1) let $S = \{1,2,3,4\}$ and $f: S \to S$ defined by $f(x) = x$.
Find...

(a) domain of f $\longrightarrow$ S

(b) codomain of f $\longrightarrow$ S

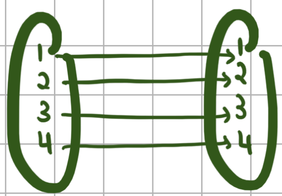
(c) image of f $\longrightarrow$ S

(d) possible mappings of f $\to$ $\{(1,1), (2,2), (3,3), (4,4)\}$

(e) the inverse of f $\longrightarrow$ ✓ one-to-one, ✓ onto $\}$ requirements

HENCE, $f^{-1}(x)$ exists

$f^{-1}(x) = x$



2) let $T = P(S) - \{\{\}\}$ and $g: T \to S$ defined by $g(x) = |X|$.
$S = \{1,2,3,4\}$
Find...

(a) domain of g $\longrightarrow$ $\{\{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\}$

(b) codomain of g $\longrightarrow$ S

(c) image of g $\longrightarrow$ S

(d) possible mappings of g $\longrightarrow$ $\{(\{1\},1), (\{2\},1), (\{3\},1), (\{4\},1), (\{1,2\},2), (\{1,3\},2), (\{1,4\},2), (\{2,3\},2), (\{2,4\},2), (\{3,4\},2), (\{1,2,3\},3), (\{1,2,4\},3), (\{2,3,4\},3), (\{1,3,4\},3), (\{1,2,3,4\},4)\}$

(e) the inverse of g $\longrightarrow$ no inverse, not one-to-one

3. Using definitions of S and T above, find whether the following functions exist

(a) $g(f(x))$
(b) $f(g(x))$
(c) $g^{-1}(f(x))$
(d) $f^{-1}(g(x))$
(e) $g(f^{-1}(x))$
(f) $f(g^{-1}(x))$
(g) $g^{-1}(f^{-1}(x))$
(h) $f^{-1}(g^{-1}(x))$

Let $S = \{1, 2, 3, 4\}$ and $f : S \to S$ defined by $f(x) = x$.

Let $T = P(S) - \{\{\}\}$ and $g : T \to S$ defined by $g(X) = |X|$.

(a) $g(f(x))$ $\longrightarrow$ $f: S \to S$, $g: T \to S$ $\}$ Don't exist

(b) $f(g(x))$ $\longrightarrow$ Exist $\to$ $g: T \to S$, $f: S \to S$ $\quad f(g(x)) = f(|x|) = |x|$

(c) $g^{-1}(f(x))$ $\longrightarrow$ Don't exist

(d) $f^{-1}(g(x))$ $\longrightarrow$ Exist $\longrightarrow$ $f^{-1}(g(x)) = f^{-1}(|x|) = |x|$

(e) $g(f^{-1}(x))$ $\longrightarrow$ Don't exist

(f) $f(g^{-1}(x))$ $\longrightarrow$ Don't exist

(g) $g^{-1}(f^{-1}(x))$ $\longrightarrow$ Don't exist

(h) $f^{-1}(g^{-1}(x))$ $\longrightarrow$ Don't exist

$g: T \to S$
$f^{-1}: S \to S$
domain : T
codomain : S

4) Find inverse of...

(a) $f(x) = \sqrt[3]{x+4} - 2$

let $x = \sqrt[3]{y+4} - 2$

$f^{-1}(x) = (x+2)^3 - 4$

(b) $f(x) = \frac{3x-7}{4x+3}$

SHORT-CUT: $f(x) = \frac{ax+b}{cx+d}$

$f^{-1}(x) = \frac{-dx+b}{cx-a}$

let $x = \frac{3y-7}{4y+3}$

$4xy + 3x = 3y - 7$

$4xy - 3y = -7 - 3x$

$y(4x-3) = -7-3x$

$y = \frac{-7-3x}{4x-3}$

$f^{-1}(x) = \frac{-7-3x}{4x-3}$ × $f^{-1}(x) = (3x+7)/(3-4y)$

5) $f(x) = x^2 - 6x + 2$

$g(x) = -2x$

$h(x) = \sqrt{x}$

(a) $f(g(x)) = f(-2x)$

$= (-2x)^2 - 6(-2x) + 2$

$= 4x^2 + 12x + 2$

(b) $g(f(x)) = -2(x^2 - 6x + 2)$

$= -2x^2 + 12x - 4$

(c) $f(g(h(x))) = f(-2\sqrt{x})$

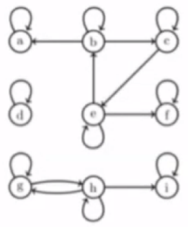
$= (-2\sqrt{x})^2 - 6(-2\sqrt{x}) + 2$

$= 4x + 12\sqrt{x} + 2$

---

1. Let P be the binary relation on the set $X = \{a, b, c, d, e, f, g, h, i\}$ pictured below. Write down whether P is reflexive, symmetric, antisymmetric, or transitive. When P does not have one of these properties give an example of why not.

[Each 'no' needs an accompanying example.]



reflexive ✓

symmetric ×

antisymmetric ×

transitive ×

1. symmetric → not antisymmetric
2. antisymmetric → not symmetric
3. not symmetric $\not\rightarrow$ antisymmetric
4. not antisymmetric $\not\rightarrow$ symmetric

symmetric / antisymmetric / not both

2) let S be the equivalence relation on $P(\{1,2,3,4\})$ defined by xSy if and only if $|x \cap \{1,2\}| = |y \cap \{1,2\}|$

list the equivalence classes of S (without repeating any class)

$P(\{1,2,3,4\}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{4\},$
$\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\},$
$\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\},$
$\{1,2,3,4\}\}$

$[0] = \{\emptyset, \{3\}, \{4\}, \{3,4\}\}$
$[1] = \{\{1\}, \{2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,3,4\}, \{2,3,4\}\}$
$[2] = \{\{1,2\}, \{1,2,3\}, \{1,2,4\}, \{1,2,3,4\}\}$

Q3 & 4 CONTINUE NEXT CLASS