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Part 1:
Task 1.1:

Advantage of 2's complement representation is that it is identical to unsigned binary numbers so addition and subtraction can be performed normally without any special considerations. Disadvantage is that overflow may be obtained if one tries to negate the lowest representable value of a number.

Advantage of sign-and-magnitude representation is that it is the simplest and most common method to represent positive and negative numbers by testing the leftmost bit where 1 represents a negative number and 0 represents a positive number. Disadvantage is that it may result in a missing bit pattern for the number as the leftmost bit of the binary numbers is used to represent if the number is positive or negative. If there is a fixed number of bits to represent a negative number, and that the unsigned version of the number just nicely accommodates the fixed number of bits, the leftmost bit must be sacrificed to be 1 to represent the number as a negative.

Advantage of floating point representation is that it can represent a large number which allows it to support a wider range of numbers. Disadvantage is that it loses precision due to rounding to cover a wider range of values.
[199 words]

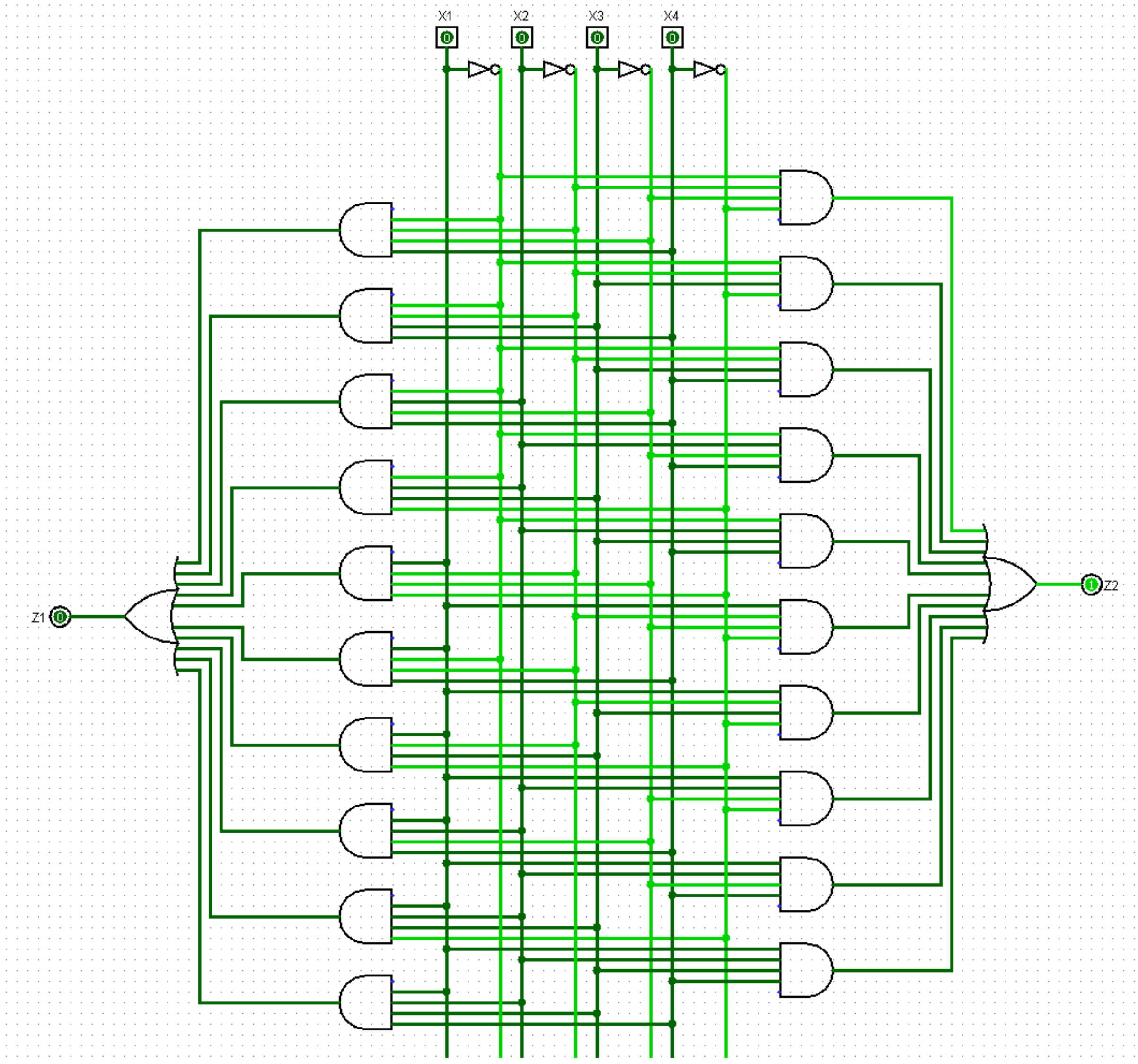
Task 1.2:
Remainder = r
$33085625 \div 16 = 2067851$ with r = 9, Hexadecimal notation = 9
$2067851 \div 16 = 129240$ with r = 11, Hexadecimal notation = B
$129240 \div 16 = 8077$ with r = 8, Hexadecimal notation = 8
$8077 \div 16 = 504$ with r = 13, Hexadecimal notation = D
$504 \div 16 = 31$ with r = 8, Hexadecimal notation = 8
$31 \div 16 = 1$ with r = 15, Hexadecimal notation = F
$1 \div 16 = 0$ with r = 1, Hexadecimal notation = 1
Hexadecimal notation of 33085625 = $1F8D8B9_{16}$
Signed Hexadecimal notation of 33085625 = $01F8D8B9_{16}$
Binary of 1F8D8B9 = 1111110001101100010111001
28-bits Two's complement notation = 0001111110001101100010111001
Hexadecimal notation of 33085625 representing 28-bits = $01F8D8B9_{16}$

Part 2:
Task 2.1:
$Z1 = (\overline{X1}\,\overline{X2}\,\overline{X3}X4)) + (\overline{X1}\,\overline{X2}X3X4) + (\overline{X1}X2\overline{X3}X4) + (\overline{X1}X2X3\overline{X4}) + (X1\overline{X2}\,\overline{X3}\,\overline{X4}) + (X1\overline{X2}\,\overline{X3}X4)$
$+ (X1\overline{X2}X3\overline{X4}) + (X1X2\overline{X3}X4) + (X1X2X3\overline{X4}) + (X1X2X3X4)$
$Z2 = (\overline{X1}\,\overline{X2}\,\overline{X3}\,\overline{X4}) + (\overline{X1}\,\overline{X2}X3\overline{X4}) + (\overline{X1}\,\overline{X2}X3X4) + (\overline{X1}X2\overline{X3}X4) + (\overline{X1}X2X3X4) + (X1\overline{X2}\,\overline{X3}\,\overline{X4})$
$+ (X1\overline{X2}X3\overline{X4}) + (X1X2\overline{X3}\,\overline{X4}) + (X1X2\overline{X3}X4) + (X1X2X3X4)$

The boolean terms from the second row above with the output labelled as $Z2$ is obtained from my truth table with the 4 input of $X1,\ X2,\ X3\ and\ X4$. The combination of the 4 inputs would return an output of $Z2 = 1$ in my truth table. The overline above some input shows that input is a negation which is shown as 0 in my truth table. $Z2 = (X1X2X3X4)$ means that

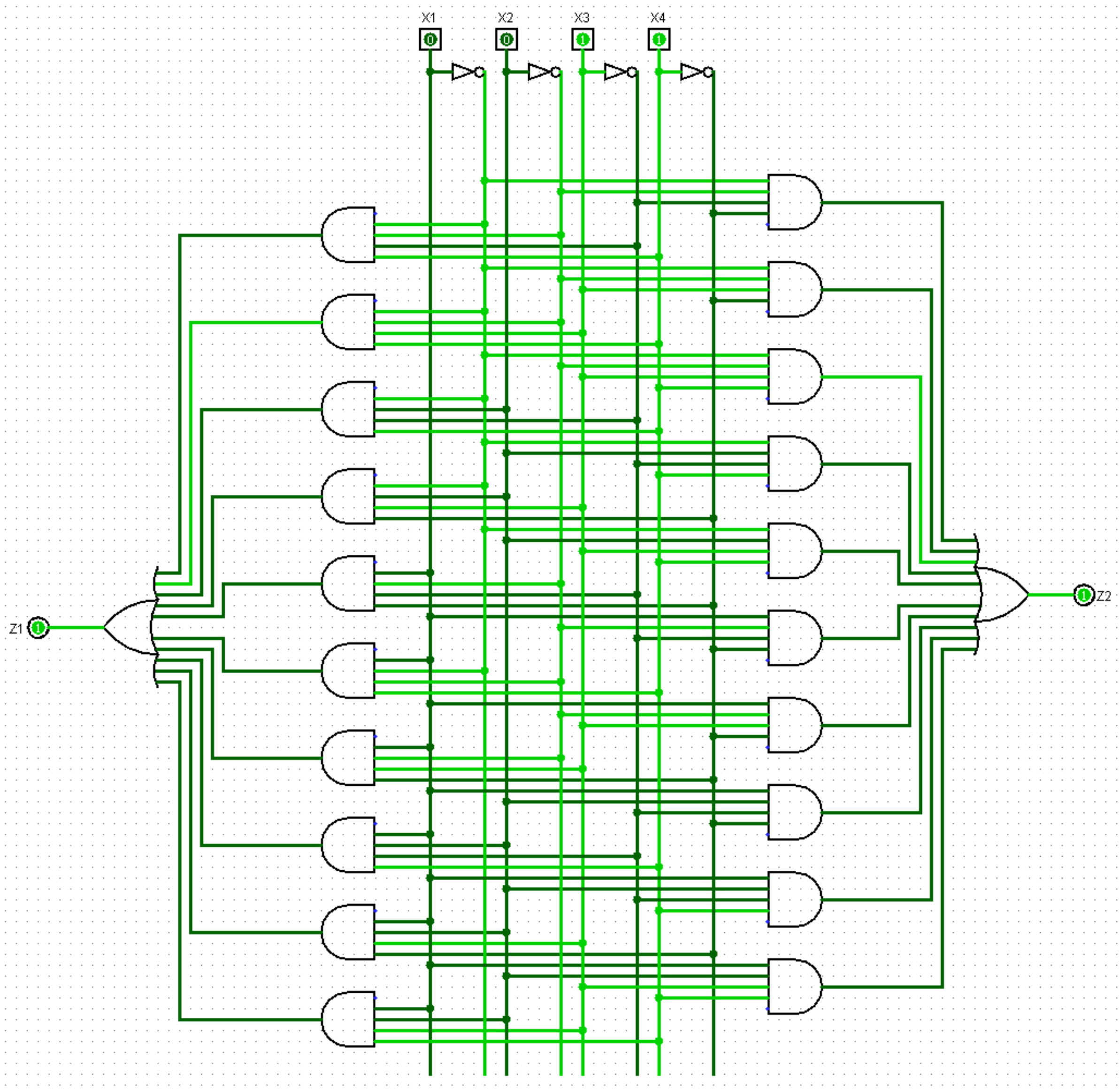
$X1X2X3X4$ all each gives an input of 1 which gives an output of $Z2 = 1$. This also means that $Z2 = (\overline{X1}\,\overline{X2}\,\overline{X3}\,\overline{X4})$ all each gives an input of 0 but also gives an output of $Z2 = 1$.

Task 2.2:

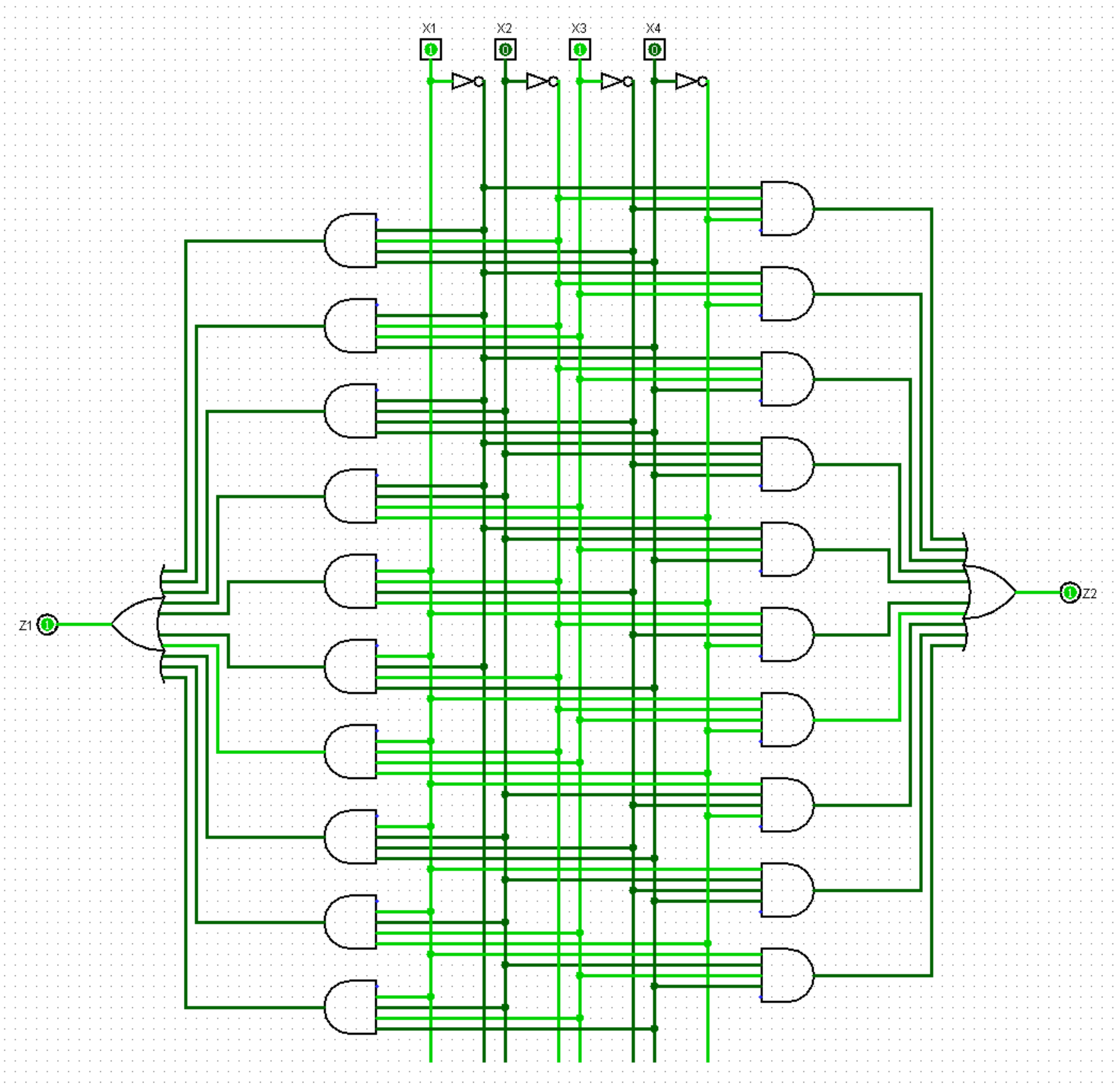


This is the original logical circuit constructed using Logisim where the input is all 0. Only AND, NOT and OR gates are used to construct this circuit. The output of $Z1$ is on the left side of the circuit whereas the output of $Z2$ is on the right side of the circuit. The input of $X1,\ X2,\ X3\ and\ X4$ is placed on top of the circuit. There is a NOT gate connected to each input for $X1,\ X2,\ X3\ and\ X4$. There are a total of 4 NOT gates, 20 AND gates and 2 OR gates in the circuit. Each AND gates is connected to the 4 inputs that can either be the negation of the input $X1,\ X2,\ X3,\ X4$ or the input $X1,\ X2,\ X3,\ X4$ without a negation.

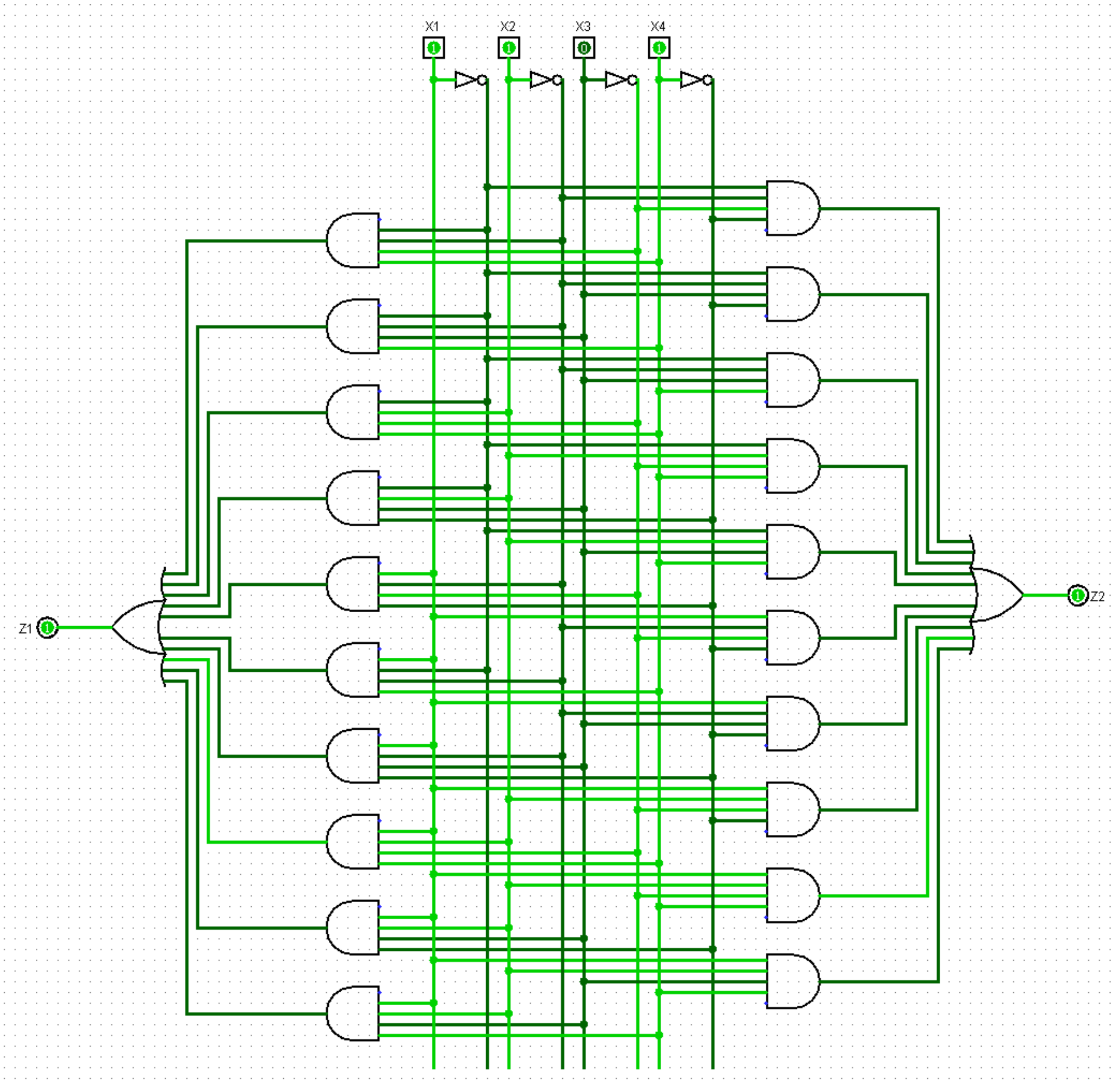
The 3 screenshot below are test-cases :



This is when the input is $(\overline{X1}\,\overline{X2}X3X4)$ which gives both $Z1$ and $Z2$ an output of 1.



This is when the input is $(X1\overline{X2}X3\overline{X4})$ which gives both $Z1$ and $Z2$ an output of 1.



This is when the input is $(X1X2\overline{X3}X4)$ which gives both $Z1$ and $Z2$ an output of 1.

Task 2.3:

K-map for output $Z1$:

X1X2 \ X3X4	00	01	11	10
00	0	1	1	0
01	0	1	0	1
11	0	1	1	1
10	1	1	0	1

K-map for output $Z2$:

X1X2 \ X3X4	00	01	11	10
00	1	0	1	1
01	0	1	1	0
11	1	1	1	0
10	1	0	0	1

Optimised function using Karnaugh maps (K-maps):

$Z1 = (\overline{X3}X4) + (X2X3\overline{X4}) + (X1X2X3) + (X1\overline{X2}\,\overline{X4}) + (\overline{X1}\,\overline{X2}X4)$

$Z2 = (\overline{X2}\,\overline{X4}) + (X2X4) + (\overline{X1}X3X4) + (X1\overline{X3}\,\overline{X4})$

The Boolean terms in the simplified Boolean Algebra Expressions are obtained from the groups from the K-maps where each group circled represents a Boolean term in the simplified Boolean Algebra Expressions.

K-map for output $Z1$:

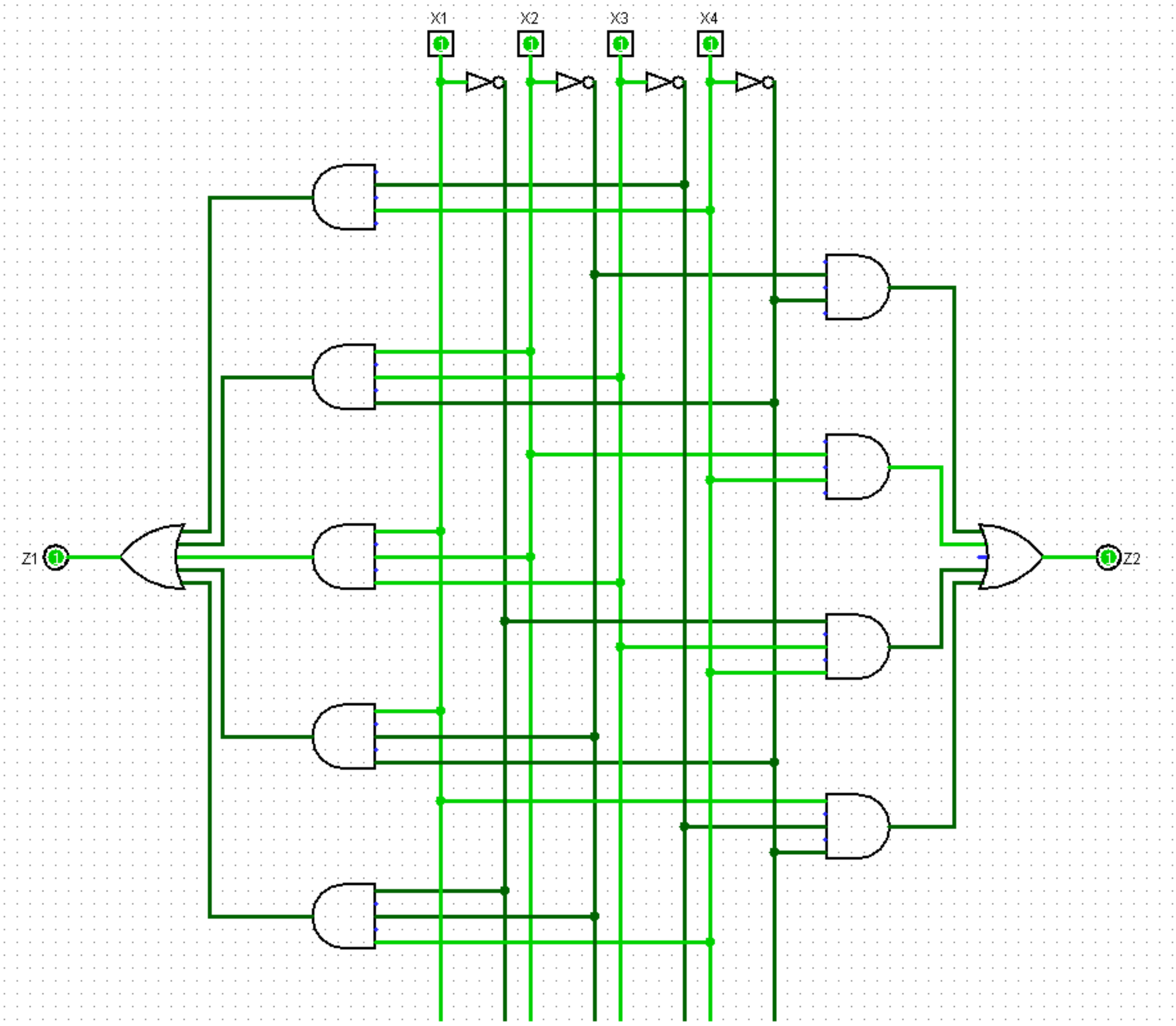
- From the *horizontal group* on **row 2**, the product term obtained is $(\overline{X1}\,\overline{X2}X4)$.
- From the *horizontal group* on **row 4**, the product term obtained is $(X1X2X3)$.
- From the *wrap-around horizontal group* on **row 5,** the product term obtained is $(X1\overline{X2}\,\overline{X4})$.
- From the *vertical group* on **column 3**, the product term obtained is $(\overline{X3}X4)$.
- From the *vertical group* on **column 5**, the product term obtained is $(X2X3\overline{X4})$.

Summing up all the product terms from the K-map for output $Z1$, the final expression for $Z1$ is $Z1 = (\overline{X3}X4) + (X2X3\overline{X4}) + (X1X2X3) + (X1\overline{X2}\,\overline{X4}) + (\overline{X1}\,\overline{X2}X4)$.

K-map for output $Z2$:

- From the *horizontal-vertical group* on **row 3, row 4, column 3 and column 4,** the product term obtained is $(X2X4)$.
- From the *wrap-around 4-corners group* on **row 2, row 5, column 2 and column 5,** the product term obtained is $(\overline{X2}\,\overline{X4})$.
- From the *vertical group* on **column 2,** the product term obtained is $(X1\overline{X3}\,\overline{X4})$.

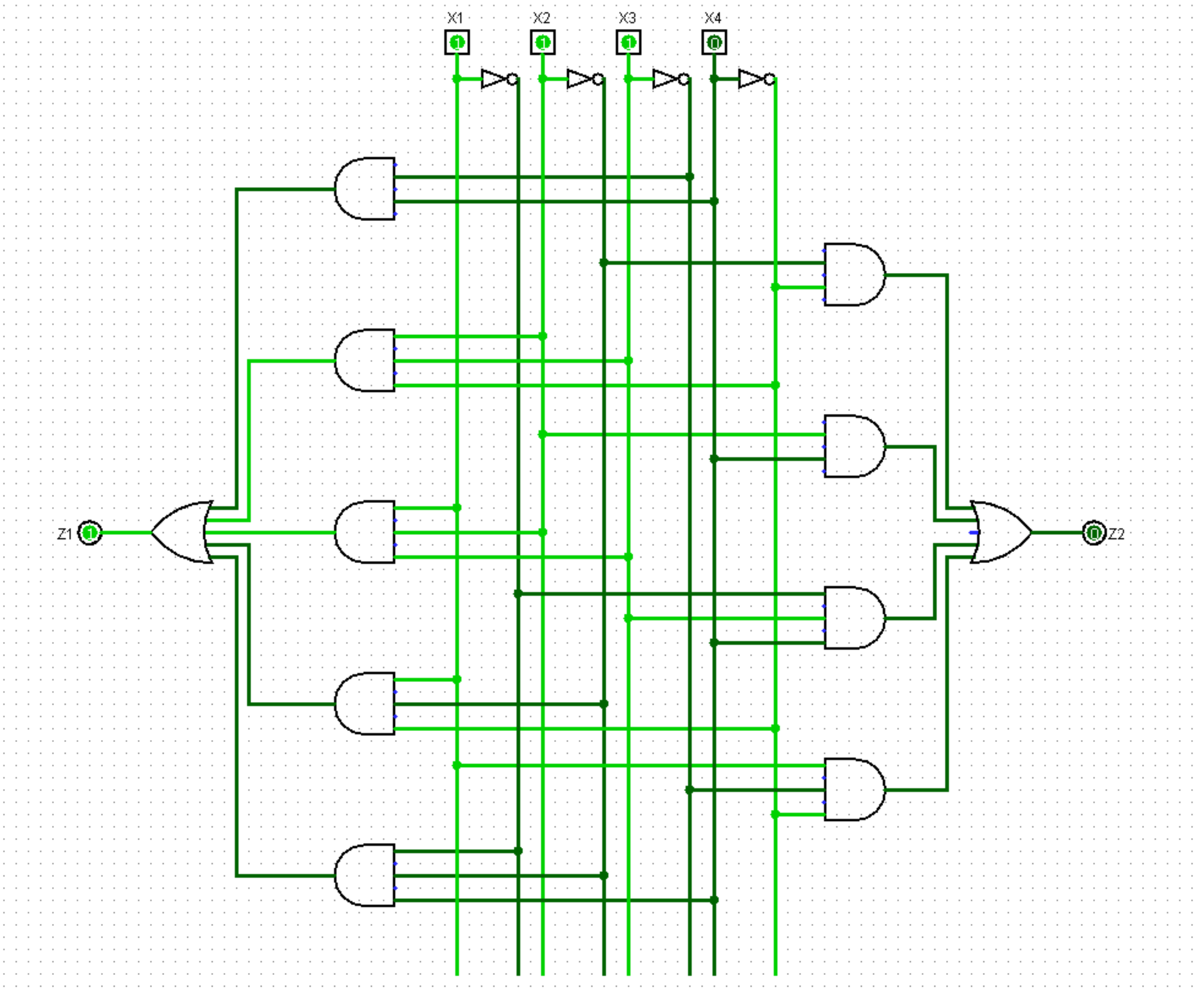
- From the *vertical group* on **column 4,** the product term obtained is $(\overline{X1}X3X4)$.
Summing up all the product terms from the K-map for output $Z2$, the final expression for $Z2$ is $Z2 = (\overline{X2}\,\overline{X4}) + (X2X4) + (\overline{X1}X3X4) + (X1\overline{X3}\,\overline{X4})$.



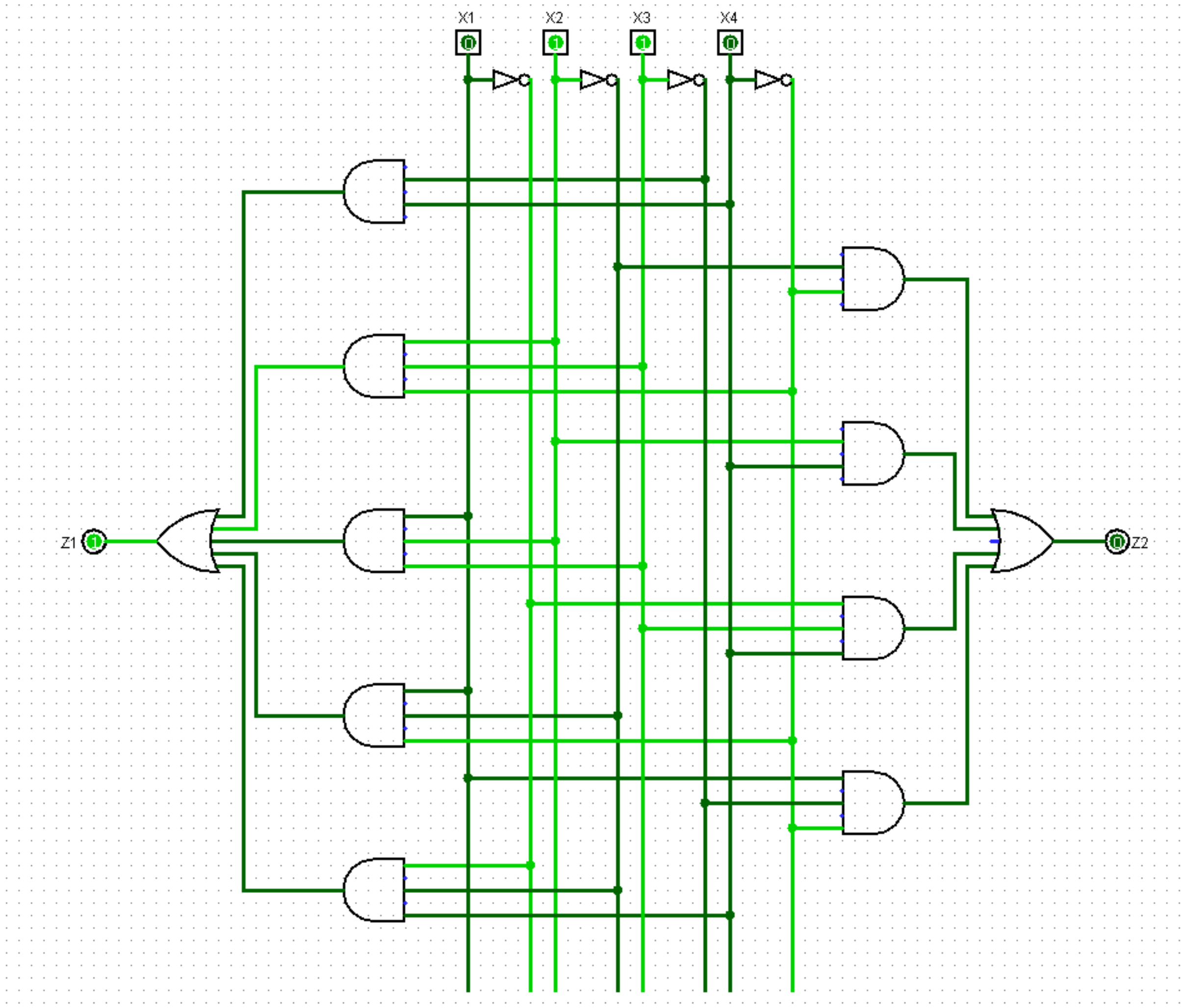
This is the original simplified logical circuit constructed using Logisim where the input is all 1. Only AND, NOT and OR gates are used to construct this circuit.

$(X1X2X3X4)$ gives $Z1$ and $Z2$ both an output of 1 in the original truth table and in this simplified logical circuit, both $Z1$ and $Z2$ lights up which shows that both $Z1$ and $Z2$ have an output of 1. This shows that both $Z1 = (X1X2X3X4)$ and $Z2 = (X1X2X3X4)$.

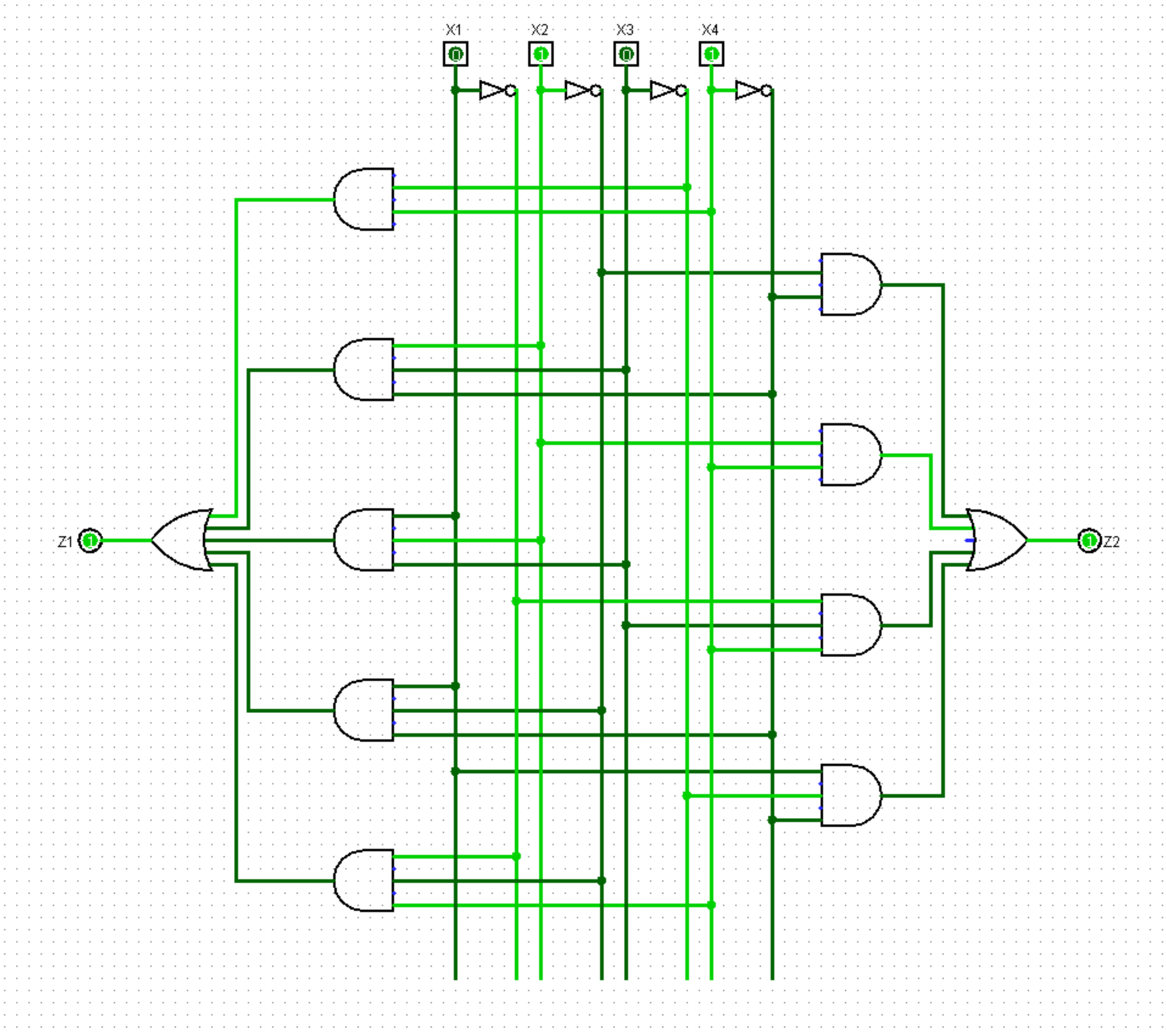
The 4 screenshot below are test-cases :



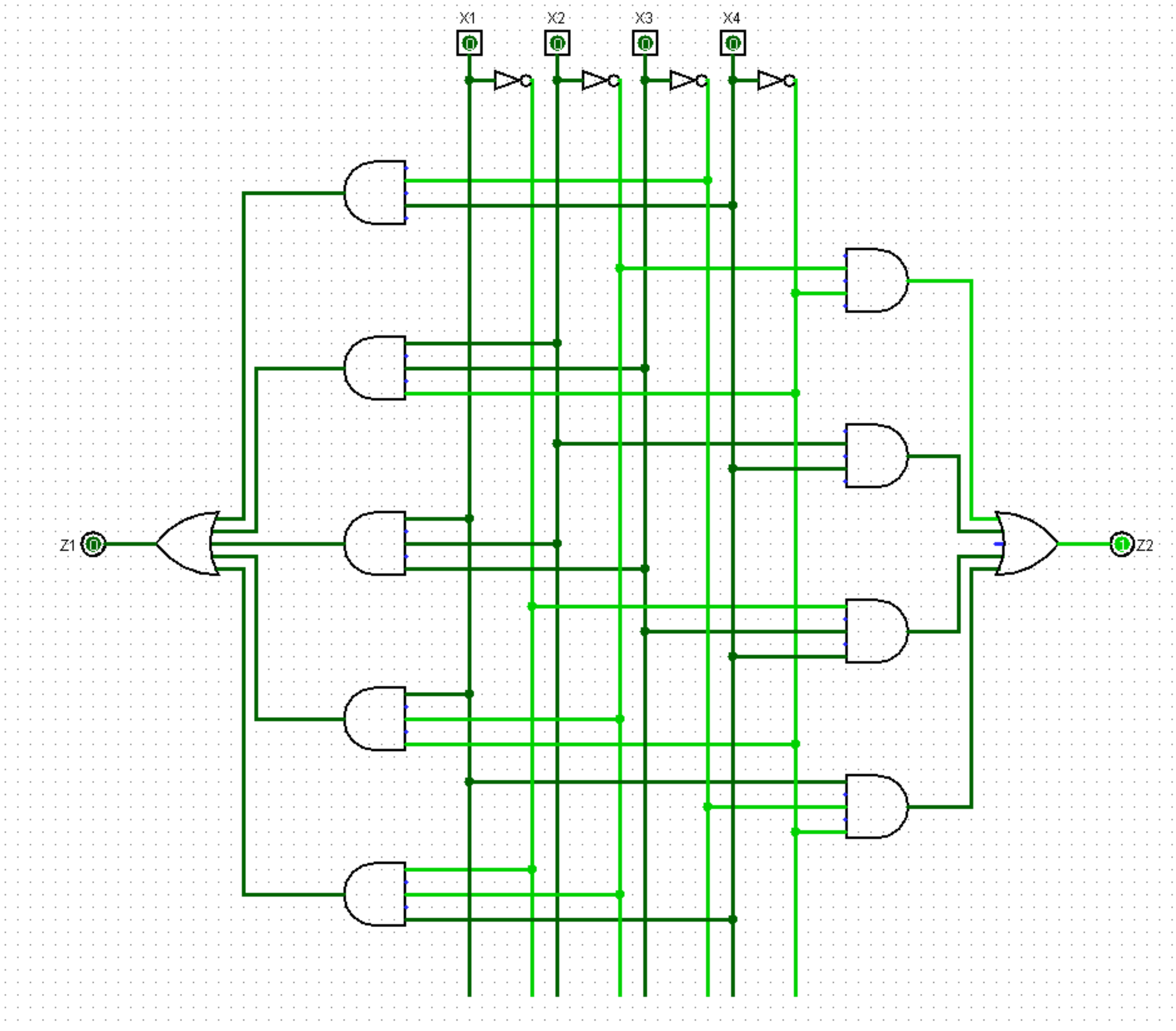
This is the testing for $Z1 = (X1X2X3)$. This does not allows $Z2$ to have an output of 1 as $(X1X2X3)$ can mean $(X1X2X3X4)$ or $(X1X2X3\overline{X4})$ but in this test-case it means $(X1X2X3\overline{X4})$ which does not allow the output of $Z2 = 1$ as referred from the original truth table. From the original truth table, $Z2 = (X1X2X3X4)$ but $Z2 \neq (X1X2X3\overline{X4})$, hence, $Z2$ does not light up. $Z1 = (X1X2X3X4) + (X1X2X3\overline{X4})$ so 2 AND gates from the $Z1$ output lights up as the other path indicates $Z1 = (X2X3\overline{X4})$ which is also the following test-case that was documented.



This is the testing for $Z1 = (X2X3\overline{X4})$. This does not allows $Z2$ to have an output of 1 as $(X2X3\overline{X4})$ can mean $(X1X2X3\overline{X4})$ or $(\overline{X1}X2X3\overline{X4})$ but in this test-case it means $(\overline{X1}X2X3\overline{X4})$ which does not allow the output of $Z2 = 1$ as referred from the original truth table. From the original truth table, $Z2 \neq (\overline{X1}X2X3\overline{X4})$, hence, $Z2$ does not light up.



This is the testing for $Z2 \;=\; (X2X4)$. This also happens for $Z1$ because from the original truth table, $(\overline{X1}X2\overline{X3}X4)$ allows both $Z1$ and $Z2$ to have an output of 1. $(X2X4)$ can mean $(X1X2X3X4)$ or $(X1X2\overline{X3}X4)$ or $(\overline{X1}X2\overline{X3}X4)$ or $(\overline{X1}X2X3X4)$ but in this test-case, this $(X2X4)$ means $(\overline{X1}X2\overline{X3}X4)$ which is true for both $Z1$ and $Z2$ according to the original truth table so both $Z1$ and $Z2$ lights up.



This is the testing for $Z2 = (\overline{X2}\,\overline{X4})$. This does not allows $Z1$ to have an output of 1 as $(\overline{X2}\,\overline{X4})$ can mean $(\overline{X1}\,\overline{X2}\,\overline{X3}\,\overline{X4})$ or $(X1\overline{X2}X3\overline{X4})$ or $(X1\overline{X2}\,\overline{X3}\,\overline{X4})$ or $(\overline{X1}\,\overline{X2}X3\overline{X4})$ but in this test-case it means $(\overline{X1}\,\overline{X2}\,\overline{X3}\,\overline{X4})$ which does not allow the output of $Z1 = 1$ as referred from the original truth table. From the original truth table, $Z1 = (X1\overline{X2}X3\overline{X4})$ but $Z1 \neq (\overline{X1}\,\overline{X2}\,\overline{X3}\,\overline{X4})$, hence, $Z1$ does not light up.