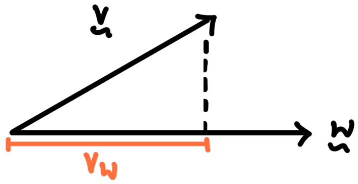
1.2.3 Scalar projections

↳ length of shadow casted

↳ $v_w = \dfrac{\underset{\sim}{v} \times \underset{\sim}{w}}{|\underset{\sim}{w}|}$



↳ eg. What is the length of scalar projection of $\underset{\sim}{v} = (1,2,7)$ in the direction of the vector $\underset{\sim}{w} = (2,3,4)$?

$$\begin{pmatrix}1\\2\\7\end{pmatrix} \times \begin{pmatrix}2\\3\\4\end{pmatrix} = \begin{pmatrix}2\\6\\28\end{pmatrix}$$

$$2+6+28 = 36$$

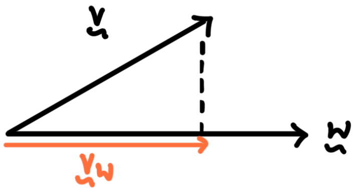
$$|\underset{\sim}{w}| = \begin{pmatrix}2\\3\\4\end{pmatrix}$$

$$v_w = \frac{2+6+28}{\sqrt{29}} = \frac{36}{\sqrt{29}} \; *$$

1.2.4 Vector projections

↳ vector shadow with length equal to scalar projection

↳ $\underset{\sim}{v}_w = \left(\dfrac{\underset{\sim}{v} \times \underset{\sim}{w}}{|\underset{\sim}{w}|^2}\right)\underset{\sim}{w}$



* $\underset{\sim}{v}_w$ & $\underset{\sim}{w}$ ARE PARALLEL → SAME UNIT VECTOR

↳ eg. Find the vector projection of $\underset{\sim}{v} = (1,2,7)$ in the direction of $\underset{\sim}{w} = (2,3,4)$

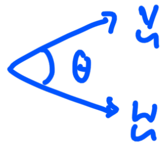
$$\begin{pmatrix}1\\2\\7\end{pmatrix} \times \begin{pmatrix}2\\3\\4\end{pmatrix} = \begin{pmatrix}2\\6\\28\end{pmatrix} \rightarrow 2+6+28 = 36$$

$$|\underset{\sim}{w}|^2 = (\sqrt{29})^2 = 29$$

$$\left(\frac{36}{29}\right)\begin{pmatrix}2\\3\\4\end{pmatrix} \; *$$

SUMMARY : (DOT PRODUCT)

↳ $\cos\theta = \dfrac{\underset{\sim}{v} \times \underset{\sim}{w}}{|\underset{\sim}{v}||\underset{\sim}{w}|}$; $0 \leq \theta \leq \pi$



↳ orthogonal if $\underset{\sim}{v} \times \underset{\sim}{w} = 0$

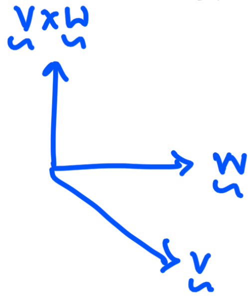
↳ scalar projection : $v_w = \dfrac{\underset{\sim}{v} \times \underset{\sim}{w}}{|\underset{\sim}{w}|}$

↳ vector projection : $\underset{\sim}{v}_w = \left(\dfrac{\underset{\sim}{v} \times \underset{\sim}{w}}{|\underset{\sim}{w}|^2}\right)\underset{\sim}{w}$

SUMMARY : (CROSS PRODUCT)

↳ orthogonal : $\underset{\sim}{v} \times \underset{\sim}{w} = -\underset{\sim}{w} \times \underset{\sim}{v}$

both $\underset{\sim}{v}$ and $\underset{\sim}{w}$ defined by right-hand rule



↳ $\sin\theta = \dfrac{|\underset{\sim}{v} \times \underset{\sim}{w}|}{|\underset{\sim}{v}||\underset{\sim}{w}|}$; $0 \leq \theta \leq \pi$

↳ parallel if $\underset{\sim}{v} \times \underset{\sim}{w} = 0$

↳ AREA OF PARALLELOGRAM ⇒ $A = |\underset{\sim}{v} \times \underset{\sim}{w}|$