Question 14 **(5 marks)**

For each of the following decision problems, indicate whether or not it is decidable.

You may assume that, when Turing machines are encoded as strings, this is done using the Code-Word Language (CWL).

Decision Problem	your answer (tick **one** box in each row)	
Input: a Turing machine M, and a positive integer k. Question: Does M accept some input string of at most k letters?	☐ Decidable	☑ Undecidable
Input: a Turing machine M, and a positive integer k. Question: Does the encoding of M have at least k letters?	☑ Decidable	☐ Undecidable
Input: a Turing machine M, and a positive integer k. Question: When M is given an encoding of itself as input, does the computation go for at least k steps?	☑ Decidable	☐ Undecidable
Input: a Turing machine M. Question: Does M eventually print an encoding of M on the tape and then halt?	☐ Decidable	☑ Undecidable
Input: a Turing machine M, and a string w. Question: Does M reject w?	☐ Decidable	☑ Undecidable

Official use only
5

Question 13 **(4 marks)**

For each of the following decision problems, indicate whether or not it is decidable.

You may assume that, when Turing machines are encoded as strings, this is done using the Code-Word Language (CWL).

Decision Problem	your answer (tick **one** box in each row)	
Input: a Turing machine M. Question: Does there exist a string w that is accepted by M in at most 7 steps?	☑ Decidable	☐ Undecidable
Input: a Turing machine M. Question: Does there exist a string w that is accepted by M?	☐ Decidable	☑ Undecidable
Input: a string w. Question: Does there exist a Turing machine that accepts w?	☑ Decidable	☐ Undecidable
Input: a Turing machine M, and a string w. Question: Is w the encoding, in CWL, of M?	☑ Decidable	☐ Undecidable

Official use only
4

Question 13 **(4 marks)**

For each of the following decision problems, indicate whether or not it is decidable.

You may assume that, when Turing machines are encoded as strings, this is done using the Code-Word Language (CWL).

Decision Problem	your answer (tick **one** box in each row)	
Input: two Turing machines M_1 and M_2. Question: Does M_1 eventually halt, when given M_2 as input?	☐ Decidable	☑ Undecidable
Input: two Turing machines M_1 and M_2. Question: Does M_1 have the same number of states as M_2?	☑ Decidable	☐ Undecidable
Input: two Turing machines M_1 and M_2. Question: Is M_1 equivalent to M_2 (i.e., do M_1 and M_2 have the same sets of accepted strings and the same sets of rejected strings)?	☐ Decidable	☑ Undecidable
Input: two Turing machines M_1 and M_2. Question: If each machine is given itself as input, does M_1 finish before M_2?	☐ Decidable	☑ Undecidable

Official use only
4

Question 12 **(3 marks)**

State the Church-Turing thesis, and give two reasons why it is believed to be true.

Any function which can defined by an algorithm can be represented by a Turing Machine.

The reasons can be any two of:

- So far, algorithms that people try to program have turned out to be programmable.
- No counterexample. (I.e., there is no algorithm which does not seem to be programmable in principle.)
- Different approaches to computability end up in agreement.
  Recursive functions, and lambda calculus, both arrive at the same class of computable functions as those captured by Turing machines.

Question 13 **(4 marks)**

For each of the following decision problems, indicate whether or not it is decidable.

You may assume that, when Turing machines are encoded as strings, this is done using the Code-Word Language (CWL).

Decision Problem	your answer (tick **one** box in each row)	
Input: a Turing machine M with at most ten states. Question: Does M eventually halt, when given itself as input?	☑ Decidable	☐ Undecidable
Input: a Turing machine M with at most ten transitions. Question: Does M eventually halt, when given itself as input?	☑ Decidable	☐ Undecidable
Input: a Turing machine M with a tape alphabet of at most ten letters. Question: Does M eventually halt, when given itself as input?	☐ Decidable	☑ Undecidable
Input: a Turing machine M that can only move to the Right. Question: Does M eventually halt, when given itself as input?	☑ Decidable	☐ Undecidable

Official use only
7