Basics

$f(x) = x^n$
$f'(x) = nx^{n-1}$

$f(x) = e^x$
$f'(x) = e^x$

$f(x) = \ln(x)$
$f'(x) = 1/x$

$f(x) = \sin(x)$
$f'(x) = \cos(x)$

$f(x) = \cos(x)$
$f'(x) = -\sin(x)$

$f(x) = \tan(x)$
$f'(x) = \sec^2(x)$

Simple Questions

$y = x^5$
$\frac{dy}{dx} = 5x^4$

$y = 10x^2$
$\frac{dy}{dx} = 20x$

$y = 20x^{-4} + 9$
$\frac{dy}{dx} = -80x^{-5}$

$y = \ln(10)$
$\frac{dy}{dx} = 0$

$y = \sin(3x+5)$
$\frac{dy}{dx} = 3\sin(3x+5)$

$y = \cos(7x+4)$
$\frac{dy}{dx} = -7\sin(7x+4)$

$y = \tan^2(x) = \tan(x) \times \tan(x)$

$u = \tan(x)$ $\quad$ $v = \tan(x)$
$\frac{du}{dx} = \sec^2(x)$ $\quad$ $\frac{dv}{dx} = \sec^2(x)$

$$\frac{dy}{dx} = \sec^2(x)\tan(x) + \sec^2(x)\tan(x) = 2\sec^2(x)\tan(x)$$

$$y = \frac{(x + \cos(x))}{\tan(x)}$$

$u = x + \cos(x)$ $\quad$ $v = \tan(x)$
$\frac{du}{dx} = 1 - \sin(x)$ $\quad$ $\frac{dv}{dx} = \sec^2(x)$

$$\frac{dy}{dx} = \frac{(1-\sin(x))(\tan(x)) - (\sec^2(x))(x + \cos(x))}{[\tan(x)]^2}$$

$$= \frac{\tan(x) - \sin(x)\tan(x) - x\sec^2(x) - \cos(x)\sec^2(x)}{\tan^2(x)}$$

$$= \frac{\tan(x) - \sin(x)\tan(x) - x\sec^2(x) - \sec^2(x)}{\tan^2(x)}$$

Section 3-3

1) $f(x) = 6x^3 - 9x + 4$

$f'(x) = 18x^2 - 9$

2) $y = 2t^4 - 10t^2 + 13t$

$\frac{dy}{dx} = 8t^3 - 20t + 13$

3) $g(z) = 4z^7 - 3z^{-7} + 9z$

$g'(z) = 28z^6 + 21z^{-8} + 9$

4) $h(y) = y^{-4} - 9y^{-3} + 8y^{-2} + 12$

$h'(y) = -4y^{-5} + 27y^{-4} - 16y^{-3}$

5) $y = \sqrt{x} + 8\sqrt[3]{x} - 2\sqrt[4]{x}$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + \frac{8}{3}x^{-2/3} - \frac{1}{2}x^{-3/4}$

6) $f(x) = 10\sqrt[5]{x^3} - \sqrt{x^7} + 6\sqrt[3]{x^8} - 3$

$f'(x) = 2x^{-2/5} - \frac{7}{2}x^{5/2} + 6\left(\frac{8}{3}x^{5/3}\right)$

7) $f(t) = 4/t - 1/6t^3 + 8/t^5$

$f'(t) = -4t^{-2} - \frac{1}{2}t^2 - 40t^{-6}$

8) $z = x(3x^2 - 9)$

$u = x \qquad v = 3x^2 - 9$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 6x$

$\frac{dz}{dx} = 3x^2 - 9 + 6x^2$

*** First Principles:**

$$\frac{df}{dx} = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$$

*** GENERAL EQUATION**

$$ax^2 + bx + c = 0$$

*** QUADRATIC FORMULA**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Using differentiation**
  - $\frac{dy}{dx}$ gives you the gradient of the curve at any point in terms of x
  - When $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$
  - Stationary/ turning point: $\frac{dy}{dx} = 0$
  - 1st Derivative = $\frac{dy}{dx} = f'(x)$
  - 2nd Derivative = $\frac{d^2y}{dx^2} = f''(x)$
  - To determine if stationary point is maximum or minimum:
    - Use 2nd derivative
      - Maximum point: $\frac{d^2y}{dx^2} < 0$
      - Minimum point: $\frac{d^2y}{dx^2} > 0$
    - Use gradients around the point
      - Input x values slightly above and below stationary point and calculate gradient

3.2 min, max points ⇒ stationary points ($f'(x)=0$)

LOCAL MAXIMUM, MINIMUM:

$f'(x) < 0 \rightarrow \searrow$
$f'(x) > 0 \rightarrow \nearrow$
$f'(x) = 0 \rightarrow \rightarrow$

min : $\searrow \rightarrow \nearrow$ decrease to increase
max : $\nearrow \rightarrow \searrow$ increase to decrease

* if tangent line is present, it's horizontal to point $x=c$

- $f'(x)$ +ve to -ve $\longrightarrow$ maximum at $x=c$
- $f'(x)$ -ve to +ve $\longrightarrow$ minimum at $x=c$

* no changes = not max, not min

GLOBAL MAXIMUM, MINIMUM:

* absolute maximum at $x=c$

$f(x) \leq f(c)$ for all x
↳ domain $[a,b]$ for $a \leq c \leq b$

* absolute minimum at $x=c$

$f(x) \geq f(c)$ for all x
↳ domain $[a,b]$ for $a \leq c \leq b$

Extreme Value Theorum :

↳ function f(x) is continuous in closed intervals
↳ f(x) obtain absolute max, min at some points in interval

* closed interval vs open intervals

↳ closed intervals
  - include end points
  - square bracket [ ]

↳ open intervals
  - don't include end points
  - round bracket ( )

absolute max / min in closed intervals

① find all critical points of the function
② find end points of the function
③ compare ① and ② for max, min

absolute max / min in open intervals

① find all critical points of the function
② find limit of function as x approach endpoint
③ compare ① and ② for max, min