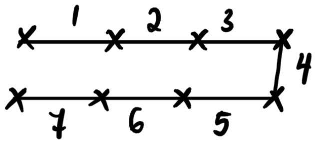
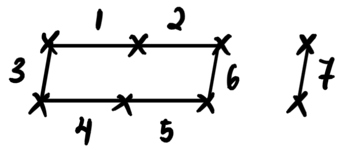
(b) (i) 8 vertices, 7 edge, not tree

impossible, simple graph of 8 vertices with 7 edges means that the vertices in the graph is connected to one another with only 1 edge which means it's connected and will be a tree. Question doesn't want a tree so unless the graph can be disconnected, it is impossible.

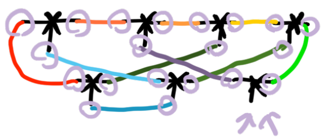
cannot be disconnected:



can be disconnected:



(ii) simple graph, 7 vertices, degree 3



$7 \times 3 = 21$

impossible, the question is basically asking for closed Euler trail but even degrees is needed and 21 is not even so it's impossible.

(iii) $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$ for all integers n and r with $n > r > 0$

Testing step: (cus I wanna see how it works...)

let $n = 7$, $r = 4$

$\binom{7}{4} = \binom{7-1}{4} + \binom{7-1}{4-1}$

$\binom{7}{4} = \binom{6}{4} + \binom{6}{3}$

$= 15 + 20$

$= 35$

$\binom{7}{4} = 35$; $\binom{6}{4} + \binom{6}{3} = 35$

$\binom{7}{3} = {}^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times \cancel{4!}}{3!\cancel{4!}} = \frac{7 \times 30}{3 \times 2 \times 1} = \frac{210}{6} = 35$

$\binom{6}{4} = {}^6C_4 = \frac{6!}{4!(6-4)!} = \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!}(2!)} = \frac{30}{2} = 15$

$\binom{6}{3} = {}^6C_3 = \frac{6!}{3!(6-3)!} = \frac{6 \times 5 \times 4}{3!} = \frac{30 \times 4}{6} = 20$

Base step: $n = 2 > r = 1 > 0$ } $P(1)$

$$\binom{2}{1} = \binom{2-1}{1} + \binom{2-1}{1-1} \qquad \binom{2}{1} = {}^2C_1 = \frac{2!}{1!(2-1)!}$$

$$\binom{2}{1} = \binom{1}{1} + \binom{1}{0} \qquad = \frac{2\times 1}{1\times 1}$$

$$\binom{2}{1} = 1 + 1 \qquad = 2$$

$$\binom{2}{1} = 2$$

since $\binom{2}{1} = 2$ and is equal to $\binom{1}{1} + \binom{1}{0}$ which also equal 2 so $P(1) \equiv$ TRUE.

Inductive step: $\binom{n+1}{r+1} = \binom{n+1-1}{r+1} + \binom{n+1-1}{r+1-1}$ is TRUE?

$P(1) \rightarrow P(2)$

if $n = 2$, $r = 1$, $P(1) \equiv T$

$$\binom{2+1}{1+1} = \binom{2+1-1}{1+1} + \binom{2+1-1}{1+1-1}$$

$$\binom{3}{2} = \binom{2}{2} + \binom{2}{1} \qquad \binom{3}{2} = {}^3C_2 = \frac{3!}{2!(3-2)!}$$

$$\binom{3}{2} = 1 + 2 \qquad = \frac{3\times \cancel{2!}}{\cancel{2!}(1)!}$$

$$\binom{3}{2} = 3 \qquad = 3$$

since $\binom{2}{2} + \binom{2}{1} = 3$ which is equal to $\binom{3}{2} = 3$

so $P(2) \equiv$ TRUE

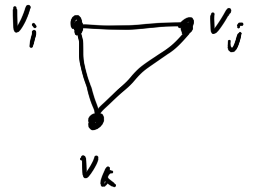
Conclusion:

$P(1) \equiv$ TRUE, $P(2) \equiv$ TRUE so $P(1) \rightarrow P(2)$

and if $P(1) \rightarrow P(2)$ is TRUE then $P(2) \rightarrow P(3) \equiv$ TRUE

so $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$ for all integers n and r with $n > r > 0$

(iv) simple graph randomly on vertices $v_1 \dots v_{20}$

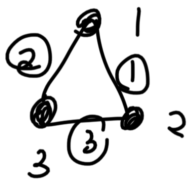


$1/3 : 2/3$
(IN) (OUT)

Expected num : $E(X)$

$v_1 \dots v_{20} = 20-2/3 = 18/3 = 6$ possible graphs?

20 vertice = max edge num $\rightsquigarrow$ ${}_{20}C_2 = 190$ edges



3 vertices, 3 edges

$190/3 = 63.\dot{3} \approx 63$ triangles.