Problem Set Four: Matrices - Inverse, Transpose and Determinant

Matrix Inverse

1. Compute the inverse of the following matrices, or state why the inverse does not exist.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 1 & 2 \end{bmatrix}$$

Verify that $A^{-1}A = I$ and $AA^{-1} = I$. What is $(A^{-1})^{-1}$?

Matrix B

① Not square matrix
matrix B has size 3×2

B has no inverse!

$AB \neq BA$

matrix A

① square matrix, 2×2

② $\det(A) = -4 - 1 = -5$

$\det(A) \neq 0$

Therefore A^{-1} exists.

matrix A is invertible.

$$A^{-1} = \frac{1}{-5}\begin{pmatrix} -4 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 4/5 & 1/5 \\ 1/5 & -1/5 \end{pmatrix}$$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Check:

$$AA^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 4/5 & 1/5 \\ 1/5 & -1/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$A^{-1}A = \begin{pmatrix} 4/5 & 1/5 \\ 1/5 & -1/5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\begin{pmatrix} 4/5 & 1/5 \\ 1/5 & -1/5 \end{pmatrix}^{-1} = \frac{1}{-\frac{4}{25} - \frac{1}{25}}\begin{pmatrix} -\frac{1}{5} & -1/5 \\ -1/5 & 4/5 \end{pmatrix} = -5\begin{pmatrix} -1/5 & -1/5 \\ -1/5 & 4/5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix} = A$$

$(A^{-1})^{-1} = A$

2. Show that the matrix $B = \begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}$ is not invertible. What are the implications of this for the following linear system?

$$2x + 5y = 7$$
$$6x + 15y = 8$$

$$\underset{B}{\begin{pmatrix} 2 & 5 \\ 6 & 15 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{C}{\begin{pmatrix} 7 \\ 8 \end{pmatrix}}$$

matrix form

$\Rightarrow \det(B) = 0 \Rightarrow$ No soln!

B^{-1} does not exist!

$\det(B) = (2\times 15) - (6\times 5)$
$= 30 - 30$

$\det(B) = 0$

B^{-1} does not exist! B is NOT invertible

$BX = C$

$X = B^{-1}C$ $\Rightarrow$ Cannot proceed

3. Calculate B^{-1}, B^2 and $(B^2)^{-1}$ for the matrix $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$.

Verify that $(B^2)^{-1} = B^{-1}B^{-1}$.

$$B^{-1} = \frac{1}{8-9}\begin{pmatrix} 4 & -3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$$

$$B^2 = BB = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 13 & 18 \\ 18 & 25 \end{pmatrix}$$

$$B^{-1}B^{-1} = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 25 & -18 \\ -18 & 13 \end{pmatrix}$$

$$(B^2)^{-1} = \frac{1}{(13\times 25) - (18\times 18)}\begin{pmatrix} 25 & -18 \\ -18 & 13 \end{pmatrix} = \begin{pmatrix} 25 & -18 \\ -18 & 13 \end{pmatrix}$$

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$(B^2)^{-1} = B^{-1}B^{-1}$

Matrix Transpose → row ↔ column

$\boxed{(A^{-1})^{-1} = A}$

4a. For the matrix $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 3 \\ 1 & 2 & -1 \end{bmatrix}$ compute A^T. What is $(A^T)^T$?

$A^T = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \\ -1 & 3 & -1 \end{pmatrix}$

$(A^T)^T = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix}$

$(A^T)^T = A$

$AA^{-1} = A^{-1}A$

4b. For the matrix $B = \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 1 & 2 \end{bmatrix}$ compute B^T. Also compute BB^T and B^TB.

$B^T = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 3 & 2 \end{pmatrix}$

$BB^T = \begin{pmatrix} 2 & 3 \\ 1 & 3 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & 1 \\ 3 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 11 & 8 \\ 11 & 10 & 7 \\ 8 & 7 & 5 \end{pmatrix}$

3×2 ✓ 2×3

$B^TB = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 11 \\ 11 & 22 \end{pmatrix}$

2×3 ✓ 3×2

$\boxed{BB^T \neq B^TB}$ $AB \neq BA$

$AA^{-1} = A^{-1}A$

5. Consider the following matrices

$$A = \begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$A^T = \begin{pmatrix} 4 & 1 \\ -2 & 5 \end{pmatrix}$ $B^T = \begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}$

Compute A^T, B^T and A^TB^T. Verify that $(AB)^T = B^TA^T$. ✓

$A^TB^T = \begin{pmatrix} 4 & 1 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ -9 & 1 \end{pmatrix}$ ✓

$AB = \begin{pmatrix} 4 & -2 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -6 \\ 12 & 4 \end{pmatrix}$

$(AB)^T = \begin{pmatrix} 4 & 12 \\ -6 & 4 \end{pmatrix}$

$B^TA^T = \begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 12 \\ -6 & 4 \end{pmatrix}$

$(AB)^T = B^TA^T$ ✓✓

$(AB)^T \neq A^TB^T$

6. For the matrices A and B from Question 5, compute the following:

(a) $(A+B)^T$

(b) $A^T + B^T$

(c) $A + A^T$ and $B + B^T$

(d) $(A + A^T)^T$ and $(B + B^T)^T$

(e) $(A^T)^{-1}$

(f) $(A^{-1})^T$

(g) $(AB)^{-1}$

(h) $B^{-1}A^{-1}$

(a) $(A+B)^T = \begin{pmatrix} 6 & -3 \\ 3 & 6 \end{pmatrix}^T = \begin{pmatrix} 6 & 3 \\ -3 & 6 \end{pmatrix}$ (b) $A^T + B^T = \begin{pmatrix} 6 & 3 \\ -3 & 6 \end{pmatrix}$

$(A+B)^T = A^T + B^T$

(c) $A + A^T = \begin{pmatrix} 8 & -1 \\ -1 & 10 \end{pmatrix}$ $B + B^T = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$ Symmetric matrix

(d) $(A + A^T)^T = \begin{pmatrix} 8 & -1 \\ -1 & 10 \end{pmatrix}$ $(B + B^T)^T = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$

$(A + A^T) = (A + A^T)^T$ $B + B^T = (B + B^T)^T$ if

(e) $(A^T)^{-1} = \frac{1}{22}\begin{pmatrix} 5 & -1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 5/22 & -1/22 \\ 2/22 & 4/22 \end{pmatrix}$

(f) $(A^{-1})^T = \left[\frac{1}{22}\begin{pmatrix} 5 & 2 \\ -1 & 4 \end{pmatrix}\right]^T = \begin{pmatrix} 5/22 & 2/22 \\ -1/22 & 4/22 \end{pmatrix}^T = \begin{pmatrix} 5/22 & -1/22 \\ 2/22 & 4/22 \end{pmatrix}$

$(A^T)^{-1} = (A^{-1})^T$

(h) $B^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ -2/4 & 2/4 \end{pmatrix}$

$B^{-1}A^{-1} = \begin{pmatrix} 1/4 & 1/4 \\ -2/4 & 2/4 \end{pmatrix}\begin{pmatrix} 5/22 & 2/22 \\ -1/22 & 4/22 \end{pmatrix} = \begin{pmatrix} 4/88 & 6/88 \\ -12/88 & 4/88 \end{pmatrix}$

(g) $(AB)^{-1} = \begin{pmatrix} 4 & -6 \\ 12 & 4 \end{pmatrix}^{-1} = \frac{1}{16+72}\begin{pmatrix} 4 & 6 \\ -12 & 4 \end{pmatrix} = \begin{pmatrix} 4/88 & 6/88 \\ -12/88 & 4/88 \end{pmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^T = B^TA^T$

$(-1)^{i+j} a_{ij} |S_{ij}|$

Laplace's Expansion

fix 1st row ($i=1$)

$\det(C) = (-1)^2 (2) \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} + (-1)^3 (3) \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} + (-1)^4 (-1) \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$

$\det(C) = 2(-7) + (-3)(-4) + (-1)(1) = -3$

Matrix Determinant

(a) $\det(A) = 14 - (-1) = 15$

$\det(B) = 2 - 3 = -1$

7. Consider the following matrices

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 3 \\ 1 & 2 & -1 \end{bmatrix}$$

a. Compute the determinant for each matrix.

b. Compute AB and find $\det(AB)$.

c. Compute BA and find $\det(BA)$.

d. What is $\det(A)\det(B)$?

e. Compute A^T. What is $\det(A^T)$?

f. Compute A^{-1}. What is $\det(A^{-1})$?

g. What is $\det(A)\det(A^{-1})$?

(b) $AB = \begin{pmatrix} 2 & -1 \\ 1 & 7 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 9 & 10 \end{pmatrix}$

$\det(AB) = 30 - 45$

$\det(AB) = -15$

$\boxed{\det(AB) = \det(A) \times \det(B) = 15 \times -1}$

(c) $BA = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} 7 & 19 \\ 3 & 6 \end{pmatrix}$ $\det(BA) = -15$

$\det(AB) = \det(BA) = \det(B)\det(A) = -1 \times 15 = -15$

(d) $\det(A) \det B = 15 \times -1 = -15$

(e) $A^T = \begin{pmatrix} 2 & 1 \\ -1 & 7 \end{pmatrix}$ $\det(A^T) = 14 - (-1) = 15$ $\det(A) = \det(A^T)$

(f) $A^{-1} = \frac{1}{15}\begin{pmatrix} 7 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 7/15 & 1/15 \\ -1/15 & 2/15 \end{pmatrix}$ $\det(A^{-1}) = \frac{14}{15^2} + \frac{1}{15^2} = \frac{15}{15^2} = \frac{1}{15}$

(g) $\det(A)\det(A^{-1}) = 15 \times \frac{1}{15} = 1$ $\det(AA^{-1}) = \det(I) = 1$

8. Write down the coefficient matrix, C, for each of the under determined systems. What do you notice about the determinant of each?

a. $\left|\begin{matrix} 1 & 2 & -1 & 1 & 2 \\ 1 & 3 & 0 & 1 & 3 \\ 2 & 5 & -1 & 2 & 5 \end{matrix}\right|$

$$x + 2y - z = 6$$
$$x + 3y + 0z = 7$$
$$2x + 5y - z = 13$$

$C = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 2 & 5 & -1 \end{pmatrix}$

$\det(C) = (-3 - 5 + 0) - (-6 + 0 - 2)$
$= -8 - (-8) = 0 \rightarrow$ not able to solve vis inverse matrix.

$\Rightarrow$ No soln. $\rightarrow$

b.

$$x + 2y - z = 6$$

$C = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 3x3 $\det(C) = 0$ C^{-1} does not exist!

$C = \begin{pmatrix} 1 & 2 & -1 \end{pmatrix}$

$BX = C \Leftrightarrow X = B^{-1}C = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Q9(a)

$$\left[\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 & 1 & 0 \\ 1 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 \leftarrow R_2 - R_3]{R_2 \leftarrow R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & 1 & -2 & 0 \\ 0 & -1 & 4 & 0 & 1 & -1 \end{array}\right]$$

$\det(C) = -3 \neq 0$

$$\xrightarrow{R_3 \leftarrow R_2 + R_3} \left[\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & 1 & -2 & 0 \\ 0 & 0 & -3 & 1 & -1 & -1 \end{array}\right]$$

9. Consider the matrix C from Question 7.

a. Use the Gauss-Jordan method to find C^{-1}. Check your answer by showing $CC^{-1} = I$.

b. You have calculated $\det(C)$ in Question 7. Use this, and the formula

$$a_{ji} = (-1)^{i+j} \frac{\det S_{ij}}{\det C}$$

to compute C^{-1}, where S_{ij} is the 2×2 minor matrix of C with column i and row j removed.

$$\xrightarrow{R_3 \leftarrow -\frac{1}{3} R_3} \left(\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1/3 & 1/3 & 1/3 \end{array}\right)$$

$$\xrightarrow[R_1 \leftarrow R_1 + R_3]{R_2 \leftarrow R_2 + 7R_3} \left(\begin{array}{ccc|ccc} 2 & 3 & 0 & 2/3 & 4/3 & 1/3 \\ 0 & 1 & 0 & -4/3 & 1/3 & 7/3 \\ 0 & 0 & 1 & -1/3 & 1/3 & 1/3 \end{array}\right)$$

$$\xrightarrow{R_1 \leftarrow R_1 - 3R_2} \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 14/3 & -2/3 & -20/3 \\ 0 & 1 & 0 & -4/3 & 1/3 & 7/3 \\ 0 & 0 & 1 & -1/3 & 1/3 & 1/3 \end{array}\right)$$

$$\xrightarrow{R_1 \leftarrow \frac{1}{2} R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/3 & -1/3 & -10/3 \\ 0 & 1 & 0 & -4/3 & 1/3 & 7/3 \\ 0 & 0 & 1 & -1/3 & 1/3 & 1/3 \end{array}\right)$$

$$C^{-1} = \begin{pmatrix} 7/3 & -1/3 & -10/3 \\ -4/3 & 1/3 & 7/3 \\ -1/3 & 1/3 & 1/3 \end{pmatrix}$$

check

$CC^{-1} = I$?

Find soln.

$$\begin{pmatrix} C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = C^{-1} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$= $ soln.

Tut #4 Q9(b) $C = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix}$

$\det(C) = (-2-2+9) - (-1+12-3)$

$= 5 - 8 = -3$

$C^{-1} = \begin{pmatrix} \frac{7}{3} & -\frac{1}{3} & -\frac{10}{3} \\ -\frac{4}{3} & \frac{1}{3} & \frac{7}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

$C = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix}$

$\boxed{i=1, j=1}$

$a_{11} \; \frac{(-1)^2 \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}}{-3} = \frac{7}{3}$

$\boxed{i=1, j=2}$

$a_{21} = \frac{(-1)^3 \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix}}{-3}$

$= -\frac{4}{3}$

$\boxed{i=1, j=3}$

$a_{31} = \frac{(-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}}{-3}$

$= -\frac{1}{3}$

2.8.1 Inverse - another method

Here is another way to compute the inverse of a matrix.

- Select the i^{th} row and j^{th} column of A.
- Compute $(-1)^{i+j}\frac{\det S_{ij}}{\det A} = a_{ji}$
- Store this entry at a_{ji} (row j and column i) in the inverse matrix.
- Repeat for all other entries in A.

That is, if

$$A = [\quad a_{ij} \quad]$$

then

$$A^{-1} = \frac{1}{\det A}[\quad (-1)^{i+j}\det S_{ji} \quad]$$

This method works but it is rather tedious.

$\boxed{i=2, j=1}$

$a_{12} = \frac{(-1)^3 \begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix}}{-3} = -\frac{1}{3}$

$\boxed{i=2, j=2}$

$a_{22} = \frac{(-1)^4 \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}}{-3} = \frac{1}{3}$

$\boxed{i=2, j=3}$

$a_{32} = \frac{(-1)^5 \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}}{-3} = \frac{1}{3}$

$\boxed{i=3, j=1}$

$a_{13} = \frac{(-1)^4 \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix}}{-3} = \frac{-10}{3}$

$\boxed{i=3, j=2}$

$a_{23} = \frac{(-1)^5 \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}}{-3} = \frac{7}{3}$

$\boxed{i=3, j=3}$

$a_{33} = \frac{(-1)^6 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}}{-3} = \frac{1}{3}$

10. Use Cramer's rule to solve the following system of equations:

$$b = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \qquad \begin{aligned} 2x + 3y - z &= 4 \\ x + y + 3z &= 1 \\ x + 2y - z &= 3 \end{aligned}$$

Q7 matrix c

$$\begin{array}{cccc} & x & y & z \\ C = & \multicolumn{3}{c}{\begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix}} \end{array} \qquad \det(C) = -3$$

$$X = \frac{\begin{vmatrix} 4 & 3 & -1 \\ 1 & 1 & 3 \\ 3 & 2 & -1 \end{vmatrix}}{-3}$$

$$X = \frac{(-4 + 27 - 2) - (-3 + 24 - 3)}{-3}$$

$$x = \frac{3}{-3}$$

$$x = -1$$

2.7.3 Cramer's rule

Recall that if a linear system $A\underset{\sim}{x} = \underset{\sim}{b}$ has a unique solution, then $\underset{\sim}{x} = A^{-1}\underset{\sim}{b}$ is this solution. If we substitute the formula for the inverse A^{-1} from the previous section (using det S_{ji}) into the product $A^{-1}\underset{\sim}{b}$ we arrive at **Cramer's rule** for solving the linear system $A\underset{\sim}{x} = \underset{\sim}{b}$.

Cramer's rule: Let $A\underset{\sim}{x} = \underset{\sim}{b}$ be a linear system with a unique solution. This means that A is a square matrix with non-zero determinant. Let A_i be the matrix that results from A by replacing the ith column of A by $\underset{\sim}{b}$. Then

x_1 $i=1$

$$x_i = \frac{detA_i}{detA}$$

x_2

Examples of Cramer's rule will be given in tutorials.

x_3

$$y = \frac{\begin{vmatrix} 2 & 4 & -1 \\ 1 & 1 & 3 \\ 1 & 3 & -1 \end{vmatrix}}{-3} = \frac{(-2 - 3 + 12) - (-1 + 18 - 4)}{-3} = \frac{-6}{-3} = 2$$

$$z = \frac{\begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}}{-3} = \frac{(6 + 8 + 3) - (4 + 4 + 9)}{-3} = \frac{0}{-3} = 0$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$$