MAT1841 Revision Discussion Questions

1. Given $\mathbf{u} = (3, 4, -1)$ and $\mathbf{v} = (1, -2, 1)$

   (a) Calculate $\mathbf{u} \cdot \mathbf{v}$.

   1 mark

   (b) Calculate $\mathbf{u} \times \mathbf{v}$ and hence find $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$

   2 mark

   (c) Calculate the vector projection of $\mathbf{u}$ onto $\mathbf{v}$, i.e. $\mathbf{u}_v$.

   2 marks

   (d) Calculate the vector form of the line through the points $(4, 0, 2)$ and $(-2, 1, 1)$.

   1 marks

(e) Consider the lines

$$r(t) = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

and

$$r(t) = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

Determine if they intersect. If yes, find the point of intersection.

2 marks

(f) Calculate the equation of the plane passing through the three points (0, 1, 1), (1, 0, 1) and (1, 1, 0), and state a normal vector to the plane.

3 marks

2. Given the following matrices

$$A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}, \ B = \begin{bmatrix} 3 & -1 \\ 0 & 2 \\ 0 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} -2 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}:$$

(a) Calculate $B^T + C$.

2 marks

(b) Calculate BB^T.

2 marks

(c) Calculate $(CB)^{-1}$

3 marks

(d) Given

$$A = \begin{bmatrix} -2 & -1 & -1 & 2 \\ 0 & 3 & 2 & -1 \\ 1 & -1 & -1 & 1 \\ -1 & 0 & -1 & 0 \end{bmatrix}$$

Calculate the determinant of A.

3 marks

3. Given the linear system

$$3x - y + 2z = 3$$
$$x + y - z = 2$$
$$2x - 2y + 3z = b$$

(a) Write the Coefficient matrix and the augmented matrix. .

2 marks

(b) Reduce the augmented matrix to echelon form using Gaussian elimination. State the rank of the coefficient matrix.

2 marks

(c) For what values if *b* will the system have
   (i) Unique solution
   (ii) Infinite solution
   (iii) No solution

3 marks

(d) Hence, solve the linear system for **x** by using result in part (c), or otherwise.

3 marks

4. Calculate the following.

(a) Using first principles, calculate the derivatives of $f(x) = \sqrt{x}$.

2 marks

(b) Calculate the derivative of $g(x) = \frac{e^{2x-1} - \ln(x^2)}{\cos(x)}$.

2 marks

(c) Consider the parametric curve defined as $x = t^2 - 2t$ and $y = t^3 + 3t + 2$. Derive an equation for the tangent line to this curve when $t = 3$.

2 marks

(d) Calculate the absolute minimum and maximum for the function $f(x) = \frac{x^2-1}{e^x}$ over the interval $[-2 \leq x \leq 2]$.

4 marks

5. (a) Calculate the first **four** non-zero terms in the Taylor series expansion of $f(x) = e^{-x} sin\,(x)$ about $x = \frac{\pi}{2}$.

6 marks

(b) The following questions relate to Taylor series. **A table of useful power series is provided in the formulae section of this paper.**

Compute the Taylor series expansions, around $x = 0$, for $\ln\,(1 + x)$ and $\ln\,(1 - x)$. Hence obtain a Taylor series for $f(x) = \ln\left(\frac{1+x}{1-x}\right)$, express in the summation form such as $\sum_{n=0}^{\infty} 2x^{n-1}$.

4 marks

6. Calculate the following indefinite integrals.

(a) $I = \int \cos(x)\, e^{3\sin(x)}\, dx$

3 marks

(b) $I = \int x\sqrt{2x-1}\, dx$

3 marks

(c) Use Integration by Parts twice to calculate $I = cos\ (x)\ e^{x} dx$.

4 marks

(d) Use a substitution and then integration by parts to solve

$$I = \int \frac{\ln(\sqrt{x})}{\sqrt{x}}\ dx$$

4 marks

7. Given the two functions $f(x) = x^2 - x$ and $g(x) = \frac{1}{2}x^2 + \frac{3}{2}$.

(a) Use the Fundamental Theorem of Calculus to calculate the area of the bounded region between the curves. Note that the curves intersect at the points (-1, 2) and (3, 6).

5 marks

(b) Approximate the area between the curves using the Trapezoidal rule with $n = 4$. Leave as a sum of terms on a common denominator.

5 marks

8. Find all first and second partial derivatives of the function $f\ (x, y\&) = x^2 \sin(x + y)$.

10 marks

9. Find the equations of the approximations to the surface $f(x, y) = x^2y + xy^2 + y^3$ at the point $(x, y) = (-2, 1)$.

(a) Find the tangent plane, $T_1(x, y)$.

4 marks

(b) Find the second order Taylor expansion, $T_2(x, y)$. You do not need to simplify.

6 marks

10. Calculate the following:

(a) Compute $\frac{df}{ds}$ for $f(x,y) = x^2y^2 + 2xy + y$, where $x = e^s$ and $y = e^{\frac{s}{2}}$.

4 marks

(b) Find the directional derivative of the function $f(x,y) = x^2y^2 + 2xy + y$ in the direction of the vector $\boldsymbol{t} = 3\boldsymbol{i} + 4\boldsymbol{j}$ at the point $(x, y) = (-2, 1)$.

6 marks

(c) A container with an open top is to have 10 m^3 capacity and be made of thin sheet metal. Calculate the dimensions of the box if it is to use the minimum possible amount of metal so that the cost is least, where the cost function is given as $M(x, y, z) = 2xy + 3xz + 3yz$.

10 marks