Recap [dot product scalar projections cross product
unit vector vector projections plane]

Questions [Mainly Intersection & Distance]

1 Dot product

$$\underset{\sim}{x} \cdot \underset{\sim}{y} = \sum_{i=1}^{n} x_i \cdot y_i$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = 1(4) + 2(5) + 3(6) = 32$$

— Length / magnitude / norm $|\underset{\sim}{v}|$

$$|\underset{\sim}{x}| = \sqrt{x \cdot x} = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

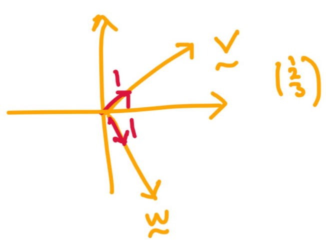
— angle

$$\underset{\sim}{x} \cdot \underset{\sim}{y} = |x||y| \cos\theta$$

$$\underset{\sim}{x} \cdot \underset{\sim}{y} = 0 \quad \text{(when perpendicular)}$$

[Important Property]

2 Unit vector

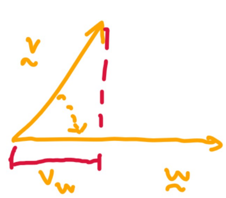


$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

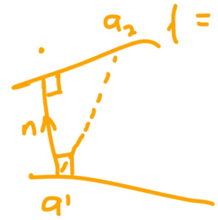
$$\hat{\underset{\sim}{v}} = \frac{\underset{\sim}{v}}{|\underset{\sim}{v}|}$$

$$= \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

3 Scalar Projections [Important in finding distance]



$$v_w = \underset{\sim}{v} \cdot \hat{\underset{\sim}{w}}$$

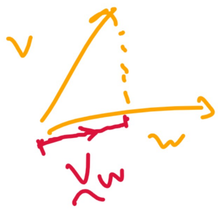


$l = \underset{\sim}{a} + t\underset{\sim}{p}$

$(a_1 - a_2) \cdot \hat{\underset{\sim}{n}}$

$\underset{\sim}{n} = b_1 \times b_2$

4. Vector



$$v_w = v \cdot \hat{\underset{\sim}{w}}$$

$$\underset{\sim}{v_w} = \underbrace{v \cdot \hat{\underset{\sim}{w}}}_{\text{scalar}} \cdot \hat{\underset{\sim}{w}}$$

$$= \underset{\sim}{v} \cdot \frac{\underset{\sim}{w}}{|\underset{\sim}{w}|} \cdot \frac{\underset{\sim}{w}}{|\underset{\sim}{w}|}$$

$$= \frac{v \cdot w}{|\underset{\sim}{w}|^2} \cdot \underset{\sim}{w}$$

5. Cross product

$$\underset{\sim}{v} \times \underset{\sim}{w} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$\begin{pmatrix}1\\2\\3\end{pmatrix}$ $\begin{pmatrix}4\\5\\6\end{pmatrix}$

$$= \begin{pmatrix} 2(6) - 3(5) \\ 3(4) - 1(6) \\ 1(5) - 2(4) \end{pmatrix} - [1(6) - 3(4)]$$

- Area

$|\underset{\sim}{v} \times \underset{\sim}{w}|$

$\frac{1}{2} |\underset{\sim}{v} \times \underset{\sim}{w}|$

6.

$rn = an$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}2\\2\\2\end{pmatrix} \cdot \begin{pmatrix}1\\2\\3\end{pmatrix}$$

$x + 2y + 3z = 2 + 4 + 6$

$x + 2y + 3z = 12$ (plane eq)

define a plane:

define a line:

$l = A + t\underset{\sim}{d}$

Questions

1.16 Find eq joining 2 lines $\left(l = A + n\left(\overrightarrow{A-B}\right)\right)$

1.19 Check 2 line intersect $(l_1 = l_2)$

1.20 parallel $(d_1 = a(d_2))$

1.16.

A B

$d = A - B$

$a = A$

$l = a + nd$

1.19

$l_1 = l_2$

$$\begin{pmatrix} 1+2n \\ 1+3n \\ 1+4n \end{pmatrix} = \begin{pmatrix} 2+4t \\ 2+1t \\ 2+5t \end{pmatrix}$$

n, t

1.20

$d_1 \quad d_2$

$d_1 = a(d_2)$

1.6.3 Point, lines, planes intersection

1.33 check point in line (substitute point in line)

1.35 check line intersection $(l_1 = l_2)$

1.37 check line & plane intersection (substitute line into plane)

1.38 find plane & plane intersection (find n using cross product, find a using simul eq)

1.39 check 3 planes intersection (simul eq)

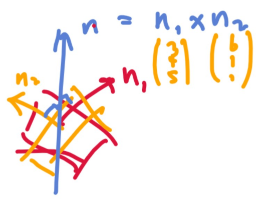
1.37

$$l = \begin{pmatrix} 1+3t \\ 2+2t \\ 3+1t \end{pmatrix} \qquad 3x + 4y + 8z = 1$$

$$3(1+3t) + 4(2+2t) + 8(3+t) = 1$$

t

1.38



$\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$

P1: $3x + 4y + 5z = 0$

P2: $6x + y + z = 1$

Let $z = 0$

$3x + 4y = 0 \qquad 6x + y = 1$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = a$

$l = a + tn$

1.39

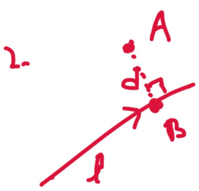
P3: $x + y + 10z = 5$

3 variables, 3 eq

1.6.4 Points, line, plane distance

1. distance between 2 points $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$
2. distance between point & line $\left((l-A)\cdot(l) = 0 \right)$
3. distance between line & line $\left(\dfrac{(a_1-a_2)\cdot(b_1 \times b_2)}{|b_1 \times b_2|} \right)$ or $\left((l_1-l_2)\cdot(l_1)=0 \right)$
4. distance between point & plane $\left(\dfrac{ax+by+cz-d}{\sqrt{a^2+b^2+c^2}} \right)$ or ($l = a + \lambda A$ substitute line to plane)

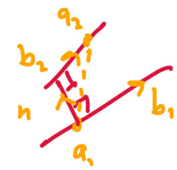
2.



$(l-A)\cdot(l) = 0$

$A-B$

3.



$n = b_1 \times b_2$

$$\boxed{\begin{aligned} &(a_1-a_2)\cdot \hat{n} \\ &(a_1-a_2)\cdot \frac{b_1\times b_2}{|b_1\times b_2|} \end{aligned}}$$

Alternative:

$(l_1-l_2)\cdot(l_1) = 0$

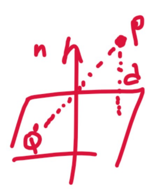
$a + tp$

$b + \lambda q$

$(t-\lambda)(t)$

Project $(\overrightarrow{a_1-a_2})$ to $\vec{n}$

4.



$p = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

plane: $ax+by+cz=d$ point: $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$3x+4y+5z=6$ $\begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$

$$d = \frac{3(2)+4(2)+5(4)-6}{\sqrt{3^2+4^2+-^2}}$$

$rn = an$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \underset{\sim}{n} = \underline{Q\cdot n} = d$

$d\ \boxed{= (P-Q)\cdot \hat{n}}$

$$d = \frac{(P\cdot \underset{\sim}{n}) - (Q\cdot \underset{\sim}{n})}{|n|}$$

$ax+by+cz=d$

Project $(\overrightarrow{P-Q})$ to $\vec{n}$ $= \dfrac{P\cdot \underset{\sim}{n} - d}{|n|}$

Alternative:

$l = P + \lambda \underset{\sim}{n}$

now we have line & plane

we find intersection and get point Q on plane

then find distance between P & Q