$\underline{u} + \underline{v} = \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$ ✓

$\underline{u} - \underline{v} = \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -4 \\ 5 \end{pmatrix}$ ✓

$\underline{u} + 2\underline{v} = \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ -3 \\ -1 \end{pmatrix}$ ✓

$\underline{v} - \underline{u} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$ ✓

$|\underline{u}| = \sqrt{\underline{u}^2}$

$= \sqrt{2^2 + (-2)^2 + 3^2}$

$= \sqrt{4 + 4 + 9}$

$= \sqrt{17}$

$|\underline{v}| = \sqrt{\underline{v}^2}$

$= \sqrt{3^2 + 2^2 + (-2)^2}$

$= \sqrt{9 + 4 + 4}$

$= \sqrt{17}$

$\hat{\underline{u}} = \frac{\underline{u}}{|\underline{u}|} = \left(\frac{1}{\sqrt{17}}\right)\begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -2/\sqrt{17} \\ -3/\sqrt{17} \\ 3/\sqrt{17} \end{pmatrix}$

$2\underline{u} + 2\underline{v} = 2(\underline{u}) + 2(\underline{v})$

$= 2(\underline{u} + \underline{v})$

LHS:

$2\underline{u} + 2\underline{v} = 2\begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ -6 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ -2 \\ 2 \end{pmatrix}$

RHS: $2(\underline{u} + \underline{v}) = 2\left[\begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}\right]$

$= 2\begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$

$= \begin{pmatrix} 10 \\ -6 \\ 2 \end{pmatrix}$

LHS = RHS

Hence $2(\underline{u}) + 2(\underline{v}) = 2(\underline{u} + \underline{v})$ ✓

$\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = 1$

$\begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -8 \end{pmatrix} = -10$

$\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} -3 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix} = -6$

$$\underset{\sim}{v} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \quad \underset{\sim}{w} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}$$

negative means it goes different direction geometrically

Find trigonometric identity

$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$$\underset{\sim}{v} \cdot \underset{\sim}{w} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \times \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta\cos\phi \\ \sin\theta\sin\phi \\ 0 \end{pmatrix}$$

$$= \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$= \cos(\theta - \phi)$$