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MONASH University

Semester Two 2016
Examination Period

Faculty of Information Technology

EXAM CODES:	**MAT1841**
TITLE OF PAPER:	**Continuous Mathematics for Computer Science – Paper 1**
EXAM DURATION:	3 hours writing time
READING TIME:	10 minutes

THIS PAPER IS FOR STUDENTS STUDYING AT: (tick where applicable)

☐ Berwick	☑ Clayton	☑ Malaysia	☐ Off Campus Learning	☐ Open Learning
☐ Caulfield	☐ Gippsland	☐ Peninsula	☐ Monash Extension	☐ Sth Africa
☐ Parkville	☐ Other (specify)			

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Instructions:

1. The exam has 10 questions with a total of 100 marks.
2. Attempt all questions in each section (Pages 2—17).
3. Answers are to be written in the spaces provided in this paper.
4. Some relevant formulae are provided at the end of the paper (pages 18—22).

AUTHORISED MATERIALS

OPEN BOOK	☐ **YES**	☑ **NO**
CALCULATORS	☐ **YES**	☑ **NO**
SPECIFICALLY PERMITTED ITEMS	☐ **YES**	☑ **NO**

Candidates must complete this section if required to write answers within this paper

STUDENT ID: __ __ __ __ __ __ __ __ DESK NUMBER: __ __ __ __ __

Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**

1. Let $\mathbf{u} = (2, 4, -3)$, $\mathbf{v} = (3, 6, 2)$ and $\mathbf{w} = (1, 0, -2)$. Find the following:

(a) $\mathbf{u} \cdot \mathbf{v}$

1 marks

(b) $\mathbf{v} \times \mathbf{w}$

2 marks

(c) The vector projection of $\mathbf{w}$ onto $\mathbf{v}$, $\mathbf{w}_v$

2 marks

(d) Find the shortest distance, d, between the line

$$\begin{aligned} x(t) &= 5 + 2t \\ y(t) &= 3 \\ z(t) &= 2 + t, \end{aligned}$$

and the line

$$\begin{aligned} x(s) &= 1 + 2s \\ y(s) &= 2 + 2s \\ z(s) &= 4 - s. \end{aligned}$$

5 marks

Total: 10 marks

2. Let A, B and C be defined as below and calculate as directed.

$$A = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 5 & 1 \end{bmatrix}, C = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

(a) $B\,A$

2 marks

(b) $A\,B^T$

2 marks

(c) $(A\,C)^{-1}$

3 marks

(d) $(B^T\,B)^{-1}$

3 marks

Total: 10 marks

3. For the matrix equation $A\mathbf{x} = \mathbf{b}$, answer the following questions when

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 1 & 0 \\ -1 & 0 & 2 & 1 \\ 3 & 0 & 3 & 2 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$

(a) Find the determinant of A.

2 marks

(b) Calculate A^{-1} using the Gauss-Jordan algorithm

6 marks

(c) Find the value of $\mathbf{x}$.

2 marks

Total: 10 marks

4. Calculate the following:

(a) $f'(x)$ when $f(x) = \dfrac{\sin(x^2+4)}{\cos(e^{-x})}$

2 marks

(b) $\dfrac{d}{dx}\left(x^2 \tan^{-1}(\ln(x+1))\right)$

3 marks

(c) The equation of the tangent line to the parametric curve $x = t^3 - 3t$ and $y = t^2$ when $t = \sqrt{3}$.

3 marks

(d) Find any critical points of the function $f(x) = e^{-(x-2)^2}$ on the interval $x \in [0, 4]$ and classify them.

2 marks
Total: 10 marks

5. The following questions relate to Taylor series.

(a) Write down the first **four** non-zero terms in the Taylor series expansion of $f(x) = e^{3x+1}$ about $x = 0$, **and** write down the Taylor expansion as an infinite sum of the form $\sum_{n=0}^{\infty} a_n x^n$.

6 marks

(b) **Deduce** the Tayor series expansion of $f(x) = e^{-x^2}$ at $x = 0$ from $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + ... = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ and write down the resulting series as an infinite sum.

4 marks
Total: 10 marks

6. Calculate the following indefinite integrals.

(a) $I = \displaystyle\int xe^x \, dx$.

2 marks

(b) $I = \displaystyle\int xe^{-(x^2+3)/3} \, dx$.

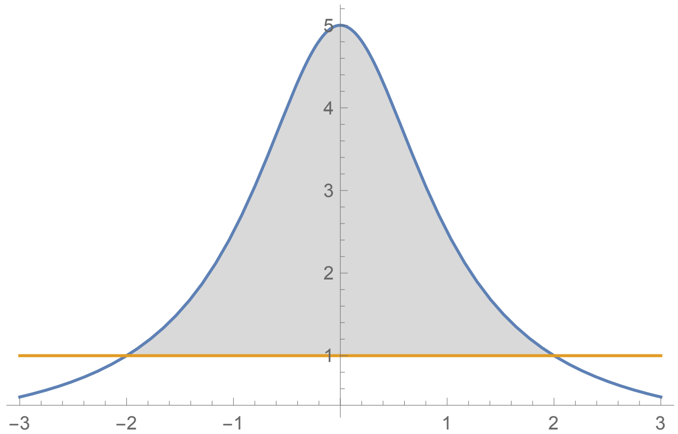
2 marks

(c) $I = \displaystyle\int \frac{\ln(\ln(x))}{x}\, dx.$

6 marks
Total: 10 marks

7. Calculate the following definite integrals or numerical estimates.

   (a) Find the area, A, of the region lying above the line $y = 1$ and below the curve $y = \dfrac{5}{x^2+1}$ as shown in the figure.



5 marks

(b) Use the trapezoidal rule with $n = 4$ to find the approximate value of $I = \displaystyle\int_0^1 \sqrt{(x+1)^3}\,dx$. Leave as a sum of terms on a common denominator.

5 marks
Total: 10 marks

8. Find all first and second partial derivatives of the function $f(x, y) = \sin(x\sqrt{y})$

Total: 10 marks

9. Find the equations of the approximations to the surfaces at the point indicated.

   (a) The tangent plane to $f(x, y) = x^2y^3$ at the point $(x, y) = (3, 1)$ in Cartesian form.

4 marks

   (b) Find $T_2(x, y)$ for the surface $f(x, y) = x^2y^3$ at the point $(x, y) = (3, 1)$ (you do not need to simplify).

6 marks
Total: 10 marks

10. Compute the following.

(a) Compute $\dfrac{df}{ds}$ for $f(x,y) = x^2y + x + 2y$, where $x(s) = 2\cos(s/2)$ and $y(s) = 2\sin(s/2)$.

4 marks

(b) Find the directional derivative of the function $f(x,y) = y^4 + 2xy^3 + x^2y^2$ in the direction $\mathbf{t} = \mathbf{i} + 2\mathbf{j}$ at the point $(x,y) = (1,1)$.

6 marks

Total: 10 marks

END OF EXAMINATION QUESTIONS

Formulae:

Scalar and vector projections:
The **scalar projection**, v_w, of $\mathbf{v}$ in the direction of $\mathbf{w}$ is given by

$$v_w = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|}$$

The **vector projection**, $\mathbf{v}_w$, of $\mathbf{v}$ in the direction of $\mathbf{w}$ is given by

$$\mathbf{v}_w = \left(\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|^2} \right) \mathbf{w}$$

Vector cross product:

The vector cross product of vectors $\mathbf{v} = (v_x, v_y, v_z)$ and $\mathbf{w} = (w_x, w_y, w_z)$ is

$$\mathbf{v} \times \mathbf{w} = (v_y w_z - v_z w_y, v_z w_x - v_x w_z, v_x w_y - v_y w_x)$$

Vector equation of a plane:

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{d}) = 0$$

Matrix inverse (2×2):

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

for $ad - bc \neq 0$.

Schematic of Gauss-Jordan algorithm:

$$[A|I] \underset{G.A}{\longrightarrow} [U|*] \underset{J.A}{\longrightarrow} [I|B] \quad \text{where } B = A^{-1}.$$

Derivative definition:
The **derivative** of $f(x)$ at the point x is defined as

$$\frac{\mathrm{d}f}{\mathrm{d}x} = f'(x) = \lim_{\Delta x \to 0} \left(\frac{\Delta f}{\Delta x} \right) = \lim_{\Delta x \to 0} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} \right).$$

Some rules for finding derivatives:

Description	**Function**	**Derivative**
Sum (or difference) of functions	$f(x) \pm g(x)$	$f'(x) \pm g'(x)$
Product of functions	$f(x)g(x)$	$f(x)g'(x) + g(x)f'(x)$
Quotient of functions	$\dfrac{f(x)}{g(x)}$	$\dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

Chain rule for composite functions

If $u = g(x)$ and $y = f(u)$ so that $y = f(g(x))$ then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} = f'(u)g'(x)$$

Derivative rule for inverse functions

$$\text{If } y = f^{-1}(x) \Leftrightarrow x = f(y), \text{ then } \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\mathrm{d}x/\mathrm{d}y} = \frac{1}{f'(y)}$$

Parametric differentiation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\Big/\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{g'(t)}{f'(t)} \quad \text{where } f'(t) = \frac{\mathrm{d}f}{\mathrm{d}t} \text{ and } g'(t) = \frac{\mathrm{d}g}{\mathrm{d}t}.$$

Taylor series at $x = 0$:

$$T_n(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n.$$

Integration by substitution:

$$I = \int f(x)dx = \int f(x(u))\frac{dx}{du}du$$

Integration by parts:

$$\int f\frac{dg}{dx}dx = fg - \int g\frac{df}{dx}dx$$

Fundamental Theorem of Calculus:

If $f(x)$ is a continuous function on the interval $[a, b]$ and there is a function $F(x)$ such that $F'(x) = f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Area between two curves. Given two continuous functions $f(x)$ and $g(x)$ where $f(x) \geq g(x)$ for all x in the interval $[a, b]$, the area of the region bounded by the curves $y = f(x)$ and $y = g(x)$, and the lines $x = a$ and $x = b$ is given by the definite integral

$$\int_a^b \big[f(x) - g(x)\big]\,dx$$

Trapezoidal rule:

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left(f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_i)\right)$$

Tangent plane to surface:

$$z = f(a, b) + f_x(a, b) \cdot (x - a) + f_y(a, b) \cdot (y - b)$$

Multivariate chain-rule:

$$\frac{df}{ds} = \frac{\partial f}{\partial x}\frac{dx}{ds} + \frac{\partial f}{\partial y}\frac{dy}{ds}$$

Directional derivative:

The *directional derivative* df/ds of a function f in the direction $\underset{\sim}{t}$ is given by

$$\frac{df}{ds} = \underset{\sim}{t} \cdot \nabla f = \nabla_{\underset{\sim}{t}} f$$

where the *gradient* ∇f is defined by

$$\nabla f = \frac{\partial f}{\partial x}\underset{\sim}{i} + \frac{\partial f}{\partial y}\underset{\sim}{j}$$

and $\underset{\sim}{t}$ is a unit vector, $\underset{\sim}{t} \cdot \underset{\sim}{t} = 1$.

Quadratic approximation to surface:

$$\begin{aligned} T_2(x, y) = f(a, b) &+ f_x(a, b) \cdot (x - a) + f_y(a, b) \cdot (y - b) \\ &+ \frac{1}{2!}\big[f_{xx}(a, b)(x - a)^2 + 2f_{xy}(a, b)(x - a)(y - b) + f_{yy}(a, b)(y - b)^2\big] \end{aligned}$$

Table of the derivatives of the basic functions of calculus	
Original function f	**Derivative function** f'
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x \equiv 1 + \tan^2 x$
$\operatorname{cosec} x \equiv 1/\sin x$	$-\operatorname{cosec} x \cdot \cot x$
$\sec x \equiv 1/\cos x$	$\sec x \cdot \tan x$
$\cot x \equiv 1/\tan x$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x \quad$ domain: $-1 \leq x \leq 1$ (ie $\|x\| \leq 1$)	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x \quad$ domain: $-1 \leq x \leq 1$ (ie $\|x\| \leq 1$)	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x \quad$ domain: $-\infty < x < \infty$	$\frac{1}{1+x^2}$
e^x	e^x
$\ln x \quad$ domain: $x > 0$	$\frac{1}{x}$

Table of Useful Power Series

Series	**Domain**
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$	$-1 < x < 1$
$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots + (-1)^n x^n + \ldots$	$-1 < x < 1$
$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots + \frac{1}{n!}x^n + \ldots$	$-\infty < x < \infty$
$\ln(1+x) \equiv \log_e(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$ $+(-1)^n \frac{x^{n+1}}{n+1} + \ldots$	$-1 < x \leq 1$
$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots$	$-\infty < x < \infty$
$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots$	$-\infty < x < \infty$

END OF EXAMINATION PAPER