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5) slithy expression: $(a \vee b) \wedge (b \vee \neg c) \wedge (c \vee \neg d)$

not slithy expression: $(a \vee b) \wedge (b \vee \neg c) \wedge (\neg a \vee \neg c) \wedge (a \vee \neg d)$

expression is considered slithy if for every positive integer k, every set of k clauses in the expression includes variable that appear only once among all the clauses present.

Base case:

Base case is when the boolean expression only have one variable which is when n=1. since there is only one variable, it means that there can only be at most one clause as the only variable will appear exactly once in that said clause. An expression is considered slithy if for every positive interger n, every set of n clauses in the expression includes a variable that appears only once among the clauses present and in the case of n=1, only the present variable will be amongst the only clause present which satisfied the slithy Boolean expression.

This is the base case where n=1 where a is the variable Here a only appear once in the clause which shows that it is indeed True that for all the clauses in the expression includes variables that only once among all the clauses present. This boolean expression is indeed evaluated as true value which satisfied the slithy aspect.

Inductive hypothesis:
Assume that for some positive integer n, for any slithy Boolean expression present in the conjunctive normal form (CNF) with at most n variables has at most n clauses and all is satisfied whereby all clauses satisfies the slithy aspect of a Boolean expression.

Induction step:
let's say the boolean expression is: $(a \vee b) \wedge (b \vee \neg c)$
where $n = 2$ is $(a \vee b)$ and $(b \vee \neg c)$
and $n+1$ is $n = 3$ where $(a \vee b)$ and $(b \vee \neg c)$ and $(c \vee \neg d)$
looking at when $n+1$ which is $n=3$ from the expression $(a \vee b) \wedge (b \vee \neg c) \wedge (c \vee \neg d)$, variable a only appeared exactly once in the boolean expression which shows that it does not affect $(b \vee \neg c) \wedge (c \vee \neg d)$ as $(b \vee \neg c) \wedge (c \vee \neg d)$ is already true and satisfied. As long as a is in normal form and not in negated form it could be true and satisfied and hence when added to $(b \vee \neg c) \wedge (c \vee \neg d)$ it is indeed true that the boolean expression will be evaluated to true as the slithy aspect of the boolean expression is satisfied.

Conclusion:
By the principle of mathematical induction, it is indeed true that all boolean expression is evaluated to true as it is indeed slithy for every positive integer n with n clauses present in the boolean expression in CNF.