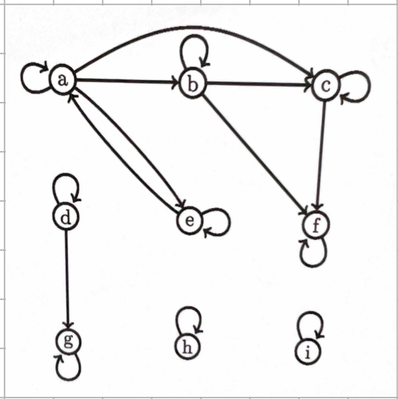
Continuation :

$A = \{a, b, c, d, e, f, g, h, i\}$

$S = \mathbb{Z} \times \mathbb{Z}$ by $(w,x)\, S\, (y,z)$ if and only if $w+x-y-z$ is even



19) reflexive = ✓

symmetric = X

20) antisymmetric : X

transitive : X ⟶ aRb , bRf but a ~~R~~ f

21) reflexive = ✓ ⟶ $(w,x)\, S\, (w,x)$ if $w=1$, $x=2$

$1+2-1-2=0$
$2+1-2-1=0$ } so ✓ reflexive

symmetric = ✓

$(w,x)\, S\, (y,z)$ if $w=1$, $x=2$, $y=3$, $z=4$

$1+2-3-4 = 3-3-4 = -4 \Rightarrow$ even ✓
$3+4-1-2 = 7-3 = 4 \Rightarrow$ even ✓ } so ✓ symmetry

22) antisymmetric = X

transitive = ✓ ⟶ for all $(u,v), (w,x), (y,z) \in \mathbb{Z} \times \mathbb{Z}$

23) S = RST

↳ equivalence relation

2 equivalence class : $\{(w,x) : (w,x) \in \mathbb{Z} \times \mathbb{Z}$ and $w+x$ is even$\}$

$\{(w,x) : (w,x) \in \mathbb{Z} \times \mathbb{Z}$ and $w+x$ is odd$\}$

24) written as sum of 1, 3, 4 only

$S_3 = 1+1+1$ && 3

25) $S_n = S_{n-1} + S_{n-3} + S_{n-4}$

because only can written as sum of 1, 3, 4 only

26) $Pr(X_1 = 2 \land Y = 2) \longrightarrow \{2,2\}; \{1,2\}$

$Pr(X_1 = 1 \land X_2 = 2) + Pr(X_1 = 2 \land X_2 = 2) = (1/3 \times 1/3) + (1/3 \times 1/3) = 2/9$

27) $E(Y) = Pr(1) + Pr(2) + Pr(3)$

$Pr(Y=1) = \{1,1\} = Pr(X_1 = 1 \land X_2 = 1) = 1/9$

$Pr(Y=2) = \{2,1\}, \{2,2\}, \{1,2\} = 3/9$

$Pr(Y=3) = \{1,3\}, \{3,3\}, \{3,1\}, \{3,2\}, \{2,3\} = 5/9$

$Pr(1) + Pr(2) + Pr(3) = 1/9 + (2 \times 3/9) + (3 \times 5/9)$

$= 22/9$

AND = ×
OR = +

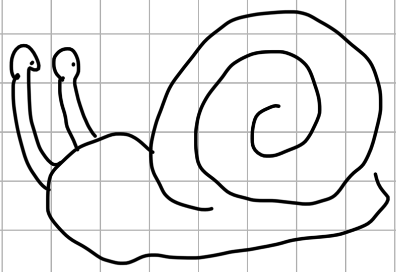
28) probability : $\{1...100\} \times \{1...100\} = 1/1000$

29) Poisson distribution , 3 calls per minute

↳ $E(X) = \lambda$

↳ $Var(X) = \lambda$

↳ $Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$



no calls in 1 min probability

$Pr(X=0) = 3^0 e^{-3} / 0!$

$= e^{-3}$

30) 2 calls in 2 min

$Pr(x=2) = e^{-3} 3^2 / 2!$

$= e^{-3}(9)/2$

$Pr(Y \geq 2) = 1 - Pr(Y=1) - Pr(Y=0)$

$= 1 - (e^{-6} 6^1 / 1!) - (e^{-6} 6^0 / 0!)$

$= 1 - 6e^{-6} - e^{-6}$

$= 1 - 7e^{-6}$

31) 8

32) Closed Euler trail $\overset{?}{\Rightarrow}$ SPRQTS

BUT ODD DEGREES so NO!

33) QS, RS must include ST OR QT AND one from PQ, PR, PS so $2 \times 3 = 6$

34)

$$\begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 1 & 1 & 1 \\ v_2 & 1 & 0 & 1 & 1 \\ v_3 & 1 & 1 & 0 & 0 \\ v_4 & 1 & 1 & 0 & 0 \end{array}$$

$\left.\begin{array}{l} v_1 v_4 \\ v_2 v_4 \end{array}\right\} 2$

← start = row

35) need m with the power of 6

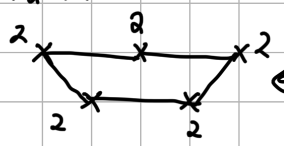
36) 100 vertices = 99 edges

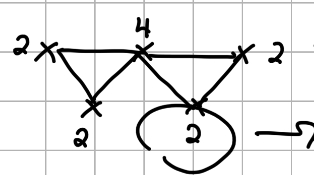
↳ disconnected = exist but no tree

38) (A) $5+3+2+2+2 = 14$ → 5 vertice, 1 vertex max 4 edge

(B) have 0 = disconnected ; $4+2+2+2+0 = 10$ don't exist

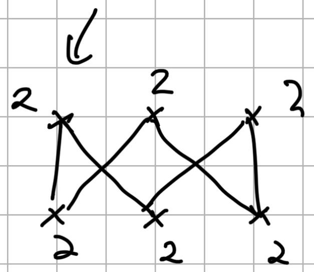
(C) $2+2+2+2+2 = 10$



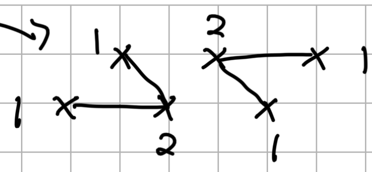


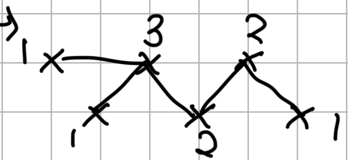
→ not ⊘

39) (A) $2+2+1+1+1+1=8$
(B) $3+2+2+1+1+1=10$
(C) $\underbrace{2+2+2}+\underbrace{2+2+2}=12$



disconnected





6 vertice + 5 edge HENCE sum of degree $= 2\times 5 = 10$ (B)

40) $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n}{n-r} \times \dfrac{(n-1)!}{r!(n-1-r)!} = \dfrac{n}{n-r}\binom{n-1}{r}$

41) $3\times 5\times\binom{4}{2} = 15\times\dfrac{4!}{2!(4-2)!}$

$= 15 \times \dfrac{4\times 3\times \cancel{2!}}{2!(\cancel{2!})}$

$= 15 \times \dfrac{\cancel{12}\ 6}{\cancel{2}}$

$= 90$ #

42) simple induction: $(19^n - 12^n)/7$ for all integers $n \geq 1$

Base step: $n=1$

$\dfrac{19^1 - 12^1}{7} = \dfrac{7}{7} = 1$ $\quad P(1) \equiv$ TRUE

Induction step: $P(1) \equiv$ TRUE

$(19^{k+1} - 12^{k+1})/7 = 2[(19^k - 12^k)/7]$

$\dfrac{19^{k+1}}{7} - \dfrac{12^{k+1}}{7} = \dfrac{2(19^k)}{7} - \dfrac{2(12^k)}{7}$

$19^{k+1} - 12^{k+1} = 2(19^k) - 2(12^k)$

$\cancel{19^k}+19 - \cancel{12^k}+12 = \cancel{2}(19^k) - \cancel{2}(12^k)$

$19 - 12 = 19^k - 12^k$

$19 + 12^k = 19^k + 12$