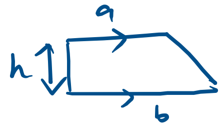
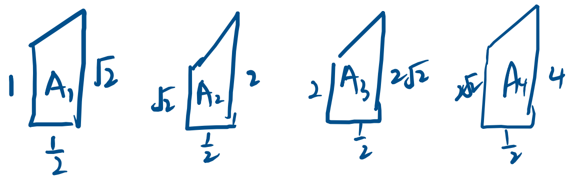


Area, Trapezium $= \frac{1}{2} h (a+b)$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left(f(a)+f(b)+2\sum_{i=1}^{n-1} f(x_i)\right)$$

$$\int_a^b f(x)\,dx = \frac{1}{2}\left(\frac{b-a}{n}\right)\left(f(a)+f(b)+2\sum_{i=1}^{n-1} f(x_i)\right)$$

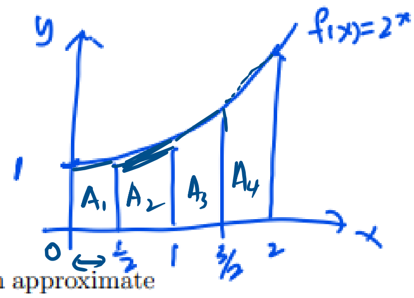


$A_1 = \frac{1}{2}\left(\frac{1}{2}\right)(1+\sqrt{2})$

$A_2 = \frac{1}{2}\left(\frac{1}{2}\right)(\sqrt{2}+2)$

$A_3 = \frac{1}{2}\left(\frac{1}{2}\right)(2+2\sqrt{2})$

$A_4 = \frac{1}{2}\left(\frac{1}{2}\right)(2\sqrt{2}+4)$



Example 4.11. Use the Trapezoidal rule with $n = 4$ to find an approximate value of $\int_0^2 2^x dx$.

width $= h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

Solution

In the interval $[0, 2]$ when $n = 4$ we have four trapezoids each of width $\frac{1}{2}$. The endpoints of our interval are $a = 0$ and $b = 2$ thus $f(a) = f(0) = 2^0 = 1$ and $f(b) = f(2) = 2^2 = 4$. Note that $\frac{b-a}{2n} = \frac{2}{2\times 4} = \frac{1}{4}$. Thus

x	$y=2^x$
0	1
$\frac{1}{2}$	$\sqrt{2}$
1	2
$\frac{3}{2}$	$2\sqrt{2}$
2	4

Total Area $= \frac{1}{2}\left(\frac{1}{2}\right)\left[(1+\sqrt{2})+(\sqrt{2}+2)+(2+2\sqrt{2})+(2\sqrt{2}+4)\right]$

$$\int_0^2 2^x dx \approx \frac{b-a}{2n}\left(f(a)+f(b)+2\sum_{i=1}^{n-1} f(x_i)\right)$$

$= \frac{1}{2}\left(\frac{b-a}{n}\right)$

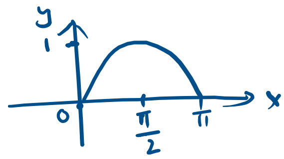
Total Area $= \frac{1}{2}\left(\frac{1}{2}\right)\left[(1+4)+2(\sqrt{2}+2+2\sqrt{2})\right]$

$$= \frac{1}{4}\left(1+4+2\sum_{i=1}^{3} 2^{x_i}\right)$$

$$= \frac{1}{4}\left(1+4+2(2^{1/2}+2^1+2^{3/2})\right)$$

$$= \frac{1}{4}\left(5+2(\sqrt{2}+2+2\sqrt{2})\right)$$

$$= \frac{1}{4}(9+6\sqrt{2})$$ exact form



Example 4.12. Use the Trapezoidal rule with $n = 5$ to find an approximate value of

$$\int_0^\pi \sqrt{\sin x}\,dx$$

width $= h = \frac{b-a}{n} = \frac{\pi - 0}{5} = \frac{\pi}{5}$

radian!

x	$y=\sqrt{\sin x}$
0	0
$\frac{\pi}{5}$	$\sqrt{\sin\left(\frac{\pi}{5}\right)}$
$\frac{2\pi}{5}$	$\sqrt{\sin\left(\frac{2\pi}{5}\right)}$
$\frac{3\pi}{5}$	$\sqrt{\sin\left(\frac{3\pi}{5}\right)}$
$\frac{4\pi}{5}$	$\sqrt{\sin\left(\frac{4\pi}{5}\right)}$
π	$\sqrt{\sin(\pi)} = 0$

sum

$A \approx \frac{1}{2}\left(\frac{\pi}{5}\right)\left[0+0+2\left[\sqrt{\sin\left(\frac{\pi}{5}\right)}+\sqrt{\sin\left(\frac{2\pi}{5}\right)}+\sqrt{\sin\left(\frac{3\pi}{5}\right)}+\sqrt{\sin\left(\frac{4\pi}{5}\right)}\right]\right]$

$= 2.19$ units^2

Example 4.13. Use the Trapezoidal rule with $n = 4$ to find an approximate value of

$$\int_0^1 \frac{1}{1+x^2}dx$$

$h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

x	$y = \frac{1}{1+x^2}$
0	1 ✓
$\frac{1}{4}$	$16/17$
$\frac{1}{2}$	$4/5$
$\frac{3}{4}$	$16/25$
1	$1/2$ ✓

Total Area $= \frac{1}{2}\left(\frac{1}{4}\right)\left[1+\frac{1}{2}+2\left(\frac{16}{17}+\frac{4}{5}+\frac{16}{25}\right)\right]$

$= \frac{5323}{6800}$ units^2

$\simeq 0.783$ units^2

Example 4.14. Use the Trapezoidal rule with $n = 4$ to find an approximate value of

$$\int_0^1 e^{x^2}dx$$

$h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

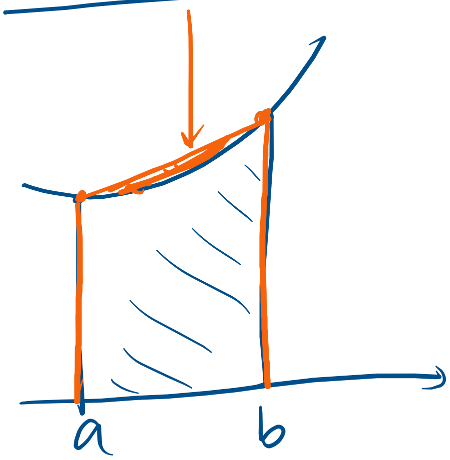
x	0	$\frac{1}{4}$	$\frac{1}{2}$	$3/4$	1
$y=e^{x^2}$	1	$e^{1/16}$	$e^{\frac{1}{4}}$	$e^{9/16}$	e

$A \approx \frac{1}{2}\left(\frac{1}{4}\right)\left[1+e+2\left(e^{1/16}+e^{1/4}+e^{9/16}\right)\right]$

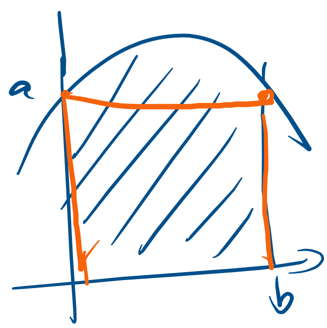
$A = 1.49$ units^2

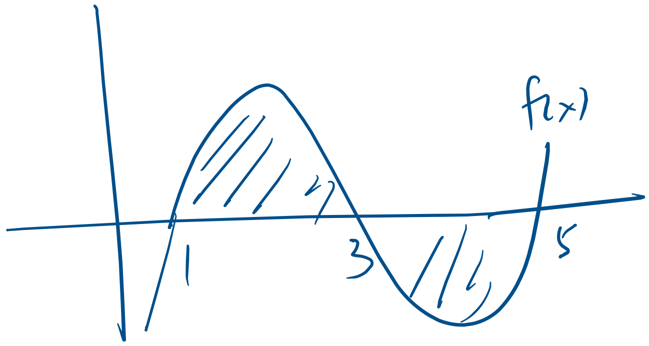
Estimation using Trapezoidal Rule

Over-Estimated



Underestimating





$\int_1^5 f(x)\,dx \Rightarrow$ Area is incorrect

$\int_1^3 f(x)\,dx + \left|\int_3^5 f(x)\,dx\right|$