# Discrete Probability Distribution


☺ Back to Discrete Probability Distributions
☺ Exercise for Discrete Probability Distribution


DRV – discrete random variable

HH → 2 heads
HT → 1 head
TH → 1 head
TT → 0 head

Table / function / piecewise

## DISCRETE PROBABILITY DISTRIBUTION

① If $X$ represents the random variable $x$. $P(X = x)$ refers to the probability that the random variable $X$ is equal to a particular value, denoted by $x$. → P(X=x)

eg; coin flipping for 2 times
X = no of successive heads obtain
$X = \{0, 1, 2\}$

### Probability distribution

② probability function

When a table or an equation links each outcome of a statistical experiment with its probability of occurrence.

P(X=x) / P(X=2) = 0.5x0.5 = 0.25

④ **Note:**

1. The sum of all probabilities must be 1 → P(X=0) + P(X=1) + P(X=2) = 1
2. Negative probabilities are not permitted → P(X=x) ≠ -ve number
3. $0 \le P(X - x) \le 1$

*** **Example 1** DRV $X = \{1, 2, 3\}$ → Table

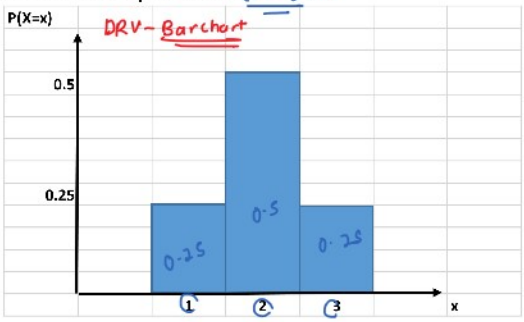
Row 1 → DRV
Row 2 → P(X=x)

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | 0.25 + | 0.50 + | 0.25 = 1 |

1. The sum of all probabilities must be 1.
2. Negative probabilities non-existent.
3. $0 \le P(X = x) \le 1$

} notes

**Discrete Probability Distribution** (Histogram)




① Discrete probability distribution
↳ a) P(X=x) ✓
b) Representation → Table ✓
→ Function ✓ → f(x) → P(X=x)
→ Piecewise Form ✓ → f(x) =?
c) Note → sum of all prob = 1
→ no (-ve) prob
$0 \le P(X=x) \le 1$
d) solving DRV distribution → venn diagram
→ tree diagram
→ sample space diagram
→ combination
→ conditional probability
→ Cumulative prob

* biased coin / unfair flipping → they will give the prob for head / tail

(4 m) Table
solve → Tree diagram → branching out

① **Example 2**

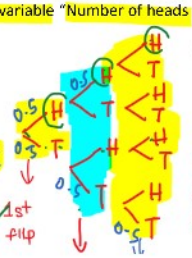
Head – H / Tail – T ? 50% of chances for you to obtain Head / Tail in each flip
0.5 0.5

Suppose that a fair coin is flipped three times. If $X$ is the discrete random variable "Number of heads obtained", complete the table below.

X = no of heads obtained

DRV distribution

| $x$ | 0 (12.5%) | 1 (37.5%) | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | 0.125 + | 0.375 + | 0.375 + | 0.125 = 1 |




① HHH → 3 heads → 0.125
② HHT → 2 heads → 0.125
③ HTH → 2 heads → 0.125
④ HTT → 1 head → 0.125
⑤ THH → 2 heads → 0.125
⑥ THT → 1 head → 0.125
⑦ TTH → 1 head → 0.125
⑧ TTT → 0 head → 0.125

① P(X=0) = P(TTT) = 0.5 x 0.5 x 0.5 = 0.125 ✓
② P(X=1) = P(HTT) + P(THT) + P(TTH) = 0.125 + 0.125 + 0.125 = 0.375 ✓
③ P(X=2) = P(HHT) + P(THH) + P(HTH) = 0.375 ✓

| | | | | |
|---|---|---|---|---|
| | 0.125 | 0.375 | 0.375 | 0.125 |

① $P(X=0) = P(TTT) = 0.5 \times 0.5 \times 0.5 = 0.125$ ✓
② $P(X=1) = P(HTT) + P(THT) + P(TTH) = 0.125 + 0.125 + 0.125 = 0.375$ ✓
③ $P(X=2) = P(HHT) + P(HTH) + P(THH) = 0.375$ ✓

**Answer:** 0.125, 0.375, 0,375,0.125


⑤ THH → 2 heads → 0.125
⑥ THT → 1 head → 0.125
⑦ TTH → 1 head → 0.125
⑧ TTT → 0 head → 0.125

combination → selecting a group from the outcomes

possibilities

**Example 3**

100 comp → 5 faulty, 95 good

① A batch of 100 components include 5 that are faulty. Four components are randomly selected from the batch without replacement. If $X$ is the number of faulty components in the selection, determine the probability distribution for $X$.

100 comp → choose 4 comp → 0 faulty, 4 good; 1 faulty, 3 good; 2 faulty, 2 good; 3 faulty, 1 good; 4 faulty, 0 good

$X$ = no of faulty from the selection

CAS → menu → 5 → 3 → nCr(n,r)

DRV

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X=x)$ | 0.8119 + | 0.1765 + | 0.01139 + | 0.000242 + | 0.00000 1275 = 1 |

0% ✓

① $P(X=0) = \dfrac{{}^5C_0 \times {}^{95}C_4}{{}^{100}C_4} = \dfrac{1 \times {}^{95}C_4}{{}^{100}C_4} = 0.8119$ ✓
(0 faulty, 4 good; ${}^{100}C_4$ = choose 4 out of 100; any number ${}^nC_0 = 1$)

② $P(X=1) = \dfrac{{}^5C_1 \times {}^{95}C_3}{{}^{100}C_4} = 0.1765$ ✓ (1 faulty, 3 good)

③ $P(X=2) = \dfrac{{}^5C_2 \times {}^{95}C_2}{{}^{100}C_4} = 0.01139$ ✓

④ $P(X=3) = \dfrac{{}^5C_3 \times {}^{95}C_1}{{}^{100}C_4} = 0.000242$ ✓

⑤ $P(X=4) = \dfrac{{}^5C_4 \times {}^{95}C_0}{{}^{100}C_4} = \dfrac{{}^5C_4 \times 1}{{}^{100}C_4} = 0.00000 1275$ ✓

**Answer:** 0.811875, 0.176495, 0.011387, 0.000242, 0.0000000001

## Probability function

If there is a defining f(x) which is associated with the probability distribution, then the probabilities and the function are known as **PROBABILITY FUNCTION**. → ① $f(x) = P(X=x)$

**Example 1**

If the function $f(x) = \dfrac{x}{15}, x = 1,2,3,4,5$

$f(x) = P(X=x)$

then the probability distribution is:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X=x)$ | 1/15 + | 2/15 + | 3/15 + | 4/15 + | 5/15 + = 1 |

hence find,

(a) $P(X \geq 2) = \dfrac{2}{15}$

(b) $P(2 < X < 5) = P(X=3) + P(X=4) = \dfrac{3}{15} + \dfrac{4}{15} = \dfrac{7}{15}$

**Answers:** 1/15, 2/15, 3/15, 4/15, 5/15 (a) 2/15 (b) 7/15

**Example 2** – piecewise wise form

The probability function for a discrete random variable X is given by

$f(x) = P(X=x) = kx$ ; $\frac{1}{15}x$

$$P(X=x) = \begin{cases} kx & for\ x = 1,2,3,4,5 \\ 0 & otherwise \end{cases}$$

→ piecewise form → $P(X=x) = \begin{cases} \frac{x}{15} & for\ x = 1,2,3,4,5 \\ 0 & otherwise \end{cases}$

(a) Find the value $k$,

$P(x=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) = 1$

$f(1) + f(2) + f(3) + f(4) + f(5) = 1$

$1k + 2k + 3k + 4k + 5k = 1$

$k = 1/15$

$f(1) + f(2) + f(3) + f(4) + f(5) = 1$

$1k + 2k + 3k + 4k + 5k = 1$

$15k = 1 \longrightarrow k = 1/15$

(b) Complete the table below

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 1/15 | 2/15 | 3/15 | 4/15 | 5/15 |

$f(x)$

(c) Determine

(i) $P(X = even) = P(X=2) + P(X=4) = 2/15 + 4/15 = 6/15 = 2/5$

(ii) $P(X < 2) = P(X=1) = 1/15$

(iii) $P(X > 2) = P(X=3) + P(X=4) + P(X=5) = \frac{3}{15} + \frac{4}{15} + \frac{5}{15} = \frac{12}{15} = \frac{4}{5}$

**Answers :** (a) 1/15 (b) 1/15, 2/15, 1/5, 4/15, 1/3 (c) (i) 2/5 (ii) 1/15 (iii) 4/5

DRV { ① General discrete probability distribution $[P(X=x)]$ ② cumulative prob dist $[P(X \leq x)]$

**Example 3** – cumulative probability distribution $P(X \leq x)$

→ accumulated probability at the $X = x$

The table below shows the cumulative probabilities for the random variable X.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(X \leq x)$ | 0.04 | 0.2 | 0.5 | 0.8 | 0.96 | 1 |

$P(0) + P(1) + P(2)$

cumulative

① $P(X \leq 0) = 0.04 = P(X=0)$

② $P(X \leq 1) = 0.2 = P(X=0) + P(X=1)$

$0.2 = 0.04 + P(X=1)$

③ $P(X \leq 4) = 0.96 = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

④ $P(X \leq 5) = 1$

Given that the possible values $X$ can take are 0, 1, 2, 3, 4, 5, copy and complete the following table.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | 0.04 + | 0.16 + | 0.3 + | 0.3 + | 0.16 + | 0.04 = 1 |

discrete prob dist

**Answers:** 0.04, 0.16, 0.3, 0.3, 0.16, 0.04

① $P(X \leq 0) = P(X=0) = 0.04$ ✓

② $P(X=1) = P(X \leq 1) - P(X=0) = 0.2 - 0.04 = 0.16$

③ $P(X=2) = P(X \leq 2) - (P(X=0) + P(X=1)) = 0.5 - 0.04 - 0.16 = 0.3$

OR

$P(X \leq 2) - P(X \leq 1) = 0.5 - 0.2 = 0.3$

④ $P(X=3) = P(X \leq 3) - P(X \leq 2) = 0.8 - 0.5 = 0.3$

⑤ $P(X=4) = P(X \leq 4) - P(X \leq 3) = 0.96 - 0.8 = 0.16$

⑥ $P(X=5) = P(X \leq 5) - P(X \leq 4) = 1 - 0.96 = 0.04$