

MAT1830 - Discrete Mathematics for Computer Science

Assignment #2

Submit by uploading a pdf to moodle by 11:55pm Wednesday in week 6

Assessment questions/solutions for this unit must not be posted on any website.

For questions 1 and 2, make sure you set out and explain your proofs clearly. To receive marks for question 3, your answers must be exactly right and use correct notation.

- (1) Prove by simple induction that, for each integer $n \geq 1$,

$$6 + 6^2 + 6^3 + \cdots + 6^n = \frac{6^{n+1} - 6}{5}. \quad [6]$$

- (2) Let $S_1, S_2, S_3, S_4, \dots$ be the sequence of sets defined by $S_1 = \{0, 1, 2\}$, $S_2 = \{0, 2, 3\}$, $S_3 = \{0, 3, 4\}$ and

$$S_i = (S_{i-3} \triangle S_{i-2}) \triangle (S_{i-1} \cup \{i-3, i+1\}) \text{ for each integer } i \geq 4.$$

Prove by strong induction that $S_n = \{0, n, n+1\}$ for each integer $n \geq 1$. [7]

- (3) Let R , S and T be sets defined as follows.

$$R = \{2, 4, 6, 7, 8\}$$

$$S = \{\{2\}, \{2, 3, 4\}, \{2, 4, 6\}, \{6, 7\}\}$$

$$T = \{x \in \mathbb{Z} : x \leq 4 \text{ or } x \geq 8\}$$

Find the following.

- (i) $R - T$
- (ii) $S - \mathcal{P}(R)$
- (iii) $\mathcal{P}(R) \cap \mathcal{P}(T)$
- (iv) $(R \cap T) \times (S - \{\{2, 3, 4\}, \{2, 4, 6\}, \{6\}\})$
- (v) $|(\mathcal{P}(R) - S) \times S|$

[No explanation required.] [5]

- (4) Let A and B be finite sets and let $a = |A|$, $b = |B|$ and $c = |A \cap B|$. Write an expression in terms of a , b and c that is equal to $|(A \times B) \cup (B \times A)|$ for every choice of A and B .

[No explanation required.] [2]