CALCULUS I

Assignment Problems

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Preface

Here are a set of assignment problems for the Calculus I notes. Please note that these problems do not have any solutions available. These are intended mostly for instructors who might want a set of problems to assign for turning in. Having solutions available (or even just final answers) would defeat the purpose the problems.

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Outline

Here is a listing of sections for which assignment problems have been written as well as a brief description of the material covered in the notes for that particular section.

Review - In this chapter we give a brief review of selected topics from Algebra and Trig that are vital to surviving a Calculus course. Included are Functions, Trig Functions, Solving Trig Equations and Equations, Exponential/Logarithm Functions and Solving Exponential/Logarithm Equations.

> **Functions** – In this section we will cover function notation/evaluation, determining the domain and range of a function and function composition.
> **Inverse Functions** – In this section we will define an inverse function and the notation used for inverse functions. We will also discuss the process for finding an inverse function.
> **Trig Functions** – In this section we will give a quick review of trig functions. We will cover the basic notation, relationship between the trig functions, the right triangle definition of the trig functions. We will also cover evaluation of trig functions as well as the unit circle (one of the most important ideas from a trig class!) and how it can be used to evaluate trig functions.
> **Solving Trig Equations** – In this section we will discuss how to solve trig equations. The answers to the equations in this section will all be one of the “standard” angles that most students have memorized after a trig class. However, the process used here can be used for any answer regardless of it being one of the standard angles or not.
> **Solving Trig Equations with Calculators, Part I** – In this section we will discuss solving trig equations when the answer will (generally) require the use of a calculator (i.e. they aren’t one of the standard angles). Note however, the process used here is identical to that for when the answer is one of the standard angles. The only difference is that the answers in here can be a little messy due to the need of a calculator. Included is a brief discussion of inverse trig functions.
> **Solving Trig Equations with Calculators, Part II** – In this section we will continue our discussion of solving trig equations when a calculator is needed to get the answer. The equations in this section tend to be a little trickier than the "normal" trig equation and are not always covered in a trig class.
> **Exponential Functions** – In this section we will discuss exponential functions. We will cover the basic definition of an exponential function, the natural exponential function, i.e. $\mathbf{e}^x$, as well as the properties and graphs of exponential functions.
> **Logarithm Functions** – In this section we will discuss logarithm functions, evaluation of logarithms and their properties. We will discuss many of the basic manipulations of logarithms that commonly occur in Calculus (and higher) classes. Included is a discussion of the natural (\(\ln(x)\)) and common logarithm (\(\log(x)\)) as well as the change of base formula.
> **Exponential and Logarithm Equations** – In this section we will discuss various methods for solving equations that involve exponential functions or logarithm functions.
> **Common Graphs** – In this section we will do a very quick review of many of the most common functions and their graphs that typically show up in a Calculus class.

Limits - In this chapter we introduce the concept of limits. We will discuss the interpretation/meaning of a limit, how to evaluate limits, the definition and evaluation of one-sided limits, evaluation of infinite limits, evaluation of limits at infinity, continuity and the Intermediate Value Theorem. We will also give a brief introduction to a precise definition of the limit and how to use it to evaluate limits.

Tangent Lines and Rates of Change – In this section we will introduce two problems that we will see time and again in this course : Rate of Change of a function and Tangent Lines to functions. Both of these problems will be used to introduce the concept of limits, although we won't formally give the definition or notation until the next section.

The Limit – In this section we will introduce the notation of the limit. We will also take a conceptual look at limits and try to get a grasp on just what they are and what they can tell us. We will be estimating the value of limits in this section to help us understand what they tell us. We will actually start computing limits in a couple of sections.

One-Sided Limits – In this section we will introduce the concept of one-sided limits. We will discuss the differences between one-sided limits and limits as well as how they are related to each other.

Limit Properties – In this section we will discuss the properties of limits that we'll need to use in computing limits (as opposed to estimating them as we've done to this point). We will also compute a couple of basic limits in this section.

Computing Limits – In this section we will looks at several types of limits that require some work before we can use the limit properties to compute them. We will also look at computing limits of piecewise functions and use of the Squeeze Theorem to compute some limits.

Infinite Limits – In this section we will look at limits that have a value of infinity or negative infinity. We'll also take a brief look at vertical asymptotes.

Limits At Infinity, Part I – In this section we will start looking at limits at infinity, i.e. limits in which the variable gets very large in either the positive or negative sense. We will concentrate on polynomials and rational expressions in this section. We'll also take a brief look at horizontal asymptotes.

Limits At Infinity, Part II – In this section we will continue covering limits at infinity. We'll be looking at exponentials, logarithms and inverse tangents in this section.

Continuity – In this section we will introduce the concept of continuity and how it relates to limits. We will also see the Intermediate Value Theorem in this section and how it can be used to determine if functions have solutions in a given interval.

The Definition of the Limit – In this section we will give a precise definition of several of the limits covered in this section. We will work several basic examples illustrating how to use this precise definition to compute a limit. We'll also give a precise definition of continuity.

Derivatives – In this chapter we introduce Derivatives. We cover the standard derivatives formulas including the product rule, quotient rule and chain rule as well as derivatives of polynomials, roots, trig functions, inverse trig functions, hyperbolic functions, exponential functions and logarithm functions. We also cover implicit differentiation, related rates, higher order derivatives and logarithmic differentiation.

The Definition of the Derivative – In this section we define the derivative, give various notations for the derivative and work a few problems illustrating how to use the definition of the derivative to actually compute the derivative of a function.

Interpretation of the Derivative – In this section we give several of the more important interpretations of the derivative. We discuss the rate of change of a function, the velocity of a moving object and the slope of the tangent line to a graph of a function.

Differentiation Formulas – In this section we give most of the general derivative formulas and properties used when taking the derivative of a function. Examples in this section concentrate mostly on polynomials, roots and more generally variables raised to powers.

Product and Quotient Rule – In this section we will give two of the more important formulas for differentiating functions. We will discuss the Product Rule and the Quotient Rule allowing us to differentiate functions that, up to this point, we were unable to differentiate.
Derivatives of Trig Functions – In this section we will discuss differentiating trig functions. Derivatives of all six trig functions are given and we show the derivation of the derivative of sin(x) and tan(x).
Derivatives of Exponential and Logarithm Functions – In this section we derive the formulas for the derivatives of the exponential and logarithm functions.
Derivatives of Inverse Trig Functions – In this section we give the derivatives of all six inverse trig functions. We show the derivation of the formulas for inverse sine, inverse cosine and inverse tangent.
Derivatives of Hyperbolic Functions – In this section we define the hyperbolic functions, give the relationships between them and some of the basic facts involving hyperbolic functions. We also give the derivatives of each of the six hyperbolic functions and show the derivation of the formula for hyperbolic sine.
Chain Rule – In this section we discuss one of the more useful and important differentiation formulas, The Chain Rule. With the chain rule in hand we will be able to differentiate a much wider variety of functions. As you will see throughout the rest of your Calculus courses a great many of derivatives you take will involve the chain rule!
Implicit Differentiation – In this section we will discuss implicit differentiation. Not every function can be explicitly written in terms of the independent variable, e.g. y = f(x) and yet we will still need to know what f'(x) is. Implicit differentiation will allow us to find the derivative in these cases. Knowing implicit differentiation will allow us to do one of the more important applications of derivatives, Related Rates (the next section).
Related Rates – In this section we will discuss the only application of derivatives in this section, Related Rates. In related rates problems we are give the rate of change of one quantity in a problem and asked to determine the rate of one (or more) quantities in the problem. This is often one of the more difficult sections for students. We work quite a few problems in this section so hopefully by the end of this section you will get a decent understanding on how these problems work
Higher Order Derivatives – In this section we define the concept of higher order derivatives and give a quick application of the second order derivative and show how implicit differentiation works for higher order derivatives.
Logarithmic Differentiation – In this section we will discuss logarithmic differentiation. Logarithmic differentiation gives an alternative method for differentiating products and quotients (sometimes easier than using product and quotient rule). More importantly, however, is the fact that logarithm differentiation allows us to differentiate functions that are in the form of one function raised to another function, i.e. there are variables in both the base and exponent of the function.

Applications of Derivatives – In this chapter we will cover many of the major applications of derivatives. Applications included are determining absolute and relative minimum and maximum function values (both with and without constraints), sketching the graph of a function without using a computational aid, determining the Linear Approximation of a function, L'Hospital's Rule (allowing us to compute some limits we could not prior to this), Newton's Method (allowing us to approximate solutions to equations) as well as a few basic Business applications.

Rates of Change – In this section we review the main application/interpretation of derivatives from the previous chapter (i.e. rates of change) that we will be using in many of the applications in this chapter.
Critical Points – In this section we give the definition of critical points. Critical points will show up in most of the sections in this chapter, so it will be important to understand them and how to find them. We will work a number of examples illustrating how to find them for a wide variety of functions.
Minimum and Maximum Values –In this section we define absolute (or global) minimum and maximum values of a function and relative (or local) minimum and maximum values of a function. It is important to understand the difference between the two types of minimum/maximum (collectively called extrema) values for many of the applications in this chapter and so we use a variety of examples to help with this. We also give the Extreme Value Theorem and Fermat's Theorem, both of which are very important in the many of the applications we'll see in this chapter.
Finding Absolute Extrema – In this section we discuss how to find the absolute (or global) minimum and maximum values of a function. In other words, we will be finding the largest and smallest values that a function will have.
The Shape of a Graph, Part I – In this section we will discuss what the first derivative of a function can tell us about the graph of a function. The first derivative will allow us to identify the relative (or local) minimum and maximum values of a function and where a function will be increasing and decreasing. We will also give the First Derivative test which will allow us to classify critical points as relative minimums, relative maximums or neither a minimum or a maximum.
The Shape of a Graph, Part II – In this section we will discuss what the second derivative of a function can tell us about the graph of a function. The second derivative will allow us to determine where the graph of a function is concave up and concave down. The second derivative will also allow us to identify any inflection points (i.e. where concavity changes) that a function may have. We will also give the Second Derivative Test that will give an alternative method for identifying some critical points (but not all) as relative minimums or relative maximums.
The Mean Value Theorem – In this section we will give Rolle's Theorem and the Mean Value Theorem. With the Mean Value Theorem we will prove a couple of very nice facts, one of which will be very useful in the next chapter.
Optimization Problems – In this section we will be determining the absolute minimum and/or maximum of a function that depends on two variables given some constraint, or relationship, that the two variables must always satisfy. We will discuss several methods for determining the absolute minimum or maximum of the function. Examples in this section tend to center around geometric objects such as squares, boxes, cylinders, etc.
More Optimization Problems – In this section we will continue working optimization problems. The examples in this section tend to be a little more involved and will often involve situations that will be more easily described with a sketch as opposed to the 'simple' geometric objects we looked at in the previous section.
L'Hospital's Rule and Indeterminate Forms – In this section we will revisit indeterminate forms and limits and take a look at L'Hospital's Rule. L'Hospital's Rule will allow us to evaluate some limits we were not able to previously.
Linear Approximations – In this section we discuss using the derivative to compute a linear approximation to a function. We can use the linear approximation to a function to approximate values of the function at certain points. While it might not seem like a useful thing to do with

when we have the function there really are reasons that one might want to do this. We give two ways this can be useful in the examples.

Differentials – In this section we will compute the differential for a function. We will give an application of differentials in this section. However, one of the more important uses of differentials will come in the next chapter and unfortunately we will not be able to discuss it until then.

Newton's Method – In this section we will discuss Newton's Method. Newton's Method is an application of derivatives that will allow us to approximate solutions to an equation. There are many equations that cannot be solved directly and with this method we can get approximations to the solutions to many of those equations.

Business Applications – In this section we will give a cursory discussion of some basic applications of derivatives to the business field. We will revisit finding the maximum and/or minimum function value and we will define the marginal cost function, the average cost, the revenue function, the marginal revenue function and the marginal profit function. Note that this section is only intended to introduce these concepts and not teach you everything about them.

Integrals – In this chapter we will give an introduction to definite and indefinite integrals. We will discuss the definition and properties of each type of integral as well as how to compute them including the Substitution Rule. We will give the Fundamental Theorem of Calculus showing the relationship between derivatives and integrals. We will also discuss the Area Problem, an important interpretation of the definite integral.

Indefinite Integrals – In this section we will start off the chapter with the definition and properties of indefinite integrals. We will not be computing many indefinite integrals in this section. This section is devoted to simply defining what an indefinite integral is and to give many of the properties of the indefinite integral. Actually computing indefinite integrals will start in the next section.

Computing Indefinite Integrals – In this section we will compute some indefinite integrals. The integrals in this section will tend to be those that do not require a lot of manipulation of the function we are integrating in order to actually compute the integral. As we will see starting in the next section many integrals do require some manipulation of the function before we can actually do the integral. We will also take a quick look at an application of indefinite integrals.

Substitution Rule for Indefinite Integrals – In this section we will start using one of the more common and useful integration techniques – The Substitution Rule. With the substitution rule we will be able integrate a wider variety of functions. The integrals in this section will all require some manipulation of the function prior to integrating unlike most of the integrals from the previous section where all we really needed were the basic integration formulas.

More Substitution Rule – In this section we will continue to look at the substitution rule. The problems in this section will tend to be a little more involved than those in the previous section.

Area Problem – In this section we start off with the motivation for definite integrals and give one of the interpretations of definite integrals. We will be approximating the amount of area that lies between a function and the \(x\)-axis. As we will see in the next section this problem will lead us to the definition of the definite integral and will be one of the main interpretations of the definite integral that we'll be looking at in this material.

Definition of the Definite Integral – In this section we will formally define the definite integral, give many of its properties and discuss a couple of interpretations of the definite integral. We will also look at the first part of the Fundamental Theorem of Calculus which shows the very close relationship between derivatives and integrals.

Computing Definite Integrals – In this section we will take a look at the second part of the Fundamental Theorem of Calculus. This will show us how we compute definite integrals without using (the often very unpleasant) definition. The examples in this section can all be done with a basic knowledge of indefinite integrals and will not require the use of the substitution rule. Included in the examples in this section are computing definite integrals of piecewise and absolute value functions.

Substitution Rule for Definite Integrals – In this section we will revisit the substitution rule as it applies to definite integrals. The only real requirements to being able to do the examples in this section are being able to do the substitution rule for indefinite integrals and understanding how to compute definite integrals in general.

Applications of Integrals – In this chapter we will take a look at some applications of integrals. We will look at Average Function Value, Area Between Curves, Volume (both solids of revolution and other solids) and Work.

Average Function Value – In this section we will look at using definite integrals to determine the average value of a function on an interval. We will also give the Mean Value Theorem for Integrals.

Area Between Curves – In this section we'll take a look at one of the main applications of definite integrals in this chapter. We will determine the area of the region bounded by two curves.

Volumes of Solids of Revolution / Method of Rings – In this section, the first of two sections devoted to finding the volume of a solid of revolution, we will look at the method of rings/disks to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the *x* or *y*-axis) around a vertical or horizontal axis of rotation.

Volumes of Solids of Revolution / Method of Cylinders – In this section, the second of two sections devoted to finding the volume of a solid of revolution, we will look at the method of cylinders/shells to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the *x* or *y*-axis) around a vertical or horizontal axis of rotation.

More Volume Problems – In the previous two sections we looked at solids that could be found by treating them as a solid of revolution. Not all solids can be thought of as solids of revolution and, in fact, not all solids of revolution can be easily dealt with using the methods from the previous two sections. So, in this section we'll take a look at finding the volume of some solids that are either not solids of revolutions or are not easy to do as a solid of revolution.

Work – In this section we will look at is determining the amount of work required to move an object subject to a force over a given distance.

Chapter 1 : Review

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Section 1-1 : Functions

For problems 1 – 6 the given functions perform the indicated function evaluations.

1. $f(x)=10x-3$
   (a) $f(-5)$ (b) $f(0)$ (c) $f(7)$
   (d) $f(t^2+2)$ (e) $f(12-x)$ (f) $f(x+h)$

2. $h(y)=4y^2-7y+1$
   (a) $h(0)$ (b) $h(-3)$ (c) $h(5)$
   (d) $h(6z)$ (e) $h(1-3y)$ (f) $h(y+k)$

3. $g(t)=\dfrac{t+5}{1-t}$
   (a) $g(0)$ (b) $g(4)$ (c) $g(-7)$
   (d) $g(x^2-5)$ (e) $g(t+h)$ (f) $g(4\sqrt{t}+9)$

4. $f(z)=\sqrt{4z+5}$
   (a) $f(0)$ (b) $f(-1)$ (c) $f(-2)$
   (d) $h(5-12y)$ (e) $f(2z^2+8)$ (f) $f(z+h)$

5. $z(x)=\dfrac{\sqrt{x^2+9}}{4x+8}$
   (a) $z(4)$ (b) $z(-4)$ (c) $z(1)$
   (d) $z(2-7x)$ (e) $z(\sqrt{3x+4})$ (f) $z(x+h)$

6. $Y(t)=\sqrt{3-t}-\dfrac{t}{2t+5}$
   (a) $Y(0)$ (b) $Y(7)$ (c) $Y(-4)$
   (d) $Y(5-t)$ (e) $Y(t^2-10)$ (f) $Y(6t-t^2)$

The **difference quotient** of a function $f\left(x\right)$ is defined to be,

$$\frac{f\left(x+h\right)-f\left(x\right)}{h}$$

For problems 7 – 13 compute the difference quotient of the given function.

7. $Q\left(t\right)=4-7t$

8. $g\left(t\right)=42$

9. $H\left(x\right)=2x^2+9$

10. $z\left(y\right)=3-8y-y^2$

11. $g\left(z\right)=\sqrt{4+3z}$

12. $y\left(x\right)=\dfrac{-4}{1-2x}$

13. $f\left(t\right)=\dfrac{t^2}{t+7}$

For problems 14 – 21 determine all the roots of the given function.

14. $y\left(t\right)=40+3t-t^2$

15. $f\left(x\right)=6x^4-5x^3-4x^2$

16. $Z\left(p\right)=6-11p-p^2$

17. $h\left(y\right)=4y^6+10y^5+y^4$

18. $g\left(z\right)=z^7+6z^4-16z$

19. $f\left(t\right)=t^{\frac{1}{2}}-8t^{\frac{1}{4}}+15$

20. $h\left(w\right)=\dfrac{w}{4w+5}+\dfrac{3w}{w-8}$

21. $g\left(w\right)=\frac{w}{w+3}-\frac{w+2}{4w-1}$

For problems 22 – 30 find the domain and range of the given function.

22. $f\left(x\right)=x^2-8x+3$

23. $z\left(w\right)=4-7w-w^2$

24. $g\left(t\right)=3t^2+2t-3$

25. $g\left(x\right)=5-\sqrt{2x}$

26. $B\left(z\right)=10+\sqrt{9+7z^2}$

27. $h\left(y\right)=1+\sqrt{6-7y}$

28. $f\left(x\right)=12-5\sqrt{2x+9}$

29. $V\left(t\right)=-6\left|5-t\right|$

30. $y\left(x\right)=12+9\left|x^2-1\right|$

For problems 31 – 51 find the domain of the given function.

31. $f\left(t\right)=\frac{4-12t+8t^2}{16t+9}$

32. $v\left(y\right)=\frac{y^3-27}{4-17y}$

33. $g\left(x\right)=\frac{3x+1}{5x^2-3x-2}$

34. $h\left(t\right)=\frac{t^3-t^2+1-1}{35t^3+2t^4-t^5}$

35. $f\left(z\right)=\frac{z^2+z}{z^3-9z^2+2z}$

36. $V(p) = \dfrac{3-p^4}{4p^2+10p+2}$

37. $g(z) = \sqrt{z^2-15}$

38. $f(t) = \sqrt{36-9t^2}$

39. $A(x) = \sqrt{15x-2x^2-x^3}$

40. $Q(y) = \sqrt{4y^3-4y^2+y}$

41. $P(t) = \dfrac{t^2+7}{\sqrt{6t-t^2}}$

42. $h(t) = \dfrac{t^2}{\sqrt{5+3t-t^2}}$

43. $h(x) = \dfrac{6}{\sqrt{x^2-7x+3}}$

44. $f(z) = \dfrac{z+1}{\sqrt{z^4-6z^3+9z^2}}$

45. $S(t) = \sqrt{8-t}+\sqrt{2t}$

46. $g(x) = \sqrt{5x-8}-2\sqrt{x+9}$

47. $h(y) = \sqrt{49-y^2}-\dfrac{y}{\sqrt{4y-12}}$

48. $A(x) = \dfrac{x+1}{x-4}+4\sqrt{x^2+10x+9}$

49. $f(t) = \dfrac{8}{t^2-3t-4}+\dfrac{3}{\sqrt{12-7t-3t^2}}$

50. $R(x) = \dfrac{3}{x^4+x^2}+\sqrt[5]{x^2-x-6}$

51. $C(z) = z^3 - \sqrt[4]{z^6 + z^2}$

For problems 52 – 55 compute $(f \circ g)(x)$ and $(g \circ f)(x)$ for each of the given pairs of functions.

52. $f(x) = 5 + 2x$, $g(x) = 8 - 23x$

53. $f(x) = \sqrt{2 - x}$, $g(x) = 2x^2 - 9$

54. $f(x) = 2x^2 + x - 4$, $g(x) = 7x - x^2$

55. $f(x) = \dfrac{x}{3 + 2x}$, $g(x) = 8 + 5x$

Section 1-2 : Inverse Functions

For each of the following functions find the inverse of the function. Verify your inverse by computing one or both of the composition as discussed in this section.

1. $f\left(x\right)=11x-8$

2. $g\left(x\right)=4-10x$

3. $Z\left(x\right)=2x^{7}-9$

4. $h\left(x\right)=7+\left(2x+1\right)^{3}$

5. $W\left(x\right)=\sqrt[7]{15x+2}$

6. $h\left(x\right)=\sqrt[3]{6-18x}$

7. $R\left(x\right)=\dfrac{2x+14}{6x+1}$

8. $g\left(x\right)=\dfrac{1-x}{9-12x}$

Section 1-3 : Trig Functions

Determine the exact value of each of the following without using a calculator.

Note that the point of these problems is not really to learn how to find the value of trig functions but instead to get you comfortable with the unit circle since that is a very important skill that will be needed in solving trig equations.

1. $\tan\left(\frac{3\pi}{4}\right)$

2. $\sin\left(\frac{7\pi}{6}\right)$

3. $\sin\left(-\frac{3\pi}{4}\right)$

4. $\cos\left(\frac{4\pi}{3}\right)$

5. $\cot\left(\frac{5\pi}{4}\right)$

6. $\sin\left(-\frac{5\pi}{6}\right)$

7. $\sec\left(-\frac{\pi}{6}\right)$

8. $\cos\left(\frac{5\pi}{4}\right)$

9. $\cos\left(\frac{11\pi}{6}\right)$

10. $\csc\left(\frac{11\pi}{6}\right)$

11. $\cot\left(-\frac{4\pi}{3}\right)$

12. $\cos\left(-\frac{\pi}{4}\right)$

13. $\csc\left(\frac{2\pi}{3}\right)$

14. $\sec\left(\frac{17\pi}{6}\right)$

15. $\sin\left(-\frac{23\pi}{3}\right)$

16. $\tan\left(\frac{31\pi}{6}\right)$

17. $\cos\left(-\frac{15\pi}{4}\right)$

18. $\sec\left(-\frac{23\pi}{4}\right)$

19. $\cot\left(\frac{11\pi}{4}\right)$

Section 1-4 : Solving Trig Equations

Without using a calculator find the solution(s) to the following equations. If an interval is given find only those solutions that are in the interval. If no interval is given find all solutions to the equation.

1. $10\cos\left(8t\right) = -5$

2. $10\cos\left(8t\right) = -5$ in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

3. $2\sin\left(\frac{z}{4}\right) = \sqrt{3}$

4. $2\sin\left(\frac{z}{4}\right) = \sqrt{3}$ in $\left[0, 16\pi\right]$

5. $2\sin\left(\frac{2t}{3}\right) + \sqrt{2} = 0$ in $\left[0, 5\pi\right]$

6. $\sqrt{6} = -\sqrt{8}\cos\left(3x\right)$ in $\left[0, \frac{5\pi}{3}\right]$

7. $10 + 7\tan\left(4x\right) = 3$ in $\left[-\pi, 0\right]$

8. $0 = 2\cos\left(\frac{y}{2}\right) - \sqrt{2}$ in $\left[-4\pi, 5\pi\right]$

9. $3\cos\left(5z\right) - 1 = 7$ in $\left[-\pi, \pi\right]$

10. $7\sqrt{3} + 7\cot\left(2w\right) = 0$ in $\left[\frac{\pi}{3}, 2\pi\right]$

11. $2\csc\left(\frac{x}{3}\right) + \sqrt{8} = 0$ in $\left[0, 2\pi\right]$

12. $3 - 4\sin\left(4t\right) = 5$ in $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$

13. $3\sec\left(\frac{y}{5}\right)+9=15$ in $\left[-3\pi,20\pi\right]$

14. $\sqrt{12}\cos\left(2z\right)-2\sin\left(2z\right)=0$ in $\left[\frac{3\pi}{2},2\pi\right]$

Section 1-5 : Solving Trig Equations with Calculators, Part I

Find the solution(s) to the following equations. If an interval is given find only those solutions that are in the interval. If no interval is given find all solutions to the equation.

1. $2-14\sin\left(\frac{t}{3}\right)=5$

2. $4\cos\left(4x\right)+8=10-\cos\left(4x\right)$

3. $2\tan\left(3w\right)+3=25$

4. $2\sin\left(\frac{3x}{5}\right)-\frac{7}{5}=\frac{1}{5}$ in $\left[0,15\right]$

5. $1=3+8\cos\left(\frac{w}{2}\right)$ in $\left[-20,5\right]$

6. $45\sin\left(\frac{x}{2}\right)-9=7\sin\left(\frac{x}{2}\right)+17$ in $\left[-10,20\right]$

7. $\frac{2}{3}=4-3\sec\left(11x\right)$ in $\left[0,1\right]$

8. $3\sin\left(4v\right)+18\cos\left(4v\right)=0$ in $\left[2,5\right]$

9. $2\left(\cos\left(\frac{2t}{7}\right)+3\right)=7\cos\left(\frac{2t}{7}\right)+6$ in $\left[-10,30\right]$

10. $\frac{1}{2}\csc\left(\frac{y}{3}\right)-\frac{10}{7}=\frac{3}{14}$ in $\left[0,32\right]$

11. $31=1+40\cos\left(\frac{t}{8}\right)$ in $\left[-50,60\right]$

12. $15\csc\left(15x\right)+14=20-12\csc\left(15x\right)$ in $\left[1,2\right]$

13. $\frac{1}{2}\cos\left(6t\right)+3=1+\frac{1}{3}\cos\left(6t\right)$ in $\left[0,5\right]$

14. $4\left(1-2\sec\left(\frac{z}{5}\right)\right)=12$ in $\left[0,15\right]$

15. $11-7\sin\left(\frac{2x}{13}\right)=23-19\sin\left(\frac{2x}{13}\right)$ in $\left[-60,60\right]$

Section 1-6 : Solving Trig Equations with Calculators, Part II

Find all the solution(s) to the following equations. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $22\cos\left(8-x\right)+10=0$

2. $10\tan\left(4x+10\right)-7=31$

3. $4\tan\left(\dfrac{w}{3}\right)\sin\left(2w\right)-\tan\left(\dfrac{w}{3}\right)=0$

4. $3\tan\left(4z\right)\sec\left(2z-1\right)+\sec\left(2z-1\right)=0$

5. $2-\sin\left(2y\right)=3\sin^2\left(2y\right)$

6. $4\cos^2\left(2t+5\right)-4\cos\left(2t+5\right)=-1$

7. $6-5\sin^2\left(\dfrac{x}{4}\right)=7\sin\left(\dfrac{x}{4}\right)$

8. $2=2\tan^2\left(8t\right)+3\tan\left(8t\right)$

9. $35\csc\left(4z\right)=z^3\csc\left(4z\right)$

10. $3t=8t\cos\left(5+t\right)$

11. $\left(5x+1\right)\sin\left(\dfrac{x-6}{2}\right)+25x+5=0$

12. $5w^2-20=\left(8-2w^2\right)\sec\left(\dfrac{4w}{9}\right)$

Section 1-7 : Exponential Functions

Sketch the graphs of each of the following functions.

1. $g\left(t\right)=7^{3-\frac{t}{2}}$

2. $f\left(x\right)=3-5^{4x+1}$

3. $h\left(x\right)=6\mathbf{e}^{2x-1}-3$

4. $f\left(t\right)=7+9\mathbf{e}^{2-\frac{3t}{5}}$

Section 1-8 : Logarithm Functions

Without using a calculator determine the exact value of each of the following.

1. $\log_7 343$

2. $\log_4 1024$

3. $\log_{\frac{3}{8}} \dfrac{27}{512}$

4. $\log_{11} \dfrac{1}{121}$

5. $\log_{0.1} 0.0001$

6. $\log_{16} 4$

7. $\log 10000$

8. $\ln \dfrac{1}{\sqrt[5]{\mathbf{e}}}$

Write each of the following in terms of simpler logarithms

9. $\log_7 \left(10a^7 b^3 c^{-8}\right)$

10. $\log\left[z^2 \left(x^2+4\right)^3\right]$

11. $\ln\left(\dfrac{w^2 \sqrt[4]{t^3}}{\sqrt{t+w}}\right)$

Combine each of the following into a single logarithm with a coefficient of one.

12. $7\ln t - 6\ln s + 5\ln w$

13. $\dfrac{1}{2}\log\left(z+1\right) - 2\log x - 4\log y - 3\log z$

14. $2\log_3\left(x+y\right)+6\log_3 x-\frac{1}{3}$

Use the change of base formula and a calculator to find the value of each of the following.

15. $\log_7 100$

16. $\log_{\frac{5}{7}}\frac{1}{8}$

Section 1-9 : Exponential and Logarithm Equations

For problems 1 – 17 find all the solutions to the given equation. If there is no solution to the equation clearly explain why.

1. $15 = 12 + 5\mathbf{e}^{10w-7}$

2. $4\mathbf{e}^{2x+x^2} - 7 = 2$

3. $8 + 3\mathbf{e}^{4-9z} = 1$

4. $4t^2 - 3t^2\mathbf{e}^{2-t} = 0$

5. $7x + 16x\mathbf{e}^{x^3-5x} = 0$

6. $3\mathbf{e}^{7t} - 12\mathbf{e}^{8t+5} = 0$

7. $2y\mathbf{e}^{y^2} - 7y\mathbf{e}^{1-5y} = 0$

8. $16 + 4\ln(x+2) = 7$

9. $3 - 11\ln\left(\dfrac{z}{3-z}\right) = 1$

10. $2\log(w) - \log(3w+7) = 1$

11. $\ln(3x+1) - \ln(x) = -2$

12. $t\log(6t+1) - 3t^2\log(6t+1) = 0$

13. $2\log(z) - \log(z^2+4z+1) = 0$

14. $\ln(x) + \ln(x-2) = 3$

15. $11 - 5^{9w-1} = 3$

16. $12 + 20^{7-2t} = 50$

17. $1 + 3^{z^2-2} = 5$

Compound Interest. If we put *P* dollars into an account that earns interest at a rate of *r* (written as a decimal as opposed to the standard percent) for *t* years then,

a. if interest is compounded *m* times per year we will have,

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$

dollars after *t* years.

b. if interest is compounded continuously we will have,

$$A = P\mathbf{e}^{rt}$$

dollars after *t* years.

18. We have \$2,500 to invest and 80 months. How much money will we have if we put the money into an account that has an annual interest rate of 9% and interest is compounded

(a) quarterly (b) monthly (c) continuously

19. We are starting with \$60,000 and we're going to put it into an account that earns an annual interest rate of 7.5%. How long will it take for the money in the account to reach \$100,000 if the interest is compounded

(a) quarterly (b) monthly (c) continuously

20. Suppose that we put some money in an account that has an annual interest rate of 10.25%. How long will it take to triple our money if the interest is compounded

(a) twice a year (b) 8 times a year (c) continuously

Exponential Growth/Decay. Many quantities in the world can be modeled (at least for a short time) by the exponential growth/decay equation.

$$Q = Q_0\mathbf{e}^{kt}$$

If *k* is positive we will get exponential growth and if *k* is negative we will get exponential decay.

21. A population of bacteria initially has 90,000 present and in 2 weeks there will be 200,000 bacteria present.

(a) Determine the exponential growth equation for this population.
(b) How long will it take for the population to grow from its initial population of 90,000 to a population of 150,000?

22. We initially have 2 kg grams of some radioactive element and in 7250 years there will be 1.5 kg left.

(a) Determine the exponential decay equation for this element.
(b) How long will it take for half of the element to decay?
(c) How long will it take until there is 250 grams of the element left?

23. For a particular radioactive element the value of *k* in the exponential decay equation is given by $k = 0.000825$.

(a) How long will it take for a quarter of the element to decay?
(b) How long will it take for half of the element to decay?
(c) How long will it take 90% of the element to decay?

Section 1-10 : Common Graphs

Without using a graphing calculator sketch the graph of each of the following.

1. $y = -2x + 7$

2. $f(x) = |x + 4|$

3. $g(x) = \sqrt{x} - 5$

4. $g(x) = \tan\left(x + \frac{\pi}{3}\right)$

5. $f(x) = \sec(x) + 2$

6. $h(x) = |x + 2| - 4$

7. $Q(x) = \mathbf{e}^{-x-3} + 6$

8. $V(x) = \sqrt{x - 6} + 3$

9. $g(x) = \sin\left(x + \frac{\pi}{6}\right) - 1$

10. $h(x) = (x + 6)^2 - 8$

11. $W(y) = (y + 5)^2 + 3$

12. $f(y) = (y - 9)^2 - 2$

13. $f(x) = (x - 1)^2 + 6$

14. $R(x) = -\ln(x)$

15. $g(x) = \ln(-x)$

16. $h(x) = x^2 + 8x - 1$

17. $Y(x) = -3x^2 - 6x + 5$

18. $f(y) = -y^2 - 4y - 2$

19. $h(y) = 2y^2 + 2y - 3$

20. $x^2 - 6x + y^2 + 8y + 24 = 0$

21. $x^2 + y^2 + 10y = -9$

22. $\dfrac{(x+4)^2}{25} + \dfrac{(y+2)^2}{25} = 1$

23. $x^2 - 2x + 4y^2 - 16y + 16 = 0$

24. $\dfrac{(x+6)^2}{4} + 16(y-5)^2 = 1$

25. $\dfrac{(y-1)^2}{25} - \dfrac{(x-3)^2}{4} = 1$

26. $(x-4)^2 - 9(y+7)^2 = 1$

Chapter 2 : Limits

Here are a set of assignment problems for the Limits chapter of the Calculus I notes. Please note that these problems do not have any solutions available. These are intended mostly for instructors who might want a set of problems to assign for turning in. Having solutions available (or even just final answers) would defeat the purpose the problems.

If you are looking for some practice problems (with solutions available) please check out the Practice Problems. There you will find a set of problems that should give you quite a bit practice.

Here is a list of all the sections for which assignment problems have been written as well as a brief description of the material covered in the notes for that particular section.

Tangent Lines and Rates of Change – In this section we will introduce two problems that we will see time and again in this course : Rate of Change of a function and Tangent Lines to functions. Both of these problems will be used to introduce the concept of limits, although we won't formally give the definition or notation until the next section.

The Limit – In this section we will introduce the notation of the limit. We will also take a conceptual look at limits and try to get a grasp on just what they are and what they can tell us. We will be estimating the value of limits in this section to help us understand what they tell us. We will actually start computing limits in a couple of sections.

One-Sided Limits – In this section we will introduce the concept of one-sided limits. We will discuss the differences between one-sided limits and limits as well as how they are related to each other.

Limit Properties – In this section we will discuss the properties of limits that we'll need to use in computing limits (as opposed to estimating them as we've done to this point). We will also compute a couple of basic limits in this section.

Computing Limits – In this section we will looks at several types of limits that require some work before we can use the limit properties to compute them. We will also look at computing limits of piecewise functions and use of the Squeeze Theorem to compute some limits.

Infinite Limits – In this section we will look at limits that have a value of infinity or negative infinity. We'll also take a brief look at vertical asymptotes.

Limits At Infinity, Part I – In this section we will start looking at limits at infinity, i.e. limits in which the variable gets very large in either the positive or negative sense. We will concentrate on polynomials and rational expressions in this section. We'll also take a brief look at horizontal asymptotes.

Limits At Infinity, Part II – In this section we will continue covering limits at infinity. We'll be looking at exponentials, logarithms and inverse tangents in this section.

Continuity – In this section we will introduce the concept of continuity and how it relates to limits. We will also see the Intermediate Value Theorem in this section and how it can be used to determine if functions have solutions in a given interval.

The Definition of the Limit – In this section we will give a precise definition of several of the limits covered in this section. We will work several basic examples illustrating how to use this precise definition to compute a limit. We'll also give a precise definition of continuity.

Section 2-1 : Tangent Lines and Rates of Change

1. For the function $f\left(x\right)=x^3-3x^2$ and the point *P* given by $x=3$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) 3.5 **(ii)** 3.1 **(iii)** 3.01 **(iv)** 3.001 **(v)** 3.0001
(vi) 2.5 **(vii)** 2.9 **(viii)** 2.99 **(ix)** 2.999 **(x)** 2.9999

(b) Use the information from **(a)** to estimate the slope of the tangent line to $f\left(x\right)$ at $x=3$ and write down the equation of the tangent line.

2. For the function $g\left(x\right)=\dfrac{x}{x^2+4}$ and the point *P* given by $x=0$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) 1 **(ii)** 0.5 **(iii)** 0.1 **(iv)** 0.01 **(v)** 0.001
(vi) -1 **(vii)** -0.5 **(viii)** -0.1 **(ix)** -0.01 **(x)** -0.001

(b) Use the information from **(a)** to estimate the slope of the tangent line to $g\left(x\right)$ at $x=0$ and write down the equation of the tangent line.

3. For the function $h\left(x\right)=2-\left(x+2\right)^2$ and the point *P* given by $x=-2$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) -2.5 **(ii)** -2.1 **(iii)** -2.01 **(iv)** -2.001 **(v)** -2.0001
(vi) -1.5 **(vii)** -1.9 **(viii)** -1.99 **(ix)** -1.999 **(x)** -1.9999

(b) Use the information from **(a)** to estimate the slope of the tangent line to $h\left(x\right)$ at $x=-2$ and write down the equation of the tangent line.

4. For the function $P(x) = \mathbf{e}^{2-8x^2}$ and the point *P* given by $x = 0.5$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) 1	**(ii)** 0.51	**(iii)** 0.501	**(iv)** 0.5001	**(v)** 0.50001
(vi) 0	**(vii)** 0.49	**(viii)** 0.499	**(ix)** 0.4999	**(x)** 0.49999

(b) Use the information from **(a)** to estimate the slope of the tangent line to $h(x)$ at $x = 0.5$ and write down the equation of the tangent line.

5. The amount of grain in a bin is given by $V(t) = \dfrac{11t+4}{t+4}$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the amount of grain in the bin between $t = 6$ and the following values of *t*.

(i) 6.5	**(ii)** 6.1	**(iii)** 6.01	**(iv)** 6.001	**(v)** 6.0001
(vi) 5.5	**(vii)** 5.9	**(viii)** 5.99	**(ix)** 5.999	**(x)** 5.9999

(b) Use the information from **(a)** to estimate the instantaneous rate of change of the volume of grain in the bin at $t = 6$.

6. The population (in thousands) of insects is given by $P(t) = 2 - \dfrac{1}{\pi}\cos(3\pi t)\sin\left(\dfrac{\pi t}{2}\right)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the population of insects between $t = 4$ and the following values of *t*. Make sure your calculator is set to radians for the computations.

(i) 4.5	**(ii)** 4.1	**(iii)** 4.01	**(iv)** 4.001	**(v)** 4.0001
(vi) 3.5	**(vii)** 3.9	**(viii)** 3.99	**(ix)** 3.999	**(x)** 3.9999

(b) Use the information from **(a)** to estimate the instantaneous rate of change of the population of the insects at $t = 4$.

7. The amount of water in a holding tank is given by $V(t) = 8t^4 - t^2 + 7$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the amount of grain in the bin between $t = 0.25$ and the following values of *t*.

(i) 1	**(ii)** 0.5	**(iii)** 0.251	**(iv)** 0.2501	**(v)** 0.25001
(vi) 0	**(vii)** 0.1	**(viii)** 0.249	**(ix)** 0.2499	**(x)** 0.24999

(b) Use the information from **(a)** to estimate the instantaneous rate of change of the volume of water in the tank at $t = 0.25$.

8. The position of an object is given by $s(t) = x^2 + \dfrac{72}{x+1}$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 5$ and the following values of *t*.

(i) 5.5	**(ii)** 5.1	**(iii)** 5.01	**(iv)** 5.001	**(v)** 5.0001
(vi) 4.5	**(vii)** 4.9	**(viii)** 4.99	**(ix)** 4.999	**(x)** 4.9999

(b) Use the information from **(a)** to estimate the instantaneous velocity of the object at $t = 5$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

9. The position of an object is given by $s(t) = 2\cos(4t - 8) - 7\sin(t - 2)$. Note that a negative position here simply means that the position is to the left of the "zero position" and is perfectly acceptable. Answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 2$ and the following values of *t*. Make sure your calculator is set to radians for the computations.

(i) 2.5	**(ii)** 2.1	**(iii)** 2.01	**(iv)** 2.001	**(v)** 2.0001
(vi) 1.5	**(vii)** 1.9	**(viii)** 1.99	**(ix)** 1.999	**(x)** 1.9999

(b) Use the information from **(a)** to estimate the instantaneous velocity of the object at $t = 2$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

10. The position of an object is given by $s(t) = t^2 - 10t + 11$. Note that a negative position here simply means that the position is to the left of the "zero position" and is perfectly acceptable. Answer each of the following questions.

(a) Determine the time(s) in which the position of the object is at $s = -5$.

(b) Estimate the instantaneous velocity of the object at each of the time(s) found in part **(a)** using the method discussed in this section.

Section 2-2 : The Limit

1. For the function $g\left(x\right)=\dfrac{x^2+6x+9}{x^2+3x}$ answer each of the following questions.

(a) Evaluate the function the following values of *x* compute (accurate to at least 8 decimal places).

(i) -2.5 **(ii)** -2.9 **(iii)** -2.99 **(iv)** -2.999 **(v)** -2.9999
(vi) -3.5 **(vii)** -3.1 **(viii)** -3.01 **(ix)** -3.001 **(x)** -3.0001

(b) Use the information from **(a)** to estimate the value of $\displaystyle\lim_{x\to -3}\frac{x^2+6x+9}{x^2+3x}$.

2. For the function $f\left(z\right)=\dfrac{10z-9-z^2}{z^2-1}$ answer each of the following questions.

(a) Evaluate the function the following values of *t* compute (accurate to at least 8 decimal places).

(i) 1.5 **(ii)** 1.1 **(iii)** 1.01 **(iv)** 1.001 **(v)** 1.0001
(vi) 0.5 **(vii)** 0.9 **(viii)** 0.99 **(ix)** 0.999 **(x)** 0.9999

(b) Use the information from **(a)** to estimate the value of $\displaystyle\lim_{z\to 1}\frac{10z-9-z^2}{z^2-1}$.

3. For the function $h\left(t\right)=\dfrac{2-\sqrt{4+2t}}{t}$ answer each of the following questions.

(a) Evaluate the function the following values of *t* compute (accurate to at least 8 decimal places). Make sure your calculator is set to radians for the computations.

(i) 0.5 **(ii)** 0.1 **(iii)** 0.01 **(iv)** 0.001 **(v)** 0.0001
(vi) -0.5 **(vii)** -0.1 **(viii)** -0.01 **(ix)** -0.001 **(x)** -0.0001

(b) Use the information from **(a)** to estimate the value of $\displaystyle\lim_{t\to 0}\frac{2-\sqrt{4+2t}}{t}$.

4. For the function $g\left(\theta\right)=\dfrac{\cos\left(\theta-4\right)-1}{2\theta-8}$ answer each of the following questions.

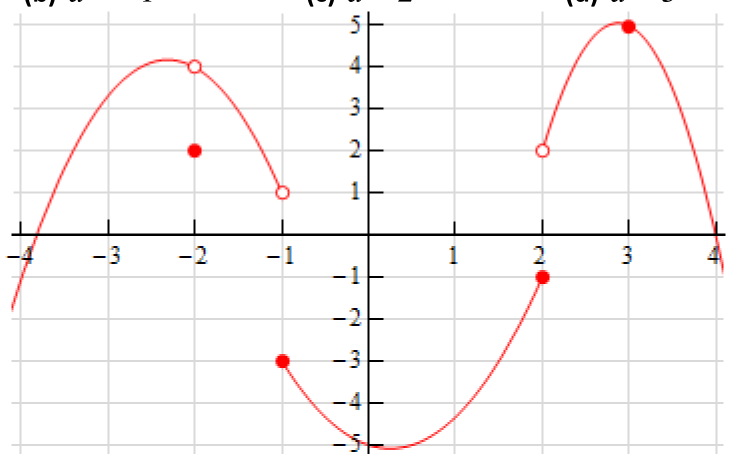
(a) Evaluate the function the following values of θ compute (accurate to at least 8 decimal places). Make sure your calculator is set to radians for the computations.

(i) 4.5 **(ii)** 4.1 **(iii)** 4.01 **(iv)** 4.001 **(v)** 4.0001
(vi) 3.5 **(vii)** 3.9 **(viii)** 3.99 **(ix)** 3.999 **(x)** 3.9999

(b) Use the information from **(a)** to estimate the value of $\lim\limits_{\theta \to 4} \dfrac{\cos(\theta - 4) - 1}{2\theta - 8}$.

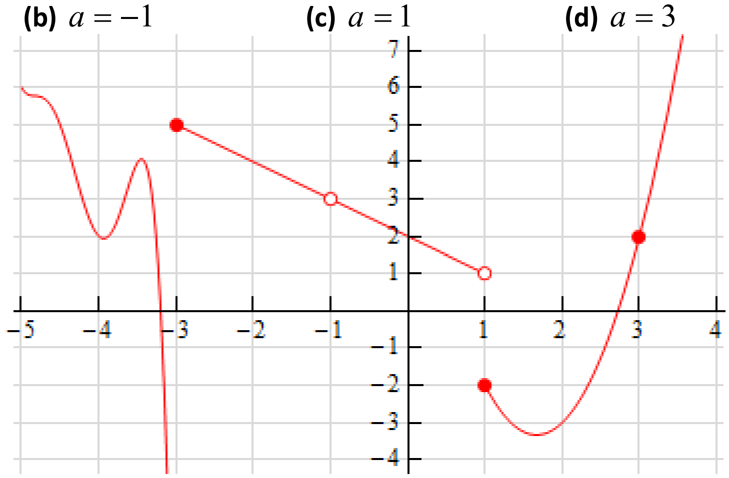
5. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim\limits_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -2$ **(b)** $a = -1$ **(c)** $a = 2$ **(d)** $a = 3$



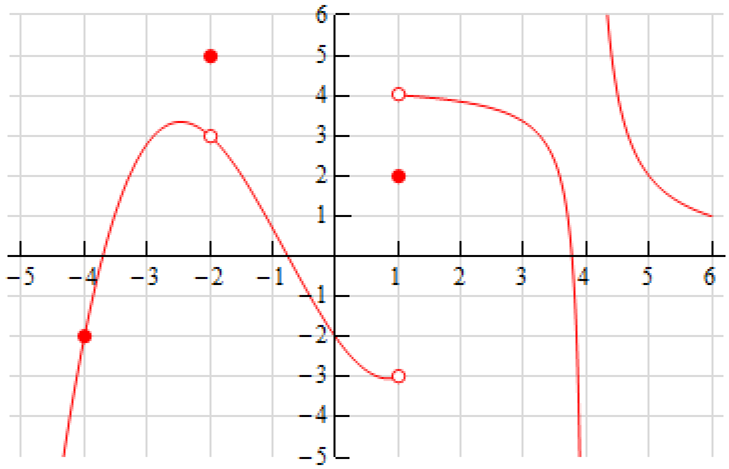
6. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim\limits_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -3$ **(b)** $a = -1$ **(c)** $a = 1$ **(d)** $a = 3$



7. Below is the graph of $f\left(x\right)$. For each of the given points determine the value of $f\left(a\right)$ and $\lim\limits_{x \to a} f\left(x\right)$. If any of the quantities do not exist clearly explain why.

(a) $a = -4$ **(b)** $a = -2$ **(c)** $a = 1$ **(d)** $a = 4$



8. Explain in your own words what the following equation means.

$$\lim_{x \to 12} f\left(x\right) = 6$$

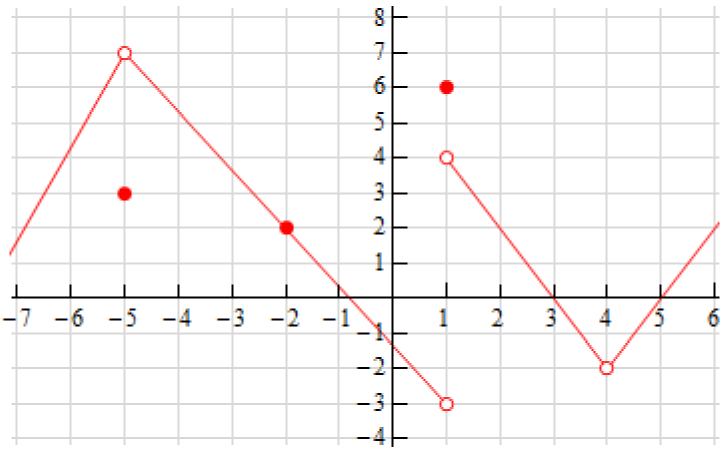
9. Suppose we know that $\lim\limits_{x \to -7} f\left(x\right) = 18$. If possible, determine the value of $f\left(-7\right)$. If it is not possible to determine the value explain why not.

10. Is it possible to have $\lim\limits_{x \to 1} f\left(x\right) = -23$ and $f\left(1\right) = 107$? Explain your answer.

Section 2-3 : One–Sided Limits

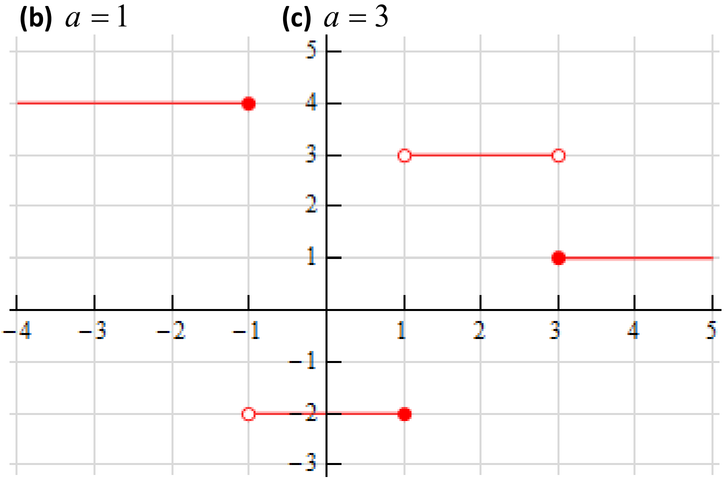
1. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim\limits_{x \to a^-} f(x)$, $\lim\limits_{x \to a^+} f(x)$, and $\lim\limits_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -5$ **(b)** $a = -2$ **(c)** $a = 1$ **(d)** $a = 4$



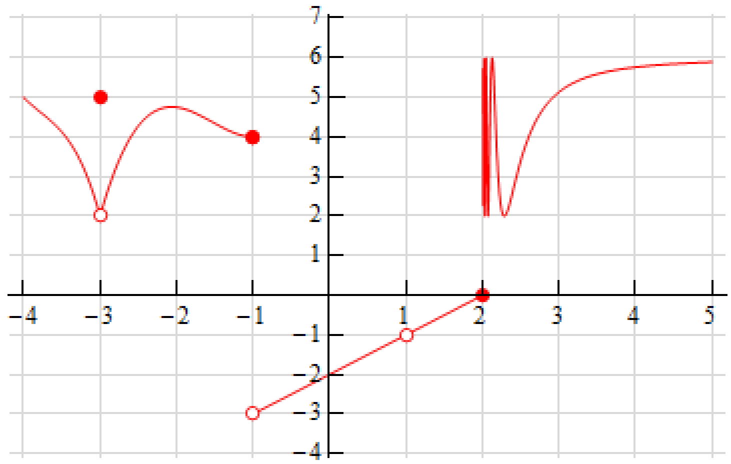
2. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim\limits_{x \to a^-} f(x)$, $\lim\limits_{x \to a^+} f(x)$, and $\lim\limits_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -1$ **(b)** $a = 1$ **(c)** $a = 3$



3. Below is the graph of $f\left(x\right)$. For each of the given points determine the value of $f\left(a\right)$, $\lim\limits_{x\to a^-} f\left(x\right)$, $\lim\limits_{x\to a^+} f\left(x\right)$, and $\lim\limits_{x\to a} f\left(x\right)$. If any of the quantities do not exist clearly explain why.

(a) $a=-3$ **(b)** $a=-1$ **(c)** $a=1$ **(d)** $a=2$



4. Sketch a graph of a function that satisfies each of the following conditions.

$$\lim_{x\to 1^-} f\left(x\right) = -2 \qquad \lim_{x\to 1^+} f\left(x\right) = 3 \qquad f\left(1\right) = 6$$

5. Sketch a graph of a function that satisfies each of the following conditions.

$$\lim_{x\to -3^-} f\left(x\right) = 1 \qquad \lim_{x\to -3^+} f\left(x\right) = 1 \qquad f\left(-3\right) = 4$$

6. Sketch a graph of a function that satisfies each of the following conditions.

$$\lim_{x\to -5^-} f\left(x\right) = -1 \qquad \lim_{x\to -5^+} f\left(x\right) = 7 \qquad f\left(-5\right) = 4$$

$$\lim_{x\to 4} f\left(x\right) = 6 \qquad f\left(4\right) \text{ does not exist}$$

7. Explain in your own words what each of the following equations mean.

$$\lim_{x\to 8^-} f\left(x\right) = 3 \qquad \lim_{x\to 8^+} f\left(x\right) = -1$$

8. Suppose we know that $\lim\limits_{x\to -7} f\left(x\right) = 18$. If possible, determine the value of $\lim\limits_{x\to -7^-} f\left(x\right)$ and the value of $\lim\limits_{x\to -7^+} f\left(x\right)$. If it is not possible to determine one or both of these values explain why not.

9. Suppose we know that $f\left(6\right) = -53$. If possible, determine the value of $\lim\limits_{x\to 6^-} f\left(x\right)$ and the value of $\lim\limits_{x\to 6^+} f\left(x\right)$. If it is not possible to determine one or both of these values explain why not.

Section 2-4 : Limit Properties

1. Given $\lim\limits_{x \to 0} f(x) = 5$, $\lim\limits_{x \to 0} g(x) = -1$ and $\lim\limits_{x \to 0} h(x) = -3$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim\limits_{x \to 0} \left[11 + 7f(x)\right]$

(b) $\lim\limits_{x \to 0} \left[6 - 4g(x) - 10h(x)\right]$

(c) $\lim\limits_{x \to 0} \left[4g(x) - 12f(x) + 3h(x)\right]$

(d) $\lim\limits_{x \to 0} \left[g(x)\left(1 + 2f(x)\right)\right]$

2. Given $\lim\limits_{x \to 12} f(x) = 2$, $\lim\limits_{x \to 12} g(x) = 6$ and $\lim\limits_{x \to 12} h(x) = 9$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim\limits_{x \to 12} \left[h(x)f(x) + \dfrac{1 + g(x)}{g(x)}\right]$

(b) $\lim\limits_{x \to 12} \left[\left(3 - f(x)\right)\left(1 + 2g(x)\right)\right]$

(c) $\lim\limits_{x \to 12} \dfrac{f(x) + 1}{3g(x) - 2h(x)}$

(d) $\lim\limits_{x \to 12} \dfrac{f(x) - 2g(x)}{7 + h(x)f(x)}$

3. Given $\lim\limits_{x \to -1} f(x) = 0$, $\lim\limits_{x \to -1} g(x) = 9$ and $\lim\limits_{x \to -1} h(x) = -7$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim\limits_{x \to -1} \left[\left(g(x)\right)^2 - \left(h(x)\right)^3\right]$

(b) $\lim\limits_{x \to -1} \sqrt{3 + 6f(x) - h(x)}$

(c) $\lim\limits_{x \to -1} \sqrt{f(x) - g(x)h(x)}$

(d) $\lim\limits_{x \to -1} \sqrt[4]{\dfrac{2 + g(x)}{1 - 10h(x)}}$

For each of the following limits use the limit properties given in this section to compute the limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

4. $\lim\limits_{x \to 4} \left(3x^2 - 9x + 2\right)$

5. $\lim\limits_{w \to -1} \left(w - \left(w^2 + 3\right)^2\right)$

6. $\lim\limits_{t \to 0} \left(t^4 - 4t^2 + 12t - 8 \right)$

7. $\lim\limits_{z \to 2} \dfrac{10 + z^2}{3 - 4z}$

8. $\lim\limits_{x \to 7} \dfrac{8x}{x^2 - 14x + 49}$

9. $\lim\limits_{y \to -3} \dfrac{y^3 - 20y + 4}{y^2 + 8y - 1}$

10. $\lim\limits_{w \to -6} \sqrt[3]{8 + 7w}$

11. $\lim\limits_{t \to 1} \left(4t^2 - \sqrt{8t + 1} \right)$

12. $\lim\limits_{x \to 8} \left(\sqrt[4]{3x - 8} + \sqrt{9 + 2x} \right)$

Section 2-5 : Computing Limits

For problems 1 – 20 evaluate the limit, if it exists.

1. $\displaystyle \lim_{x \to -9} \left(1 - 4x^3\right)$

2. $\displaystyle \lim_{y \to 1} \left(6y^4 - 7y^3 + 12y + 25\right)$

3. $\displaystyle \lim_{t \to 0} \frac{t^2 + 6}{t^2 - 3}$

4. $\displaystyle \lim_{z \to 4} \frac{6z}{2 + 3z^2}$

5. $\displaystyle \lim_{w \to -2} \frac{w + 2}{w^2 - 6w - 16}$

6. $\displaystyle \lim_{t \to -5} \frac{t^2 + 6t + 5}{t^2 + 2t - 15}$

7. $\displaystyle \lim_{x \to 3} \frac{5x^2 - 16x + 3}{9 - x^2}$

8. $\displaystyle \lim_{z \to 1} \frac{10 - 9z - z^2}{3z^2 + 4z - 7}$

9. $\displaystyle \lim_{x \to -2} \frac{x^3 + 8}{x^2 + 8x + 12}$

10. $\displaystyle \lim_{t \to 8} \frac{t(t - 5) - 24}{t^2 - 8t}$

11. $\displaystyle \lim_{w \to -4} \frac{w^2 - 16}{(w - 2)(w + 3) - 6}$

12. $\displaystyle \lim_{h \to 0} \frac{(2 + h)^3 - 8}{h}$

13. $\displaystyle \lim_{h \to 0} \frac{(1 + h)^4 - 1}{h}$

14. $\displaystyle\lim_{t\to 25}\frac{5-\sqrt{t}}{t-25}$

15. $\displaystyle\lim_{x\to 2}\frac{x-2}{\sqrt{2}-\sqrt{x}}$

16. $\displaystyle\lim_{z\to 6}\frac{z-6}{\sqrt{3z-2}-4}$

17. $\displaystyle\lim_{z\to -2}\frac{3-\sqrt{1-4z}}{2z+4}$

18. $\displaystyle\lim_{t\to 3}\frac{3-t}{\sqrt{t+1}-\sqrt{5t-11}}$

19. $\displaystyle\lim_{x\to 7}\frac{\frac{1}{7}-\frac{1}{x}}{x-7}$

20. $\displaystyle\lim_{y\to -1}\frac{\frac{1}{4+3y}+\frac{1}{y}}{y+1}$

21. Given the function

$$f(x)=\begin{cases}15 & x<-4\\ 6-2x & x\geq -4\end{cases}$$

Evaluate the following limits, if they exist.

(a) $\displaystyle\lim_{x\to -7} f(x)$ (b) $\displaystyle\lim_{x\to -4} f(x)$

22. Given the function

$$g(t)=\begin{cases}t^2-t^3 & t<2\\ 5t-14 & t\geq 2\end{cases}$$

Evaluate the following limits, if they exist.

(a) $\displaystyle\lim_{t\to -3} g(t)$ (b) $\displaystyle\lim_{t\to 2} g(t)$

23. Given the function

$$h(w) = \begin{cases} 2w^2 & w \le 6 \\ w-8 & w > 6 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim\limits_{w \to 6} h(w)$ (b) $\lim\limits_{w \to 2} h(w)$

24. Given the function

$$g(x) = \begin{cases} 5x+24 & x < -3 \\ x^2 & -3 \le x < 4 \\ 1-2x & x \ge 4 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim\limits_{x \to -3} g(x)$ (b) $\lim\limits_{x \to 0} g(x)$ (c) $\lim\limits_{x \to 4} g(x)$ (d) $\lim\limits_{x \to 12} g(x)$

For problems 25 – 30 evaluate the limit, if it exists.

25. $\lim\limits_{z \to -10} \left(|t+10| + 3 \right)$

26. $\lim\limits_{x \to 4} \left(9 + |8-2x| \right)$

27. $\lim\limits_{h \to 0} \dfrac{|h|}{h}$

28. $\lim\limits_{t \to 2} \dfrac{2-t}{|t-2|}$

29. $\lim\limits_{w \to -5} \dfrac{|2w+10|}{w+5}$

30. $\lim\limits_{x \to 4} \dfrac{|x-4|}{x^2-16}$

31. Given that $3+2x \le f(x) \le x-1$ for all *x* determine the value of $\lim\limits_{x \to -4} f(x)$.

32. Given that $\sqrt{x+7} \le f(x) \le \dfrac{x-1}{2}$ for all *x* determine the value of $\lim\limits_{x \to 9} f(x)$.

33. Use the Squeeze Theorem to determine the value of $\lim\limits_{x \to 0} x^4 \cos\left(\dfrac{3}{x}\right)$.

34. Use the Squeeze Theorem to determine the value of $\lim\limits_{x \to 0} x \cos\left(\dfrac{1}{x}\right)$.

35. Use the Squeeze Theorem to determine the value of $\lim\limits_{x \to 1} \left(x-1\right)^2 \cos\left(\dfrac{1}{x-1}\right)$.

Section 2-6 : Infinite Limits

For problems 1 – 8 evaluate the indicated limits, if they exist.

1. For $g\left(x\right)=\dfrac{-4}{\left(x-1\right)^{2}}$ evaluate,

(a) $\lim\limits_{x\to 1^{-}} g\left(x\right)$ (b) $\lim\limits_{x\to 1^{+}} g\left(x\right)$ (c) $\lim\limits_{x\to 1} g\left(x\right)$

2. For $h\left(z\right)=\dfrac{17}{\left(4-z\right)^{3}}$ evaluate,

(a) $\lim\limits_{z\to 4^{-}} h\left(z\right)$ (b) $\lim\limits_{z\to 4^{+}} h\left(z\right)$ (c) $\lim\limits_{z\to 4} h\left(z\right)$

3. For $g\left(t\right)=\dfrac{4t^{2}}{\left(t+3\right)^{7}}$ evaluate,

(a) $\lim\limits_{t\to -3^{-}} g\left(t\right)$ (b) $\lim\limits_{t\to -3^{+}} g\left(t\right)$ (c) $\lim\limits_{t\to -3} g\left(t\right)$

4. For $f\left(x\right)=\dfrac{1+x}{x^{3}+8}$ evaluate,

(a) $\lim\limits_{x\to -2^{-}} f\left(x\right)$ (b) $\lim\limits_{x\to -2^{+}} f\left(x\right)$ (c) $\lim\limits_{x\to -2} f\left(x\right)$

5. For $f\left(x\right)=\dfrac{x-1}{\left(x^{2}-9\right)^{4}}$ evaluate,

(a) $\lim\limits_{x\to 3^{-}} f\left(x\right)$ (b) $\lim\limits_{x\to 3^{+}} f\left(x\right)$ (c) $\lim\limits_{x\to 3} f\left(x\right)$

6. For $W\left(t\right)=\ln\left(t+8\right)$ evaluate,

(a) $\lim\limits_{t\to -8^{-}} W\left(t\right)$ (b) $\lim\limits_{t\to -8^{+}} W\left(t\right)$ (c) $\lim\limits_{t\to -8} W\left(t\right)$

7. For $h\left(z\right)=\ln\left|z\right|$ evaluate,

(a) $\lim\limits_{z\to 0^{-}} h\left(z\right)$ (b) $\lim\limits_{z\to 0^{+}} h\left(z\right)$ (c) $\lim\limits_{z\to 0} h\left(z\right)$

8. For $R\left(y\right)=\cot\left(y\right)$ evaluate,

(a) $\lim\limits_{y\to \pi^{-}} R\left(y\right)$ (b) $\lim\limits_{y\to \pi^{+}} R\left(y\right)$ (c) $\lim\limits_{y\to \pi} R\left(y\right)$

For problems 9 – 12 find all the vertical asymptotes of the given function.

9. $h(x) = \dfrac{-6}{9-x}$

10. $f(x) = \dfrac{x+8}{x^2(5-2x)^3}$

11. $g(t) = \dfrac{5t}{t(t+7)(t-12)}$

12. $g(z) = \dfrac{z^2+1}{(z^2-1)^5(z+15)^6}$

Section 2-7 : Limits at Infinity, Part I

1. For $f\left(x\right)=8x+9x^3-11x^5$ evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to-\infty} f\left(x\right)$ **(b)** $\displaystyle\lim_{x\to\infty} f\left(x\right)$

2. For $h\left(t\right)=10t^2+t^4+6t-2$ evaluate each of the following limits.

(a) $\displaystyle\lim_{t\to-\infty} h\left(t\right)$ **(b)** $\displaystyle\lim_{t\to\infty} h\left(t\right)$

3. For $g\left(z\right)=7+8z+\sqrt[3]{z^4}$ evaluate each of the following limits.

(a) $\displaystyle\lim_{z\to-\infty} g\left(z\right)$ **(b)** $\displaystyle\lim_{z\to\infty} g\left(z\right)$

For problems 4 – 17 answer each of the following questions.

(a) Evaluate $\displaystyle\lim_{x\to-\infty} f\left(x\right)$

(b) Evaluate $\displaystyle\lim_{x\to\infty} f\left(x\right)$

(c) Write down the equation(s) of any horizontal asymptotes for the function.

4. $f\left(x\right)=\dfrac{10x^3-6x}{7x^3+9}$

5. $f\left(x\right)=\dfrac{12+x}{3x^2-8x+23}$

6. $f\left(x\right)=\dfrac{5x^8-9}{x^3+10x^5-3x^8}$

7. $f\left(x\right)=\dfrac{2-6x-9x^2}{15x^2+x-4}$

8. $f\left(x\right)=\dfrac{5x+7x^4}{4-x^2}$

9. $f\left(x\right)=\dfrac{4x^3-3x^2+2x-1}{10-5x+x^3}$

10. $f\left(x\right)=\dfrac{5-x^8}{2x^3-7x+1}$

11. $f\left(x\right)=\dfrac{1+4\sqrt[3]{x^2}}{9+10x}$

12. $f\left(x\right)=\dfrac{25x+7}{\sqrt{5x^2+2}}$

13. $f\left(x\right)=\dfrac{\sqrt{8+11x^2}}{-9-x}$

14. $f\left(x\right)=\dfrac{\sqrt{9x^4+2x^2+3}}{5x-2x^2}$

15. $f\left(x\right)=\dfrac{6+x^3}{\sqrt{8+4x^6}}$

16. $f\left(x\right)=\dfrac{\sqrt[3]{2-8x^3}}{4+7x}$

17. $f\left(x\right)=\dfrac{1+x}{\sqrt[4]{5+2x^4}}$

Section 2-8 : Limits At Infinity, Part II

For problems 1 – 11 evaluate **(a)** $\lim\limits_{x \to -\infty} f\left(x\right)$ and **(b)** $\lim\limits_{x \to \infty} f\left(x\right)$.

1. $f\left(x\right) = \mathbf{e}^{x^4 + 8x}$

2. $f\left(x\right) = \mathbf{e}^{2x + 4x^2 + 2x^5}$

3. $f\left(x\right) = \mathbf{e}^{\frac{3 - x^3}{x + x^2}}$

4. $f\left(x\right) = \mathbf{e}^{\frac{5 - 9x}{7 + 3x}}$

5. $f\left(x\right) = \mathbf{e}^{\frac{5 + 2x^6}{x - 8x^4}}$

6. $f\left(x\right) = \mathbf{e}^{x} + 12\mathbf{e}^{-3x} - 2\mathbf{e}^{-10x}$

7. $f\left(x\right) = 9\mathbf{e}^{2x} - 7\mathbf{e}^{-14x} - \mathbf{e}^{x}$

8. $f\left(x\right) = 20\mathbf{e}^{-8x} - \mathbf{e}^{5x} + 3\mathbf{e}^{2x} - \mathbf{e}^{-7x}$

9. $f\left(x\right) = \dfrac{6\mathbf{e}^{4x} + \mathbf{e}^{-15x}}{11\mathbf{e}^{4x} + 6\mathbf{e}^{-15x}}$

10. $f\left(x\right) = \dfrac{\mathbf{e}^{3x} + 9\mathbf{e}^{-x} - 4\mathbf{e}^{10x}}{2\mathbf{e}^{7x} - \mathbf{e}^{-x}}$

11. $f\left(x\right) = \dfrac{3\mathbf{e}^{-14x} - \mathbf{e}^{18x}}{\mathbf{e}^{-x} - 2\mathbf{e}^{20x} - \mathbf{e}^{-9x}}$

For problems 12 – 19 evaluate the given limit.

12. $\lim\limits_{x \to \infty} \ln\left(5x^2 + 12x - 6\right)$

13. $\lim\limits_{y \to -\infty} \ln\left(5 - 7y^5\right)$

14. $\lim\limits_{x \to \infty} \ln\left(\dfrac{3+x}{1+5x^3}\right)$

15. $\lim\limits_{t \to -\infty} \ln\left(\dfrac{2t-5t^3}{4+3t^2}\right)$

16. $\lim\limits_{z \to \infty} \ln\left(\dfrac{10z+8z^2}{z^2-1}\right)$

17. $\lim\limits_{x \to -\infty} \tan^{-1}\left(7+4x-x^3\right)$

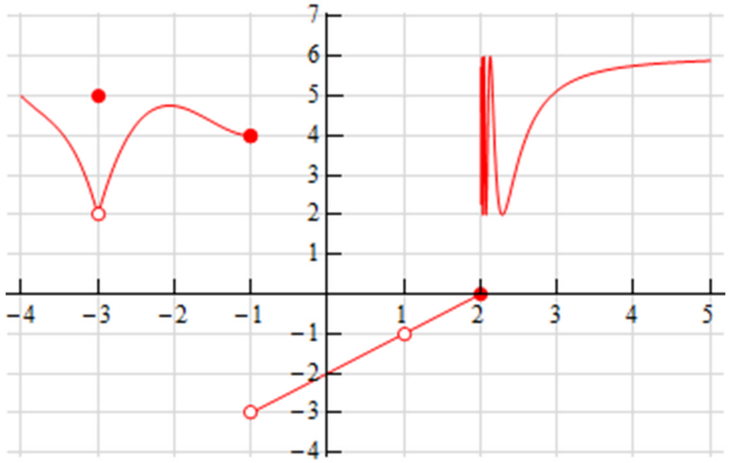
18. $\lim\limits_{w \to \infty} \tan^{-1}\left(4w^2-w^6\right)$

19. $\lim\limits_{t \to \infty} \tan^{-1}\left(\dfrac{4t^3+t^2}{1+3t}\right)$

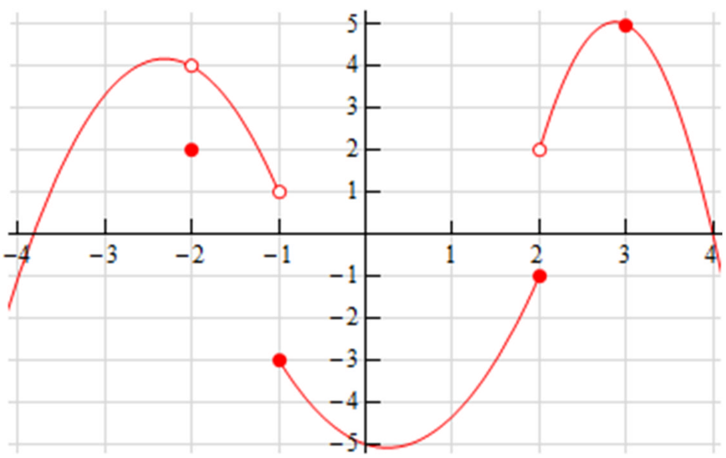
20. $\lim\limits_{z \to -\infty} \tan^{-1}\left(\dfrac{z^4+4}{3z^2+5z^3}\right)$

Section 2-9 : Continuity

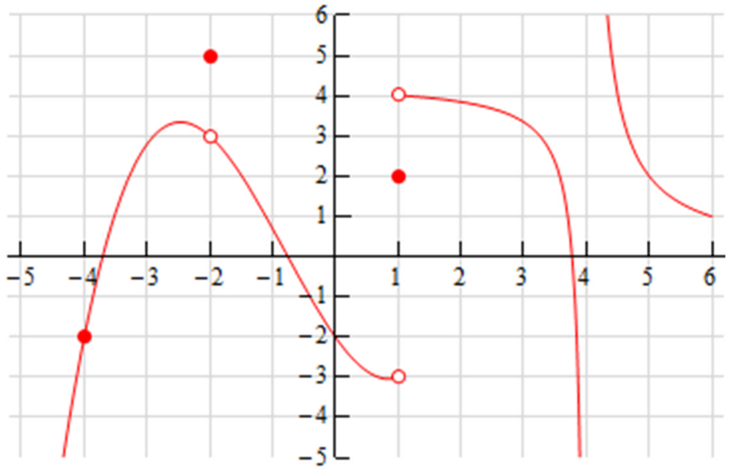
1. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.



2. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.



3. The graph of $f\left(x\right)$ is given below. Based on this graph determine where the function is discontinuous.



For problems 4 – 13 using only Properties 1- 9 from the **Limit Properties** section, one-sided limit properties (if needed) and the definition of continuity determine if the given function is continuous or discontinuous at the indicated points.

4. $f\left(x\right)=\dfrac{6+2x}{7x-14}$

(a) $x=-3$, **(b)** $x=0$, **(c)** $x=2$

5. $R\left(y\right)=\dfrac{2y}{y^2-25}$

(a) $y=-5$, **(b)** $y=-1$, **(c)** $y=3$

6. $g\left(z\right)=\dfrac{5z-20}{z^2-12z}$

(a) $z=-1$, **(b)** $z=0$, **(c)** $z=4$

7. $W\left(x\right)=\dfrac{2+x}{x^2+6x-7}$

(a) $x=-7$, **(b)** $x=0$, **(c)** $x=1$

8. $h\left(z\right)=\begin{cases} 2z^2 & z<-1 \\ 4z+6 & z\geq -1 \end{cases}$

(a) $z=-6$, **(b)** $z=-1$

9. $g\left(x\right)=\begin{cases} x+\mathbf{e}^x & x<0 \\ x^2 & x\geq 0 \end{cases}$

(a) $x=0$, **(b)** $x=4$

10. $Z(t)=\begin{cases} 8 & t<5 \\ 1-6t & t\ge 5 \end{cases}$

(a) $t=0$, **(b)** $t=5$

11. $h(z)=\begin{cases} z+2 & z<-4 \\ 0 & z=-4 \\ 18-z^2 & z>-4 \end{cases}$

(a) $z=-4$, **(b)** $z=2$

12. $f(x)=\begin{cases} 1-x^2 & x<2 \\ -3 & x=2 \\ 2x-7 & 2<x<7 \\ 0 & x=7 \\ x^2 & x>7 \end{cases}$

(a) $x=2$, **(b)** $x=7$

13. $g(w)=\begin{cases} 3w & w<0 \\ 0 & w=0 \\ w+6 & 0<w<8 \\ 14 & w=8 \\ 22-w & w>8 \end{cases}$

(a) $w=0$, **(b)** $w=8$

For problems 14 – 22 determine where the given function is discontinuous.

14. $f(x)=\dfrac{11-2x}{2x^2-13x-7}$

15. $Q(z)=\dfrac{3}{2z^2+3z-4}$

16. $h(t)=\dfrac{t^2-1}{t^3+6t^2+t}$

17. $f(z)=\dfrac{4z+1}{5\cos\left(\frac{z}{2}\right)+1}$

18. $h(x)=\dfrac{1-x}{x\sin(x-1)}$

19. $f\left(x\right)=\dfrac{3}{4\mathbf{e}^{x-7}-1}$

20. $R\left(w\right)=\dfrac{\mathbf{e}^{w^2+1}}{\mathbf{e}^{w}-2\mathbf{e}^{1-w}}$

21. $g\left(x\right)=\cot\left(4x\right)$

22. $f\left(t\right)=\sec\left(\sqrt{t}\right)$

For problems 23 – 27 use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.

23. $1+7x^3-x^4=0$ on $\left[4,8\right]$

24. $z^2+11z=3$ on $\left[-15,-5\right]$

25. $\dfrac{t^2+t-15}{t-8}=0$ on $\left[-5,1\right]$

26. $\ln\left(2t^2+1\right)-\ln\left(t^2+4\right)=0$ on $\left[-1,2\right]$

27. $10=w^3+w^2\mathbf{e}^{-w}-5$ on $\left[0,4\right]$

For problems 28 – 33 assume that $f\left(x\right)$ is continuous everywhere unless otherwise indicated in some way. From the given information is it possible to determine if there is a root of $f\left(x\right)$ in the given interval?

If it is possible to determine that there is a root in the given interval clearly explain how you know that a root must exist. If it is not possible to determine if there is a root in the interval sketch a graph of two functions each of which meets the given information and one will have a root in the given interval and the other will not have a root in the given interval.

28. $f\left(-5\right)=12$ and $f\left(0\right)=-3$ on the interval $\left[-5,0\right]$.

29. $f\left(1\right)=30$ and $f\left(9\right)=6$ on the interval $\left[1,9\right]$.

30. $f\left(20\right)=-100$ and $f\left(40\right)=-100$ on the interval $\left[20,40\right]$.

31. $f\left(-4\right)=-10$, $f\left(5\right)=17$, $\lim\limits_{x\to 1^-} f\left(x\right)=-2$, and $\lim\limits_{x\to 1^+} f\left(x\right)=4$ on the interval $\left[-4,5\right]$.

32. $f\left(-8\right)=2$, $f\left(1\right)=23$, $\lim\limits_{x\to -4^-} f\left(x\right)=35$, and $\lim\limits_{x\to -4^+} f\left(x\right)=1$ on the interval $\left[-8,1\right]$.

33. $f\left(0\right)=-1$, $f\left(9\right)=10$, $\lim\limits_{x\to 2^-} f\left(x\right)=-12$, and $\lim\limits_{x\to 2^+} f\left(x\right)=-3$ on the interval $\left[0,9\right]$.

Section 2-10 : The Definition of the Limit

Use the definition of the limit to prove the following limits.

1. $\lim\limits_{x \to -4} \left(2x\right) = -8$

2. $\lim\limits_{x \to 1} \left(-7x\right) = -7$

3. $\lim\limits_{x \to 3} \left(2x + 8\right) = 14$

4. $\lim\limits_{x \to 2} \left(5 - x\right) = 3$

5. $\lim\limits_{x \to -2} x^2 = 4$

6. $\lim\limits_{x \to 4} x^2 = 16$

7. $\lim\limits_{x \to 1} \left(x^2 + x + 6\right) = 8$

8. $\lim\limits_{x \to -2} \left(x^2 + 3x - 1\right) = -3$

9. $\lim\limits_{x \to 1} x^4 = 1$

10. $\lim\limits_{x \to -6} \dfrac{1}{\left(x+6\right)^2} = \infty$

11. $\lim\limits_{x \to 0} \dfrac{-3}{x^2} = -\infty$

12. $\lim\limits_{x \to 0^+} \dfrac{1}{x} = \infty$

13. $\lim\limits_{x \to 1^-} \dfrac{1}{x-1} = -\infty$

14. $\lim\limits_{x \to -\infty} \dfrac{1}{x^2} = 0$

15. $\lim\limits_{x\to\infty}\dfrac{1}{x^3}=0$

Chapter 3 : Derivatives

Here are a set of assignment problems for the Derivatives chapter of the Calculus I notes. Please note that these problems do not have any solutions available. These are intended mostly for instructors who might want a set of problems to assign for turning in. Having solutions available (or even just final answers) would defeat the purpose the problems.

If you are looking for some practice problems (with solutions available) please check out the Practice Problems. There you will find a set of problems that should give you quite a bit practice.

Here is a list of all the sections for which assignment problems have been written as well as a brief description of the material covered in the notes for that particular section.

The Definition of the Derivative **–** In this section we define the derivative, give various notations for the derivative and work a few problems illustrating how to use the definition of the derivative to actually compute the derivative of a function.

Interpretation of the Derivative – In this section we give several of the more important interpretations of the derivative. We discuss the rate of change of a function, the velocity of a moving object and the slope of the tangent line to a graph of a function.

Differentiation Formulas – In this section we give most of the general derivative formulas and properties used when taking the derivative of a function. Examples in this section concentrate mostly on polynomials, roots and more generally variables raised to powers.

Product and Quotient Rule – In this section we will give two of the more important formulas for differentiating functions. We will discuss the Product Rule and the Quotient Rule allowing us to differentiate functions that, up to this point, we were unable to differentiate.

Derivatives of Trig Functions – In this section we will discuss differentiating trig functions. Derivatives of all six trig functions are given and we show the derivation of the derivative of sin(x) and tan(x).

Derivatives of Exponential and Logarithm Functions – In this section we derive the formulas for the derivatives of the exponential and logarithm functions.

Derivatives of Inverse Trig Functions – In this section we give the derivatives of all six inverse trig functions. We show the derivation of the formulas for inverse sine, inverse cosine and inverse tangent.

Derivatives of Hyperbolic Functions – In this section we define the hyperbolic functions, give the relationships between them and some of the basic facts involving hyperbolic functions. We also give the derivatives of each of the six hyperbolic functions and show the derivation of the formula for hyperbolic sine.

Chain Rule – In this section we discuss one of the more useful and important differentiation formulas, The Chain Rule. With the chain rule in hand we will be able to differentiate a much wider variety of functions. As you will see throughout the rest of your Calculus courses a great many of derivatives you take will involve the chain rule!

Implicit Differentiation – In this section we will discuss implicit differentiation. Not every function can be explicitly written in terms of the independent variable, e.g. y = f(x) and yet we will still need to know what f'(x) is. Implicit differentiation will allow us to find the derivative in these cases. Knowing implicit differentiation will allow us to do one of the more important applications of derivatives, Related Rates (the next section).

Related Rates – In this section we will discuss the only application of derivatives in this section, Related Rates. In related rates problems we are give the rate of change of one quantity in a problem and asked to determine the rate of one (or more) quantities in the problem. This is often one of the more difficult sections for students. We work quite a few problems in this section so hopefully by the end of this section you will get a decent understanding on how these problems work.

Higher Order Derivatives – In this section we define the concept of higher order derivatives and give a quick application of the second order derivative and show how implicit differentiation works for higher order derivatives.

Logarithmic Differentiation – In this section we will discuss logarithmic differentiation. Logarithmic differentiation gives an alternative method for differentiating products and quotients (sometimes easier than using product and quotient rule). More importantly, however, is the fact that logarithm differentiation allows us to differentiate functions that are in the form of one function raised to another function, i.e. there are variables in both the base and exponent of the function.

Section 3-1 : The Definition of the Derivative

Use the definition of the derivative to find the derivative of the following functions.

1. $g(x) = 10$

2. $T(y) = -8$

3. $f(x) = 5x + 7$

4. $Q(t) = 1 - 12t$

5. $f(z) = z^2 + 3$

6. $R(w) = w^2 - 8w + 20$

7. $V(t) = 6t - t^2$

8. $Q(t) = 2t^2 - 8t + 10$

9. $g(z) = 1 + 10z - 7z^2$

10. $f(x) = 5x - x^3$

11. $Y(t) = 2t^3 + 9t + 5$

12. $Z(x) = 2x^3 - x^2 - x$

13. $f(t) = \dfrac{2}{t-3}$

14. $g(x) = \dfrac{x+2}{1-x}$

15. $Q(t) = \dfrac{t^2}{t+2}$

16. $f(w) = \sqrt{w+8}$

17. $V(t) = \sqrt{14+3t}$

18. $G(x) = \sqrt{2-5x}$

19. $Q(t) = \sqrt{1+4t}$

20. $f(x) = \sqrt{x^2+1}$

21. $W(t) = \dfrac{1}{\sqrt{t}}$

22. $g(x) = \dfrac{4}{\sqrt{1-x}}$

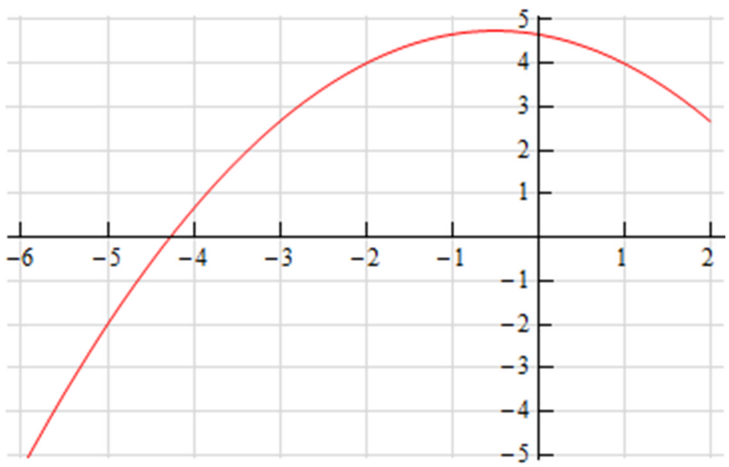
23. $f(x) = x + \sqrt{x}$

24. $f(x) = x + \dfrac{1}{x}$

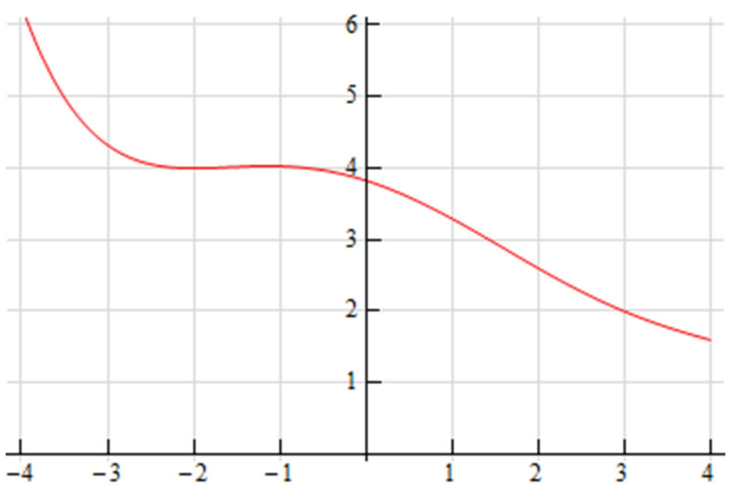
Section 3-2 : Interpretation of the Derivative

For problems 1 – 3 use the graph of the function, $f(x)$, estimate the value of $f'(a)$ for the given values of *a*.

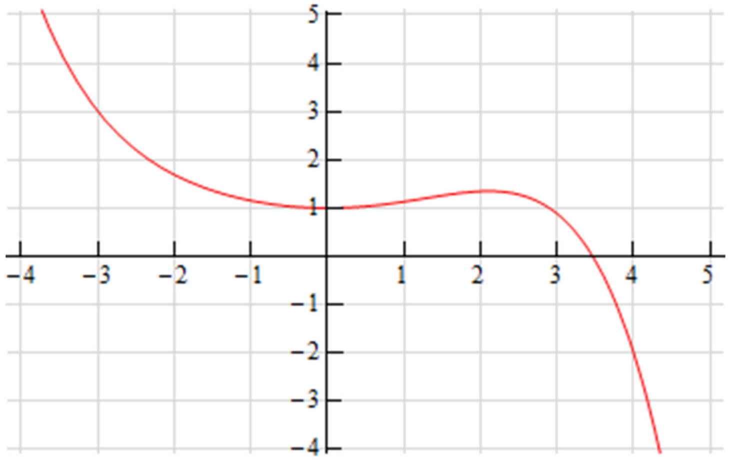
1. **(a)** $a = -5$ **(b)** $a = 1$



2. **(a)** $a = -2$ **(b)** $a = 3$



3. **(a)** $a=-3$ **(b)** $a=4$



For problems 4 – 6 sketch the graph of a function that satisfies the given conditions.

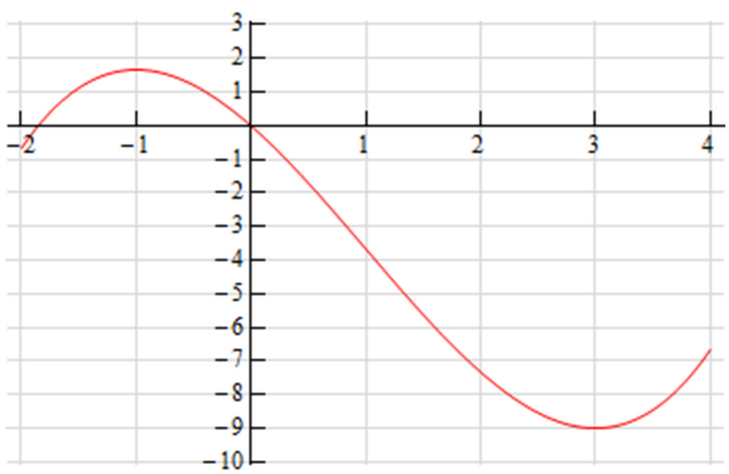
4. $f(-7)=5$, $f'(-7)=-3$, $f(4)=-1$, $f'(4)=1$

5. $f(1)=2$, $f'(1)=4$, $f(6)=2$, $f'(6)=3$

6. $f(-1)=-9$, $f'(-1)=0$, $f(2)=-1$, $f'(2)=3$, $f(5)=4$, $f'(5)=-1$

For problems 7 – 9 the graph of a function, $f(x)$, is given. Use this to sketch the graph of the derivative, $f'(x)$.

7.



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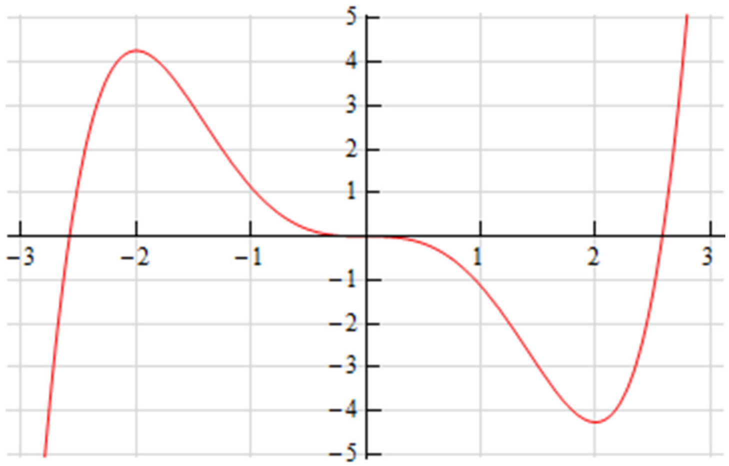

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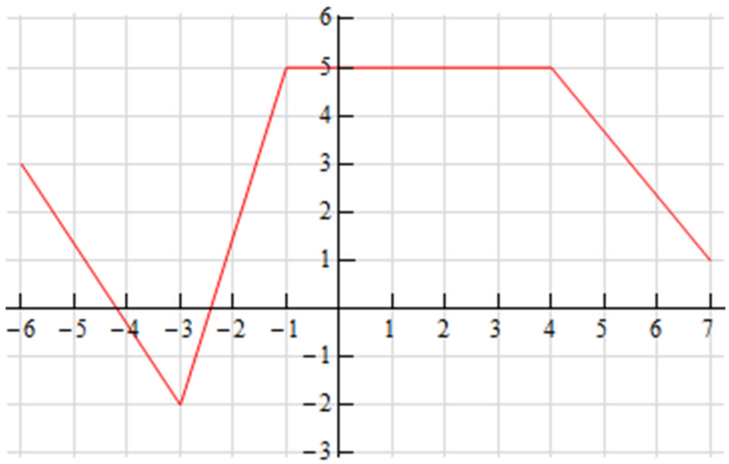


10. Answer the following questions about the function $g(z) = 1 + 10z - 7z^2$.

(a) Is the function increasing or decreasing at $z = 0$?
(b) Is the function increasing or decreasing at $z = 2$?
(c) Does the function ever stop changing? If yes, at what value(s) of *z* does the function stop changing?

11. What is the equation of the tangent line to $f(x) = 5x - x^3$ at $x = 1$.

12. The position of an object at any time *t* is given by $s(t) = 2t^2 - 8t + 10$.

(a) Determine the velocity of the object at any time *t*.
(b) Is the object moving to the right or left at $t = 1$?
(c) Is the object moving to the right or left at $t = 4$?
(d) Does the object ever stop moving? If so, at what time(s) does the object stop moving?

13. Does the function $R(w) = w^2 - 8w + 20$ ever stop changing? If yes, at what value(s) of *w* does the function stop changing?

14. Suppose that the volume of air in a balloon for $0 \le t \le 6$ is given by $V(t) = 6t - t^2$.

(a) Is the volume of air increasing or decreasing at $t = 2$?
(b) Is the volume of air increasing or decreasing at $t = 5$?
(c) Does the volume of air ever stop changing? If yes, at what times(s) does the volume stop changing?

15. What is the equation of the tangent line to $f(x) = 5x + 7$ at $x = -4$?

16. Answer the following questions about the function $Z(x) = 2x^3 - x^2 - x$.

(a) Is the function increasing or decreasing at $x = -1$?
(b) Is the function increasing or decreasing at $x = 2$?
(c) Does the function ever stop changing? If yes, at what value(s) of *x* does the function stop changing?

17. Determine if the function $V(t) = \sqrt{14 + 3t}$ increasing or decreasing at the given points.

(a) $t = 0$
(b) $t = 5$
(c) $t = 100$

18. Suppose that the volume of water in a tank for $t \ge 0$ is given by $Q(t) = \dfrac{t^2}{t+2}$.

(a) Is the volume of water increasing or decreasing at $t = 0$?
(b) Is the volume of water increasing or decreasing at $t = 3$?
(c) Does the volume of water ever stop changing? If so, at what times(s) does the volume stop changing?

19. What is the equation of the tangent line to $g(x) = 10$ at $x = 16$?

20. The position of an object at any time *t* is given by $Q(t) = \sqrt{1 + 4t}$.

(a) Determine the velocity of the object at any time *t*.
(b) Does the object ever stop moving? If so, at what time(s) does the object stop moving?

21. Does the function $Y(t) = 2t^3 + 9t + 5$ ever stop changing? If yes, at what value(s) of *t* does the function stop changing?

Section 3-3 : Differentiation Formulas

For problems 1 – 20 find the derivative of the given function.

1. $g(x) = 8 - 4x^3 + 2x^8$

2. $f(z) = z^{10} - 7z^5 + 2z^3 - z^2$

3. $y = 8x^4 - 10x^3 - 9x + 4$

4. $f(x) = 3x^{-4} + x^4 - 3x$

5. $R(t) = 9t^{10} + 8t^{-10} + 12$

6. $h(y) = 3y^{-6} - 8y^{-3} + 9y^{-1}$

7. $g(t) = t^{-7} + 2t^{-3} - 6t^{-2} + 8t^4 - 1$

8. $z = \sqrt[6]{x} - 7\sqrt[4]{x} + 3\sqrt{x}$

9. $f(x) = 7\sqrt[9]{x^4} - 2\sqrt[2]{x^7} + \sqrt[3]{x^4}$

10. $h(y) = 6\sqrt{y} + \sqrt[6]{y^5} + \dfrac{7}{\sqrt[9]{y^2}}$

11. $g(z) = \dfrac{4}{z^2} + \dfrac{1}{7z^5} - \dfrac{1}{2z}$

12. $y = \dfrac{2}{3t^9} + \dfrac{1}{7t^3} - 9t^2 - \sqrt{t^3}$

13. $W(x) = x^3 - \dfrac{1}{x^6} + \dfrac{1}{\sqrt[5]{x^2}}$

14. $g(w) = (w-5)(w^2+1)$

15. $h(x) = \sqrt{x}\left(1 - 9x^3\right)$

16. $f(t) = (3 - 2t^3)^2$

17. $g(x) = (1 + 2x)(2 - x + x^2)$

18. $y = \dfrac{4 - 8x + 2x^2}{x}$

19. $Y(t) = \dfrac{t^4 - 2t^2 + 7t}{t^3}$

20. $S(w) = \dfrac{w^2(2 - w) + w^5}{3w}$

For problems 21 – 26 determine where, if anywhere, the function is not changing.

21. $f(x) = 2x^3 - 9x^2 - 108x + 14$

22. $u(t) = 45 + 300t^2 + 20t^3 - 3t^4$

23. $Q(t) = t^3 - 9t^2 + t - 10$

24. $h(w) = 2w^3 + 3w^2 + 4w + 5$

25. $g(x) = 9 + 8x^2 + 3x^3 - x^4$

26. $G(z) = z^2(z - 1)^2$

27. Find the tangent line to $f(x) = 3x^5 - 4x^2 + 9x - 12$ at $x = -1$.

28. Find the tangent line to $g(x) = \dfrac{x^2 + 1}{x}$ at $x = 2$.

29. Find the tangent line to $h(x) = 2\sqrt{x} - 8\sqrt[4]{x}$ at $x = 16$.

30. The position of an object at any time *t* is given by $s(t) = 3t^4 - 44t^3 + 108t^2 + 20$.

(a) Determine the velocity of the object at any time *t*.
(b) Does the position of the object ever stop changing?
(c) When is the object moving to the right and when is the object moving to the left?

31. The position of an object at any time *t* is given by $s(t) = 1 - 150t^3 + 45t^4 - 2t^5$.

(a) Determine the velocity of the object at any time *t*.
(b) Does the position of the object ever stop changing?
(c) When is the object moving to the right and when is the object moving to the left?

32. Determine where the function $f(x) = 4x^3 - 18x^2 - 336x + 27$ is increasing and decreasing.

33. Determine where the function $g(w) = w^4 + 2w^3 - 15w^2 - 9$ is increasing and decreasing.

34. Determine where the function $V(t) = t^3 - 24t^2 + 192t - 50$ is increasing and decreasing.

35. Determine the percentage of the interval $[-6,4]$ on which $f(x) = 7 + 10x^3 - 5x^4 - 2x^5$ is increasing.

36. Determine the percentage of the interval $[-5,2]$ on which $f(x) = 3x^4 - 8x^3 - 144x^2$ is decreasing.

37. Is $h(x) = 3 - x + x^2 + 2x^3$ increasing or decreasing more on the interval $[-1,1]$?

38. Determine where, if anywhere, the tangent line to $f(x) = 12x^2 - 9x + 3$ is parallel to the line $y = 1 - 7x$.

39. Determine where, if anywhere, the tangent line to $f(x) = 8 + 4x + x^2 - 2x^3$ is perpendicular to the line $y = -\frac{1}{4}x + \frac{8}{3}$.

40. Determine where, if anywhere, the tangent line to $f(x) = \sqrt[3]{x} - 8x$ is perpendicular to the line $y = 2x - 11$.

41. Determine where, if anywhere, the tangent line to $f(x) = \frac{13x}{9} + \frac{1}{x}$ is parallel to the line $y = x$.

Section 3-4 : Product and Quotient Rule

For problems 1 – 7 use the Product Rule or the Quotient Rule to find the derivative of the given function.

1. $h(z) = \left(2 - \sqrt{z}\right)\left(3 + 8\sqrt[3]{z^2}\right)$

2. $f(x) = \left(x - \dfrac{2}{x}\right)\left(7 - 2x^3\right)$

3. $y = \left(x^2 - 5x + 1\right)\left(12 + 2x - x^3\right)$

4. $g(x) = \dfrac{\sqrt[3]{x}}{1 + x^2}$

5. $Z(y) = \dfrac{4y - y^2}{6 - y}$

6. $V(t) = \dfrac{1 - 10t + t^2}{5t + 2t^3}$

7. $f(w) = \dfrac{(1 - 4w)(2 + w)}{3 + 9w}$

For problems 8 – 12 use the fact that $f(-3) = 12$, $f'(-3) = 9$, $g(-3) = -4$, $g'(-3) = 7$, $h(-3) = -2$ and $h'(-3) = 5$ determine the value of the indicated derivative.

8. $(f\,g)'(-3)$

9. $\left(\dfrac{h}{g}\right)'(-3)$

10. $\left(\dfrac{f\,g}{h}\right)'(-3)$

11. If $y = \left[x - f(x)\right]h(x)$ determine $\left.\dfrac{dy}{dx}\right|_{x=-3}$.

12. If $y = \dfrac{1 - g(x)h(x)}{x + f(x)}$ determine $\left.\dfrac{dy}{dx}\right|_{x=-3}$.

13. Find the equation of the tangent line to $f(x) = (8 - x^2)(1 + x + x^2)$ at $x = -2$.

14. Find the equation of the tangent line to $f(x) = \dfrac{4 - x^3}{x + 2x^2}$ at $x = 1$.

15. Determine where $g(z) = \dfrac{2 - z}{12 + z^2}$ is increasing and decreasing.

16. Determine where $R(x) = (3 - x)(1 - 2x + x^2)$ is increasing and decreasing.

17. Determine where $h(t) = \dfrac{7t - t^2}{1 + 2t^2}$ is increasing and decreasing.

18. Determine where $f(x) = \dfrac{1 + x}{1 - x}$ is increasing and decreasing.

19. Derive the formula for the Product Rule for four functions.

$$(f\,g\,h\,w)' = f'\,g\,h\,w + f\,g'\,h\,w + f\,g\,h'\,w + f\,g\,h\,w'$$

Section 3-5 : Derivatives of Trig Functions

For problems 1 – 6 evaluate the given limit.

1. $\displaystyle\lim_{t \to 0} \frac{3t}{\sin(t)}$

2. $\displaystyle\lim_{w \to 0} \frac{\sin(9w)}{10w}$

3. $\displaystyle\lim_{\theta \to 0} \frac{\sin(2\theta)}{\sin(17\theta)}$

4. $\displaystyle\lim_{x \to -4} \frac{\sin(x+4)}{3x+12}$

5. $\displaystyle\lim_{x \to 0} \frac{\cos(x)-1}{9x}$

6. $\displaystyle\lim_{z \to 0} \frac{\cos(8z)-1}{2z}$

For problems 6 – 10 differentiate the given function.

6. $h(x) = x^4 - 9\sin(x) + 2\tan(x)$

7. $g(t) = 8\sec(t) + \cos(t) - 4\csc(t)$

8. $y = 6\cot(w) - 8\cos(w) + 9$

9. $f(x) = 8\sec(x)\csc(x)$

10. $h(t) = 8 - t^9 \tan(t)$

11. $R(x) = 6\sqrt[5]{x^2} + 8x\sin(x)$

12. $h(z) = 3z - \dfrac{\cos(z)}{z^3}$

13. $Y(x) = \dfrac{1+\cos(x)}{1-\sin(x)}$

14. $f(w) = 3w - \dfrac{\sec(w)}{1+9\tan(w)}$

15. $g(t) = \dfrac{t\cot(t)}{t^2+1}$

16. Find the tangent line to $f(x) = 2\tan(x) - 4x$ at $x = 0$.

17. Find the tangent line to $f(x) = x\sec(x)$ at $x = 2\pi$.

18. Find the tangent line to $f(x) = \cos(x) + \sec(x)$ at $x = \pi$.

19. The position of an object is given by $s(t) = 9\sin(t) + 2\cos(t) - 7$ determine all the points where the object is not changing.

20. The position of an object is given by $s(t) = 8t + 10\sin(t)$ determine where in the interval $[0,12]$ the object is moving to the right and moving to the left.

21. Where in the range $[-6,6]$ is the function $f(z) = 3z - 8\cos(z)$ is increasing and decreasing.

22. Where in the range $[-3,5]$ is the function $R(w) = 7\cos(w) - \sin(w) + 3$ is increasing and decreasing.

23. Where in the range $[0,10]$ is the function $h(t) = 9 - 15\sin(t)$ is increasing and decreasing.

24. Using the definition of the derivative prove that $\dfrac{d}{dx}\left(\cos(x)\right) = -\sin(x)$.

25. Prove that $\dfrac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$.

26. Prove that $\dfrac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$.

27. Prove that $\dfrac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cot(x)$.

Section 3-6 : Derivatives of Exponential and Logarithm Functions

For problems 1 – 12 differentiate the given function.

1. $g\left(z\right)=10^{z}-9^{z}$

2. $f\left(x\right)=9\log_{4}\left(x\right)+12\log_{11}\left(x\right)$

3. $h\left(t\right)=6^{t}-4\mathbf{e}^{t}$

4. $R\left(x\right)=20\ln\left(x\right)+\log_{123}\left(x\right)$

5. $Q\left(t\right)=\left(t^{2}-6t+3\right)\mathbf{e}^{t}$

6. $y=v+8^{v}\,9^{v}$

7. $U\left(z\right)=\log_{4}\left(z\right)-z^{6}\ln\left(z\right)$

8. $h\left(x\right)=\log_{3}\left(x\right)\log\left(x\right)$

9. $f\left(w\right)=\dfrac{1-\mathbf{e}^{w}}{1+7\mathbf{e}^{w}}$

10. $f\left(t\right)=\dfrac{1+4\ln\left(t\right)}{5t^{3}}$

11. $g\left(r\right)=\dfrac{r^{2}+\log_{7}\left(r\right)}{7^{r}}$

12. $V\left(t\right)=\dfrac{t^{4}\mathbf{e}^{t}}{\ln\left(t\right)}$

13. Find the tangent line to $f\left(x\right)=\left(1-8x\right)\mathbf{e}^{x}$ at $x=-1$.

14. Find the tangent line to $f\left(x\right)=3x^{2}\ln\left(x\right)$ at $x=1$.

15. Find the tangent line to $f\left(x\right)=3\mathbf{e}^{x}+8\ln\left(x\right)$ at $x=2$.

16. Determine if $U\left(y\right)=4^{y}-3\mathbf{e}^{y}$ is increasing or decreasing at the following points.

(a) $y=-2$ (b) $y=0$ (c) $y=3$

17. Determine if $y\left(z\right)=\dfrac{z^{2}}{\ln\left(z\right)}$ is increasing or decreasing at the following points.

(a) $z=\dfrac{1}{2}$ (b) $z=2$ (c) $z=6$

18. Determine if $h\left(x\right)=x^{2}\mathbf{e}^{x}$ is increasing or decreasing at the following points.

(a) $x=-1$ (b) $x=0$ (c) $x=2$

Section 3-7 : Derivatives of Inverse Trig Functions

For each of the following problems differentiate the given function.

1. $f\left(x\right) = \sin\left(x\right) + 9\sin^{-1}\left(x\right)$

2. $C\left(t\right) = 5\sin^{-1}\left(t\right) - \cos^{-1}\left(t\right)$

3. $g\left(z\right) = \tan^{-1}\left(z\right) + 4\cos^{-1}\left(z\right)$

4. $h\left(t\right) = \sec^{-1}\left(t\right) - t^3\cos^{-1}\left(t\right)$

5. $f\left(w\right) = \left(w - w^2\right)\sin^{-1}\left(w\right)$

6. $y = \left(x - \cot^{-1}\left(x\right)\right)\left(1 + \csc^{-1}\left(x\right)\right)$

7. $Q\left(z\right) = \dfrac{z+1}{\tan^{-1}\left(z\right)}$

8. $A\left(t\right) = \dfrac{1 + \sin^{-1}\left(t\right)}{1 - \cos^{-1}\left(t\right)}$

Section 3-8 : Derivatives of Hyperbolic Functions

For each of the following problems differentiate the given function.

1. $h(w) = w^2 - 3\sinh(w)$

2. $g(x) = \cos(x) + \cosh(x)$

3. $H(t) = 3\operatorname{csch}(t) + 7\sinh(t)$

4. $A(r) = \tan(r)\tanh(r)$

5. $f(x) = \mathbf{e}^x \cosh(x)$

6. $f(z) = \dfrac{\operatorname{sech}(z) + 1}{1 - z}$

7. $Q(w) = \dfrac{\coth(w)}{w + \sinh(w)}$

Section 3-9 : Chain Rule

For problems 1 – 51 differentiate the given function.

1. $g\left(x\right)=\left(3-8x\right)^{11}$

2. $g\left(z\right)=\sqrt[7]{9z^{3}}$

3. $h\left(t\right)=\left(9+2t-t^{3}\right)^{6}$

4. $y=\sqrt{w^{3}+8w^{2}}$

5. $R\left(v\right)=\left(14v^{2}-3v\right)^{-2}$

6. $H\left(w\right)=\dfrac{2}{\left(6-5w\right)^{8}}$

7. $f\left(x\right)=\sin\left(4x+7x^{4}\right)$

8. $T\left(x\right)=\tan\left(1-2\mathbf{e}^{x}\right)$

9. $g\left(z\right)=\cos\left(\sin\left(z\right)+z^{2}\right)$

10. $h\left(u\right)=\sec\left(u^{2}-u\right)$

11. $y=\cot\left(1+\cot\left(x\right)\right)$

12. $f\left(t\right)=\mathbf{e}^{1-t^{2}}$

13. $J\left(z\right)=\mathbf{e}^{12z-z^{6}}$

14. $f\left(z\right)=\mathbf{e}^{z+\ln\left(z\right)}$

15. $B\left(x\right)=7^{\cos\left(x\right)}$

16. $z=3^{x^{2}-9x}$

17. $R(z) = \ln\left(6z + \mathbf{e}^{z}\right)$

18. $h(w) = \ln\left(w^7 - w^5 + w^3 - w\right)$

19. $g(t) = \ln\left(1 - \csc(t)\right)$

20. $f(v) = \tan^{-1}(3 - 2v)$

21. $h(t) = \sin^{-1}(9t)$

22. $A(t) = \cos(t) - \sqrt[6]{1 - \sin(t)}$

23. $H(z) = \ln(6z) - 4\sec(z)$

24. $f(x) = \tan^4(x) + \tan\left(x^4\right)$

25. $f(u) = \mathbf{e}^{4u} - 6\mathbf{e}^{-u} + 7\mathbf{e}^{u^2 - 8u}$

26. $g(z) = \sec^8(z) + \sec\left(z^8\right)$

27. $k(w) = \left(w^4 - 1\right)^5 + \sqrt{2 + 9w}$

28. $h(x) = \sqrt[3]{x^2 - 5x + 1} + (9x + 4)^{-7}$

29. $T(x) = \left(2x^3 - 1\right)^5 (5 - 3x)^4$

30. $w = \left(z^2 + 4z\right)\sin(1 - 2z)$

31. $Y(t) = t^8 \cos^4(t)$

32. $f(x) = \sqrt{6 - x^4}\ \ln(10x + 3)$

33. $A(z) = \sec(4z)\tan\left(z^2\right)$

34. $h(v) = \sqrt{5v} + \ln\left(v^4\right)\mathbf{e}^{6+9v}$

35. $f(x) = \dfrac{\mathbf{e}^{x^2+8x}}{\sqrt{x^4+7}}$

36. $g(x) = \dfrac{(4x+1)^3}{(x^2-x)^6}$

37. $g(t) = \dfrac{\csc(1-t)}{1+\mathbf{e}^{-t}}$

38. $V(z) = \dfrac{\sin^2(z)}{1+\cos(z^2)}$

39. $U(w) = \ln\left(\mathbf{e}^{w}\cos(w)\right)$

40. $h(t) = \tan\left(\left(5-t^2\right)\ln(t)\right)$

41. $z = \ln\left(\dfrac{3+x}{2-x^2}\right)$

42. $g(v) = \sqrt{\dfrac{\mathbf{e}^{v}}{7+2v}}$

43. $f(x) = \sqrt{x^2+\sqrt{1+4x}}$

44. $u = \left(6+\cos(8w)\right)^5$

45. $h(z) = \left(7z - z^2 + \mathbf{e}^{5z^2+z}\right)^{-4}$

46. $A(y) = \ln\left(7y^3 + \sin^2(y)\right)$

47. $g(x) = \csc^6(8x)$

48. $V(w) = \sqrt[4]{\cos\left(9-w^2\right)+\ln(6w+5)}$

49. $h(t) = \sin\left(t^3\mathbf{e}^{-6t}\right)$

50. $B(r) = \left(\mathbf{e}^{\sin(r)} - \sin\left(\mathbf{e}^{r}\right)\right)^{8}$

51. $f(z) = \cos^2\left(1 + \cos^2(z)\right)$

52. Find the tangent line to $f(x) = \left(2 - 4x^2\right)^5$ at $x = 1$.

53. Find the tangent line to $f(x) = \mathbf{e}^{2x+4} - 8\ln\left(x^2 - 3\right)$ at $x = -2$.

54. Determine where $A(t) = t^3(9 - t)^4$ is increasing and decreasing.

55. Is $h(x) = (2x+1)^4(2-x)^5$ increasing or decreasing more in the interval $[-2,3]$?

56. Determine where $U(w) = 3\cos\left(\frac{w}{2}\right) + w - 3$ is increasing and decreasing in the interval $[-10,10]$.

57. If the position of an object is given by $s(t) = 4\sin(3t) - 10t + 7$. Determine where, if anywhere, the object is not moving in the interval $[0,4]$.

58. Determine where $f(x) = 6\sin(2x) - 7\cos(2x) - 3$ is increasing and decreasing in the interval $[-3,2]$.

59. Determine where $H(w) = \left(w^2 - 1\right)\mathbf{e}^{2-w^2}$ is increasing and decreasing.

60. What percentage of $[-3,5]$ is the function $g(z) = \mathbf{e}^{z^2-8} + 3\mathbf{e}^{1-2z^2}$ decreasing?

61. The position of an object is given by $s(t) = \ln\left(2t^3 - 21t^2 + 36t + 200\right)$. During the first 10 hours of motion (assuming the motion starts at $t = 0$) what percentage of the time is the object moving to the right?

62. For the function $f(x) = 1 - \frac{x}{2} - \ln\left(2 + 9x - x^2\right)$ determine each of the following.

(a) The interval on which the function is defined.
(b) Where the function is increasing and decreasing.

Section 3-10 : Implicit Differentiation

For problems 1 – 6 do each of the following.

(a) Find y' by solving the equation for *y* and differentiating directly.

(b) Find y' by implicit differentiation.

(c) Check that the derivatives in **(a)** and **(b)** are the same.

1. $x^2 y^9 = 2$

2. $\dfrac{6x}{y^7} = 4$

3. $1 = x^4 + 5y^3$

4. $8x - y^2 = 3$

5. $4x - 6y^2 = xy^2$

6. $\ln\left(x\,y\right) = x$

For problems 7 – 21 find y' by implicit differentiation.

7. $y^2 - 12x^3 = 8y$

8. $3y^7 + x^{10} = y^{-2} - 6x^3 + 2$

9. $y^{-3} + 4x^{-1} = 8y^{-1}$

10. $10x^4 - y^{-6} = 7y^3 + 4x^{-3}$

11. $\sin\left(x\right) + \cos\left(y\right) = \mathbf{e}^{4y}$

12. $x + \ln\left(y\right) = \sec\left(y\right)$

13. $y^2\left(4 - x^2\right) = y^7 + 9x$

14. $6x^{-2} - x^3 y^2 + 4x = 0$

15. $8xy + 2x^4 y^{-3} = x^3$

16. $yx^3 - \cos(x)\sin(y) = 7x$

17. $\mathbf{e}^x \cos(y) + \sin(xy) = 9$

18. $x^2 + \sqrt{x^3 + 2y} = y^2$

19. $\tan(3x + 7y) = 6 - 4x^{-1}$

20. $\mathbf{e}^{x^2 + y^2} = \mathbf{e}^{x^2 y^2} + 1$

21. $\sin\left(\dfrac{x}{y}\right) + x^3 = 2 - y^4$

For problems 22 - 24 find the equation of the tangent line at the given point.

22. $3x + y^2 = x^2 - 19$ at $(-4,3)$

23. $x^2 y = y^2 - 6x$ at $(2,6)$

24. $2\sin(x)\cos(y) = 1$ at $\left(\dfrac{\pi}{4}, -\dfrac{\pi}{4}\right)$

For problems 25 – 27 determine if the function is increasing, decreasing or not changing at the given point.

25. $x^2 - y^3 = 4y + 9$ at $(2,-1)$

26. $\mathbf{e}^{1-x}\mathbf{e}^{y^2} = x^3 + y$ at $(1,0)$

27. $\sin(\pi - x) + y^2\cos(x) = y$ at $\left(\dfrac{\pi}{2}, 1\right)$

For problems 28 - 31 assume that $x = x(t)$, $y = y(t)$ and $z = z(t)$ and differentiate the given equation with respect to *t*.

28. $x^4 - 6z = 3 - y^2$

29. $x\,y^4 = y^2 z^3$

30. $z^7 \mathbf{e}^{6y} = \left(y^2 - 8x\right)^{10} + z^{-4}$

31. $\cos\left(z^2 x^3\right) + \sqrt{y^2 + x^2} = 0$

Section 3-11 : Related Rates

1. In the following assume that *x* and *y* are both functions of *t*. Given $x = 3$, $y = 2$ and $y' = 7$ determine x' for the following equation.

$$x^3 - y^4 = x^2 y - 7$$

2. In the following assume that *x* and *y* are both functions of *t*. Given $x = \frac{\pi}{6}$, $y = -4$ and $x' = 12$ determine y' for the following equation.

$$x^2\left(y^2 - 16\right) - 6\cos\left(2x\right) = 1 + y$$

3. In the following assume that *x*, *y* and *z* are all functions of *t*. Given $x = -1$, $y = 8$, $z = 2$, $x' = -4$ and $y' = 7$ determine z' for the following equation.

$$x^4 + \frac{y}{z} = 2x^2 z^2 - 3$$

4. In the following assume that *x*, *y* and *z* are all functions of *t*. Given $x = -2$, $y = 3$, $z = 4$, $y' = 6$ and $z' = 0$ determine x' for the following equation.

$$x\,y^2 z^2 = x^3 - z^4 - 8y$$

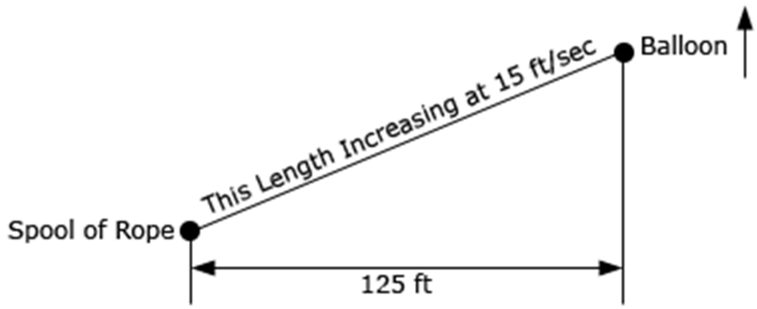
5. The sides of a square are increasing at a rate of 10 cm/sec. How fast is the area enclosed by the square increasing when the area is 150 cm^2.

6. The sides of an equilateral triangle are decreasing at a rate of 3 in/hr. How fast is the area enclosed by the triangle decreasing when the sides are 2 feet long?

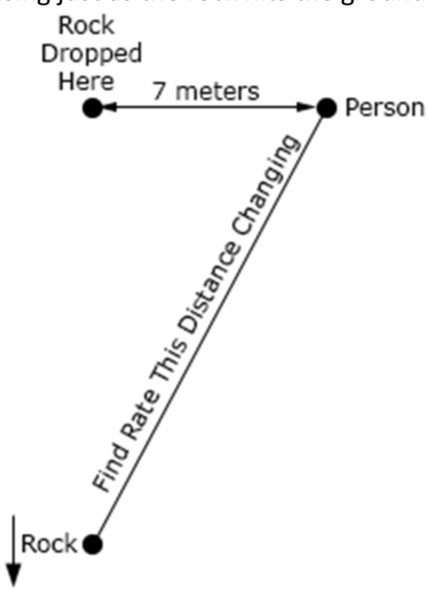
7. A spherical balloon is being filled in such a way that the surface area is increasing at a rate of 20 cm^2/sec when the radius is 2 meters. At what rate is air being pumped in the balloon when the radius is 2 meters?

8. A cylindrical tank of radius 2.5 feet is being drained of water at a rate of 0.25 ft^3/sec. How fast is the height of the water decreasing?

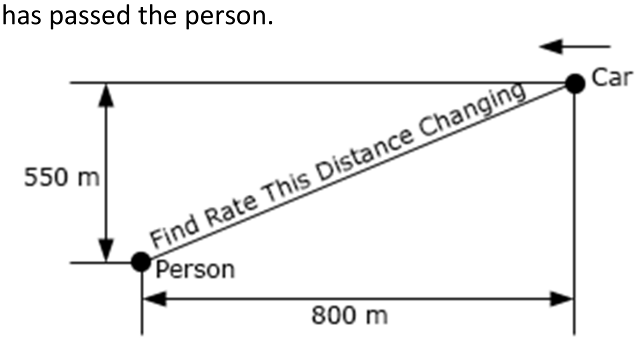
9. A hot air balloon is attached to a spool of rope that is 125 feet away from the balloon when it is on the ground. The hot air balloon rises straight up in such a way that the length of rope increases at a rate of 15 ft/sec. How fast is the hot air balloon rising 20 seconds after it lifts off?



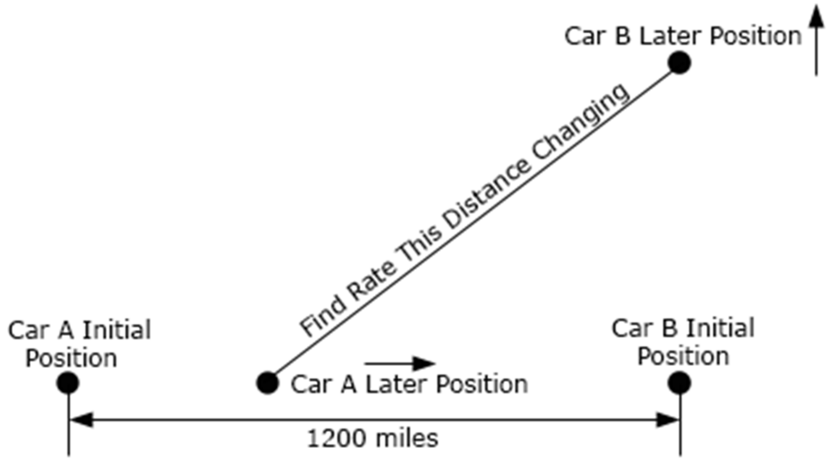
10. A rock is dropped straight off a bridge that is 50 meters above the ground and falls at a speed of 10 m/sec. Another person is 7 meters away on the same bridge. At what rate is the distance between the rock and the second person increasing just as the rock hits the ground?



11. A person is 550 meters away from a road and there is a car that is initially 800 meters away approaching the person at a speed of 45 m/sec. At what rate is the distance between the person and the car changing **(a)** 5 seconds after the start, **(b)** when the car is directly in front of the person and **(c)** 10 seconds after the car has passed the person.

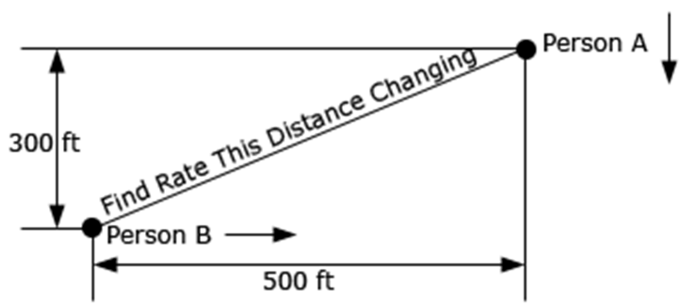


12. Two cars are initially 1200 miles apart. At the same time Car A starts driving at 35 mph to the east while Car B starts driving at 55 mph to the north (see sketch below for this initial setup). At what rate is the distance between the two cars changing after **(a)** 5 hours of travel, **(b)** 20 hours of travel and **(c)** 40 hours of travel?



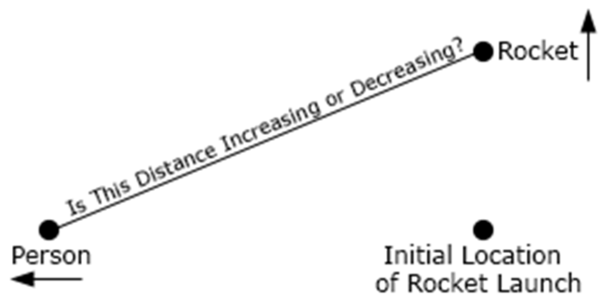
13. Repeat problem **12** above except for this problem assume that Car A starts traveling 4 hours after Car B starts traveling. For parts **(a)**, **(b)** and **(c)** assume that these are travel times for Car B.

14. Two people are on a city block. See the sketch below for placement and distances. Person A is on the northeast corner and Person B is on the southwest corner. Person A starts walking towards the southeast corner at a rate of 3 ft/sec. Four seconds later Person B starts walking towards the southeast corner at a rate of 2 ft/sec. At what rate is the distance between them changing **(a)** 10 seconds after Person A starts walking and **(b)** after Person A has covered half the distance?

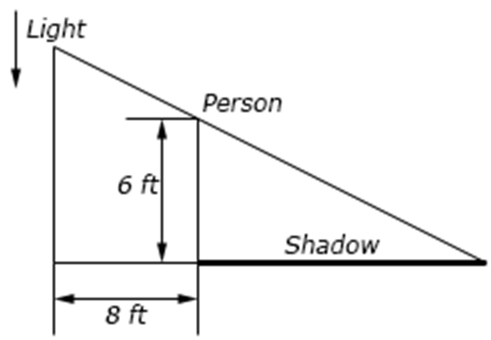


15. A person is standing 75 meters away from a kite and has a spool of string attached to the kite. The kite starts to rise straight up in the air at a rate of 2 m/sec and at the same time the person starts to move towards the kites launch point at a rate of 0.75 m/sec. Is the length string increasing or decreasing after **(a)** 4 seconds and **(b)** 20 seconds.

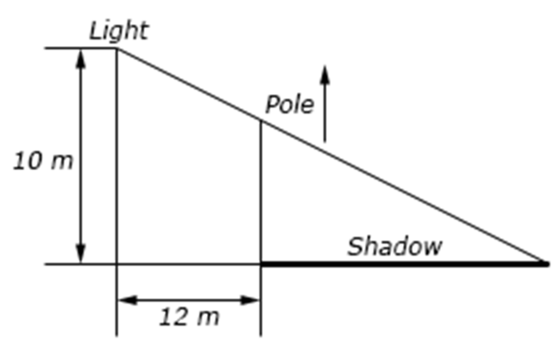
16. A person lights the fuse on a model rocket and starts to move away from the rocket at a rate of 3 ft/sec. Five seconds after lighting the fuse the rocket launches straight up into the air at a rate of 10 ft/sec. Is the distance between the person and the rocket increasing or decreasing **(a)** 6 seconds after launch and **(b)** 12 seconds after launch?



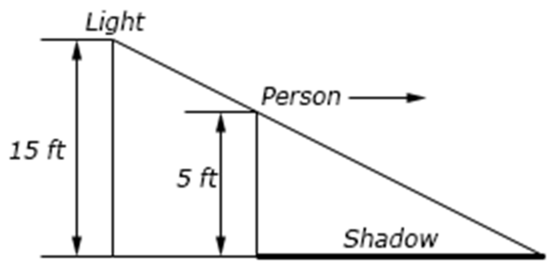
17. A light is on a pole and is being lowered towards the ground at a rate of 9 in/sec. A 6 foot tall person is on the ground and 8 feet away from the pole. At what rate is the persons shadow increasing then the light is 15 feet above the ground?



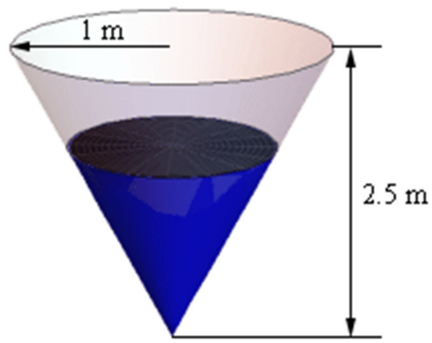
18. A light is fixed on a wall 10 meters above the floor. Twelve meters away from the wall a pole is being raised straight up at a rate of 45 cm/sec. When the pole is 6 meters tall at what rate is the tip of the shadow moving **(a)** away from the pole and **(b)** away from the wall? Note the sketch below is not to scale…



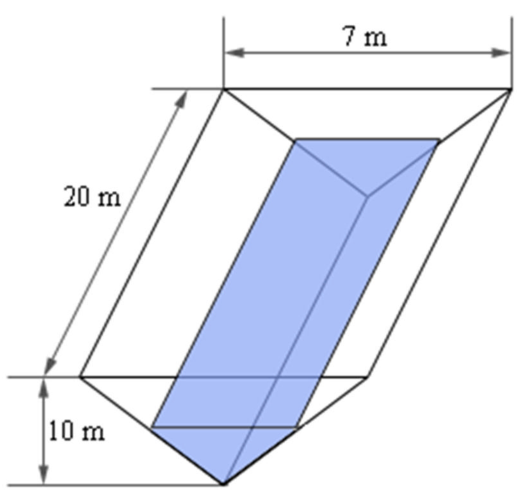
19. A light is on the top of a 15 foot tall pole. A 5 foot tall person starts at the pole and moves away from the pole at a rate of 2.5 ft/sec. After moving for 8 seconds at what rate is the tip of the shadow moving **(a)** away from the person and **(b)** away from the pole?



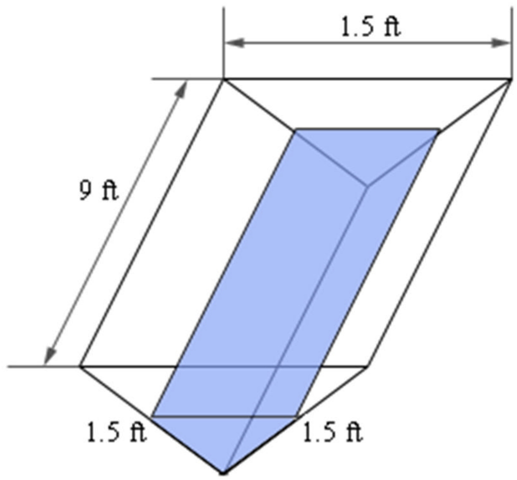
20. A tank of water is in the shape of a cone (assume the "point" of the cone is pointing downwards) and is leaking water at a rate of 35 cm^3/sec. The base radius of the tank is 1 meter and the height of the tank is 2.5 meters. When the depth of the water is 1.25 meters at what rate is the **(a)** depth changing and **(b)** the radius of the top of the water changing?



21. A trough of water is 20 meters in length and its ends are in the shape of an isosceles triangle whose width is 7 meters and height is 10 meters. Assume that the two equal length sides of the triangle are the sides of the water tank and the other side of the triangle is the top of the tank and is parallel to the ground. Water is being pumped into the tank at a rate of 2 m^3/min. When the water is 6 meters deep at what rate is **(a)** depth changing and **(b)** the width of the top of the water changing? Note the sketch below is not to scale…

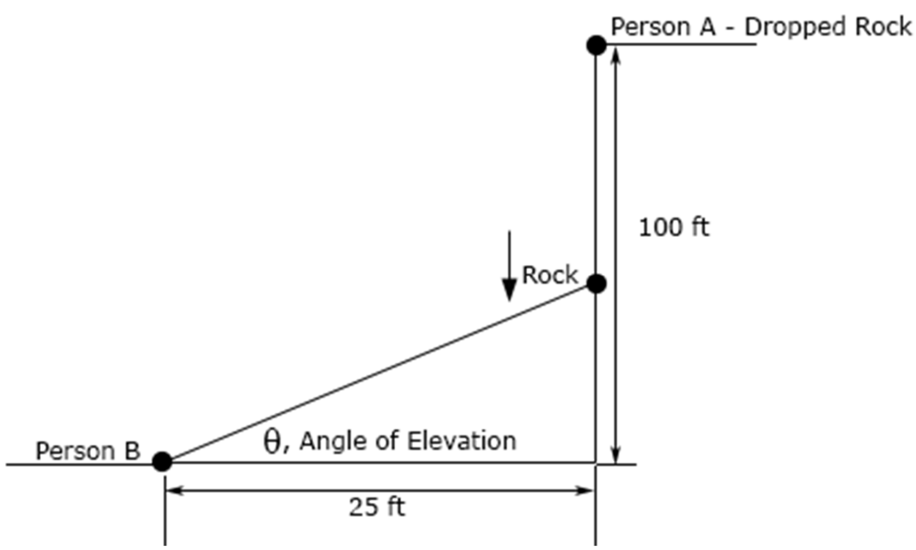


22. A trough of water is 9 feet long and its ends are in the shape of an equilateral triangle whose sides are 1.5 feet long. Assume that the top of the tank is parallel to the ground. If water is being pumped out of the tank at a rate of 2 ft^3/s at what rate is the depth of the water changing when the depth is 0.75 feet?

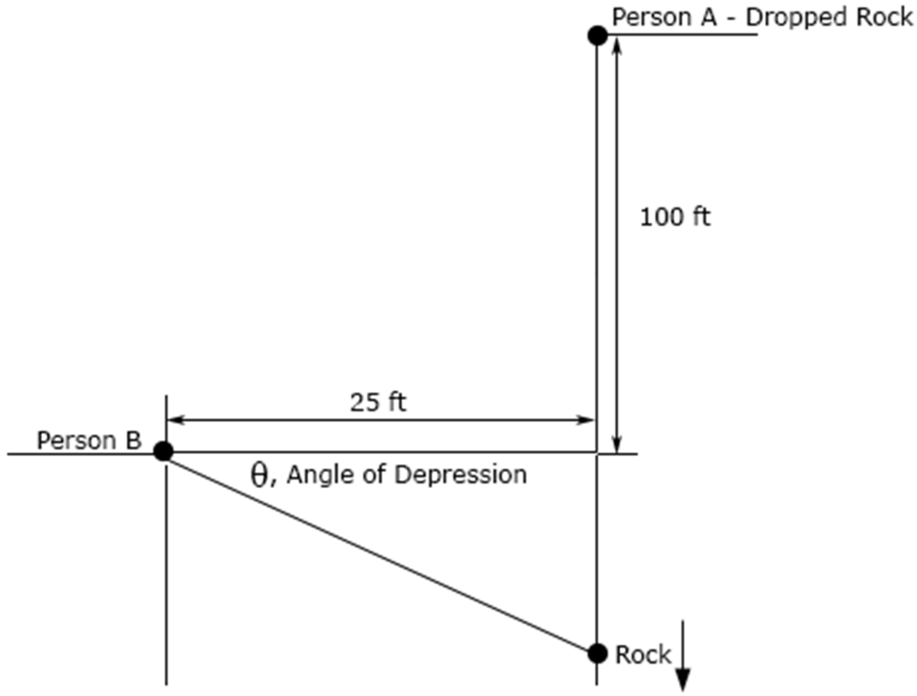


23. The angle of elevation (depression) is the angle formed by a horizontal line and a line joining the observer's eye to an object above (below) the horizontal line. Two people are on the roof of buildings separated by at 25 foot wide road. Person A is 100 feet above Person B and drops a rock off the roof of their building and it falls at a rate of 3 ft/sec. The sketches below are not to scale….

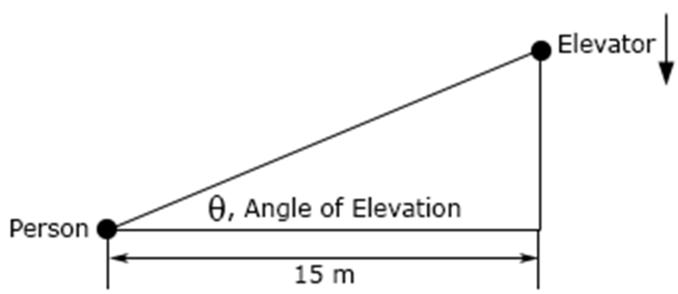
(a) At what rate is the angle of elevation changing as Person B watches the rock fall when the rock is 35 feet above Person B?



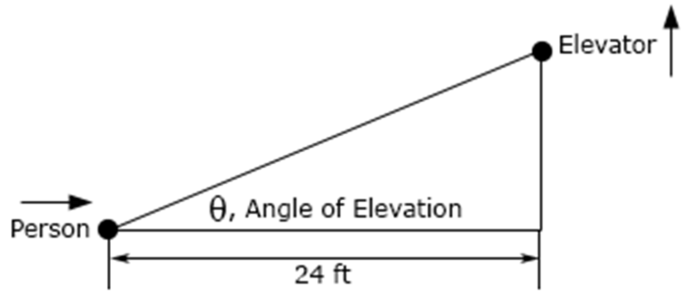
(b) At what rate is the angle of depression changing as Person B watches the rock fall when the rock is 65 feet below Person B?



24. The angle of elevation is the angle formed by a horizontal line and a line joining the observer's eye to an object above the horizontal line. A person is standing 15 meters away from a building and watching an outside elevator move down the face of the building. When the angle of elevation is 1 radians it is changing at a rate of 0.15 radians/sec. At this point in time what is the speed of the elevator?



25. The angle of elevation is the angle formed by a horizontal line and a line joining the observer's eye to an object above the horizontal line. A person is 24 feet away from a building and watching an outside elevator move up the face of the building. The elevator is moving up at a rate of 4 ft/sec and the person is moving towards the building at a rate of 0.75 ft/sec. Assuming that the elevator started moving from the ground at the same time that the person started walking is the angle of elevation increasing or decreasing after 10 seconds?



Section 3-12 : Higher Order Derivatives

For problems 1 – 9 determine the fourth derivative of the given function.

1. $f\left(z\right)=z^8+2z^6-7z^4+20z^2-3$

2. $y=6t^4-5t^3+4t^2-3t+2$

3. $V\left(t\right)=6t^{-2}+7t^{-3}-t^{-4}$

4. $g\left(x\right)=\dfrac{3}{x}-\dfrac{1}{4x^3}+\dfrac{3}{2x^5}$

5. $h\left(x\right)=8\sqrt{x}-\sqrt[3]{x}+5\sqrt[4]{x^9}$

6. $h\left(y\right)=\sqrt[3]{y^2}-\dfrac{32}{\sqrt[4]{y}}+\dfrac{1}{3\sqrt{y^5}}$

7. $y=9\sin\left(z\right)-\sin\left(4z\right)+7\cos\left(\frac{2z}{3}\right)$

8. $R\left(x\right)=2\mathbf{e}^{-x}-3\mathbf{e}^{1+8x}+9\ln\left(6x\right)$

9. $f\left(t\right)=\ln\left(t^6\right)-\cos\left(4t\right)+9\sin\left(2t\right)+\mathbf{e}^{7t}$

For problems 10 – 20 determine the second derivative of the given function.

10. $Q\left(w\right)=\cos\left(2-7w^2\right)$

11. $f\left(z\right)=\sin\left(1+\mathbf{e}^{2x}\right)$

12. $y=\tan\left(3x\right)$

13. $z=\csc\left(8w\right)$

14. $f\left(u\right)=\mathbf{e}^{4u^2+9u}$

15. $h\left(x\right)=\ln\left(x^2-3x\right)$

16. $g\left(z\right)=\ln\left(3+\cos\left(z\right)\right)$

17. $f\left(x\right)=\dfrac{1}{\sqrt{6x+x^4}}$

18. $f\left(x\right)=\left[3\sin\left(x\right)+8\cos\left(2x\right)\right]^{-3}$

19. $f\left(t\right)=\sin^3\left(2t\right)$

20. $A\left(w\right)=\tan^4\left(w\right)$

For problems 21 – 23 determine the third derivative of the given function.

21. $g\left(x\right)=\sec\left(3x\right)$

22. $y=\mathbf{e}^{1-2t^3}$

23. $h\left(w\right)=\cos\left(w-w^2\right)$

For problems 24 - 27 determine the second derivative of the given function.

24. $6y-y^2=3x^4+9x$

25. $y^3-4x^2=11x-2y^2$

26. $\mathbf{e}^y+4x=y^3-1$

27. $y\cos\left(x\right)=3+4y^2$

Section 3-13 : Logarithmic Differentiation

For problems 1 – 6 use logarithmic differentiation to find the first derivative of the given function.

1. $h(x) = x^8 \cos(3x)\left(6+3x^2\right)^4$

2. $f(w) = \sqrt{4+2w-9w^2}\ \sqrt[5]{7w+2w^3+w^5}$

3. $h(z) = \dfrac{\left(1+7z^2\right)^3}{\left(2+3z+4z^2\right)^4}$

4. $g(x) = \dfrac{\sqrt{1+\sin(2x)}}{2x-\tan(x)}$

5. $h(t) = \dfrac{(9-3t)^{10}}{t^2\sin(7t)}$

6. $y = \dfrac{3+8x}{\left(1+2x^2\right)^4}\ \dfrac{\cos(1-x)}{\left(5x+x^2\right)^7}$

For problems 7 – 10 find the first derivative of the given function.

7. $y = x^{\ln(x)}$

8. $R(t) = \left[\sin(4t)\right]^{6t}$

9. $h(w) = \left(6-w^2\right)^{2+8w+w^3}$

10. $g(z) = z^2\left[3+z\right]^{1-z^2}$

Chapter 4 : Applications of Derivatives

Here are a set of assignment problems for the Applications of Derivatives chapter of the Calculus I notes. Please note that these problems do not have any solutions available. These are intended mostly for instructors who might want a set of problems to assign for turning in. Having solutions available (or even just final answers) would defeat the purpose the problems.

If you are looking for some practice problems (with solutions available) please check out the Practice Problems. There you will find a set of problems that should give you quite a bit practice.

Here is a list of all the sections for which assignment problems have been written as well as a brief description of the material covered in the notes for that particular section.

Rates of Change – In this section we review the main application/interpretation of derivatives from the previous chapter (i.e. rates of change) that we will be using in many of the applications in this chapter.

Critical Points – In this section we give the definition of critical points. Critical points will show up in most of the sections in this chapter, so it will be important to understand them and how to find them. We will work a number of examples illustrating how to find them for a wide variety of functions.

Minimum and Maximum Values – In this section we define absolute (or global) minimum and maximum values of a function and relative (or local) minimum and maximum values of a function. It is important to understand the difference between the two types of minimum/maximum (collectively called extrema) values for many of the applications in this chapter and so we use a variety of examples to help with this. We also give the Extreme Value Theorem and Fermat's Theorem, both of which are very important in the many of the applications we'll see in this chapter.

Finding Absolute Extrema – In this section we discuss how to find the absolute (or global) minimum and maximum values of a function. In other words, we will be finding the largest and smallest values that a function will have.

The Shape of a Graph, Part I – In this section we will discuss what the first derivative of a function can tell us about the graph of a function. The first derivative will allow us to identify the relative (or local) minimum and maximum values of a function and where a function will be increasing and decreasing. We will also give the First Derivative test which will allow us to classify critical points as relative minimums, relative maximums or neither a minimum or a maximum.

The Shape of a Graph, Part II – In this section we will discuss what the second derivative of a function can tell us about the graph of a function. The second derivative will allow us to determine where the graph of a function is concave up and concave down. The second derivative will also allow us to identify any inflection points (i.e. where concavity changes) that a function may have. We will also give the Second Derivative Test that will give an alternative method for identifying some critical points (but not all) as relative minimums or relative maximums.

The Mean Value Theorem – In this section we will give Rolle's Theorem and the Mean Value Theorem. With the Mean Value Theorem we will prove a couple of very nice facts, one of which will be very useful in the next chapter.

Optimization Problems – In this section we will be determining the absolute minimum and/or maximum of a function that depends on two variables given some constraint, or relationship, that the two variables must always satisfy. We will discuss several methods for determining the absolute minimum or maximum of the function. Examples in this section tend to center around geometric objects such as squares, boxes, cylinders, etc.

More Optimization Problems – In this section we will continue working optimization problems. The examples in this section tend to be a little more involved and will often involve situations that will be more easily described with a sketch as opposed to the 'simple' geometric objects we looked at in the previous section.

L'Hospital's Rule and Indeterminate Forms – In this section we will revisit indeterminate forms and limits and take a look at L'Hospital's Rule. L'Hospital's Rule will allow us to evaluate some limits we were not able to previously.

Linear Approximations – In this section we discuss using the derivative to compute a linear approximation to a function. We can use the linear approximation to a function to approximate values of the function at certain points. While it might not seem like a useful thing to do with when we have the function there really are reasons that one might want to do this. We give two ways this can be useful in the examples.

Differentials – In this section we will compute the differential for a function. We will give an application of differentials in this section. However, one of the more important uses of differentials will come in the next chapter and unfortunately we will not be able to discuss it until then.

Newton's Method – In this section we will discuss Newton's Method. Newton's Method is an application of derivatives that will allow us to approximate solutions to an equation. There are many equations that cannot be solved directly and with this method we can get approximations to the solutions to many of those equations.

Business Applications – In this section we will give a cursory discussion of some basic applications of derivatives to the business field. We will revisit finding the maximum and/or minimum function value and we will define the marginal cost function, the average cost, the revenue function, the marginal revenue function and the marginal profit function. Note that this section is only intended to introduce these concepts and not teach you everything about them.

Section 4-1 : Rates of Change

As noted in the text for this section the purpose of this section is only to remind you of certain types of applications that were discussed in the previous chapter. As such there aren't any problems written for this section. Instead here is a list of links (note that these will only be active links in the web version and not the pdf version) to problems from the relevant sections from the previous chapter.

Each of the following sections has a selection of increasing/decreasing problems towards the bottom of the problem set.

Differentiation Formulas
Product & Quotient Rules
Derivatives of Trig Functions
Derivatives of Exponential and Logarithm Functions
Chain Rule

Related Rates problems are in the Related Rates section.

Section 4-2 : Critical Points

For problems 1 – 43 determine the critical points of each of the following functions. Note that a couple of the problems involve equations that may not be easily solved by hand and as such may require some computational aids. These are marked are noted below.

1. $R(x) = 8x^3 - 18x^2 - 240x + 2$

2. $f(z) = 2z^4 - 16z^3 + 20z^2 - 7$

3. $g(z) = 8 - 12z^5 - 25z^6 + \frac{90}{7}z^7$

4. $g(t) = 3t^4 - 20t^3 - 132t^2 + 672t - 4$
   Note : Depending upon your factoring skills this may require some computational aids.

5. $h(x) = 10x^2 - 15x^3 + \frac{15}{2}x^4 - x^5$
   Note : Depending upon your factoring skills this may require some computational aids.

6. $P(w) = w^3 - 4w^2 - 7w - 1$

7. $A(t) = 7t^3 - 3t^2 + t - 15$

8. $a(t) = 4 - 2t^2 - 6t^3 - 3t^4$

9. $f(x) = 3x^4 - 20x^3 + 6x^2 + 120x + 5$
   Note : This problem will require some computational aids.

10. $h(v) = v^5 + v^4 + 10v^3 - 15$

11. $g(z) = (z-3)^5(2z+1)^4$

12. $R(q) = (q+2)^4(q^2-8)^2$

13. $f(t) = (t-2)^3(t^2+1)^2$

14. $f(w) = \dfrac{w^2 + 2w + 1}{3w - 5}$

15. $h\left(t\right)=\dfrac{3-4t}{t^2+1}$

16. $R\left(y\right)=\dfrac{y^2-y}{y^2+3y+8}$

17. $Y\left(x\right)=\sqrt[3]{x-7}$

18. $f\left(t\right)=\left(t^3-25t\right)^{\frac{2}{3}}$

19. $h\left(x\right)=\sqrt[5]{x}\ \left(2x+8\right)^2$

20. $Q\left(w\right)=\left(6-w^2\right)\ \sqrt[3]{w^2-4}$

21. $Q\left(t\right)=7\sin\left(\frac{t}{4}\right)-2$

22. $g\left(x\right)=3\cos\left(2x\right)-5x$

23. $f\left(x\right)=7\cos\left(x\right)+2x$

24. $h\left(t\right)=6\sin\left(2t\right)+12t$

25. $w\left(z\right)=\cos^3\left(\frac{z}{5}\right)$

26. $U\left(z\right)=\tan\left(z\right)-4z$

27. $h\left(x\right)=x\cos\left(x\right)-\sin\left(x\right)$

28. $h\left(x\right)=2\cos\left(x\right)-\cos\left(2x\right)$

29. $f\left(w\right)=\cos^2\left(w\right)-\cos^4\left(w\right)$

30. $F\left(w\right)=\mathbf{e}^{14w+3}$

31. $g\left(z\right)=z^2\mathbf{e}^{1-z}$

32. $A\left(x\right)=\left(3-2x\right)\mathbf{e}^{x^2}$

33. $P(t) = (6t+1)\mathbf{e}^{8t-t^2}$

34. $f(x) = \mathbf{e}^{3+x^2} - \mathbf{e}^{2x^2-4}$

35. $f(z) = \mathbf{e}^{z^2-4z} + \mathbf{e}^{8z-2z^2}$

36. $h(y) = \mathbf{e}^{6y^3-8y^2}$

37. $g(t) = \mathbf{e}^{2t^3+4t^2-t}$

38. $Z(t) = \ln(t^2+t+3)$

39. $G(r) = r - \ln(r^2+1)$

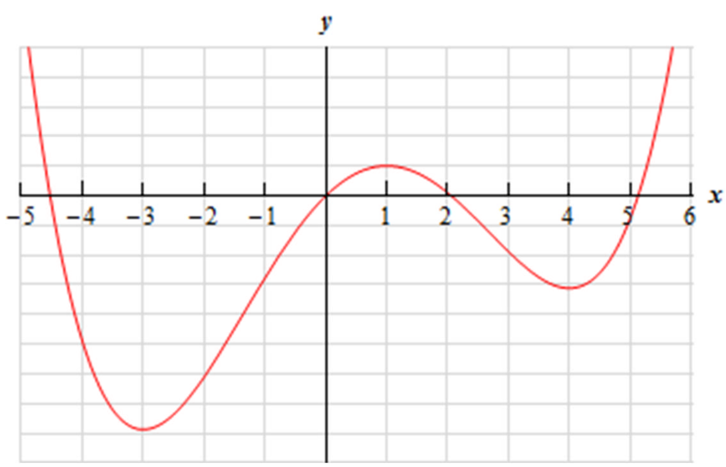
40. $A(z) = 2 - 6z + \ln(8z+1)$

41. $f(x) = x - 4\ln(x^2+x+2)$

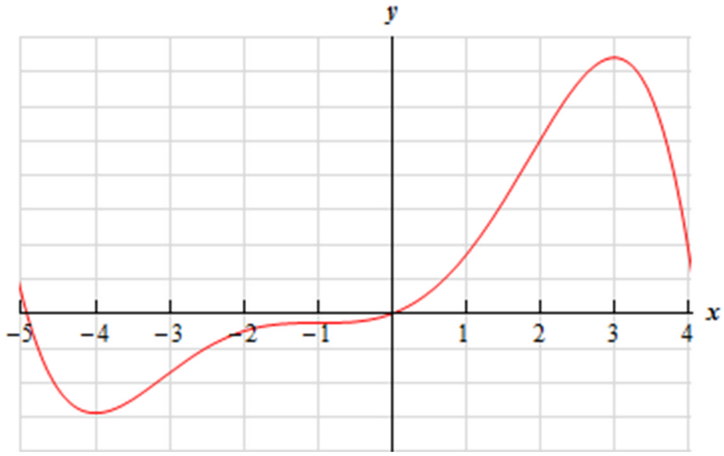
42. $g(x) = \ln(4x+2) - \ln(x+4)$

43. $h(t) = \ln(t^2-t+1) + \ln(4-t)$

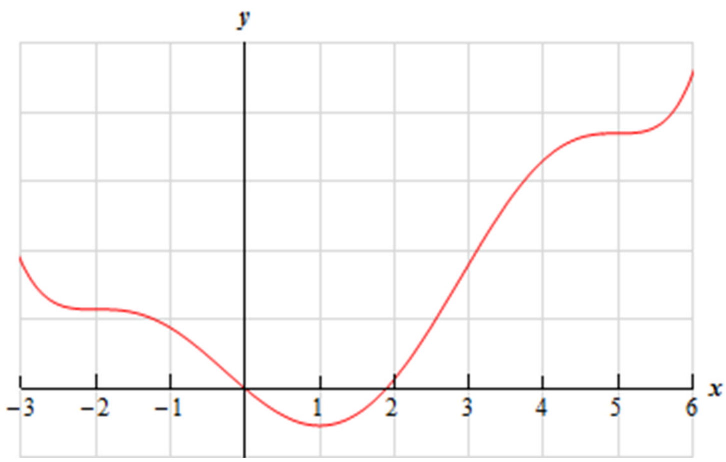
44. The graph of some function, $f(x)$, is shown. Based on the graph, estimate the location of all the critical points of the function.



45. The graph of some function, $f(x)$, is shown. Based on the graph, estimate the location of all the critical points of the function.

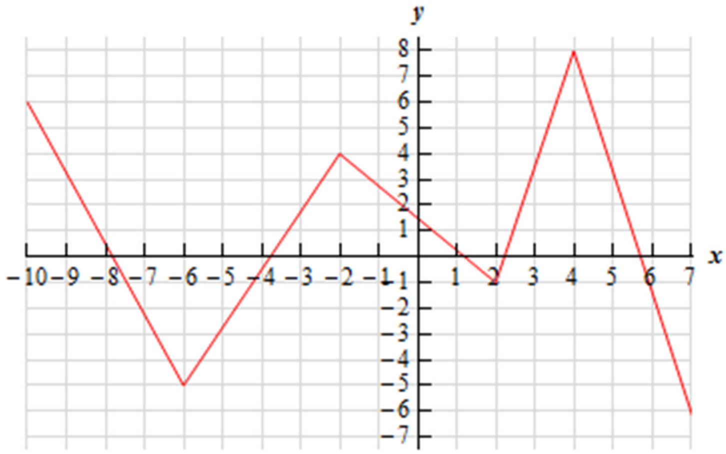


46. The graph of some function, $f(x)$, is shown. Based on the graph, estimate the location of all the critical points of the function.

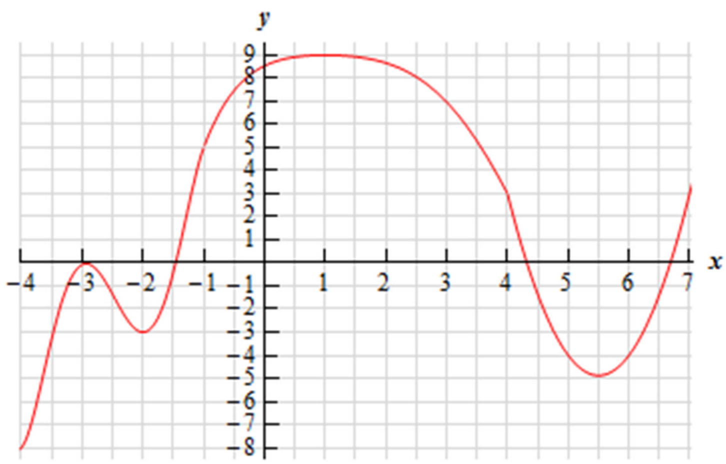


Section 4-3 : Minimum and Maximum Values

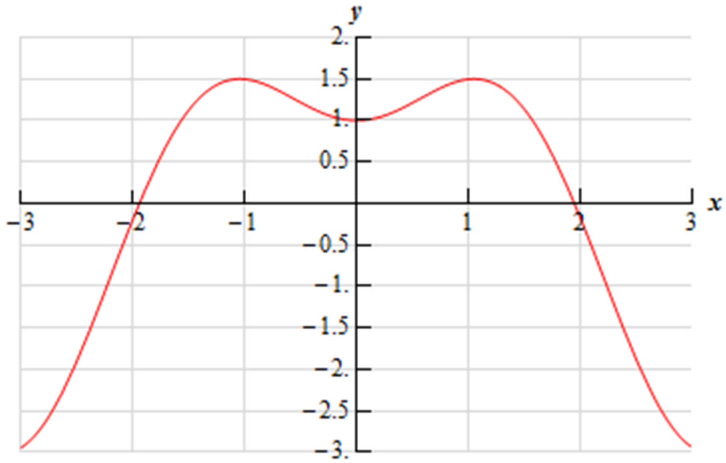
1. Below is the graph of some function, $f(x)$. Identify all of the relative extrema and absolute extrema of the function.



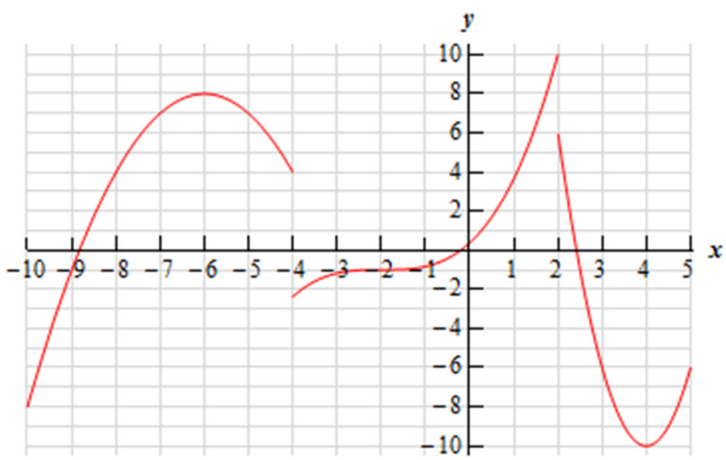
2. Below is the graph of some function, $f(x)$. Identify all of the relative extrema and absolute extrema of the function.



3. Below is the graph of some function, $f\left(x\right)$. Identify all of the relative extrema and absolute extrema of the function.



4. Below is the graph of some function, $f\left(x\right)$. Identify all of the relative extrema and absolute extrema of the function.



5. Sketch the graph of $f\left(x\right)=3-\frac{1}{2}x$ and identify all the relative extrema and absolute extrema of the function on each of the following intervals.

(a) $\left(-\infty,\infty\right)$

(b) $\left[-3,2\right]$

(c) $\left[-4,1\right)$

(d) $\left(0,5\right)$

6. Sketch the graph of $g\left(x\right)=\left(x-2\right)^2+1$ and identify all the relative extrema and absolute extrema of the function on each of the following intervals.

(a) $\left(-\infty,\infty\right)$

(b) $\left[0,3\right]$

(c) $\left[-1,5\right]$

(d) $\left[-1,1\right]$

(e) $\left[1,3\right)$

(f) $\left(2,4\right)$

7. Sketch the graph of $h\left(x\right)=\mathbf{e}^{3-x}$ and identify all the relative extrema and absolute extrema of the function on each of the following intervals.

(a) $\left(-\infty,\infty\right)$

(b) $\left[-1,3\right]$

(c) $\left[-6,-1\right]$

(d) $\left(1,4\right]$

8. Sketch the graph of $h\left(x\right)=\cos\left(x\right)+2$ and identify all the relative extrema and absolute extrema of the function on each of the following intervals. Do, all work for this problem in radians.

(a) $\left(-\infty,\infty\right)$

(b) $\left[-\frac{\pi}{3},\frac{\pi}{4}\right]$

(c) $\left[-\frac{\pi}{2},2\pi\right]$

(d) $\left[\frac{1}{2},1\right]$

9. Sketch the graph of a function on the interval $\left[3,9\right]$ that has an absolute maximum at $x=5$ and an absolute minimum at $x=4$.

10. Sketch the graph of a function on the interval $\left[0,10\right]$ that has an absolute minimum at $x=5$ and an absolute maximums at $x=0$ and $x=10$.

11. Sketch the graph of a function on the interval $\left(-\infty,\infty\right)$ that has a relative minimum at $x=-7$, a relative maximum at $x=2$ and no absolute extrema.

12. Sketch the graph of a function that meets the following conditions :
    - **(a)** Has at least one absolute maximum.
    - **(b)** Has one relative minimum.
    - **(c)** Has no absolute minimum.

13. Sketch the graph of a function that meets the following conditions :
    - **(a)** Graphed on the interval $\left[2,9\right]$.
    - **(b)** Has a discontinuity at some point interior to the interval.
    - **(c)** Has an absolute maximum at the discontinuity in part **(b)**.
    - **(d)** Has an absolute minimum at the discontinuity in part **(b)**.

14. Sketch the graph of a function that meets the following conditions :
    - **(a)** Graphed on the interval $\left[-4,10\right]$.
    - **(b)** Has no relative extrema.
    - **(c)** Has an absolute maximum at one end point.
    - **(d)** Has an absolute minimum at the other end point.

15. Sketch the graph of a function that meets the following conditions :
    - **(a)** Has a discontinuity at some point.
    - **(b)** Has an absolute maximum and an absolute minimum.
    - **(c)** Neither absolute extrema occurs at the discontinuity.

Section 4-4 : Finding Absolute Extrema

For each of the following problems determine the absolute extrema of the given function on the specified interval.

1. $f(z) = 2z^4 - 16z^3 + 20z^2 - 7$ on $[-2, 6]$

2. $f(z) = 2z^4 - 16z^3 + 20z^2 - 7$ on $[-2, 4]$

3. $f(z) = 2z^4 - 16z^3 + 20z^2 - 7$ on $[0, 2]$

4. $Q(w) = 20 + 280w^3 + 75w^4 - 12w^5$ on $[-3, 2]$

5. $Q(w) = 20 + 280w^3 + 75w^4 - 12w^5$ on $[-1, 8]$

6. $g(z) = 8 - 12z^5 - 25z^6 + \frac{90}{7}z^7$ on $[-1, 1]$

7. $g(t) = 3t^4 - 20t^3 - 132t^2 + 672t - 4$ on $[-5, 8]$
   Note : Depending upon your factoring skills this may require some computational aids.

8. $g(t) = 3t^4 - 20t^3 - 132t^2 + 672t - 4$ on $[-2, 8]$
   Note : Depending upon your factoring skills this may require some computational aids.

9. $V(x) = 14x^3 + 11x^2 - 4x + 3$ on $[-1, 1]$

10. $a(t) = 4 - 2t^2 - 6t^3 - 3t^4$ on $[-2, 1]$

11. $h(x) = 8 + 3x + 7x^2 - x^3$ on $[-1, 5]$

12. $f(x) = 3x^4 - 20x^3 + 6x^2 + 120x + 5$ on $[-1, 5]$
    Note : This problem will require some computational aids.

13. $h(v) = v^5 + v^4 + 10v^3 - 15$ on $[-3, 2]$

14. $g(z) = (z-3)^5 (2z+1)^4$ on $[-1, 3]$

15. $R(q) = (q+2)^4 (q^2 - 8)^2$ on $[-4, 1]$

16. $h(t) = \dfrac{3-4t}{t^2+1}$ on $[-2,4]$

17. $g(x) = \dfrac{6+9x+x^2}{1+x+x^2}$ on $[-6,0]$

18. $f(t) = \left(t^3 - 25t\right)^{\frac{2}{3}}$ on $[2,6]$

19. $F(t) = 2 + t^{\frac{2}{5}}\left(1+t+t^2\right)$ on $[-2,1]$

20. $Q(w) = \left(6-w^2\right)\sqrt[3]{w^2-4}$ on $\left[-5,\frac{1}{2}\right]$

21. $g(x) = 3\cos(2x) - 5x$ on $[0,6]$

22. $s(w) = 3w - 10\sin\left(\frac{w}{3}\right)$ on $[10,38]$

23. $f(x) = 7\cos(x) + 2x$ on $[-5,4]$

24. $h(x) = x\cos(x) - \sin(x)$ on $[-15,-5]$

25. $g(z) = z^2\mathbf{e}^{1-z}$ on $\left[-\frac{1}{2},\frac{5}{2}\right]$

26. $P(t) = (6t+1)\mathbf{e}^{8t-t^2}$ on $[-1,3]$

27. $f(x) = \mathbf{e}^{5+9x} + \mathbf{e}^{1-3x} + 6$ on $[-1,0]$

28. $h(y) = \mathbf{e}^{6y^3-8y^2}$ on $\left[-\frac{1}{2},1\right]$

29. $Z(t) = \ln\left(t^2+t+3\right)$ on $[-2,2]$

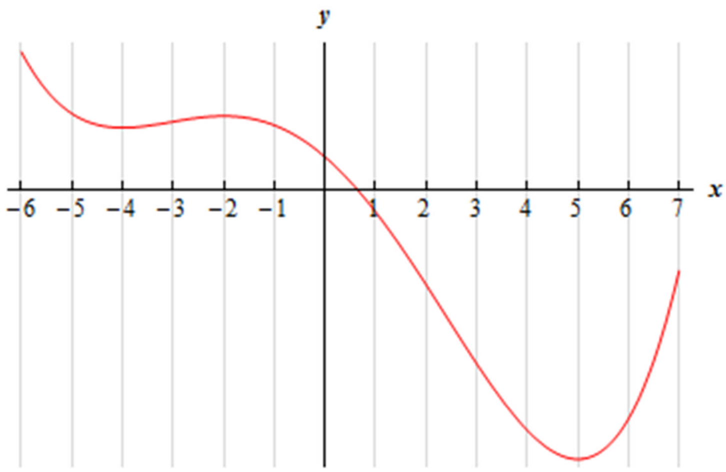
30. $f(x) = x - 4\ln\left(x^2+x+2\right)$ on $[-1,9]$

31. $h(t) = \ln\left(t^2-t+1\right) + \ln(4-t)$ on $[1,3]$

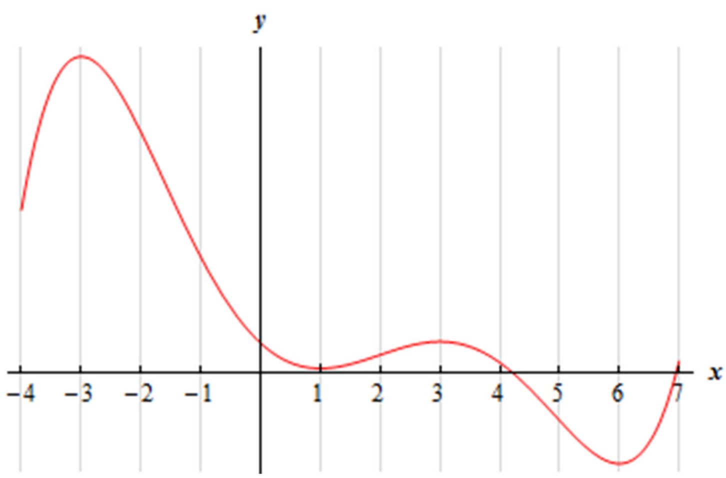
Section 4-5 : The Shape of a Graph, Part I

For problems 1 – 4 the graph of a function is given. Determine the intervals on which the function increases and decreases.

1.



2.



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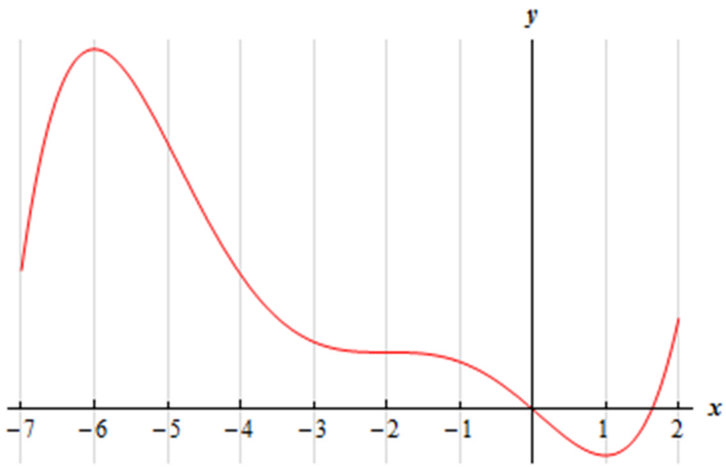
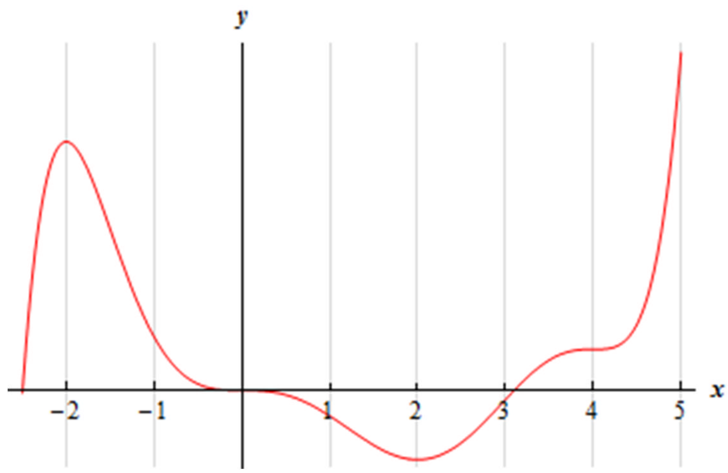
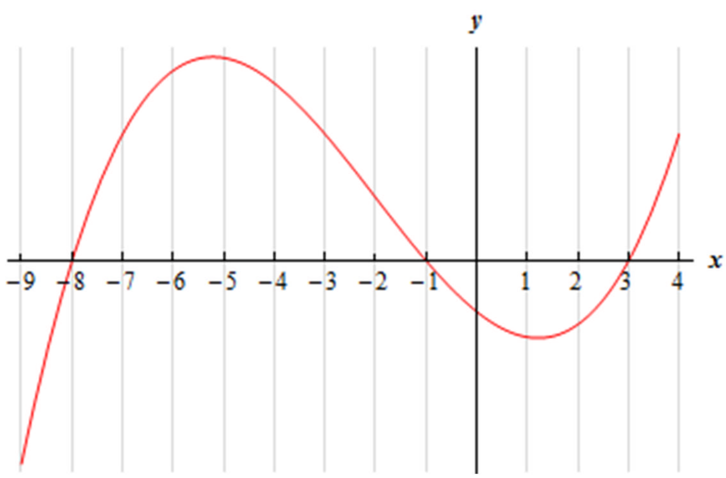


4.

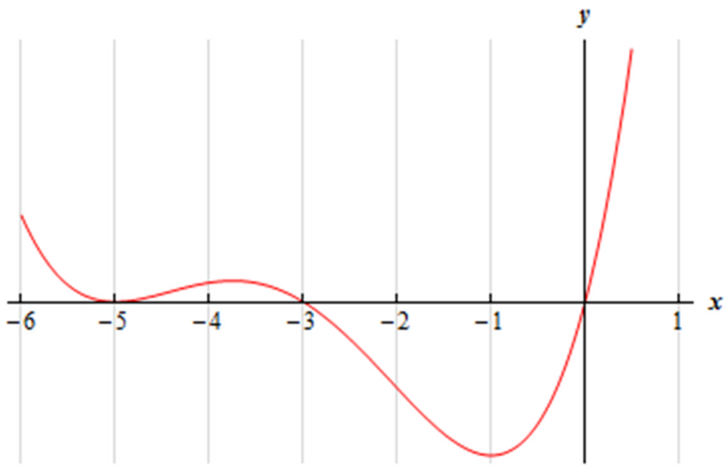


For problems 5 – 7 the graph of the **derivative** of a function is given. From this graph determine the intervals in which the **function** increases and decreases.

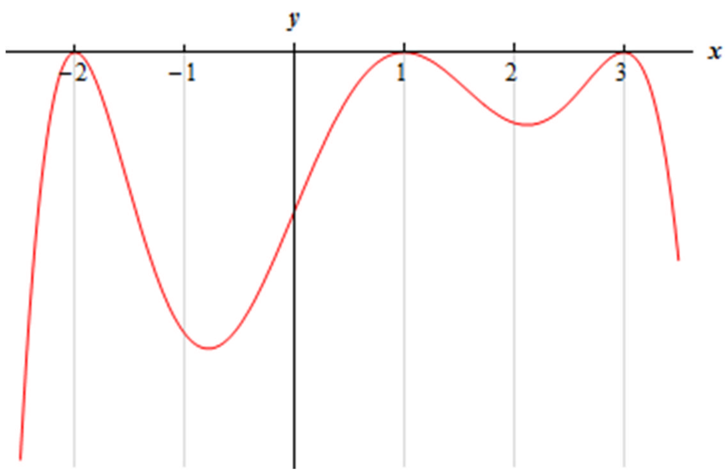
5.



6.



7.



For problems 8 – 10 The known information about the derivative of a function is given. From this information answer each of the following questions.

(a) Identify the critical points of the function.
(b) Determine the intervals on which the function increases and decreases.
(c) Classify the critical points as relative maximums, relative minimums or neither.

8.

$$f'(1)=0 \qquad f'(3)=0 \qquad f'(8)=0$$
$$f'(x)<0 \quad \text{on} \quad (-\infty,1),\ (3,8) \qquad\qquad f'(x)>0 \quad \text{on} \quad (1,3),\ (8,\infty)$$

9.

$$g'(-2)=0 \qquad g'(0)=0 \qquad g'(3)=0 \qquad g'(6)=0$$
$$g'(x)<0 \quad \text{on} \quad (0,3),\ (6,\infty) \qquad\qquad g'(x)>0 \quad \text{on} \quad (-\infty,-2),\ (-2,0),\ (3,6)$$

10.

$$h'(-1)=0 \qquad h'(2)=0 \qquad h'(5)=0$$

$$h'(x)<0 \quad \text{on} \quad (-\infty,-1),\ (-1,2) \qquad\qquad h'(x)>0 \quad \text{on} \quad (2,5),\ (5,\infty)$$

For problems 11 – 28 answer each of the following.

(a) Identify the critical points of the function.

(b) Determine the intervals on which the function increases and decreases.

(c) Classify the critical points as relative maximums, relative minimums or neither.

11. $f(t)=t^3-15t^2+63t+3$

12. $g(x)=20+8x^2+4x^3-x^4$

13. $Q(w)=8w^3-18w^2-24w-10$

14. $f(x)=x^5+\frac{5}{4}x^4-20x^3-7$

15. $P(x)=5-4x-9x^2-3x^3$

16. $R(z)=z^5+z^4-6z^3+5$

17. $h(z)=1-12z^2-9z^3-2z^4$

18. $Q(t)=7-t+\sin(4t)$ on $\left[-\frac{3}{2},\frac{3}{2}\right]$

19. $f(z)=6z-20\cos\left(\frac{z}{2}\right)$ on $[0,22]$

20. $g(x)=24\cos\left(\frac{x}{3}\right)+8x+2$ on $[-30,25]$

21. $h(w)=9w-5\sin(2w)$ on $[-5,0]$

22. $h(x)=\sqrt[5]{x}\ (x+7)$

23. $W(z)=(10-w^2)(w+2)^{\frac{2}{3}}$

24. $f(t)=(t^2-8)\ \sqrt[3]{t^2-4}$

25. $f(x)=\mathbf{e}^{\frac{1}{3}x^3-x^2-3x}$

26. $h(z) = (z^2 - 8)\mathbf{e}^{3-z}$

27. $A(t) = \ln(t^2 + 5t + 8)$

28. $g(x) = x - 3 + \ln(1 + x + x^2)$

29. Answer each of the following questions.
   (a) What is the minimum degree of a polynomial that has exactly one relative extrema?
   (b) What is the minimum degree of a polynomial that has exactly two relative extrema?
   (c) What is the minimum degree of a polynomial that has exactly three relative extrema?
   (d) What is the minimum degree of a polynomial that has exactly *n* relative extrema?

30. For some function, $f(x)$, it is known that there is a relative minimum at $x = -4$. Answer each of the following questions about this function.
   (a) What is the simplest form that the derivative of this function? Note : There really are many possible forms of the derivative so to make the rest of this problem as simple as possible you will want to use the simplest form of the derivative.
   (b) Using your answer from **(a)** determine the most general form that the function itself can take.
   (c) Given that $f(-4) = 6$ find a function that will have a relative minimum at $x = -4$. Note : There are many possible answers here so just give one of them.

31. For some function, $f(x)$, it is known that there is a relative maximum at $x = -1$. Answer each of the following questions about this function.
   (a) What is the simplest form that the derivative of this function? Note : There really are many possible forms of the derivative so to make the rest of this problem as simple as possible you will want to use the simplest form of the derivative.
   (b) Using your answer from **(a)** determine the most general form that the function itself can take.
   (c) Given that $f(-1) = 3$ find a function that will have a relative maximum at $x = -1$. Note : There are many possible answers here so just give one of them.

32. For some function, $f(x)$, it is known that there is a critical point at $x = 3$ that is neither a relative minimum or a relative maximum. Answer each of the following questions about this function.
   (a) What is the simplest form that the derivative of this function? Note : There really are many possible forms of the derivative so to make the rest of this problem as simple as possible you will want to use the simplest form of the derivative.
   (b) Using your answer from **(a)** determine the most general form that the function itself can take.
   (c) Given that $f(3) = 2$ find a function that will have a critical point at $x = 3$ that is neither a relative minimum or a relative maximum. Note : There are many possible answers here so just give one of them.

33. For some function, $f(x)$, it is known that there is a relative maximum at $x=1$ and a relative minimum at $x=4$. Answer each of the following questions about this function.

- **(a)** What is the simplest form that the derivative of this function? Note : There really are many possible forms of the derivative so to make the rest of this problem as simple as possible you will want to use the simplest form of the derivative.
- **(b)** Using your answer from **(a)** determine the most general form that the function itself can take.
- **(c)** Given that $f(1)=6$ and $f(4)=-2$ find a function that will have a relative maximum at $x=1$ and a relative minimum at $x=4$. Note : There are many possible answers here so just give one of them.

34. Given that $f(x)$ and $g(x)$ are increasing functions will $h(x)=f(x)-g(x)$ always be an increasing function? If so, prove that $h(x)$ will be an increasing function. If not, find increasing functions, $f(x)$ and $g(x)$, so that $h(x)$ will be a decreasing function and find a different set of increasing functions so that $h(x)$ will be an increasing function.

35. Given that $f(x)$ is an increasing function. There are several possible conditions that we can impose on $g(x)$ so that $h(x)=f(x)-g(x)$ will be an increasing function. Determine as many of these possible conditions as you can.

36. For a function $f(x)$ determine a set of conditions on $f(x)$, different from those given in **#15** in the practice problems, for which $h(x)=\left[f(x)\right]^2$ will be an increasing function.

37. For a function $f(x)$ determine a single condition on $f(x)$ for which $h(x)=\left[f(x)\right]^3$ will be an increasing function.

38. Given that $f(x)$ and $g(x)$ are positive functions. Determine a set of conditions on them for which $h(x)=f(x)g(x)$ will be an increasing function. Note that there are several possible sets of conditions here but try to determine the "simplest" set of conditions.

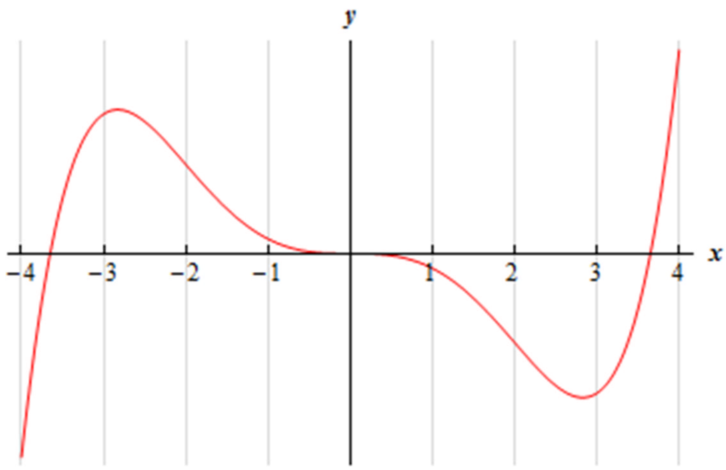
39. Repeat **#38** for $h(x)=\dfrac{f(x)}{g(x)}$.

40. Given that $f(x)$ and $g(x)$ are increasing functions prove that $h(x)=f(g(x))$ will also be an increasing function.

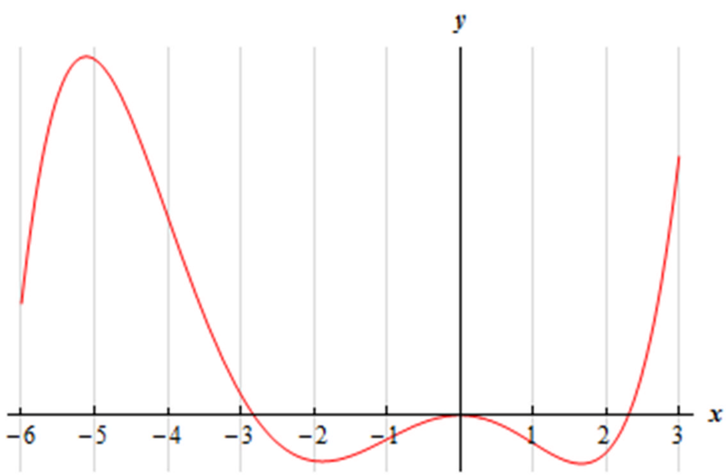
Section 4-6 : The Shape of a Graph, Part II

For problems 1 & 2 the graph of a function is given. Determine the intervals on which the function is concave up and concave down.

1.

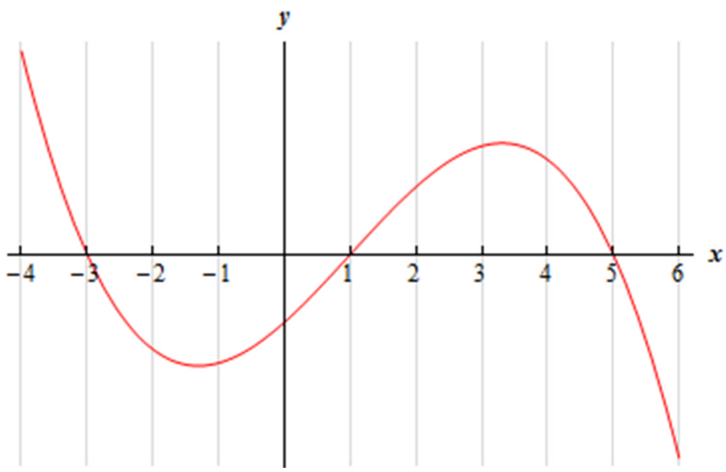


2.

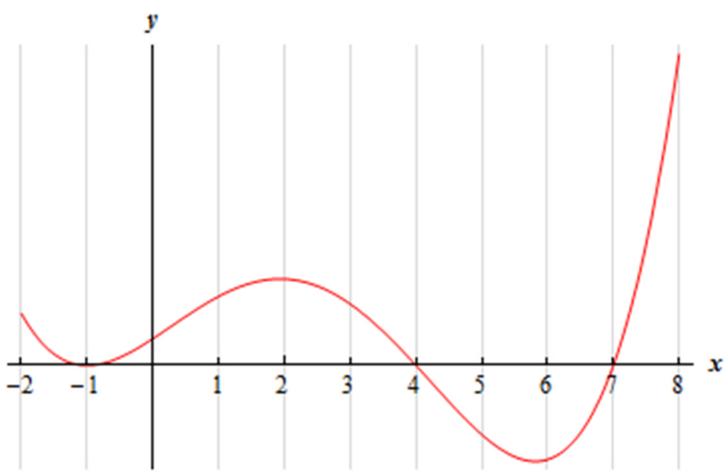


For problems 3 – 5 the graph of the **2nd derivative** of a function is given. From this graph determine the intervals in which the **function** is concave up and concave down.

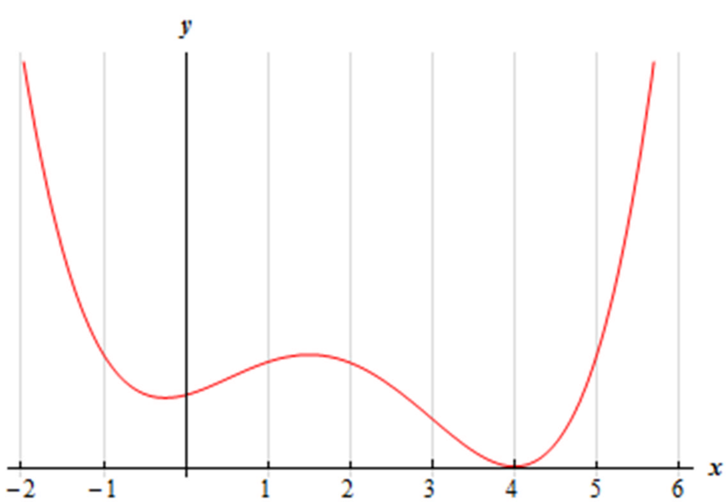
3.



4.



5.



For problems 6 – 18 answer each of the following.

(a) Determine the intervals on which the function is concave up and concave down.

(b) Determine the inflection points of the function.

6. $f\left(x\right)=x^3+9x^2+24x-6$

7. $Q\left(t\right)=t^4-2t^3-120t^2-84t+35$

8. $h\left(z\right)=3z^5-20z^4+40z^3$

9. $g\left(w\right)=5w^4-2w^3-18w^2+108w-12$

10. $g\left(x\right)=10+360x+20x^4+3x^5-x^6$

11. $A\left(x\right)=9x-3x^2-160\sin\left(\frac{x}{4}\right)$ on $\left[-20,11\right]$

12. $f\left(x\right)=3\cos\left(2x\right)-x^2-14$ on $\left[0,6\right]$

13. $h\left(t\right)=1+2t^2-\sin\left(2t\right)$ on $\left[-2,4\right]$

14. $R\left(v\right)=v\left(v-8\right)^{\frac{1}{3}}$

15. $g\left(x\right)=\left(x-1\right)\left(x+3\right)^{\frac{2}{5}}$

16. $f\left(x\right)=\mathbf{e}^{4x}-\mathbf{e}^{-x}$

17. $h\left(w\right)=w^2\mathbf{e}^{-w}$

18. $A\left(w\right)=w^2-\ln\left(w^2+1\right)$

For problems 19 – 33 answer each of the following.

(a) Identify the critical points of the function.

(b) Determine the intervals on which the function increases and decreases.

(c) Classify the critical points as relative maximums, relative minimums or neither.

(d) Determine the intervals on which the function is concave up and concave down.

(e) Determine the inflection points of the function.

(f) Use the information from steps **(a)** – **(e)** to sketch the graph of the function.

19. $f\left(x\right)=10-30x^2+2x^3$

20. $G(t) = 14 + 4t^3 - t^4$

21. $h(w) = w^4 + 4w^3 - 18w^2 - 9$

22. $g(z) = 10z^3 + 10z^4 + 3z^5$

23. $f(z) = z^6 - 9z^5 + 20z^4 + 10$

24. $Q(t) = 3t - 5\sin(2t)$ on $[-1, 4]$

25. $g(x) = \frac{1}{2}x + \cos\left(\frac{1}{3}x\right)$ on $[-25, 0]$

26. $h(x) = x(x-4)^{\frac{1}{3}}$

27. $f(t) = t\sqrt{t^2 + 1}$

28. $A(z) = z^{\frac{4}{5}}(z - 27)$

29. $g(w) = \mathbf{e}^{4w} - \mathbf{e}^{6w}$

30. $P(t) = 3t\mathbf{e}^{1 - \frac{1}{4}t^2}$

31. $g(x) = (x+1)^3 \mathbf{e}^{-x}$

32. $h(z) = \ln(z^2 + z + 1)$

33. $f(w) = 2w - 8\ln(w^2 + 4)$

34. Answer each of the following questions.
   (a) What is the minimum degree of a polynomial that has exactly two inflection points.
   (b) What is the minimum degree of a polynomial that has exactly three inflection points.
   (c) What is the minimum degree of a polynomial that has exactly *n* inflection points.

35. For some function, $f(x)$, it is known that there is an inflection point at $x = 3$. Answer each of the following questions about this function.
   (a) What is the simplest form that the 2nd derivative of this function?
   (b) Using your answer from **(a)** determine the most general form that the function itself can take.
   (c) Given that $f(0) = -6$ and $f(3) = 1$ find a function that will have an inflection point at $x = 3$.

For problems 36 – 39 $f(x)$ is a polynomial. Given the 2nd derivative of the function, classify, if possible, each of the given critical points as relative minimums or relative maximum. If it is not possible to classify the critical point(s) clearly explain why they cannot be classified.

36. $f''(x) = 3x^2 - 4x - 15$. The critical points are : $x = -3$, $x = 0$ and $x = 5$.

37. $f''(x) = 4x^3 - 21x^2 - 24x + 68$. The critical points are : $x = -2$, $x = 4$ and $x = 7$.

38. $f''(x) = 23 + 18x - 9x^2 - 4x^3$. The critical points are : $x = -4$, $x = -1$ and $x = 3$.

39. $f''(x) = 216 - 410x + 249x^2 - 60x^3 + 5x^4$. The critical points are : $x = 1$, $x = 4$ and $x = 5$.

40. Use $f(x) = (x+1)^3 (x-1)^4$ for this problem.

(a) Determine the critical points for the function.

(b) Use the 2nd derivative test to classify the critical points as relative minimums or relative maximums. If it is not possible to classify the critical point(s) clearly explain why they cannot be classified.

(c) Use the 1st derivative test to classify the critical points as relative minimums, relative maximums or neither.

41. Given that $f(x)$ and $g(x)$ are concave down functions. If we define $h(x) = f(x) + g(x)$ show that $h(x)$ is a concave down function.

42. Given that $f(x)$ is a concave up function. Determine a condition on $g(x)$ for which $h(x) = f(x) + g(x)$ will be a concave up function.

43. For a function $f(x)$ determine conditions on $f(x)$ for which $h(x) = \left[f(x)\right]^2$ will be a concave up function. Note that there are several sets of conditions that can be used here. How many of them can you find?

Section 4-7 : The Mean Value Theorem

For problems 1 – 4 determine all the number(s) *c* which satisfy the conclusion of Rolle's Theorem for the given function and interval.

1. $f(x) = x^3 - 4x^2 + 3$ on $[0,4]$

2. $Q(z) = 15 + 2z - z^2$ on $[-2,4]$

3. $h(t) = 1 - \mathbf{e}^{t^2 - 9}$ on $[-3,3]$

4. $g(w) = 1 + \cos[\pi\, w]$ on $[5,9]$

For problems 5 – 8 determine all the number(s) *c* which satisfy the conclusion of the Mean Value Theorem for the given function and interval.

5. $f(x) = x^3 - x^2 + x + 8$ on $[-3,4]$

6. $g(t) = 2t^3 + t^2 + 7t - 1$ on $[1,6]$

7. $P(t) = \mathbf{e}^{2t} - 6t - 3$ on $[-1,0]$

8. $h(x) = 9x - 8\sin\left(\frac{x}{2}\right)$ on $[-3,-1]$

9. Suppose we know that $f(x)$ is continuous and differentiable on the interval $[-2,5]$, that $f(5) = 14$ and that $f'(x) \le 10$. What is the smallest possible value for $f(-2)$?

10. Suppose we know that $f(x)$ is continuous and differentiable on the interval $[-6,-1]$, that $f(-6) = -23$ and that $f'(x) \ge -4$. What is the smallest possible value for $f(-1)$?

11. Suppose we know that $f(x)$ is continuous and differentiable on the interval $[-3,4]$, that $f(-3) = 7$ and that $f'(x) \le -17$. What is the largest possible value for $f(4)$?

12. Suppose we know that $f(x)$ is continuous and differentiable on the interval $[1,9]$, that $f(9) = 0$ and that $f'(x) \ge 8$. What is the largest possible value for $f(1)$?

13. Show that $f(x) = x^7 + 2x^5 + 3x^3 + 14x + 1$ has exactly one real root.

14. Show that $f\left(x\right)=6x^3-2x^2+4x-3$ has exactly one real root.

15. Show that $f\left(x\right)=20x-\mathbf{e}^{-4x}$ has exactly one real root.

Section 4-8 : Optimization

1. Find two positive numbers whose sum of six times one of them and the second is 250 and whose product is a maximum.

2. Find two positive numbers whose sum of twice the first and seven times the second is 600 and whose product is a maximum.

3. Let *x* and *y* be two positive numbers whose sum is 175 and $\left(x+3\right)\left(y+4\right)$ is a maximum. Determine *x* and *y*.

4. Find two positive numbers such that the sum of the one and the square of the other is 200 and whose product is a maximum.

5. Find two positive numbers whose product is 400 and such that the sum of twice the first and three times the second is a minimum.

6. Find two positive numbers whose product is 250 and such that the sum of the first and four times the second is a minimum.

7. Let *x* and *y* be two positive numbers such that $y\left(x+2\right)=100$ and whose sum is a minimum. Determine *x* and *y*.

8. Find a positive number such that the sum of the number and its reciprocal is a minimum.

9. We are going to fence in a rectangular field and have 200 feet of material to construct the fence. Determine the dimensions of the field that will enclose the maximum area.

10. We are going to fence in a rectangular field. Starting at the bottom of the field and moving around the field in a counter clockwise manner the cost of material for each side is $6/ft, $9/ft, $12/ft and $14/ft respectively. If we have $1000 to buy fencing material determine the dimensions of the field that will maximize the enclosed area.

11. We are going to fence in a rectangular field that encloses 75 ft^2. Determine the dimensions of the field that will require the least amount of fencing material to be used.

12. We are going to fence in a rectangular field that encloses 200 m^2. If the cost of the material for of one pair of parallel sides is $3/m and cost of the material for the other pair of parallel sides is $8/m determine the dimensions of the field that will minimize the cost to build the fence around the field.

13. Show that a rectangle with a fixed area and minimum perimeter is a square.

14. Show that a rectangle with a fixed perimeter and a maximum area is a square.

15. We have 350 m^2 of material to build a box whose base width is four times the base length. Determine the dimensions of the box that will maximize the enclosed volume.

16. We have \$1000 to buy the materials to build a box whose base length is seven times the base width and has no top. If the material for the sides cost \$10/cm^2 and the material for the bottom cost \$15/cm^2 determine the dimensions of the box that will maximize the enclosed volume.

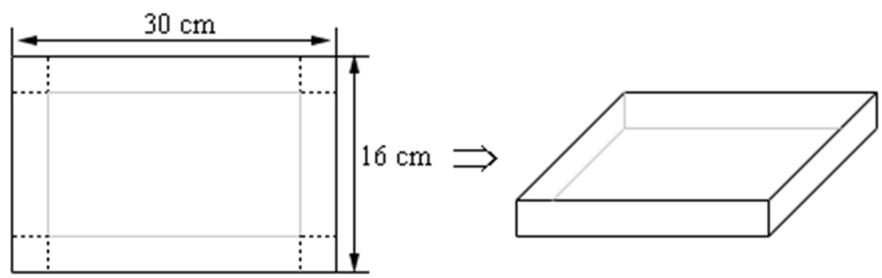
17. We want to build a box whose base length is twice the base width and the box will enclose 80 ft^3. The cost of the material of the sides is \$0.5/ft^2 and the cost of the top/bottom is \$3/ft^2. Determine the dimensions of the box that will minimize the cost.

18. We want to build a box whose base is a square, has no top and will enclose 100 m^3. Determine the dimensions of the box that will minimize the amount of material needed to construct the box.

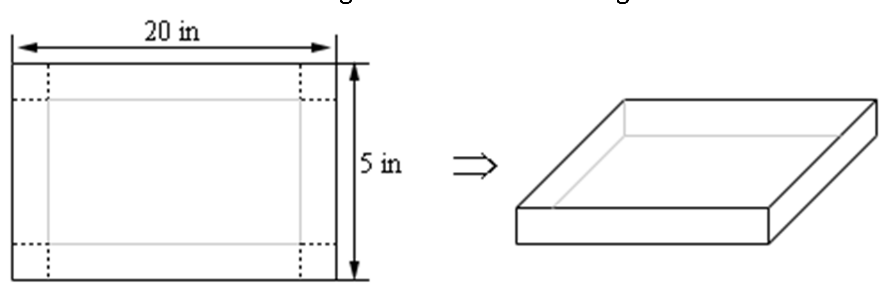
19. We want to construct a cylindrical can with a bottom but no top that will have a volume of 65 in^3. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.

20. We want to construct a cylindrical can whose volume is 105 mm^3. The material for the wall of the can costs \$3/mm^2, the material for the bottom of the can costs \$7/mm^2 and the material for the top of the can costs \$2/mm^2. Determine the dimensions of the can that will minimize the cost of the materials needed to construct the can.

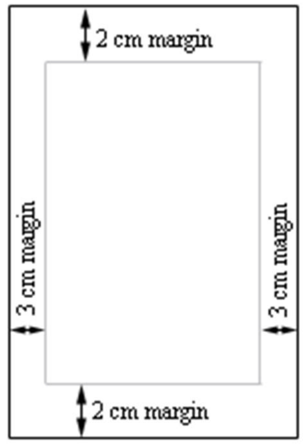
21. We have a piece of cardboard that is 30 cm by 16 cm and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.



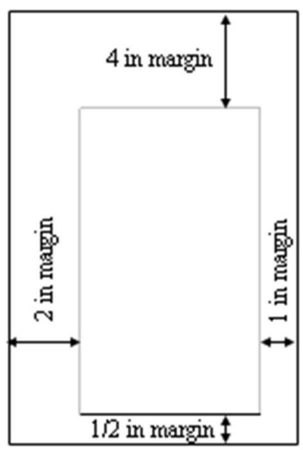
22. We have a piece of cardboard that is 5 in by 20 in and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.



23. A printer needs to make a poster that will have a total of 500 cm^2 that will have 3 cm margins on the sides and 2 cm margins on the top and bottom. What dimensions of the poster will give the largest printed area?



24. A printer needs to make a poster that will have a total of 125 in^2 that will have ½ inch margin on the bottom, 1 inch margin on the right, 2 inch margin on the left and 4 inch margin on the top. What dimensions of the poster will give the largest printed area?



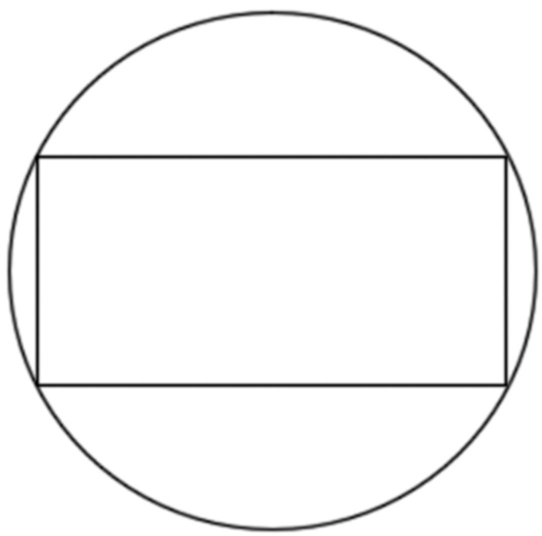
Section 4-9 : More Optimization

1. We want to construct a window whose bottom is a rectangle and the top of the window is an equilateral triangle. If we have 75 inches of framing material what are the dimensions of the window that will let in the most light?

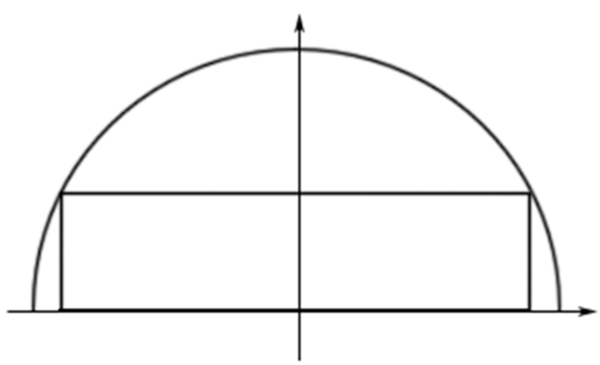
2. We want to construct a window whose middle is a rectangle and the top and bottom of the window are equilateral triangles. If we have 4 feet of framing material what are the dimensions of the window that will let in the most light?

3. We want to construct a window whose middle is a rectangle, the top of the window is a semicircle and the bottom of the window is an equilateral triangle. If we have 1500 cm of framing material what are the dimensions of the window that will let in the most light?

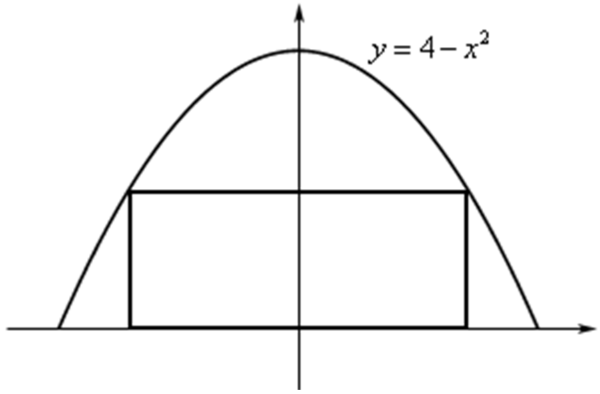
4. Determine the area of the largest rectangle that can be inscribed in a circle of radius 5.



5. Determine the area of the largest rectangle whose base is on the x-axis and the top two corners lie on semicircle of radius 16.



6. Determine the area of the largest rectangle whose base is on the *x*-axis and the top two corners lie $y = 4 - x^2$.



7. Find the point(s) on $\frac{x^2}{4} + \frac{y^2}{36} = 1$ that are closest to $(0,1)$.

8. Find the point(s) on $x = y^2 - 8$ that are closest to $(5,0)$.

9. Find the point(s) on $y = 2 - x^2$ that are closest to $(0,-3)$.

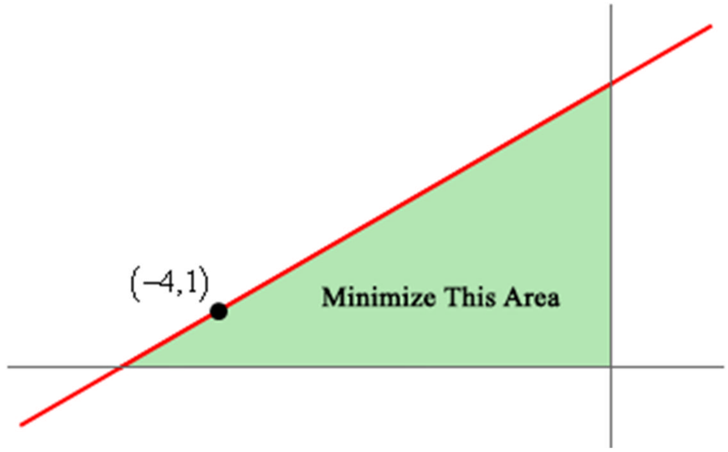
10. A 6 ft piece of wire is cut into two pieces. One piece is bent into an equilateral triangle and the other will be bent into a rectangle with one side twice the length of the other side. Determine where, if anywhere, the wire should be cut to minimize the area enclosed by the two figures.

11. A 250 cm piece of wire is cut into two pieces. One piece is bent into an equilateral triangle and the other will be bent into circle. Determine where, if anywhere, the wire should be cut to maximize the area enclosed by the two figures.

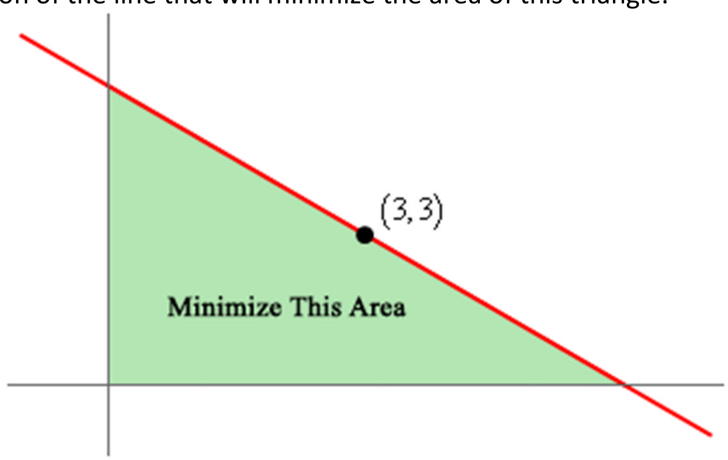
12. A 250 cm piece of wire is cut into two pieces. One piece is bent into an equilateral triangle and the other will be bent into circle. Determine where, if anywhere, the wire should be cut to minimize the area enclosed by the two figures.

13. A 4 m piece of wire is cut into two pieces. One piece is bent into a circle and the other will be bent into a rectangle with one side three times the length of the other side. Determine where, if anywhere, the wire should be cut to maximize the area enclosed by the two figures.

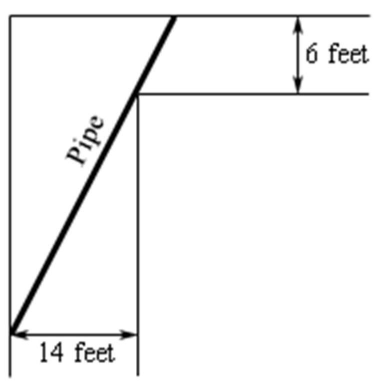
14. A line through the point $\left(-4,1\right)$ forms a right triangle with the x-axis and y-axis in the 2nd quadrant. Determine the equation of the line that will minimize the area of this triangle.



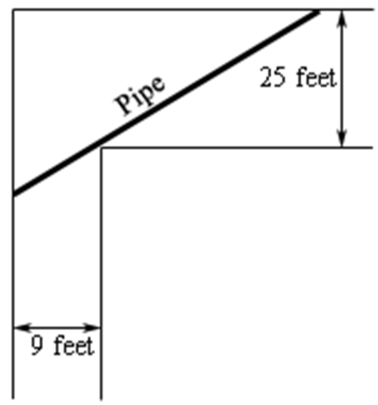
15. A line through the point $\left(3,3\right)$ forms a right triangle with the x-axis and y-axis in the 1st quadrant. Determine the equation of the line that will minimize the area of this triangle.



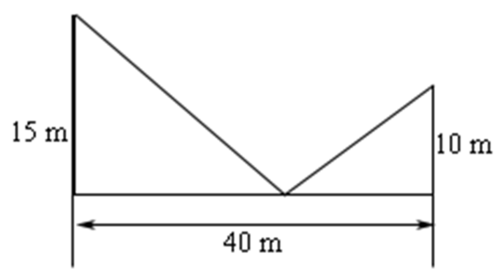
16. A piece of pipe is being carried down a hallway that is 14 feet wide. At the end of the hallway there is a right-angled turn and the hallway narrows down to 6 feet wide. What is the longest pipe (always keeping it horizontal) that can be carried around the turn in the hallway?



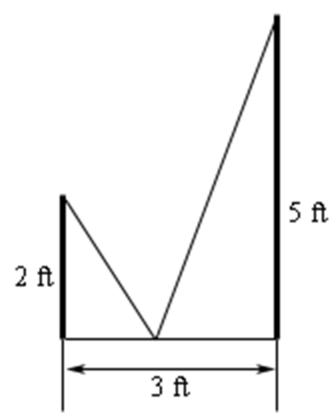
17. A piece of pipe is being carried down a hallway that is 9 feet wide. At the end of the hallway there is a right-angled turn and the hallway widens up to 21 feet wide. What is the longest pipe (always keeping it horizontal) that can be carried around the turn in the hallway?



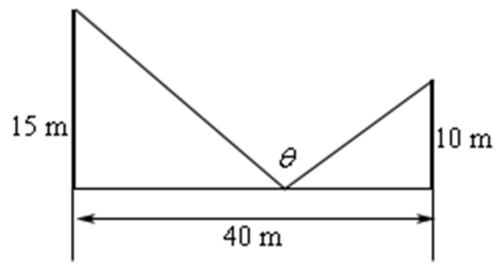
18. Two poles, one 15 meters tall and one 10 meters tall, are 40 meters apart. A length of wire is attached to the top of each pole and it is staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?



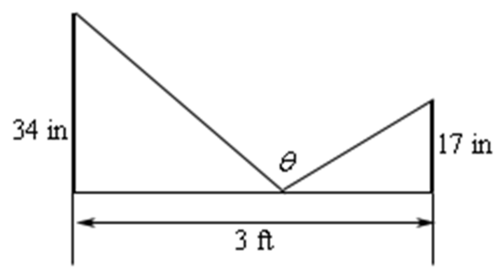
19. Two poles, one 2 feet tall and one 5 feet tall, are 3 feet apart. A length of wire is attached to the top of each pole and it is staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?



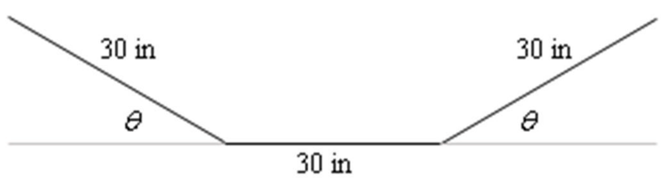
20. Two poles, one 15 meters tall and one 10 meters tall, are 40 meters apart. A length of wire is attached to the top of each pole and it is staked to the ground somewhere between the two poles. Where should the wire be staked so that the angle formed by the two pieces of wire at the stake is a maximum?



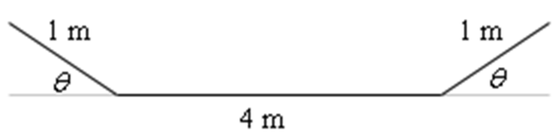
21. Two poles, one 34 inches tall and one 17 inches tall, are 3 feet apart. A length of wire is attached to the top of each pole and it is staked to the ground somewhere between the two poles. Where should the wire be staked so that the angle formed by the two pieces of wire at the stake is a maximum?



22. A trough for holding water is to be formed as shown in the figure below. Determine the angle θ that will maximize the amount of water that the trough can hold.



23. A trough for holding water is to be formed as shown in the figure below. Determine the angle θ that will maximize the amount of water that the trough can hold.



Section 4-10 : L'Hospital's Rule and Indeterminate Forms

For problems 1 – 18 use L'Hospital's Rule to evaluate the given limit.

1. $\displaystyle\lim_{x\to -4} \frac{x^3+6x^2-32}{x^3+5x^2+4x}$

2. $\displaystyle\lim_{w\to -\infty} \frac{\mathbf{e}^{-6w}}{4+\mathbf{e}^{-3w}}$

3. $\displaystyle\lim_{t\to 0} \frac{\sin(6t)}{\sin(11t)}$

4. $\displaystyle\lim_{x\to 1} \frac{x^2+8x-9}{x^3-2x^2-5x+6}$

5. $\displaystyle\lim_{t\to 2} \frac{t^3-7t^2+16t-12}{t^4-4t^3+4t^2}$

6. $\displaystyle\lim_{w\to -\infty} \frac{w^2-4w+1}{3w^2+7w-4}$

7. $\displaystyle\lim_{y\to \infty} \frac{y^2-\mathbf{e}^{6y}}{4y^2+\mathbf{e}^{7y}}$

8. $\displaystyle\lim_{x\to 0} \frac{2\cos(4x)-4x^2-2}{\sin(2x)-x^2-2x}$

9. $\displaystyle\lim_{x\to -3} \frac{3\mathbf{e}^{2x+6}+x^2-12}{x^3+6x^2+9x}$

10. $\displaystyle\lim_{z\to 6} \frac{\sin(\pi z)}{\ln(z-5)}$

11. $\displaystyle\lim_{w\to \infty} \left[w\ln\left(1-\frac{2}{3w}\right)\right]$

12. $\displaystyle\lim_{t\to 0^+} \left[\ln(t)\sin(t)\right]$

13. $\lim\limits_{z \to -\infty} z^2 \mathbf{e}^z$

14. $\lim\limits_{x \to \infty} \left[x \sin\left(\dfrac{7}{x} \right) \right]$

15. $\lim\limits_{z \to 0^+} \left[z^2 \left(\ln z \right)^2 \right]$

16. $\lim\limits_{x \to 0^+} x^{1/x}$

17. $\lim\limits_{t \to 0^+} \left[\mathbf{e}^t + t \right]^{1/t}$

18. $\lim\limits_{x \to \infty} \left[\mathbf{e}^{-2x} + 3x \right]^{1/x}$

19. Suppose that we know that $f'\left(x\right)$ is a continuous function. Use L'Hospital's Rule to show that,

$$\lim_{h \to 0} \frac{f\left(x+h\right) - f\left(x-h\right)}{2h} = f'\left(x\right)$$

20. Suppose that we know that $f''\left(x\right)$ is a continuous function. Use L'Hospital's Rule to show that,

$$\lim_{h \to 0} \frac{f\left(x+h\right) - 2f\left(x\right) + f\left(x-h\right)}{h^2} = f''\left(x\right)$$

Section 4-11 : Linear Approximations

For problems 1 – 4 find a linear approximation to the function at the given point.

1. $f\left(x\right)=\cos\left(2x\right)$ at $x=\pi$

2. $h\left(z\right)=\ln\left(z^2+5\right)$ at $z=2$

3. $g\left(x\right)=2-9x-3x^2-x^3$ at $x=-1$

4. $g\left(t\right)=\mathbf{e}^{\sin(t)}$ at $t=-4$

5. Find the linear approximation to $h\left(y\right)=\sin\left(y+1\right)$ at $y=0$. Use the linear approximation to approximate the value of $\sin\left(2\right)$ and $\sin\left(15\right)$. Compare the approximated values to the exact values.

6. Find the linear approximation to $R\left(t\right)=\sqrt[5]{t}$ at $t=32$. Use the linear approximation to approximate the value of $\sqrt[5]{31}$ and $\sqrt[5]{3}$. Compare the approximated values to the exact values.

7. Find the linear approximation to $h\left(x\right)=\mathbf{e}^{1-x}$ at $x=1$. Use the linear approximation to approximate the value of $\mathbf{e}$ and $\mathbf{e}^{-4}$. Compare the approximated values to the exact values.

For problems 8 – 10 estimate the given value using a linear approximation and without using any kind of computational aid.

8. $\ln\left(1.1\right)$

9. $\sqrt{8.9}$

10. $\sec\left(0.1\right)$

Section 4-12 : Differentials

For problems 1 – 5 compute the differential of the given function.

1. $f(x) = 3x^6 - 8x^3 + x^2 - 9x - 4$

2. $u = t^2 \cos(2t)$

3. $y = \mathbf{e}^{\cos(z)}$

4. $g(z) = \sin(3z) - \cos(1-z)$

5. $R(x) = \sqrt[4]{6x + \mathbf{e}^{-x}}$

6. Compute dy and Δy for $y = \sin(x)$ as *x* changes from 6 radians to 6.05 radians.

7. Compute dy and Δy for $y = \ln(x^2 + 1)$ as *x* changes from -2 to -2.1.

8. Compute dy and Δy for $y = \dfrac{1}{x-2}$ as *x* changes from 3 to 3.02.

9. Compute dy and Δy for $y = x\,\mathbf{e}^{\frac{1}{4}x}$ as *x* changes from -10 to -9.99.

10. The sides of a cube are found to be 6 feet in length with a possible error of no more than 1.5 inches. What is the maximum possible error in the surface area of the cube if we use this value of the length of the side to compute the surface area?

11. The radius of a circle is found to be 7 cm in length with a possible error of no more than 0.04 cm. What is the maximum possible error in the area of the circle if we use this value of the radius to compute the area?

12. The radius of a sphere is found to be 22 cm in length with a possible error of no more than 0.07 cm. What is the maximum possible error in the volume of the sphere if we use this value of the radius to compute the volume?

13. The radius of a sphere is found to be ½ foot in length with a possible error of no more than 0.03 inches. What is the maximum possible error in the surface area of the sphere if we use this value of the radius to compute the surface area?

Section 4-13 : Newton's Method

For problems 1 – 3 use Newton's Method to determine x_2 for the given function and given value of x_0.

1. $f(x) = 7x^3 - 8x + 4$, $x_0 = -1$

2. $f(x) = \cos(3x) - \sin(x)$, $x_0 = 0$

3. $f(x) = 7 - \mathbf{e}^{2x-3}$, $x_0 = 5$

For problems 4 – 8 use Newton's Method to find the root of the given equation, accurate to six decimal places, that lies in the given interval.

4. $x^5 = 6$ in $[1,2]$

5. $2x^3 - 9x^2 + 17x + 20 = 0$ in $[-1,1]$

6. $3 - 12x - 4x^3 - 3x^4 = 0$ in $[-3,-1]$

7. $\mathbf{e}^x = 4\cos(x)$ in $[-1,1]$

8. $x^2 = \mathbf{e}^{2-x^2}$ in $[0,2]$

For problems 9 – 12 use Newton's Method to find all the roots of the given equation accurate to six decimal places.

9. $2x^3 + 5x^2 - 10x - 4 = 0$

10. $x^4 + 4x^3 - 54x^2 - 92x + 105 = 0$

11. $\frac{3}{2} - \mathbf{e}^{-x^2} = \cos(x)$

12. $\ln(x) = 2\cos(x)$

13. Suppose that we want to find the root to $x^3 - 7x^2 + 8x - 3 = 0$. Is it possible to use $x_0 = 4$ as the initial point? What can you conclude about using Newton's Method to approximate roots from this example?

14. Use the function $f(x) = \cos^2(x) - \sin(x)$ for this problem.

(a) Plot the function on the interval $[0,9]$.

(b) Use $x_0 = 4$ to find one of the roots of this function to six decimal places. Did you get the root you expected to?

(c) Use $x_0 = 5$ to find one of the roots of this function to six decimal places. Did you get the root you expected to?

(d) Use $x_0 = 6$ to find one of the roots of this function to six decimal places. Did you get the root you expected to?

(e) What can you conclude about choosing values of x_0 to find roots of equations using Newton's Method.

15. Use $x_0 = 0$ to find one of the roots of $2x^5 - 7x^3 + 3x - 1 = 0$ accurate to six decimal places. Did we choose a good value of x_0 for this problem?

Section 4-14 : Business Applications

1. A company can produce a maximum of 2500 widgets in a year. If they sell *x* widgets during the year then their profit, in dollars, is given by,

$$P(x) = 500,000,000 - 1,540,000x + 1450x^2 - \tfrac{1}{3}x^3$$

How many widgets should they try to sell in order to maximize their profit?

2. A company can produce a maximum of 25 widgets in a day. If they sell *x* widgets during the day then their profit, in dollars, is given by,

$$P(x) = 3000 - 40x + 11x^2 - \tfrac{1}{3}x^3$$

How many widgets should they try to sell in order to maximize their profit?

3. A management company is going to build a new apartment complex. They know that if the complex contains *x* apartments the maintenance costs for the building, landscaping etc. will be,

$$C(x) = 70,000 + \tfrac{2736}{5}x - \tfrac{211}{50}x^2 + \tfrac{1}{150}x^3$$

The land they have purchased can hold a complex of at most 400 apartments. How many apartments should the complex have in order to minimize the maintenance costs?

4. The production costs of producing *x* widgets is given by,

$$C(x) = 2000 + 4x + \frac{90,000}{x}$$

If the company can produce at most 200 widgets how many should they produce to minimize the production costs?

5. The production costs, in dollars, per day of producing *x* widgets is given by,

$$C(x) = 400 - 3x + 2x^2 + 0.002x^3$$

What is the marginal cost when $x = 20$ and $x = 75$? What do your answers tell you about the production costs?

6. The production costs, in dollars, per month of producing *x* widgets is given by,

$$C(x) = 10,000 + 14x - \frac{8,000,000}{x^2}$$

What is the marginal cost when $x = 80$ and $x = 150$? What do your answers tell you about the production costs?

7. The production costs, in dollars, per week of producing *x* widgets is given by,

$$C(x) = 65,000 + 4x + 0.2x^2 - 0.00002x^3$$

and the demand function for the widgets is given by,

$$p(x) = 5000 - 0.5x$$

What is the marginal cost, marginal revenue and marginal profit when $x = 2000$ and $x = 4800$? What do these numbers tell you about the cost, revenue and profit?

8. The production costs, in dollars, per week of producing *x* widgets is given by,

$$C(x) = 800 + 0.008x^2 + \frac{56,000}{x}$$

and the demand function for the widgets is given by,

$$p(x) = 350 - 0.05x - 0.001x^2$$

What is the marginal cost, marginal revenue and marginal profit when $x = 175$ and $x = 325$? What do these numbers tell you about the cost, revenue and profit?

Chapter 5 : Integrals

Here are a set of assignment problems for the Integrals chapter of the Calculus I notes. Please note that these problems do not have any solutions available. These are intended mostly for instructors who might want a set of problems to assign for turning in. Having solutions available (or even just final answers) would defeat the purpose the problems.

If you are looking for some practice problems (with solutions available) please check out the Practice Problems. There you will find a set of problems that should give you quite a bit practice.

Here is a list of all the sections for which assignment problems have been written as well as a brief description of the material covered in the notes for that particular section.

Indefinite Integrals – In this section we will start off the chapter with the definition and properties of indefinite integrals. We will not be computing many indefinite integrals in this section. This section is devoted to simply defining what an indefinite integral is and to give many of the properties of the indefinite integral. Actually computing indefinite integrals will start in the next section.

Computing Indefinite Integrals – In this section we will compute some indefinite integrals. The integrals in this section will tend to be those that do not require a lot of manipulation of the function we are integrating in order to actually compute the integral. As we will see starting in the next section many integrals do require some manipulation of the function before we can actually do the integral. We will also take a quick look at an application of indefinite integrals.

Substitution Rule for Indefinite Integrals – In this section we will start using one of the more common and useful integration techniques – The Substitution Rule. With the substitution rule we will be able integrate a wider variety of functions. The integrals in this section will all require some manipulation of the function prior to integrating unlike most of the integrals from the previous section where all we really needed were the basic integration formulas.

More Substitution Rule – In this section we will continue to look at the substitution rule. The problems in this section will tend to be a little more involved than those in the previous section.

Area Problem – In this section we start off with the motivation for definite integrals and give one of the interpretations of definite integrals. We will be approximating the amount of area that lies between a function and the \(x\)-axis. As we will see in the next section this problem will lead us to the definition of the definite integral and will be one of the main interpretations of the definite integral that we'll be looking at in this material.

Definition of the Definite Integral – In this section we will formally define the definite integral, give many of its properties and discuss a couple of interpretations of the definite integral. We will also look at the first part of the Fundamental Theorem of Calculus which shows the very close relationship between derivatives and integrals.

Computing Definite Integrals – In this section we will take a look at the second part of the Fundamental Theorem of Calculus. This will show us how we compute definite integrals without using (the often very unpleasant) definition. The examples in this section can all be done with a basic knowledge of indefinite integrals and will not require the use of the substitution rule. Included in the examples in this section are computing definite integrals of piecewise and absolute value functions.

Substitution Rule for Definite Integrals – In this section we will revisit the substitution rule as it applies to definite integrals. The only real requirements to being able to do the examples in this section are being able to do the substitution rule for indefinite integrals and understanding how to compute definite integrals in general.

Section 5-1 : Indefinite Integrals

1. Evaluate each of the following indefinite integrals.
   (a) $\int 10x^9 - 12x^3 - 5\,dx$
   (b) $\int 10x^9 - 12x^3\,dx - 5$

2. Evaluate each of the following indefinite integrals.
   (a) $\int t^7 + 33t^2 + 8t\,dt$
   (b) $\int t^7\,dt + 33t^2 + 8t$

3. Evaluate each of the following indefinite integrals.
   (a) $\int 6x^5 - 7x^3 + 12x^2 - 10\,dx$
   (b) $\int 6x^5 - 7x^3\,dx + 12x^2 - 10$
   (c) $\int 6x^5\,dx - 7x^3 + 12x^2 - 10$

4. Evaluate each of the following indefinite integrals.
   (a) $\int 21x^6 - 9x^5 - x^3 - x\,dx$
   (b) $\int 21x^6 - 9x^5 - x^3\,dx - x$
   (c) $\int 21x^6 - 9x^5\,dx - x^3 - x$

For problems 5 – 9 evaluate the indefinite integral.

5. $\int 8t^5 - 15t^2 - 1\,dt$

6. $\int 120y^9 - 24y^5 - 4y^3\,dy$

7. $\int dw$

8. $\int x^9 + 14x^6 - 10x^3 + 13x\,dx$

9. $\int 8x^6 - x^4 - 7x^2 + 11x - 12\,dx$

10. Determine $f(x)$ given that $f'(x) = 16x^4 - 9x^2 - x$.

11. Determine $g(t)$ given that $g'(t) = 4t^5 + 16t^2 - 18t + 72$.

12. Determine $R(z)$ given that $R'(z) = 4z^{15} + 121z^{10} + 20z^5 + z - 4$.

13. Determine $f(x)$ given that $f''(x) = 8x^3 - 12x + 3$.

Section 5-2 : Computing Indefinite Integrals

For problems 1 – 43 evaluate the given integral.

1. $\int 7x^5 - 5x^4 + 6x^2 - 14x + 3\,dx$

2. $\int t^4 - 9t^3 + 12t^2 - 7t\,dt$

3. $\int 4 - 18w^{11} - 9w^9 + 8w^7 + 2w^5\,dw$

4. $\int x^9 - 6x^4 - 21x^2 - 1 + 9x\,dx$

5. $\int -7\,dz$

6. $\int 4\,dw$

7. $\int 10z^{-6} + 8z^{-5} - z^{-2} + 1\,dz$

8. $\int y^{-16} + 24y^{-12} - 14y^{-8} - 2y^{-4}\,dy$

9. $\int 2x^{-9} + 12x^{-5} + 7x^{-3} - x^{-2}\,dx$

10. $\int 5z^{-4} + 5z^4 - 9dz$

11. $\int 6t^3 + 8t^{-6} + t^{-10}\,dt$

12. $\int x^{-3} + 9x^2 + 11x^8 - 7x^{-12}\,dx$

13. $\int \sqrt[7]{w^2} + 3 - 9\sqrt[3]{w^7}\,\,dw$

14. $\int w^5 + \sqrt{w^5} - \sqrt[5]{w}\,dw$

15. $\int 6\sqrt[3]{v^2} - 7\sqrt[4]{v}\,dv$

16. $\int \frac{6}{y^3} - \frac{1}{7y^6} + \frac{1}{y^2}\,dy$

17. $\displaystyle\int 8+u^5-\frac{1}{u^5}+\frac{1}{6u^5}\,du$

18. $\displaystyle\int \frac{12}{x^5}+\frac{1}{4x^8}+\frac{6}{7x^2}\,dx$

19. $\displaystyle\int \sqrt[3]{t^5}-\frac{1}{\sqrt{t^9}}+t^4\,dt$

20. $\displaystyle\int \frac{2}{z^6}-\frac{1}{5\sqrt[7]{z^8}}+9\,dz$

21. $\displaystyle\int x^3+\frac{1}{x^3}-\sqrt{x^3}\,dx$

22. $\displaystyle\int x^6\left(1-4x^2+x^3\right)dx$

23. $\displaystyle\int \left(6-2u\right)^2\,du$

24. $\displaystyle\int 2-\left(3+y\right)\left(4-y^3\right)dy$

25. $\displaystyle\int \sqrt{w}\left(\sqrt[3]{w}-\sqrt[4]{w}\right)dw$

26. $\displaystyle\int 3v\left(v^2-\frac{1}{6v^2}+\sqrt[3]{v^2}\right)dv$

27. $\displaystyle\int \frac{8x^5-2x^3+7}{x^2}\,dx$

28. $\displaystyle\int \frac{9-z+2z^4+10z^6}{z^4}\,dz$

29. $\displaystyle\int \frac{2\sqrt{t}-4t+\sqrt[3]{t}}{t^2}\,dt$

30. $\displaystyle\int \frac{\left(1-x\right)\left(2+x\right)}{x}\,dx$

31. $\int 6\sin(t) - 2\cos(t)\,dt$

32. $\int \sec^2(u) + 7\sec(u)\tan(u)\,du$

33. $\int \csc^2(y) - \sec^2(y)\,dy$

34. $\int 8\cos(z) - 3\csc(z)\cot(z)\,dz$

35. $\int \tan(x)\left[\cot(x) - \cos(x)\right]dx$

36. $\int \frac{\cos^3(v) + \sin(v)}{\cos^2(v)}\,dv$

37. $\int w^2 + 2\mathbf{e}^w\,dw$

38. $\int \mathbf{e}^t + \frac{2}{t}\,dt$

39. $\int \frac{14}{x} - \frac{3}{x^2}\,dx$

40. $\int \mathbf{e}^{-u}\left(\mathbf{e}^{2u} + \mathbf{e}^u\right)du$

41. $\int \frac{1}{7z} + \frac{1}{\mathbf{e}^{-z}} + \frac{1}{4z^8}\,dz$

42. $\int 1 + w^2 - \frac{6}{1+w^2}\,dw$

43. $\int \frac{5}{1+t^2} + \frac{1}{10\sqrt{1-t^2}}\,dt$

44. Determine $f(x)$ given that $f'(x) = 12x^5 + 30x^2$ and $f(4) = -23$.

45. Determine $h(z)$ given that $h'(z) = 12z^3 - 14z^2 + 10$ and $h(-1) = 8$.

46. Determine $g(v)$ given that $g'(v) = \frac{1}{2}v^{-\frac{1}{2}} - \frac{1}{4}v^{-\frac{3}{4}}$ and $g(16) = 1$.

47. Determine $P(t)$ given that $P'(t) = 6\mathbf{e}^{t} - 4 - 10t$ and $P(0) = -6$.

48. Determine $g(x)$ given that $g''(x) = 12x^2 - 30x + 4$, $g(-1) = 7$ and $g(2) = 3$.

49. Determine $f(u)$ given that $f''(u) = 60u^4 - 60u^2$, $f(-1) = 14$ and $f'(1) = 6$.

50. Determine $h(t)$ given that $h''(t) = 6t - 14 + 9\mathbf{e}^{t}$, $h(0) = 4$ and $h(3) = 9\mathbf{e}^{3} + 8$.

Section 5-3 : Substitution Rule for Indefinite Integrals

For problems 1 – 31 evaluate the given integral.

1. $\int 12v\left(7+6v^2\right)^9\,dv$

2. $\int \left(4x^3-12x\right)\left(x^4-6x^2\right)^{-3}\,dx$

3. $\int \left(z^2-4\right)\left(12z-z^3\right)^4\,dz$

4. $\int 7z^2\left(14+8z^3\right)^{-5}\,dz$

5. $\int 3\left(y^6-4y^{-3}\right)\left(y^7+14y^{-2}-7\right)^6\,dy$

6. $\int \left(\frac{1}{2}x^3-1\right)\sqrt{8x-x^4}\,dx$

7. $\int \left(6w^{-4}+12w^{-7}\right)\sqrt[4]{w^{-3}+w^{-6}}\,dw$

8. $\int \cos\left(7t\right)dt$

9. $\int \left(v-2v^3\right)\cos\left(v^2-v^4\right)dv$

10. $\int \sqrt{z}\sin\left(1+\sqrt{z^3}\right)dz$

11. $\int \csc^2\left(1+2x\right)dx$

12. $\int 7w^{-5}\sec\left(w^{-4}\right)\tan\left(w^{-4}\right)dw$

13. $\int \left(2-t^2\right)\mathbf{e}^{6t-t^3}\,dt$

14. $\int 12z^{-2}\mathbf{e}^{4+z^{-1}}\,dz$

15. $\int \frac{1}{4-9w}\,dw$

16. $\displaystyle\int \frac{9y}{y^2+3}\,dy$

17. $\displaystyle\int \frac{6x^2-10x^4}{x^5-x^3}\,dx$

18. $\displaystyle\int \frac{1}{t}\sin\left(1-\ln\left(t\right)\right)dt$

19. $\displaystyle\int \left[6v-18\sin\left(6v\right)\right]\sqrt[5]{v^2+\cos\left(6v\right)}\,dv$

20. $\displaystyle\int \mathbf{e}^{-3z}\sec\left(\mathbf{e}^{-3z}\right)\tan\left(\mathbf{e}^{-3z}\right)dz$

21. $\displaystyle\int \left(\cos\left(x\right)+\sin\left(x\right)\right)\mathbf{e}^{\sin(x)-\cos(x)}\,dx$

22. $\displaystyle\int \frac{\left[\ln\left(w^2\right)\right]^4}{w}\,dw$

23. $\displaystyle\int \cos\left(v\right)\cos\left(1+\sin\left(v\right)\right)dv$

24. $\displaystyle\int \frac{y+\sin\left(2y\right)}{y^2-\cos\left(2y\right)}\,dy$

25. $\displaystyle\int \sec^7\left(t\right)\tan\left(t\right)dt$

26. $\displaystyle\int \mathbf{e}^{z}\sec^2\left(\mathbf{e}^{z}\right)\left[1+\tan\left(\mathbf{e}^{z}\right)\right]^{-3}dz$

27. $\displaystyle\int \frac{7}{1+5x^2}\,dx$

28. $\displaystyle\int \frac{2}{3+4t^2}\,dt$

29. $\displaystyle\int \frac{1}{\sqrt{16-y^2}}\,dy$

30. $\displaystyle\int \frac{3}{\sqrt{7-4v^2}}\,dv$

31. $\displaystyle\int \frac{x}{1+x^4}\,dx$

32. Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{1}{3+x}\,dx$

(b) $\displaystyle\int \frac{x}{3+x^2}\,dx$

(c) $\displaystyle\int \frac{x}{\left(3+x^2\right)^7}\,dx$

(d) $\displaystyle\int \frac{1}{3+x^2}\,dx$

33. Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{4w}{25+9w^2}\,dw$

(b) $\displaystyle\int \frac{4w}{\left(25+9w^2\right)^3}\,dw$

(c) $\displaystyle\int \frac{4}{25+9w^2}\,dw$

Section 5-4 : More Substitution Rule

Evaluate each of the following integrals.

1. $\int 3x\cos\left(4-x^2\right)-8x\sqrt{4-x^2}\,dx$

2. $\displaystyle\int \frac{4}{\left(9+6t\right)^5}+\frac{13}{9+6t}\,dt$

3. $\int \left(6-5w\right)\mathbf{e}^{12w-5w^2}+\left(20w-24\right)\sec^2\left(12w-5w^2\right)dw$

4. $\displaystyle\int \frac{\sin\left(1+\ln\left(2x\right)\right)-\sqrt{1+\ln\left(2x\right)}}{x}\,dx$

5. $\int 17\left(x\mathbf{e}^x+\mathbf{e}^x\right)\sin\left(x\mathbf{e}^x\right)-14\sin\left(x\right)dx$

6. $\displaystyle\int \frac{1}{3t}+\sec\left(9t\right)\tan\left(9t\right)\mathbf{e}^{\sec(9t)}\,dt$

7. $\displaystyle\int 8w^2+\frac{\sin\left(w\right)+\cos\left(w\right)}{\sin\left(w\right)-\cos\left(w\right)}\,dw$

8. $\int 8+\left(3+x^6\right)\cos\left(21x+x^7\right)+9x^2-4\sqrt{x}\,dx$

9. $\int \sin\left(y\right)\cos\left(y\right)\sqrt{3+\sin^2\left(y\right)}+5\mathbf{e}^y\,dy$

10. $\int \sin\left(2-t\right)+8\cos\left(5t\right)-\mathbf{e}^{3t}\,dt$

11. $\displaystyle\int \frac{4x^2-1}{\sqrt[4]{6x-8x^3}}+9x\mathbf{e}^{x^2}\,dx$

12. $\int z^3+\sqrt{4-3z}-4\sec\left(8z\right)\tan\left(8z\right)dz$

13. $\displaystyle\int \frac{17}{6-w}+\sin\left(w\right)\sin\left[1+\cos\left(w\right)\right]dw$

14. $\displaystyle\int \frac{\sqrt{1+2\ln(7x)}}{x} + \frac{10x^3}{x^4+9}\,dx$

15. $\displaystyle\int x\sin\left(x^2\right)\left[\cos^4\left(x^2\right)+8\cos^2\left(x^2\right)-10\right]dx$

16. $\displaystyle\int \csc\left(\tfrac{t}{2}\right)\cot\left(\tfrac{t}{2}\right)\left[\csc^6\left(\tfrac{t}{2}\right)+3\csc^4\left(\tfrac{t}{2}\right)-8\csc\left(\tfrac{t}{2}\right)\right]dt$

17. $\displaystyle\int \frac{3+7y}{y^2+3}\,dy$

18. $\displaystyle\int \frac{15z+27}{100z^2+11}\,dz$

19. $\displaystyle\int \frac{8x+1}{\sqrt{16-x^2}}\,dx$

20. $\displaystyle\int \frac{2-w}{\sqrt{25-2w^2}}\,dw$

21. $\displaystyle\int \frac{9z^5}{2+3z^3}\,dz$

22. $\displaystyle\int 4t^{15}\sqrt{1-t^8}\,dt$

23. $\displaystyle\int \cot(x)\,dx$

24. $\displaystyle\int \csc(x)\,dx$

25. $\displaystyle\int \frac{x}{1+x^4}\,dx$

26. $\displaystyle\int \mathbf{e}^{8t}\left(4+\mathbf{e}^{4t}\right)^{-3}dt$

27. $\displaystyle\int x^8\sqrt{2-x^3}\,dx$

Section 5-5 : Area Problem

For problems 1 – 5 estimate the area of the region between the function and the *x*-axis on the given interval using $n = 6$ and using,

(a) the right end points of the subintervals for the height of the rectangles,
(b) the left end points of the subintervals for the height of the rectangles and,
(c) the midpoints of the subintervals for the height of the rectangles.

1. $f\left(x\right) = 15 + 4x - x^3$ on $\left[1,3\right]$

2. $g\left(x\right) = -3x^2 + 2x - 1$ on $\left[-4,0\right]$

3. $h\left(x\right) = 8\ln\left(x\right) - x$ on $\left[2,6\right]$

4. $f\left(x\right) = \sin^2\left(\frac{x}{2}\right)$ on $\left[0,3\right]$

5. $g\left(x\right) = \sin\left(x\right)\cos\left(x\right) - 1$ on $\left[-2,1\right]$

For problems 6 – 8 estimate the net area between the function and the *x*-axis on the given interval using $n = 8$ and the midpoints of the subintervals for the height of the rectangles. Without looking at a graph of the function on the interval does it appear that more of the area is above or below the *x*-axis?

6. $h\left(x\right) = 8x - \sqrt{x + 4}$ on $\left[-3,2\right]$

7. $g\left(x\right) = 5 + x - x^2$ on $\left[0,4\right]$

8. $f\left(x\right) = x\mathbf{e}^{-x^2}$ on $\left[-1,1\right]$

Section 5-6 : Definition of the Definite Integral

For problems 1 – 4 use the definition of the definite integral to evaluate the integral. Use the right end point of each interval for x_i^*.

1. $\int_{-2}^{1} 7-4x\,dx$

2. $\int_{0}^{2} 3x^2+4x\,dx$

3. $\int_{-1}^{1} \left(x-3\right)^2\,dx$

4. $\int_{0}^{3} 8x^3+3x-2\,dx$

5. Evaluate : $\int_{-123}^{-123} \cos^6\left(2x\right)-\sin^8\left(4x\right)dx$

For problems 6 – 8 determine the value of the given integral given that $\int_{-2}^{5} f\left(x\right)dx=1$ and $\int_{-2}^{5} g\left(x\right)dx=8$.

6. $\int_{-2}^{5} -3g\left(x\right)dx$

7. $\int_{-2}^{5} 7f\left(x\right)-\frac{1}{4}g\left(x\right)dx$

8. $\int_{5}^{-2} 12g\left(x\right)-3f\left(x\right)dx$

9. Determine the value of $\int_{7}^{-1} f\left(x\right)dx$ given that $\int_{13}^{7} f\left(x\right)dx=-9$ and $\int_{13}^{-1} f\left(x\right)dx=-12$.

10. Determine the value of $\int_{0}^{6} 4f\left(x\right)dx$ given that $\int_{0}^{5} f\left(x\right)dx=10$ and $\int_{5}^{6} f\left(x\right)dx=3$.

11. Determine the value of $\int_{2}^{10} f\left(x\right)dx$ given that $\int_{2}^{4} f\left(x\right)dx=-1$, $\int_{4}^{7} f\left(x\right)dx=3$ and $\int_{10}^{7} f\left(x\right)dx=-8$.

12. Determine the value of $\int_{-5}^{-1} f\left(x\right)dx$ given that $\int_{2}^{-5} f\left(x\right)dx = 56$, $\int_{7}^{2} f\left(x\right)dx = -90$ and $\int_{-1}^{7} f\left(x\right)dx = 45$.

For problems 13 – 17 sketch the graph of the integrand and use the area interpretation of the definite integral to determine the value of the integral.

13. $\int_{-2}^{1} 12 - 5x\,dx$

14. $\int_{0}^{4} \sqrt{16 - x^2}\,dx$

15. $\int_{-3}^{3} 5 - \sqrt{9 - x^2}\,dx$

16. $\int_{-1}^{3} 8x - 3\,dx$

17. $\int_{1}^{6} \left|x - 3\right|dx$

For problems 18 – 23 differentiate each of the following integrals with respect to *x*.

18. $\int_{-8}^{x} \mathbf{e}^{\cos(t)}\,dt$

19. $\int_{2}^{x^2} \sqrt{\cos\left(t\right) + 3}\,dt$

20. $\displaystyle\int_{0}^{\mathbf{e}^{3x}} \frac{1}{t^4 + t^2 + 1}\,dt$

21. $\displaystyle\int_{\sin(9x)}^{8} \frac{\mathbf{e}^t}{7t}\,dt$

22. $\int_{x^3}^{x} \cos^4\left(t\right) - \sin^2\left(t\right)dt$

23. $\displaystyle\int_{9x}^{\tan(x)} \frac{\cos\left(t\right) + 2}{\sin\left(t\right) + 4}\,dt$

24. Evaluate the limit : $\displaystyle\lim_{x \to 0} \frac{\int_0^x \mathbf{e}^{t^2}\,dt}{x}$

Section 5-7 : Computing Definite Integrals

1. Evaluate each of the following integrals.

a. $\displaystyle\int 3z^2 - 4 + \frac{4}{z^2}\,dz$

b. $\displaystyle\int_1^4 3z^2 - 4 + \frac{4}{z^2}\,dz$

c. $\displaystyle\int_{-2}^1 3z^2 - 4 + \frac{4}{z^2}\,dz$

2. Evaluate each of the following integrals.

a. $\displaystyle\int 6x + \frac{1}{3x}\,dx$

b. $\displaystyle\int_0^7 6x + \frac{1}{3x}\,dx$

c. $\displaystyle\int_3^7 6x + \frac{1}{3x}\,dx$

3. Evaluate each of the following integrals.

a. $\int \sin\left(y\right) + \sec^2\left(y\right)dy$

b. $\int_0^{\frac{\pi}{4}} \sin\left(y\right) + \sec^2\left(y\right)dy$

c. $\int_0^{\frac{2\pi}{3}} \sin\left(y\right) + \sec^2\left(y\right)dy$

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

4. $\int_0^3 10t - 6t^2 + 9\,dt$

5. $\int_{-1}^4 24z^2 + 5z^4 dz$

6. $\int_1^0 9w - 3w^2 + 4w^3\,dw$

7. $\int_{-3}^{-1} 15t^2 - 10t - 2dt$

8. $\int_{-2}^4 v^3 - 7v^2 + 3v\,dv$

9. $\int_0^{16} 9\sqrt{x} + 10\sqrt[4]{x}\,dx$

10. $\int_{-1}^{2} 8\sqrt[3]{z} - 12\sqrt[5]{z}\,dz$

11. $\int_1^4 \sqrt{y^5} - \frac{1}{\sqrt[3]{y}}\,dy$

12. $\int_1^4 \frac{6}{x^3} - \frac{1}{3x^2}\,dx$

13. $\int_6^{-3} 8w^3 - 25w^4 + \frac{4}{3w^5}\,dw$

14. $\int_{-1}^{-3} \frac{4}{3z^2} - \frac{6}{z^3}\,dz$

15. $\int_0^6 (3-t)(2t^2+3)\,dt$

16. $\int_4^1 \sqrt{x}\left(x - 2x^2 + 1\right)dx$

17. $\int_2^5 \frac{6z^5 - 8z^4 + 2z^2}{z^4}\,dz$

18. $\int_{-2}^{-4} \frac{9x^4 - 8x^3 + x}{3x^2}\,dx$

19. $\int_{-8}^{2} \frac{7v^{10} + 4v^6 - 3v^2}{v^5}\,dv$

20. $\int_1^2 \frac{(y-2)(y+2)}{y^2}\,dy$

21. $\int_0^{\frac{\pi}{4}} 8\sec^2(t) + 2\sec(t)\tan(t)\,dt$

22. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} 3\cos(w) + \sin(w)\,dw$

23. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 12\sec^2\left(y\right) - 9\csc^2\left(y\right)dy$

24. $\int_{\frac{2\pi}{3}}^{\frac{\pi}{4}} 3\sin\left(v\right) + 8\csc\left(v\right)\cot\left(v\right)dv$

25. $\int_{-3}^{1} 4x - 7\mathbf{e}^x \, dx$

26. $\displaystyle\int_{-2}^{1} \frac{4\mathbf{e}^{2w} + 4w\mathbf{e}^w}{\mathbf{e}^w} dw$

27. $\displaystyle\int_{0}^{\frac{1}{2}} \frac{3}{\sqrt{1-x^2}} + \frac{7}{x^2+1} dx$

28. $\displaystyle\int_{-2}^{3} 5\sin\left(t\right) + \frac{1}{\sqrt{1-t^2}} dt$

29. $\displaystyle\int_{6}^{10} \frac{4}{z} + \frac{1}{2z^2} dz$

30. $\displaystyle\int_{1}^{6} 2x^3 + \frac{3}{8x} dx$

31. $\int_{-4}^{-1} f\left(t\right)dt$ where $f\left(t\right) = \begin{cases} 9+6t^2 & t > -3 \\ 8t & t \le -3 \end{cases}$

32. $\int_{-2}^{4} g\left(x\right)dx$ where $g\left(x\right) = \begin{cases} 9-2\mathbf{e}^x & x > 0 \\ 8\sin\left(x\right) & x \le 0 \end{cases}$

33. $\int_{4}^{9} h\left(w\right)dw$ where $h\left(w\right) = \begin{cases} 4 & w > 6 \\ 3w+1 & w \le 6 \end{cases}$

34. $\int_{-1}^{7} f\left(x\right)dx$ where $f\left(x\right) = \begin{cases} 9x^2 & x > 5 \\ -7 & 1 < x \le 5 \\ 3-8x & x \le 1 \end{cases}$

35. $\int_{-3}^{1} \left|8+4x\right| dx$

36. $\int_{2}^{8} \left|3v - 12\right| dv$

37. $\int_{0}^{6} \left|10 - 2z\right| dz$

38. $\int_{-3}^{6} \left|t^2 - 4\right| dt$

Section 5-8 : Substitution Rule for Definite Integrals

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

1. $\displaystyle\int_{-2}^{3} \frac{4}{(5+2x)^3}\,dx$

2. $\displaystyle\int_{1}^{0} 10\left(1-2w^2\right)\sqrt[4]{7-3w+2w^3}\,dw$

3. $\displaystyle\int_{-1}^{4} (t-2)\mathbf{e}^{t^2-4t}\,dt$

4. $\displaystyle\int_{1}^{6} 7\cos\left(\tfrac{\pi z}{2}\right)\left(4+\sin\left(\tfrac{\pi z}{2}\right)\right)^5 dz$

5. $\displaystyle\int_{0}^{1} \frac{w^3}{6w^4+3}\,dw$

6. $\displaystyle\int_{-1}^{1} x^2\cos\left(x^3+2\right)-x^2\mathbf{e}^{x^3+2}\,dx$

7. $\displaystyle\int_{0}^{\frac{\pi}{3}} \frac{4\sin(3t)}{2+\cos(3t)}+\frac{7\sin(3t)}{\left(2+\cos(3t)\right)^2}\,dt$

8. $\displaystyle\int_{0}^{\pi} \sec^2(y)\sqrt{2+\tan(y)}\,dy$

9. $\displaystyle\int_{1}^{9} \sqrt{x^5}+\frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

10. $\displaystyle\int_{0}^{1} \sec^2(w)-\frac{2}{4w^2+1}\,dw$

11. $\displaystyle\int_{3}^{0} \mathbf{e}^{-4t}\sqrt{2+\mathbf{e}^{-4t}}+8\mathbf{e}^{t}\,dt$

12. $\displaystyle\int_{3}^{7} \frac{9\mathbf{e}^x}{\mathbf{e}^x+4}+\frac{\left[\ln(2x)\right]^2}{x}\,dx$

13. $\int_0^{\pi} \sin\left(\frac{v}{2}\right)\left[6+3\cos^2\left(\frac{v}{2}\right)-4\cos^4\left(\frac{v}{2}\right)\right]dv$

14. $\int_1^2 \mathbf{e}^{-t}+3t\mathbf{e}^{5-t^2}\,dt$

15. $\displaystyle\int_0^6 \frac{8t^3}{2t^4+1}-\frac{7t}{t^2-9}\,dt$

16. $\int_2^6 \sqrt{1+2y}+\left(4-y\right)\left(y^2-8y+5\right)^4\,dy$

17. $\int_0^1 \mathbf{e}^{2z}\sin\left(\mathbf{e}^{2z}-1\right)+\sin\left(z\right)\mathbf{e}^{2-\cos(z)}\,dz$

Chapter 6 : Applications of Integrals

Here are a set of assignment problems for the Applications of Integrals chapter of the Calculus I notes. Please note that these problems do not have any solutions available. These are intended mostly for instructors who might want a set of problems to assign for turning in. Having solutions available (or even just final answers) would defeat the purpose the problems.

If you are looking for some practice problems (with solutions available) please check out the Practice Problems. There you will find a set of problems that should give you quite a bit practice.

Here is a list of all the sections for which assignment problems have been written as well as a brief description of the material covered in the notes for that particular section.

Average Function Value – In this section we will look at using definite integrals to determine the average value of a function on an interval. We will also give the Mean Value Theorem for Integrals.

Area Between Curves – In this section we'll take a look at one of the main applications of definite integrals in this chapter. We will determine the area of the region bounded by two curves.

Volumes of Solids of Revolution / Method of Rings – In this section, the first of two sections devoted to finding the volume of a solid of revolution, we will look at the method of rings/disks to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the x or y-axis) around a vertical or horizontal axis of rotation.

Volumes of Solids of Revolution / Method of Cylinders – In this section, the second of two sections devoted to finding the volume of a solid of revolution, we will look at the method of cylinders/shells to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the x or y-axis) around a vertical or horizontal axis of rotation.

More Volume Problems – In the previous two sections we looked at solids that could be found by treating them as a solid of revolution. Not all solids can be thought of as solids of revolution and, in fact, not all solids of revolution can be easily dealt with using the methods from the previous two sections. So, in this section we'll take a look at finding the volume of some solids that are either not solids of revolutions or are not easy to do as a solid of revolution.

Work – In this section we will look at is determining the amount of work required to move an object subject to a force over a given distance.

Section 6-1 : Average Function Value

For problems 1 – 4 determine f_{avg} for the function on the given interval.

1. $f\left(x\right)=8x^4-7x^3+2$ on $\left[-2,1\right]$

2. $f\left(x\right)=\left(4-x\right)\mathbf{e}^{x^2-8x}$ on $\left[1,4\right]$

3. $f\left(x\right)=6x-\dfrac{4x}{x^2+1}$ on $\left[-3,0\right]$

4. $f\left(x\right)=\cos\left(3x\right)\left[2+\sin\left(3x\right)\right]^4$ on $\left[0,\frac{\pi}{6}\right]$

For problems 5 – 8 find f_{avg} for the function on the given interval and determine the value of *c* in the given interval for which $f\left(c\right)=f_{\text{avg}}$.

5. $f\left(x\right)=10-4x-6x^2$ on $\left[2,6\right]$

6. $f\left(x\right)=7x^2+2x-3$ on $\left[-1,1\right]$

7. $f\left(x\right)=9-2\mathbf{e}^{4x+1}$ on $\left[-1,2\right]$

8. $f\left(x\right)=8-\cos\left(\frac{x}{4}\right)$ on $\left[0,4\pi\right]$

Section 6-2 : Area Between Curves

1. Determine the area below $f(x) = 8x - 2x^2$ and above the *x*-axis.

2. Determine the area above $f(x) = 3x^2 + 6x - 9$ and below the *x*-axis.

3. Determine the area to the right of $g(y) = y^2 + 4y - 5$ and to the left of the *y*-axis.

4. Determine the area to the left of $g(y) = -4y^2 + 24y - 20$ and to the right of the *y*-axis.

5. Determine the area below $f(x) = 10 - 2x^2$ and above the line $y = 3$.

6. Determine the area above $f(x) = x^2 + 2x + 3$ and below the line $y = 11$.

7. Determine the area to the right of $g(y) = y^2 + 2y - 4$ and to the left of the line $x = -1$.

8. Determine the area to the left of $g(y) = 2 + 4y - y^2$ and to the right of the line $x = -1$.

For problems 9 – 26 determine the area of the region bounded by the given set of curves.

9. $y = x^3 + 2$, $y = 1$ and $x = 2$.

10. $y = x^2 - 6x + 10$ and $y = 5$.

11. $y = x^2 - 6x + 10$, $x = 1$, $x = 5$ and the *x*-axis.

12. $x = y^2 + 2y + 4$ and $x = 4$.

13. $y = 5 - \sqrt{x}$, $x = 1$, $x = 4$ and the *x*-axis.

14. $x = \mathbf{e}^y$, $x = 1$, $y = 1$ and $y = 2$.

15. $x = 4y - y^2$ and the *y*-axis.

16. $y = x^2 + 2x + 4$, $y = 3x + 6$, $x = -3$ and $x = 3$.

17. $x = 6y - y^2$, $x = 2y$, $y = -2$ and $y = 5$.

18. $y = x^2 + 8$, $y = 3x^2$, $x = -3$ and $x = 4$.

19. $x = y^2$, $x = y^3$ and $y = 2$.

20. $y = \dfrac{7}{x}$, $y = \dfrac{1}{x} - 3$, $x = -1$ and $x = -4$.

21. $y = 2x^2 + 1$, $y = 7 - x$, $x = 4$ and the *y*-axis.

22. $y = \sin\left(\frac{1}{2}x\right)$, $y = 3 + \cos\left(2x\right)$, $x = 0$ and $x = \frac{\pi}{4}$.

23. $x = \sqrt{2y+6}$, $x = y - 1$, $y = 1$ and $y = 6$.

24. $y = 2 - \mathbf{e}^{2-x}$, $y = x^2 - 4x + 7$, $x = 3$ and the *y*-axis. Note : These functions do not intersect.

25. $y = \mathbf{e}^{2x-1}$, $y = \mathbf{e}^{5-x}$, $x = 0$ and $x = 3$.

26. $x = \cos\left(\pi y\right)$, $x = 3$, $y = 0$ and $y = 4$.

Section 6-3 : Volumes of Solids of Revolution / Method of Rings

For problems 1 – 16 use the method disks/rings to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $y = 2x^2$, $y = 8$ and the *y*-axis about the *y*-axis.

2. Rotate the region bounded by $y = 2x^2$, $y = 8$ and the *y*-axis about the *x*-axis.

3. Rotate the region bounded by $y = 2x^2$, $x = 2$ and the *x*-axis about the *x*-axis.

4. Rotate the region bounded by $y = 2x^2$, $x = 2$ and the *x*-axis about the *y*-axis.

5. Rotate the region bounded by $x = y^3$, $x = 8$ and the *x*-axis about the *x*-axis.

6. Rotate the region bounded by $x = y^3$, $x = 8$ and the *x*-axis about the *y*-axis.

7. Rotate the region bounded by $x = y^3$, $y = 2$ and the *y*-axis about the *x*-axis.

8. Rotate the region bounded by $x = y^3$, $y = 2$ and the *y*-axis about the *y*-axis.

9. Rotate the region bounded by $y = \dfrac{1}{x^2}$, $y = 9$, $x = -2$, $x = -\frac{1}{3}$ about the *y*-axis.

10. Rotate the region bounded by $y = \dfrac{1}{x^2}$, $y = 9$, $x = -2$, $x = -\frac{1}{3}$ about the *x*-axis.

11. Rotate the region bounded by $y = 4 + 3\mathbf{e}^{-x}$, $y = 2$, $x = \frac{1}{2}$ and $x = 3$ about the *x*-axis.

12. Rotate the region bounded by $x = 5 - y^2$ and $x = 4$ about the *y*-axis.

13. Rotate the region bounded by $y = 6 - 2x$, $y = 3 + x$ and $x = 3$ about the *x*-axis.

14. Rotate the region bounded by $y = 6 - 2x$, $y = 3 + x$ and $y = 6$ about the *y*-axis.

15. Rotate the region bounded by $y = x^2 - 2x + 4$ and $y = x + 14$ about the *x*-axis.

16. Rotate the region bounded by $x = \left(y - 3\right)^2$ and $x = 16$ about the *y*-axis.

17. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2$, $y = 8$ and the *y*-axis about the

(a) line $x = 3$ **(b)** line $x = -2$
(c) line $y = 11$ **(d)** line $y = -4$

18. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $x = y^2 - 6y + 9$ and $x = -y^2 + 6y - 1$ about the

(a) line $x = 10$ **(b)** line $x = -3$

19. Use the method of disks/rings to determine the volume of the solid obtained by rotating the triangle with vertices $(3,2)$, $(7,2)$ and $(7,14)$ about the

(a) line $x = 12$ **(b)** line $x = 2$ **(c)** line $x = -1$
(d) line $y = 14$ **(e)** line $y = 1$ **(f)** line $y = -3$

20. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = 4 + 3\mathbf{e}^{-x}$, $y = 2$, $x = \frac{1}{2}$ and $x = 3$ about the

(a) line $y = 7$ **(b)** line $y = 1$ **(c)** line $y = -3$

21. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $x = 3 + y^2$ and $x = 2y + 11$ about the

(a) line $x = 23$ **(b)** line $x = 2$ **(c)** line $x = -1$

22. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = 5 + \sqrt{x}$, $y = 5$ and $x = 4$ about the

(a) line $y = 8$ **(b)** line $y = 2$ **(c)** line $y = -2$

23. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = 10 - 2x$, $y = x + 1$ and $y = 7$ about the

(a) line $x = 8$ **(b)** line $x = 1$ **(c)** line $x = -4$

24. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = -x^2 - 2x - 5$ and $y = 2x - 17$ about the

(a) line $y = 3$ **(b)** line $y = -1$ **(c)** line $y = -34$

25. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $x = -2y^2 - 3$ and $x = -5$ about the

(a) line $x = 4$ **(b)** line $x = -2$ **(c)** line $x = -9$

Section 6-4 : Volumes of Solids of Revolution / Method of Cylinders

For problems 1 – 14 use the method cylinders to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $y = 2x^2$, $y = 8$ and the *y*-axis about the *y*-axis.

2. Rotate the region bounded by $y = 2x^2$, $y = 8$ and the *y*-axis about the *x*-axis.

3. Rotate the region bounded by $y = 2x^2$, $x = 2$ and the *x*-axis about the *x*-axis.

4. Rotate the region bounded by $y = 2x^2$, $x = 2$ and the *x*-axis about the *y*-axis.

5. Rotate the region bounded by $x = y^3$, $x = 8$ and the *x*-axis about the *x*-axis.

6. Rotate the region bounded by $x = y^3$, $x = 8$ and the *x*-axis about the *y*-axis.

7. Rotate the region bounded by $x = y^3$, $y = 2$ and the *y*-axis about the *x*-axis.

8. Rotate the region bounded by $x = y^3$, $y = 2$ and the *y*-axis about the *y*-axis.

9. Rotate the region bounded by $y = \dfrac{1}{x}$, $y = \dfrac{1}{3}$ and $x = \dfrac{1}{2}$ about the *y*-axis.

10. Rotate the region bounded by $y = \dfrac{1}{x}$, $y = \dfrac{1}{3}$ and $x = \dfrac{1}{2}$ about the *x*-axis.

11. Rotate the region bounded by $y = 6 - 2x$, $y = 3 + x$ and $x = 3$ about the *y*-axis.

12. Rotate the region bounded by $y = 6 - 2x$, $y = 3 + x$ and $y = 6$ about the *x*-axis.

13. Rotate the region bounded by $y = x^2 - 6x + 11$ and $y = 6$ about the *y*-axis.

14. Rotate the region bounded by $x = y^2 - 8y + 19$ and $x = 2y + 3$ about the *x*-axis.

15. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2$, $y = 8$ and the *y*-axis about the

(a) line $x = 3$ **(b)** line $x = -2$
(c) line $y = 11$ **(d)** line $y = -4$

16. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $x = y^3$, $x = 8$ and the *x*-axis about the

(a) line $x = 10$ **(b)** line $x = -3$
(c) line $y = 3$ **(d)** line $y = -4$

17. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $x = y^2 - 6y + 9$ and $x = -y^2 + 6y - 1$ about the

(a) line $y = 7$ **(b)** line $y = -2$

18. Use the method of cylinders to determine the volume of the solid obtained by rotating the triangle with vertices $(3,2)$, $(7,2)$ and $(7,14)$ about the

(a) line $x = 12$ **(b)** line $x = 2$ **(c)** line $x = -1$ **(d)** line $y = 14$
(e) line $y = 1$ **(f)** line $y = -3$

19. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $y = 4 + 3\mathbf{e}^{-x}$, $y = 2$, $x = \frac{1}{2}$ and $x = 3$ about the

(a) line $x = 5$ **(b)** line $x = \frac{1}{4}$ **(c)** line $x = -1$

20. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $x = y^2 - 8y + 19$ and $x = 2y + 3$ about the

(a) line $y = 9$ **(b)** line $y = 1$ **(c)** line $y = -3$

21. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $y = 5 + \sqrt{x-3}$, $y = 5$ and $x = 4$ about the

(a) line $x = 9$ **(b)** line $x = 2$ **(c)** line $x = -1$

22. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $y = -x^2 - 10x + 6$ and $y = 2x + 26$ about the

(a) line $x = 2$ **(b)** line $x = -1$ **(c)** line $x = -14$

23. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $x = y^2 - 10y + 27$ and $x = 11$ about the

(a) line $y = 10$ **(b)** line $y = 1$ **(c)** line $y = -3$

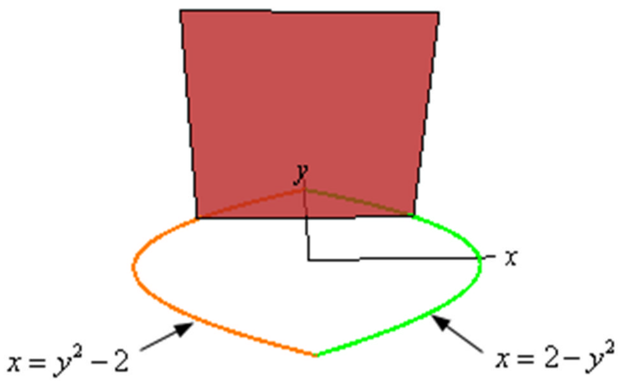
24. Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2 + 1$, $y = 7 - x$, $x = 3$ and $x = \frac{3}{2}$ about the

(a) line $x = 6$ **(b)** line $x = 1$ **(c)** line $x = -2$

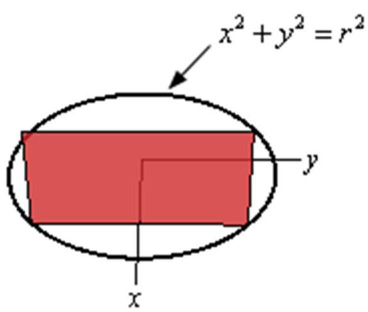
Section 6-5 : More Volume Problems

1. Use the method of finding volume from this section to determine the volume of a sphere of radius *r*.

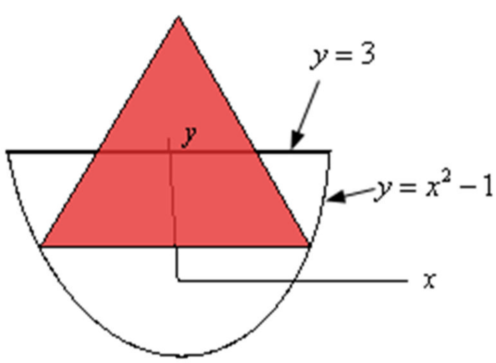
2. Find the volume of the solid whose base is the region bounded by $x = 2 - y^2$ and $x = y^2 - 2$ and whose cross-sections are squares with the base perpendicular to the *y*-axis. See figure below to see a sketch of the cross-sections.



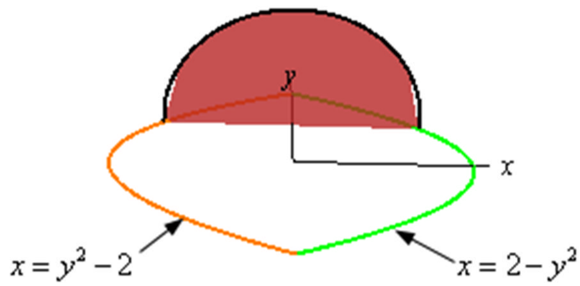
3. Find the volume of the solid whose base is a disk of radius *r* and whose cross-sections are rectangles whose height is half the length of the base and whose base is perpendicular to the *x*-axis. See figure below to see a sketch of the cross-sections (the positive *x*-axis and positive *y*-axis are shown in the sketch).



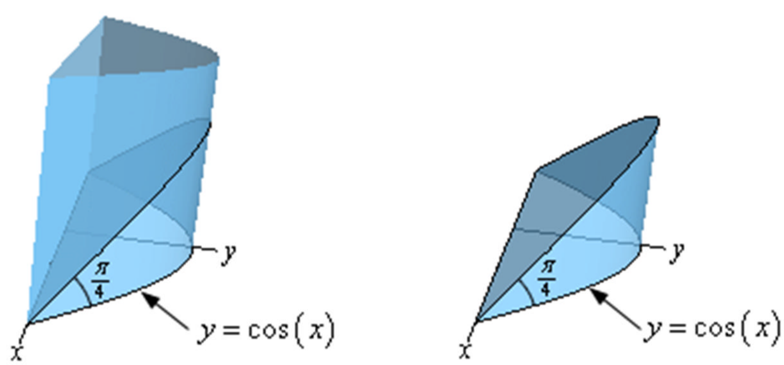
4. Find the volume of the solid whose base is the region bounded by $y = x^2 - 1$ and $y = 3$ and whose cross-sections are equilateral triangles with the base perpendicular to the *y*-axis. See figure below to see a sketch of the cross-sections.



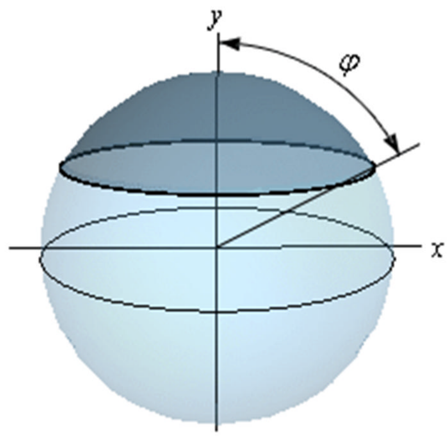
5. Find the volume of the solid whose base is the region bounded by $x = 2 - y^2$ and $x = y^2 - 2$ and whose cross-sections are the upper half of the circle centered on the *y*-axis. See figure below to see a sketch of the cross-sections.



6. Find the volume of a wedge cut out of a “cylinder” whose base is the region bounded by $y = \cos\left(x\right)$ and the *x*-axis between $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. The angle between the top and bottom of the wedge is $\frac{\pi}{4}$. See the figure below for a sketch of the “cylinder” and the wedge (the positive *x*-axis and positive *y*-axis are shown in the sketch).



7. For a sphere of radius *r* find the volume of the cap which is defined by the angle φ where φ is the angle formed by the *y*-axis and the line from the origin to the bottom of the cap. See the figure below for an illustration of the angle φ .



Section 6-6 : Work

1. A force of $F\left(x\right)=x\mathbf{e}^{-2x^2}+6x-2$ acts on an object. What is the work required to move the object from $x=1$ to $x=4$?

2. A force of $F\left(x\right)=4\cos\left(2x\right)-7\sin\left(\frac{1}{2}x\right)$, *x* is in meters, acts on an object. What is the work required to move the object 10 meters to the right of $x=2$?

3. A force of $F\left(x\right)=\sin\left(x\right)\mathbf{e}^{\cos\left(x\right)}-4x+1$, *x* is in meters, acts on an object. What is the work required to move the object 6.5 meters to the left of $x=9$?

4. A spring has a natural length of 25 cm and a force of 3.5 N is required to stretch and hold the spring to a length of 32 cm. What is the work required to stretch the spring from a length of 30 cm to a length of 45 cm?

5. A spring has a natural length of 9 inches and a force of 7 lbs is required to stretch and hold the spring to a length of 21 inches. What is the work required to stretch the spring from a length of 12 inches to a length of 30 inches?

6. A cable with mass 2 kg/meter is lifting a load of 50 kg that is initially at the bottom of a 75 meter shaft. How much work is required to lift the load 40 meters?

7. A cable with mass 1.5 kg/meter is attached to a bucket that has mass 75 kg. Initially there is 500 kg of grain in the bucket and as the bucket is raised 2 kg of grain leaks out of a hole in the bucket for every meter the bucket is raised. The bucket is 200 meters below a bridge. How much work is required to raise the bucket to the top of the bridge?

8. A tank of water is in the shape of a cylinder of height 25 meters and radius of 7 meters. If the tank is completely filled with water how much work is required to pump all of the water to the top of the tank. Assume that the density of water is 1000 kg/m^3.

9. A tank of water is in the shape of an inverted pyramid that is 18 feet tall and whose top is a square with sides 4 feet long. If there is initially 12 feet of water in the tank determine the amount of work needed to pump all of the water to the top of the tank. Assume that the density of water is 62 lb/ft^3.

10. A tank of is the shape of the lower half of a sphere of radius 6 meters. If the initial depth of the water is 4 meters how much work is required to pump all the water to the top of the tank. Assume that the density of water is 1000 kg/m^3.

