips if AC > 0
Subroutine	JnS X	Jumps and Store: Stores value of PC at address X then increments PC to X+1
	Jumpl X	Uses the value at X as the address to jump to

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Indirect Addressing	Storel	Stores value in AC at the indirect address. e.g. Storel addresspointer Gets value from addresspointer, stores the AC value into the address
	Loadl	Loads value from indirect address into AC e.g. LoadI addresspointer Gets address value from addresspointer, loads value at the address into AC
	Halt	End the program

1. What does the following MARIE program do?

Load X Add Y Output Halt

X, Dec 2
Y, Dec 3

• It performs the addition between X (value 2) and Y (value 3) then outputs the result.

2. Write a program that subtracts value Y from X, then store it in a new variable and output the variable.

Load X
Subt Y
Store Z
Load Z
Output
Halt

X, Dec 8
Y, Dec 2

Z, Dec 0

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3. Write a program that accepts two 2 number inputs then output the larger number.

Pseudocode:

```
x = input()
y = input()
if x > y:
     print(x)
else:
     print(y)
MARIE code:
     //x = input()
     Input
     Store X
     //y = input()
     Input
     Store Y
     //if x > y: ## x - y > 0
     Load X
     Subt Y
     Skipcond 800 // skip next line if > 0 == if x > y: goto next
     line
     Jump PrintY
     Jump PrintX
     PrintX, Load X
     Output
     Halt
     PrintY, Load Y
     Output
     Halt
     X, Dec 0
     Y, Dec 0
```

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Extra! Simplify F(A,B,C) = A'B + BC' + BC + AB'C'.

F= A'B +BC' +BC +AB'C'

$$x = x+x \text{ (idempotent)}$$

A'B +(Bc' +Bc') +Bc +AB'c'

A'B +(Bc' +Bc) +Bc' +AB'c'

B'B +B(C+c') +C'(B+AB')

B'B +BA+B'A

B'B +BA+B'A

B+A(B+B')

B(A'+1) +C'(B+A)

B'A'

B'B +BC' +AC'

B(1+C')+AC'

B+AC'