NO: Cheat sheet DATE:

** $P(A) + P(A') = 1$ **	$P(A \cup B)$	$P(A \cap B)$	$P(A\|B)$	$P(B\|A)$
normal event	$P(A) + P(B) - P(A \cap B)$	$P(A\|B) \times P(B)$	$\frac{P(A \cap B)}{P(B)}$	$\frac{P(B \cap A)}{P(A)}$
mutually exclusive event	$P(A) + P(B)$	0	0	0
independant event	$P(A)+P(B)-[P(A) \times P(B)]$	$P(A) \times P(B)$	$P(A)$	$P(B)$

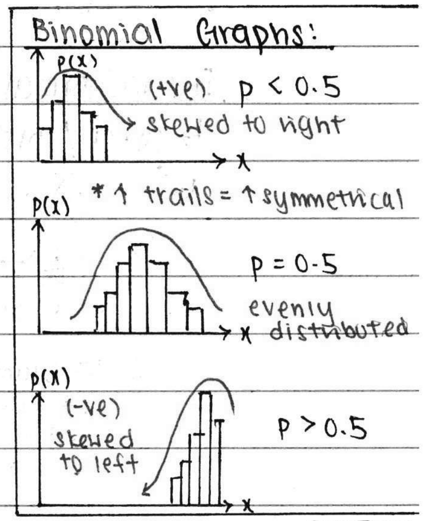
DRV:

i. sum of all probability = 1
ii. x -ve probability
iii. $0 \leq P(X=x) \leq 1$
iv. $E(X) = \mu = \sum x_i p_i$
v. $Var(X) = \sigma^2 = \sum [p_i (x_i - \mu)^2]$
vi. $SD(X) = |\sigma| = |\sqrt{Var(X)}|$

Change of origin & scale:
↳ $Y = aX + b$

i. $E(Y) = aE(X) + b$
ii. $Var(Y) = a^2 Var(X)$
iii. $SD(Y) = |a| \cdot SD(X)$

Binomial Graphs:



Bernoulli distribution:

	Failure	Success
x	0	1
$P(X=x)$	$1-p$	p

i. $E(X) = \mu = p$
ii. $Var(X) = p(1-p)$
iii. $SD(X) = \sqrt{p(1-p)}$

Binomial distribution: $P(X=x) = {}^nC_x \, p^x (1-p)^{n-x}$

i. $P(X=3)$ = BinomPDF
** Always write: $X \sim Bi(n, p)$
ii. $P(X \geq 3)$ = BinomCDF
iii. $E(X) = np$
iv. $Var(X) = np(1-p)$
v. $SD(X) = \sqrt{np(1-p)}$

CRV:

i. measure (eg. height, weight, distance)
ii. interval range ($a < x < b$)
** $P(X=x) = 0$
iii. > & ≥ same range / probability
iv. < & ≤ same range / probability
↳ $P(X \geq a) = P(X > a)$

Probability Density Function:

i. $f(x) \Rightarrow a < x < b$ ⟶ area of $f(x) = \int_a^b f(x)\,dx = P(\text{total}) = 1$
ii. area of $f(x)$ must not drop below x-axis (if not -ve)

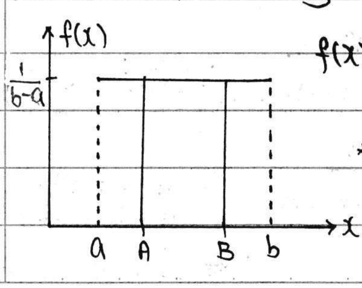
Piecewise form:

$$f(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Cumulative Distribution Function ($F(x)$):

i. $F(x) = \int_a^x f(x)\,dx$
ii. $P(a < x < b) = F(b) - F(a)$

Uniform probability distribution:

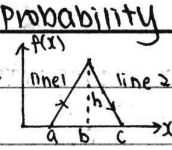


$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

* $P(A \leq X \leq B)$ = area under graph

Triangular Probability Distribution:

Symmetrical:

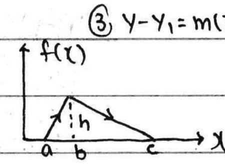


① Find height, h
② Find gradient
③ $y - y_1 = m(x - x_1)$

Non-symmetrical:

$f_1(x) = \frac{2(x-a)}{(c-a)(b-a)}$

$f_2(x) = \frac{2(c-x)}{(c-a)(c-b)}$



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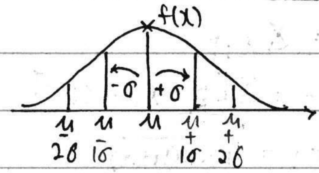
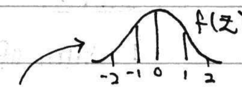
	$E(x)$	$Var(x)$
uniform distribution	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
symmetrical Δ	$\frac{a+b}{2}$	$\frac{(b-a)^2}{24}$
non-symmetrical Δ	$\int_a^b x f_1(x)\,dx + \int_b^c x f_2(x)\,dx$	$\int_a^b (x-\mu)^2 f_1(x)\,dx + \int_b^c (x-\mu)^2 f_2(x)\,dx$

Normal distribution: * $X \sim N(\mu, \sigma^2)$ μ = mean, σ^2 = varience, $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$



$\boxed{Z = \frac{x-\mu}{\sigma}}$ $\Leftarrow$ Standardisation



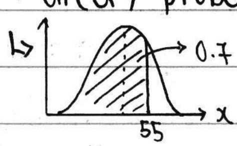
* Standard normal distribution:

$Z \sim N(0, 1^2)$, $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

* 68.3% = 1σ ; 95.4% = 2σ ; 99.7% = 3σ

Inverse normal $\Rightarrow$ Quantile:

↳ area / probability on left side only

↳ 

$\Rightarrow$ 55 is the 70^{th} percentile

↳ eg. $X \sim N(20, 3^2)$

(a) 0.5 quantile : $P(X < a) = 0.5$
a $\Rightarrow$ inversenormal
CAS: invNorm(0.5, 20, 3)
↳ $a = 20$

(b) the 24^{th} percentile
$P(X < a) = 0.24$
CAS: invNorm(0.24, 20, 3)
↳ $a = 17.9$

Random sampling:

- sample = part of population
- parameter = characteristics eg. weight
- statistic = mean, max, min, range
- survey = ↳ obtain same information / same criteria / questions
- census = data from whole population
- sample $\Rightarrow$ i) fair ii) biased iii) random
- bias $\Rightarrow$ selection, design flaw, interviewer, recall, completion, non-response

Random:
- simple random sampling
- stratified random sampling
- cluster sampling
- systematic sampling

Non-random:
- convenience
- purposive
- judgement
- quota

Variability of random sampling:

i) ↑ sample size = ↑ accurate / closely representation of population

ii) ↑ sample size = mean & SD & other statistic of each sample closer to population statistics

iii) sample mean ≠ population mean (when ↓ sample size)

sample proportions ($\hat{p}$):

i) $\hat{p} = \frac{x}{n}$ ii) $E(\hat{p}) = p$ iii) $Var(\hat{p}) = \frac{p(1-p)}{n}$ iv) $SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$

central limit theorem:

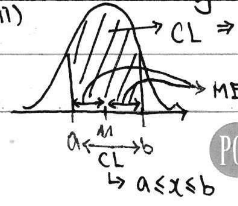
i) sample size, $n \geq 30$ ii) $np \geq 10$, $nq \geq 10$

sample proportion & standard normal distribution:

i) $Z = \frac{x-\mu}{\sigma} = (\hat{p} - p) \div \left(\sqrt{\frac{p(1-p)}{n}}\right)$ ii) must write : $X \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}^2\right)$ & normal CDF

Confidence level (CL) & Margin of Error (ME):

i) 90% → $Z = 1.645$
95% → $Z = 1.960$
98% → $Z = 2.326$
99% → $Z = 2.576$

ii) 

CL $\Rightarrow$ P(CI) = area under curve
↳ $P(a \leq x \leq b)$ = %

iii) need find :
n, x, CL, Z

** CL is usually in %

Probability

For any event A and its complement A'	$P(A') = 1 - P(A)$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	$P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$

Random variables and probability distributions	Mean	Variance
Bernoulli: mean is the sample proportion $\hat{p}$	$\mu = p$	$\sigma^2 = p(1-p)$
Binomial distribution: $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
Discrete random variable: $P(X = x) = P(x)$	$\mu = E(X) = \sum x p(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
Continuous random variable: $P(a \le X \le b) = \int_a^b p(x)dx$		
Expected value: $\mu = E(X) = \int_{-\infty}^{\infty} x\,p(x)dx$	Variance: $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\,dx$	

Expected value for uniform distribution: $E(X) = \frac{a+b}{2}$ variance of uniform distribution : $Var(X) = \frac{(b-a)^2}{12}$

Sample proportions	$\hat{p} = \dfrac{X}{n}$
Mean: $E(\hat{p}) = p$	Standard deviation: $\sigma = \sqrt{\dfrac{p(1-p)}{n}}$
Margin of error: $E = z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$	Confidence interval: $\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.

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