1)(a) Use the Euclidean algorithm to find the greatest common divisor of 504 and 385.

$504 = 1(385) + 119$

$385 = 3(119) + 28$

$119 = 4(28) + 7$

$28 = 4(7) + 0$ HENCE, GCD of 504 and 385 is 7 ✓

(b) Is it possible to find an integer y such that 504y is congruent to 10(mod 385)? If it is, find one. If it isn't, explain why not.

$504y \equiv 10 \pmod{385}$

$10/385 = 2/77 \approx 0.025974$

$0 \times 385 = 0$

$10 - 0 = 10$

$504y = 10$

$y = 10/504$

$y = 2/252 \approx 0.01984126984$

$2/252$ is not an integer so it's not possible. ✓

(c) Is it possible to find an integer z such that 504z is congruent to 7(mod 385)? If it is, find one. If it isn't, explain why not.

$504z \equiv 7 \pmod{385}$

$7/385 = 1/55 \approx 0.01818182$

$385 \times 0 = 0$

$7 - 0 = 7$

$504z = 7$

$z = 7/504$

$z = 1/72 \approx 0.0138889$

$1/72$ is not an integer so it's not possible. X WRONG

(d) Prove using induction that, for each integer n >= 1,

$5 + 5^2 + 5^3 + \ldots + 5^n = (5^{n+1} - 5)/4$

Base step: $n = 1$

LHS $\rightarrow$ $(5^{1+1} - 5)/4 = (5^2 - 5)/4$ RHS $\rightarrow$ $5^1 = 5$

$= 20/4$

$= 5$ LHS = RHS so P(1) is TRUE

Inductive step: prove $5^{n+1} = (5^{n+1+1} - 5)/4$

$5^{n+1} = (5^{n+2} - 5)/4$

$5^1 + 5^2 + 5^3 + \ldots + 5^{k+1} = \underline{5 + 5^2 + 5^3 + \ldots + 5^k} + 5^{k+1}$

$= (5^{k+1} - 5)/4 + (5^{k+1})$

$= [(5^{k+1} - 5) + 4(5^{k+1})]/4$

$= [5(5^{k+1}) - 5]/4$

$= [(5^{k+1+1}) - 5]/4$

$= [5^{k+2} - 5]/4$ $\therefore$ Hence, P(k+1) is TRUE

Conclusion:

Since P(1) is TRUE

$P(1) \rightarrow P(2)$ is TRUE

$\vdots$

P(n) is true for all $n \geq 1$

2)(a) $\neg((p \to \neg q) \land r)$ and $(p \land q) \lor \neg r$ logically equivalent ?

$\neg((p \to \neg q) \land r)$
$= \neg(((\neg p) \land \neg q) \land r)$
$= \neg((\neg p \land \neg q) \land r)$
$= \neg(\neg(p \land q) \land r)$
$= (p \land q) \lor \neg r$

equals to $(p \land q) \lor \neg r$

Hence, they are logically equivalent

(b) $p \Rightarrow x \in A$ ; $q \Rightarrow x \in B$ , $x \in A \cup B$ in terms of p and q ?

$A \cup B = p \land q$
$x \in A \cup B = p \land q$ X $p \lor q$

(c) $(\exists x(P(x) \land Q(x)) \leftrightarrow ((\exists x P(x)) \land (\exists x Q(x))$ valid ?

OR Valid, for some P(x) and Q(x) it also means some P(x) and some Q(x)

if $p = (\exists x(P(x) \land Q(x))$
if $q = ((\exists x P(x)) \land (\exists x Q(x))$
then $(\exists x(P(x) \land Q(x)) \leftrightarrow ((\exists x P(x)) \land (\exists x Q(x)) =$

$((\exists x(P(x) \land Q(x)) \to ((\exists x P(x)) \land (\exists x Q(x))) \land (((\exists x P(x)) \land (\exists x Q(x)) \to (\exists x(P(x) \land Q(x)))$
$= \neg((\exists x(P(x) \land Q(x))) \lor (((\exists x P(x)) \land (\exists x Q(x))) \land (\neg(((\exists x P(x)) \land (\exists x Q(x))) \to (\exists x(P(x) \land Q(x)))$

⋮

.... idk what i doing dy WRONG Q7 in prac exam

(d)(i) $\exists x \forall y P(x,y)$

is it true that x,y range over +ve integers & P(x,y) is "$x \leq y$" ?

False ; some x is smaller than all y if x,y are +ve integers

X

(ii) $\forall y \exists x P(x,y)$

is it true that x,y range over +ve integers & P(x,y) is "$x \leq y$" ?

TRUE ; For all y, there's some x that's smaller than y for all +ve integer

X

3)(a) P = prime number set. T = natural number divisible by 3 set ; $A = \{3,4,5,6\}$

↳ eg. $P = \{2,3,5,7,11,13,17\}$ ↳ eg. $T = \{3,6,9,12,15,18,21 \ldots\}$

(i) $P \cap T = \{3\}$ ✓

(ii) $T \cap A = \{3,6\}$ ✓

(iii) $T \cup \mathbb{N}$ = natural number set $= \{1,2,3,4,5,6,7 \ldots\}$ ✓ $\mathbb{N}$

(iv) $P(A \times \{1,2\})$

$= P(\{3,4,5,6\} \times \{1,2\})$ $\quad A \times \{1,2\} = 4 \times 2 = 8$

$= P(2^4)$

$= P(16)$

$= 16^2$

$= 32$ ✗ $\quad 2^8$

(b) A = set of all non-empty subsets of $\{1,2,\ldots,10\}$

$f : A \to \mathbb{Z}$ defined by $f(x) = a - b$

↳ a = largest element of x, b = smallest element of x

$g : A \to A$ defined by $g(x) = x \cup \{1,2\}$

(i) $f(\{2,3,6\})$ & $g(\{2,7,10\})$

$f(\{2,3,6\})$ $\qquad g(\{2,7,10\})$

$= 6 - 2$ $\qquad = \{1,2,7,10\}$ ✓

$= 4$ ✓

(ii) f is ~~not~~ one-to-one, g is ~~one-to-many~~ → means not 1-to-1

(iii) How to find range of a function?

$\mathbb{Z} \times \mathbb{Z} - \{0,0\}$

(iv) $f \circ g = f(g(x))$ does exist

$f(g(\{9\})) = f(\{1,2,9\})$

$= 9 - 1$

$= 8$ ✓

(v) $g \circ f = g(f(x))$ doesn't exist as when $g(f(\{9\}))$, biggest element is 9, smallest element is also 9 if 9-9 then $g(\{0\})$ but not in range so it doesn't exist

4)(a) $A = \{a, b, c, d, e, f, g, h, i\}$

(i) ✓ reflective $\longrightarrow$ aRa … iRi ✓

✗ symmetric $\longrightarrow$ dRg but g$\not R$d ✓

✗ antisymmetric $\longrightarrow$ aRe, eRa ✓

✗ transive $\longrightarrow$ bRc $\wedge$ cRf but b$\not R$f ✓

(ii) ✗ partial, ✗ equivalence

(iii) $S = \mathbb{Z} \times \mathbb{Z}$ by $(u,x)\,S\,(y,z)$ if and only if $u+x-y-z$ is even.

| | | |
|---|---|---|
| $(1+2)-3-4$ | $(3+5)-7-9$ | $(2+4)-6-10$ |
| $=3-3-4$ | $=8-7-9$ | $=6-6-10$ |
| $=-4 \rightarrow$ is even | $=-8 \rightarrow$ is even | $=-10 \rightarrow$ is even |
| $(3+4)-1-2$ | $(7+9)-3-5$ | $(10+6)-2-4$ |
| $=7-3$ | $=16-8$ | $=16-6$ |
| $=-4 \rightarrow$ is even | $=8 \rightarrow$ is even | $=10 \rightarrow$ is even |

Symmetric:

✓ reflexive, ✓ ✓ symmetric, ✓ ✗ antisymmetric ✓, ✗ transitive ✗

(iv) ✗ partial ✓ ✗ equivalence ✗ ✓RST $\rightarrow$ equivalence $\rightarrow$ for all $(u,v), (w,x), (y,z) \in \mathbb{Z} \times \mathbb{Z}$

(v) ✗ R and S are neither partial nor equivalence

$\rightarrow$ for all $(w,x) \in \mathbb{Z} \times \mathbb{Z}$, $w+x-w-x = 0$ and $0$ is even so $(w,x)\,S\,(w,x)$

$\rightarrow$ equivalence classes:

$\{(x,x) : (w,x) \in \mathbb{Z} \times \mathbb{Z}$ and $w+x$ is even$\}$

$\{(x,x) : (w,x) \in \mathbb{Z} \times \mathbb{Z}$ and $w+x$ is odd$\}$

(b) (i) $(999)^6 + 7 \times 81$ divisible by 9?

$(999)^6 + 567 = (999)^6/9 + 63$

$= (111)^6 + 63$

$$\begin{array}{r} 63 \\ 9\,\overline{)\,567} \\ \underline{54} \\ 27 \\ \underline{27} \\ - \end{array}$$

Hence, $(999)^6 + 7 \times 81$ is divisible by 9

(ii) $x, y$ = integers

$x \equiv 3 \pmod{12}$, $y \equiv 7 \pmod{18}$ then $x+y \equiv 4 \pmod 6$

from $x \equiv 3 \pmod{12}$, $y \equiv 7 \pmod{18}$,

$x+y = 3 \pmod{12} + 7 \pmod{18}$

$= 10$

| | | |
|---|---|---|
| | | $10 \equiv 4 \pmod 6$ |
| $3/12 = 1/4$ | $7/18 = 0$ R $7$ | $4 \pmod 6 = 4$ |
| $1/4 \times 12 = 3$ | $0 \times 18 + 7 = 7$ | $10 \equiv 4$ # |
| $3 \pmod{12} = 3$ | $7 \bmod 18 = 7$ | |

OR

$3/12 = 0$ R3 $\rightarrow$ remainder

$0 \times 12 + 3 = 3$

$\rightarrow$ (quotient) $\times$ (divisor) + (remainder)

= dividend

* numbers are congruent when they leave the same remainder when divided by a 3rd number