1. Compute the first four non-zero terms in the Taylor series for the following functions centred about the specified point $a$. **[5 + 5 = 10 marks]**

   a. $f(x) = \ln(x^3 + 1)$, $a = 1$

   b. $f(x) = \tan^{-1}(e^x - 1)$, $a = 0$

$x = a$

$$T_n(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$f(x) = \ln(x^3+1)$ $\qquad f(1) = \ln 2$

$f'(x) = \dfrac{3x^2}{x^3+1}$ $\qquad f'(1) = \dfrac{3}{2}$

$f''(x) = \dfrac{6x(x^3+1) - 3x^2(3x^2)}{(x^3+1)^2} = \dfrac{-3x^4+6x}{(x^3+1)^2}$ $\qquad f''(1) = \dfrac{3}{4}$

$f^3(x) = \cdots$

$$\ln(x^3+1) \simeq \ln(2) + \frac{\left(\frac{3}{2}\right)}{1!}(x-1) + \frac{\left(\frac{3}{4}\right)}{2!}(x-1)^2 +$$
$$= \ln 2 + \frac{3}{2}(x-1) + \frac{3}{8}(x-1)^2 +$$

3. Use integration by parts to calculate the following integrals. **[4 + 4 = 8 marks]**

   a. $I = \int x^3 \ln(2x)\, dx$

   b. $I = \int e^{2x} \sin(x)\, dx$

(a) let $u = \ln(2x)$ $\qquad \dfrac{dv}{dx} = x^3$ $\qquad$ "LATE"

$\dfrac{du}{dx} = \dfrac{1}{x}$ $\qquad v = \dfrac{x^4}{4}$ $\qquad$ "LIATE"

$$I = \int x^3 \ln(2x)\,dx = \frac{x^4}{4}\ln(2x) - \int x^2\,dx$$
$$= \frac{x^4}{4}\ln(2x) - \frac{x^3}{3} + c$$

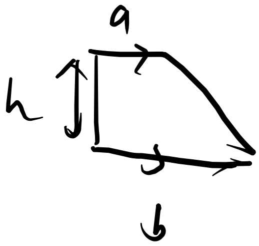
4. Consider the area bounded by the two functions $y = (x^3 - 5x^2 + 4x)/5$ and $y = x^2 - 4x$ over the domain $0 \leq x \leq 4$. **[2 + 4 + 4 + 4 = 14 marks]**

a. Sketch these two curves, noting the area bounded between them.

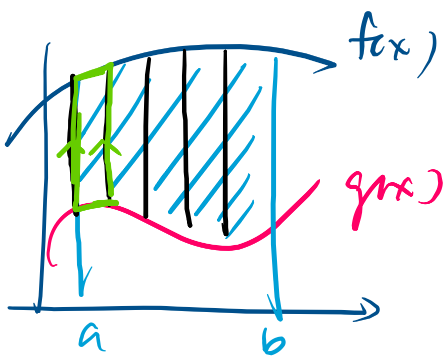
b. Use the Fundamental Theorem of Calculus to calculate the area bounded between the two curves.

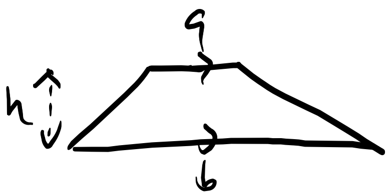
c. Approximate the area between the curves using the Trapezoidal rule with n = 4.

d. Approximate the area between the curves using the Trapezoidal rule with n = 8.




Area $= \frac{1}{2}h(a+b)$




parallel length → $f(x) - g(x)$