MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #6 and Additional Practice Questions

Tutorial Questions

1. Let $X = \{a, b, c, d, e, f\}$.

   (a) Make up a relation on X which is reflexive and transitive but is not symmetric and is not antisymmetric. Draw its arrow diagram.

   (b) Make up a relation on X which is symmetric but is not reflexive and is not transitive. Draw its arrow diagram.

   (c) Make up an equivalence relation on X. Draw its arrow diagram.

2. Let S and T be binary relations defined as follows.

   - S is defined on $\mathcal{P}(\{1,2,3,4\}) - \{\emptyset\}$ by ASB if and only if $\min(A) = \min(B)$ (where $\min(A)$ means the smallest element of the nonempty set A).
   - T is defined on the set of finite binary strings by cTd if and only if $c = d$ or c can be obtained from d by deleting some bits (for example 0,1,01,10 and 11 can all be obtained from 101).

   For each relation S and T, state whether the relation is reflexive, symmetric, antisymmetric and transitive, and explain why in each case.

3. State which (if any) of S and T are equivalence relations and which (if any) are partial order relations. For those that are equivalence relations, describe the equivalence classes. For those that are partial order relations, state whether they are total order relations and whether they are well-order relations, and explain why.

4. Let Q, R, S and T be relations on $\{1,2,3,4,5,6,7,8,9\}$.

   (a) If $3Q4$ and $4\not{Q}3$, can Q be reflexive? symmetric? antisymmetric? transitive?

   (b) If $1R2$, $2R1$ and $1\not{R}1$, can R be reflexive? symmetric? antisymmetric? transitive?

   (c) Suppose S is antisymmetric and transitive. If $5S6$, $6S7$ and $6S8$, then what else can we definitely say about S?

   (d) If $7T8$, $8T4$ and $4T7$ can T be both transitive and antisymmetric? Can it be an equivalence relation?

(See over for practice questions.)

Practice Questions

1. For the following (informally defined) relations on the set of all people in Australia, decide whether they are reflexive, symmetric, antisymmetric and transitive. If they are equivalence relations find their equivalence classes and if they are partial order relations find whether they are total order relations and well-order relations.

   (a) x is related to y if and only if x has shaken hands with y.

   (b) x is related to y if and only if x is a (biological) ancestor of y.

   (c) x is related to y if and only if x and y were born in the same month.

   (d) x is related to y if and only if x has ever beaten y at chess.

2. Let $X = \{a, b, c, d\}$.

   (a) How many possible relations are there on X?

   (b) How many of these are reflexive?

   (c) How many of these are reflexive and symmetric?

   (d) How many of these are equivalence relations?

   (e) What would the answers to (a), (b) and (c) be if $|X| = n$ instead of $|X| = 4$?

3. Let A and B be sets such that $A \cap B = \emptyset$. Suppose R is a well-order relation on A and S is a well-order relation on B.

   (a) Can you find a well-order relation on the set $A \cup B$?

   (b) Can you find a well-order relation on the set $A \times B$?

Practice Questions (Tutorial sheet 6)

1) (a) YES: symmetric ; NO: reflexive, antisymmetric, transitive
↳ $xRy = yRx$

(b) YES: transitive, antisymmetric ; NO: symmetric, reflexive
↳ $xRy \wedge yRx \rightarrow xRy$

(c) YES: symmetric, transitive, reflexive ; NO: antisymmetric
↳ $xRy \rightarrow yRx$; $xRy \wedge yRx \rightarrow xRy$; xRx, yRy for all x / y
↳ equivalence relations

(d) NO: symmetric, transitive, reflexive, antisymmetric

2) $X = \{a, b, c, d\}$

(a) $2^{4\times4} = 2^{16}$

(b) 2^{12}

(c) 2^{6}

(d) * equivalence relation = reflexive, symmetric, transitive *
15 [(4), 4(3,1), 3(2,2), 6(2,1,1), 1(1,1,1,1)] ⇒ Bell number

(e) if $|X| = n$ then (a) $2^{(n^2)}$
(b) $2^{n(n-1)}$
(c) $2^{n(n-1)/2}$

3) $\underbrace{A \cap B = \emptyset}_{\text{independant events}}$; R = well-ordered relation on A ; S = well-ordered relation on B

(a) let $A \cup B = a \sqcup b$ where ⊔ is a relation
↳ $a, b \in A$ and aRb
↳ $a, b \in B$ and aSb
↳ $a \in A$; $b \in B$

(b) let $A \times B = a \sqcup b$ where $(x, a) \sqcup (y, b)$ with ⊔ as a relation
↳ $x \sqcup b \Rightarrow x \neq b$
↳ $x = b \Rightarrow xSy$