Week 10 pass mat1830

1. You roll a fair die of 6 sides. (to prove discrete uniform distribution formula)
a. Find probability of getting a 2
b. find the expected value

2. A biased coin flips heads with probability 3/7 and tails with probability 4/7 . The coin is flipped 80 times. What is the probability that heads is flipped exactly 30 times?

3.Cars pass through a road junction according to a distribution. An average of 7 cars per minute pass through the junction.
(a) What is the probability that exactly one car passes through the junction in a certain minute?
(b) What is the expected number of cars to pass through in three minutes?
(c) What is the probability that exactly the expected number from (b) pass through in a certain three minute period?

4. Suppose a batter has probability 1/3 to hit the ball. What is the chance that he misses the ball less than 3 times before succeeding?
5.

Example 1. Suppose we have a six-sided die marked with five 5 3's and one 6. (This was the red one from our non-transitive dice.) What would you expect the average of 6000 rolls to be?

6.A fair coin is flipped until a head appears. What is the probability that the coin needs to be flipped more than 5 times?
7.

Write down the first five values of each of the following recursive sequences.

(a) $r_0 = 2, \qquad r_n = (r_{n-1})^2 - n - 1 \quad$ for all integers $n \geq 1$.
(b) $s_0 = 2, \qquad s_n = (s_{n-1})^2 + (s_{n-2})^2 + \cdots + (s_0)^2 \quad$ for all integers $n \geq 1$.

8.Rewrite with sigma or pi notation

a. $(2x-1)(5x-4)(8x-9)(11x-16)(14x-25)(17x-36)(20x-49)$

b. $\frac{2}{3-0} + \frac{4}{4+5} + \frac{8}{5-10} + \frac{16}{6+15} + \frac{32}{7-20} + \frac{64}{8+25}$

9.

For each integer n >= 1, let t_n be the number of strings of n letters that can be produced by concatenating copies of strings “a”, “bb”, “cc”.
Find the recurrence relation for t_n.
t_1 = |{a}| = 1
t_2 = |{aa, bb, cc}| = 3
t_3 = |{aaa, abb, acc, bba, cca}| = 5

10.

Astronomers estimate that the average number of asteroids that hit the Eearth's surface each day is 17.

Passan Distribution

(i) Of the distributions that we studied in Lecture 25, which is likely to be the best model for the number of asteroids that hit the Earth's surface each day? Assume for subsequent parts that this is the distribution.

(ii) What is the probability that no asteroids hit the Earth's surface tomorrow

(iii) What is the probability that no asteroids hit the Earth's surface within the next two days

(ii) $\lambda = 17, k = 0$

$$Pr(X=0) = \frac{17^0 e^{-17}}{0!}$$

(iii) $\lambda = 34, k = 0$

$$Pr(X=0) = \frac{34^0 (e^{-34})}{0!}$$

May

NOTES :

W10 Notes

1. Discrete Uniform Distribution
   - ↳ choose 1 of a set of consecutive integers
   - $Pr(X=k) = \frac{1}{b-a+1}$
   - $E[X] = \frac{a+b}{2}$
   - $Var[X] = \frac{(b-a+1)^2 - 1}{12}$

2. Bernoulli Distribution
   - ↳ single process with success probability p and fails otherwise
   - $Pr(X=k) = \begin{cases} p & \text{for } k=1 \\ 1-p & \text{for } k=0 \end{cases}$
   - $E[X] = p$
   - $Var[X] = p(1-p)$

3. Geometric Distribution
   - ↳ k failures before a success
   - $Pr(X=k) = p(1-p)^k$
   - $E[X] = \frac{1-p}{p}$
   - $Var[X] = \frac{1-p}{p^2}$

start 5:10

4. Binomial Distribution
   - ↳ exactly k successes
   - $Pr(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$
   - $E[X] = np$
   - $Var[X] = np(1-p)$

5. Poisson Distribution
   - ↳ know avg of λ events occur per time period, count probability that k events occur in a time period
   - $Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$
   - $E[X] = \lambda$
   - $Var[X] = \lambda$

6. Σ sigma (sum), e.g. $\sum_{k=1}^{n} k = 1+2+\ldots+n$

   Π pi (product), e.g. $\prod_{k=1}^{n} k = 1 \times 2 \times \ldots \times n$

7. arithmetic progression $a_n = a_1 + (n-1)d$, $S_n = \frac{n}{2}(a_1 + a_n)$

   geometric progression $a_n = a_1 \cdot r^{n-1}$, $S_n = \frac{a_1 - a_1 \cdot r^n}{1-r}$

1) Fair dice of 6 sides
↳ discrete uniform distribution

(a) $Pr(X=2) = 1/(6-1+1) = 1/6$
(b) $E(X) = 3.5$

2) $3/7$ = heads, $4/7$ = tails
coin flipped 80 times

probability that heads is flipped 30 times

↳ Binomial Distribution

success ⇒ flip head

$$\left.\begin{array}{l} n=80 \\ k=30 \\ p=3/7 \end{array}\right\} Pr(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$\rightarrow {}^{80}C_{30}$

$$Pr(X=30) = \binom{80}{30} 3/7^{30} (4/7)^{50}$$

$=$

3) 7 cars per minute
Possan distribution

(i) $\lambda = 7$ $k=1$ (want 1 car pass through)
$Pr(X=1) = 7e^{-7}$ ✓

(ii) $E(X)$ in 3 mins
$E(X) = \lambda$
$\lambda = 7 \times 3$
$E(X) = 21$ ✓

(iii) $Pr(X=3) = 21e^{-21}$
$\lambda = 21$, $k = 21$ $\frac{21^{21} e^{-21}}{21!}$

4) 1/3 chance to hit ball
Probability miss ball < 3
geometric distribution

$$\left.\begin{array}{l} p = 1/3 \\ k = 3 \end{array}\right\} P(X<3)$$
$= P(X=0) +$
$P(X=1) +$
$P(X=2)$

5) Binomial Distribution
let x = roll 3
let y = roll 6
let n = num of rolls

$E(X \mid n=6000) = 5/6 \times 6000 = 5$
$E(Y \mid n=6000) = 1/6 \times 6000 = 1$
$$\frac{(5000 \times 3) \div 6(1000)}{6000}$$

6) Geometric Distribution

$Pr(X > 5) = 1 - P(X \leq 5)$
$= 1 - [P(X=0) +$
$P(X=1) + P(X=2)$
$+ P(X=3) +$
$P(X=4) +$
$P(X=5)]$
$= 1 - (1/2 + 1/2 +$
$1/2 + (\frac{1}{2})^2 \frac{1}{2} +$
$(\frac{1}{2})^3 (\frac{1}{2}) +$
$\ldots (1/2)^5 (\frac{1}{2})$

Assignment 4 Q3

* X : elements in subset → can be $X = \{0,1,2,\ldots 8\}$
* Y : 0 / min num of subset → can be $Y = \{0,1,2\ldots 8\}$

$P([1,2,3,4,5,6,7,8])$

(i) $P(X=1, Y=3)$

$Pr(X=1 \wedge Y=3) \overset{?}{=} Pr(X=1) * Pr(Y=3)$

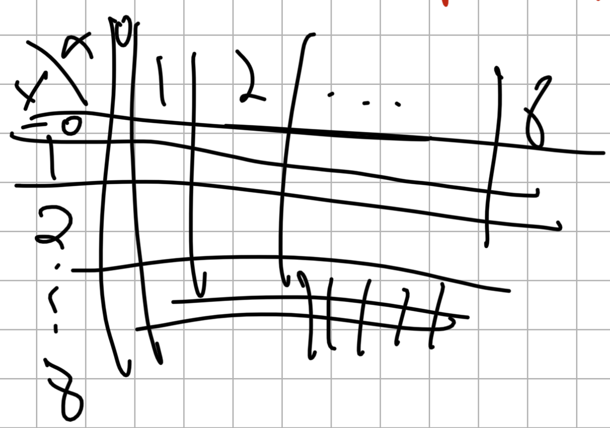
Num of subset, 1 element with 3 as smallest
↳ only 1 : $\{3\}$
↳ $1/256$ } probability

$$\frac{1}{256} \overset{?}{=} \frac{8}{256} * \frac{32}{256} \rightarrow \text{how many of 256 have min 3}$$

↳ how many of 256 have 1 member = 8

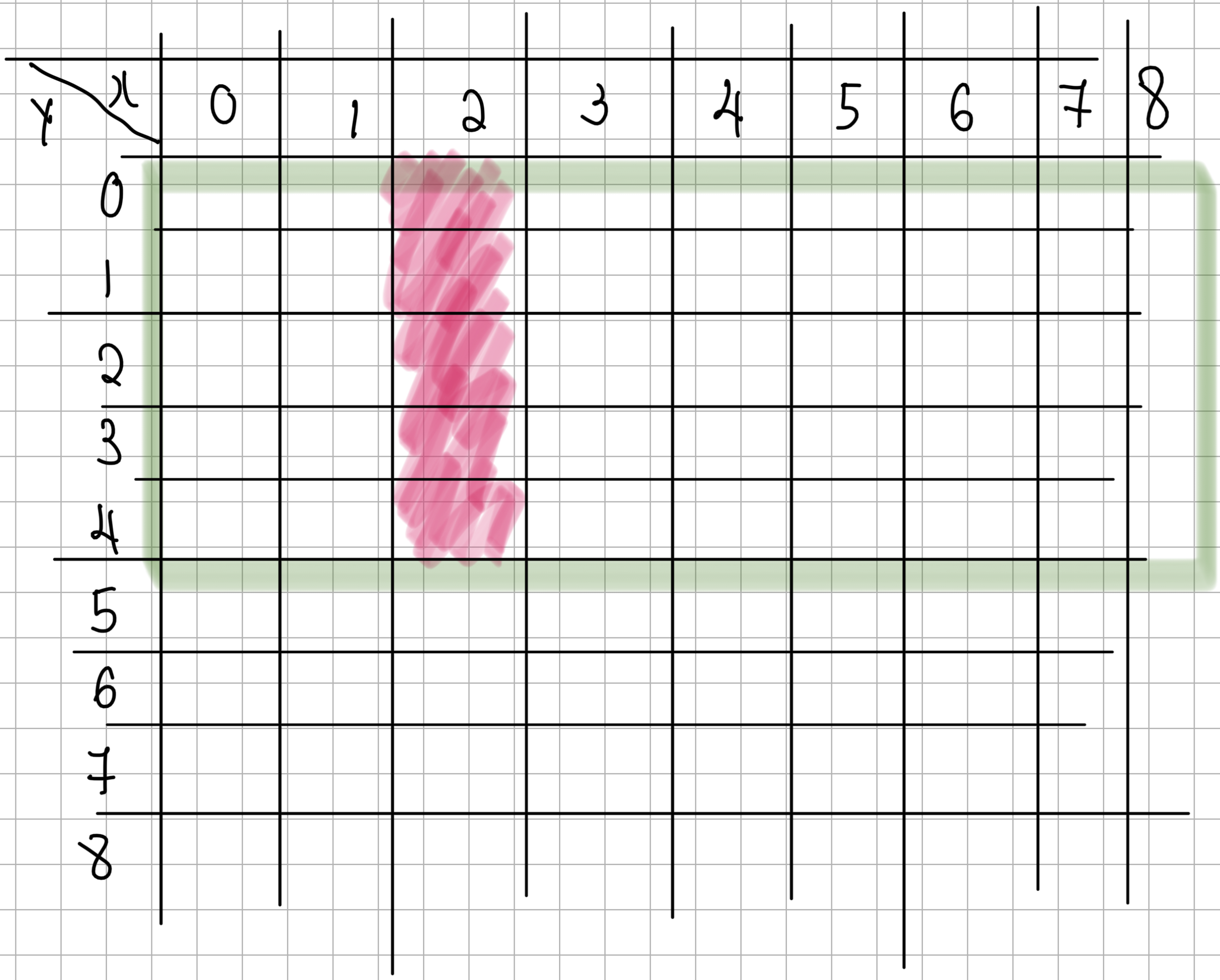
- $\{3\}$ $5C_0$
- $\{3, x\}$ $5C_1$
- $\{3, x, x_2\}$ $5C_2$
- $\{3, x, x_2, x_3\}$ $5C_3$
- $\{3, x_1, x_2, x_3, x_4\}$ $5C_4$
- $\{3, x_1, x_2, x_3, x_4, x_5\}$ $5C_5$

if = then independant

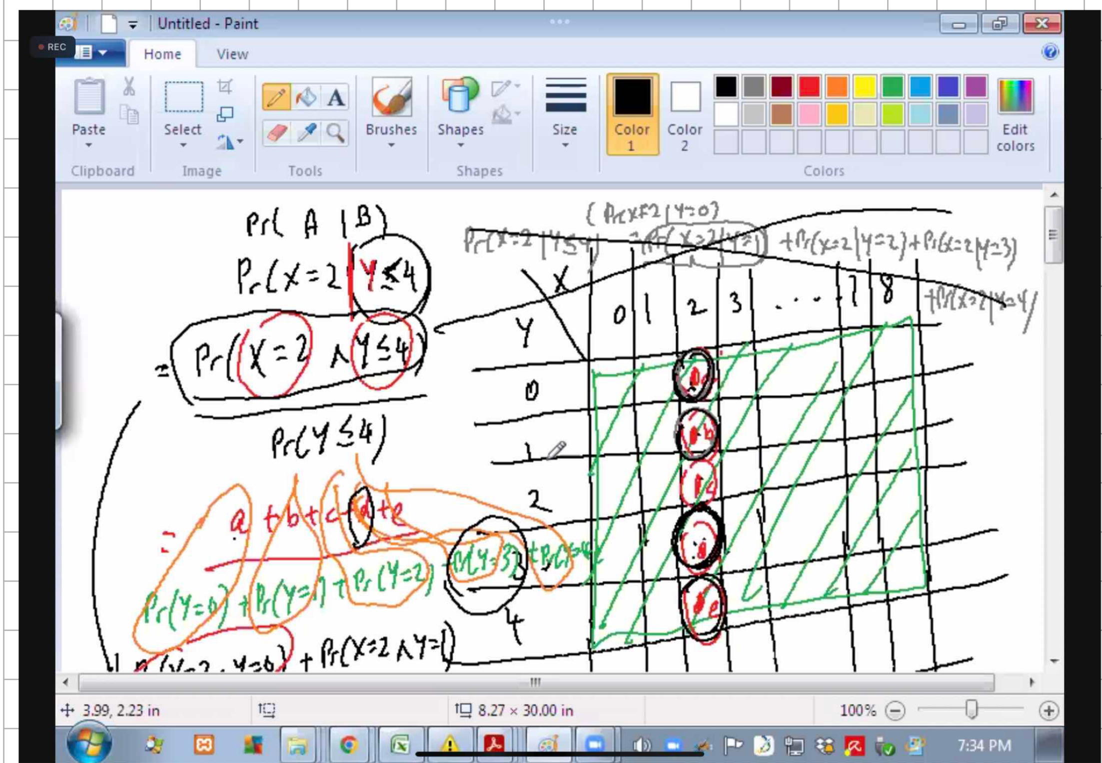


$$\frac{Pr(X=2 \wedge Y \leq 4)}{Pr(Y \leq 4)} = Pr(Y=0) + Pr(Y=1) + Pr(Y=2) + Pr(Y=3) + Pr(Y=4)$$

$$\frac{Pr(X=2 \wedge Y=0) + Pr(X=2 \wedge Y=1) + Pr(X=2 \wedge Y=2) + Pr(X=2 \wedge Y=3) + Pr(X=2 \wedge Y=4)}{Pr(Y=0) + Pr(Y=1) + Pr(Y=2) + Pr(Y=3) + Pr(Y=4)}$$



sum of pink divide by sum of green



Assignment 3

3) (i) X = elements in the subset
Y = 0 OR the minimum value in the subset
P = ([1, 2, 3, 4, 5, 6, 7, 8])

$P(X=1, Y=3)$
$P(X=1 \wedge Y=3) \stackrel{?}{=} P(X=1) * P(Y=3)$

1 number in subset $\Rightarrow X=1$
element with 3 as smallest $\Rightarrow Y=3$
so only {3} is eligible HENCE probability is 1/256

* total number of subsets with 1 element within 256 possibilities = 8
* total number of subsets with element number 3 as smallest value 3, 4, 5, 6, 7, 8 = 32

$\{3\} \longrightarrow {}^5C_0$
$\{3, x\} \longrightarrow {}^5C_1$
$\{3, x, x_2\} \longrightarrow {}^5C_2$
$\{3, x, x_2, x_3\} \longrightarrow {}^5C_3$
$\{3, x, x_2, x_3, x_4\} \longrightarrow {}^5C_4$
$\{3, x, x_2, x_3, x_4, x_5\} \rightarrow {}^5C_5$

${}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 32$

$1/256 \stackrel{?}{=} 8/256 * 32/256$

$8/256 = 1/32$
$32/256 = 1/8$

$1/32 \times 1/8 = 1/256$
HENCE, the events "$x=1$" and "$Y=3$" are independent.

3) (ii) X = elements in the subset
Y = 0 OR the minimum value in the subset
P = ([1,2,3,4,5,6,7,8])

P(X=2, Y=6)
$P(X=2 \wedge Y=6) \overset{?}{=} P(X=2) * P(Y=6)$

2 elements in a subset ⇒ X=2
subset element with 6 as smallest element ⇒ Y=6

6, 7, 8

{6,7} and {6,8} are the only eligible subsets so probability is 2/256

* total number of subsets with 2 element within 256 possibilities
= 36

01 02 03 04 05 06 07 08 → 8
12 13 14 15 16 17 18 → 7
23 24 25 26 27 28 → 6
34 35 36 37 38 → 5
45 46 47 48 → 4
56 57 58 → 3
67 68 → 2
78 → 1

8+7+6+5+4+3+2+1 = 36

* total number of subsets with element number 6 as smallest value
= 4

6, 7, 8
{6} → $_2C_0$
{6, x} → $_2C_1$
{6, x, x₂} → $_2C_2$

$_2C_0 + {_2C_1} + {_2C_2} = 4$

$2/256 \overset{?}{=} 36/256 * 4/256$

36/256 = 9/64
4/256 = 1/64

9/64 * 1/64 = 9/4096

2/256 ≠ 9/4096 HENCE the events "x=2" and "y=6" are NOT independant

3)(iii) $Pr(X=2 \mid Y \leq 4)$?

$$Pr(X=2 \mid Y \leq 4) = \frac{Pr(X=2 \wedge Y \leq 4)}{Pr(Y \leq 4)}$$

$$= \frac{a+b+c+d+e}{Pr(Y=0)+Pr(Y=1)+Pr(Y=2)+Pr(Y=3)+Pr(Y=4)}$$

$$= \frac{Pr(X=2 \wedge Y=0) + Pr(X=2 \wedge Y=1) + Pr(X=2 \wedge Y=2) + Pr(X=2 \wedge Y=3) + Pr(X=2 \wedge Y=4)}{Pr(Y=0)+Pr(Y=1)+Pr(Y=2)+Pr(Y=3)+Pr(Y=4)}$$

$$Pr(Y=4) \Rightarrow \frac{{}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4}{256} = \frac{16}{256} = \frac{1}{16}$$

$$Pr(Y=3) \Rightarrow \frac{{}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5}{256} = \frac{32}{256} = \frac{1}{8}$$

$$Pr(Y=2) \Rightarrow \frac{{}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6}{256} = \frac{64}{256} = \frac{1}{4}$$

$$Pr(Y=1) \Rightarrow \frac{{}^7C_0 + {}^7C_1 + {}^7C_2 + {}^7C_3 + {}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7}{256} = \frac{128}{256} = \frac{1}{2}$$

$$Pr(Y=0) \Rightarrow \frac{{}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8}{256} = \frac{256}{256} = 1/256$$

↳ NO NO, probability of $Y=0$ is $1/256$

$$Pr(Y=0)+Pr(Y=1)+Pr(Y=2)+Pr(Y=3)+Pr(Y=4) = 1/16 + 1/8 + 1/4 + 1/2 + 1/256$$
$$= 15/16 + 1/256$$
$$= 241/256$$

denominator

Expected : $23/241$

$$23/241 = \frac{x}{241/256}$$

$$x = (23/241) \times (241/256)$$
$$= 23/256$$

expected numerator

X = elements in the subset

Y = 0 OR the minimum value in the subset

$$\frac{22/256}{241/256} = 22/241$$

$Pr(X=2 \wedge Y=0) \Rightarrow 0 \rightarrow 0$

$Pr(X=2 \wedge Y=1) \Rightarrow 12\ 13\ 14\ 15\ 16\ 17\ 18 \rightarrow 7$

$Pr(X=2 \wedge Y=2) \Rightarrow 23\ 24\ 25\ 26\ 27\ 28 \rightarrow 6$

$Pr(X=2 \wedge Y=3) \Rightarrow 34\ 35\ 36\ 37\ 38 \rightarrow 5$

$Pr(X=2 \wedge Y=4) \Rightarrow 45\ 46\ 47\ 48 \rightarrow 4$

3

2

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