Monash University
Faculty of Information Technology
2^{nd} Semester 2023

FIT2014
Exercises 4
Kleene's Theorem II and FA State Minimisation

ASSESSED PREPARATION: Question 4.
You must provide a serious attempt at this entire question at the start of your tutorial.

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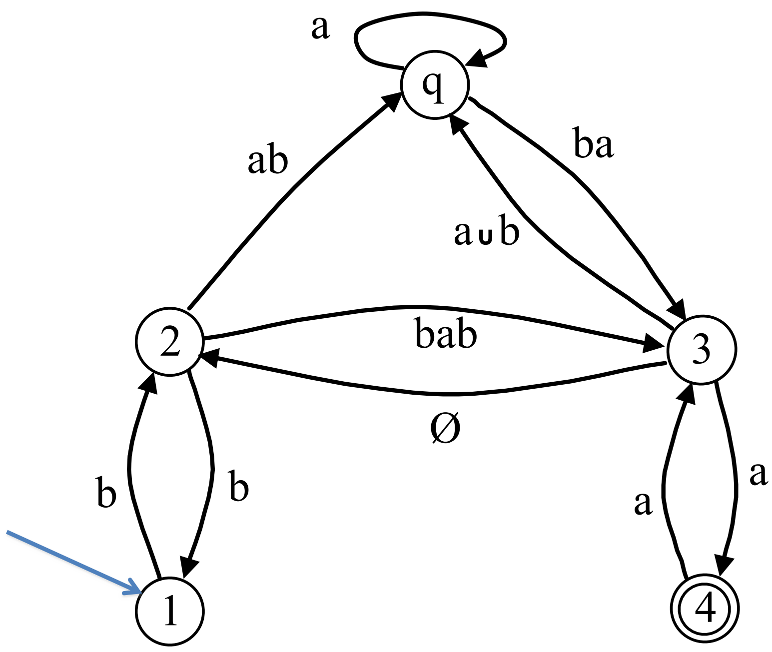
1. *Based on a question from FIT2014 Final Exam, 2nd semester 2015:*

Consider the following Generalised Nondeterministic Finite Automaton (GNFA). Construct an equivalent GNFA with the top state, q, removed.



2. [1]

You are in a maze represented by a rectangular grid. One cell is the start cell, another is the cell you want to get to. Some pairs of adjacent cells have a wall between them, preventing you from moving directly from one to the other. There is at least one path from the start to the end. Suppose that the characters U, D, L, R represent moving up, down, left and right by one grid cell. A string of these characters represents a sequence of movements through the maze, but is only valid if it does not make you bump into a wall. It's ok to visit cells more than once.

[1]Thanks to FIT2014 tutor Nathan Companez for this question.

Describe an algorithm for converting a given maze into a regular expression over the alphabet {U,D,L,R} which matches exactly those strings which correspond to sequences of moves which solve the maze.

Your algorithm may call any algorithm presented in lectures; if you do this, you do not have to list the steps in the algorithm from lectures that you're calling.

3. Consider the five-state Finite Automaton represented by the following table.

	state	a	b
Start	1	2	3
	2	1	5
	3	1	4
Final	4	2	5
Final	5	3	5

Convert this into an equivalent FA with the minimum possible number of states.

4. Consider the six-state Finite Automaton represented by the following table.

	state	a	b
Start	1	2	4
	2	2	6
	3	3	4
Final	4	5	3
Final	5	5	5
Final	6	5	1

Convert this into an equivalent FA with the minimum possible number of states.

5. For each Finite Automaton you found in last week's Question 7 (in Exercise Sheet 3), find the corresponding minimum state Finite Automaton.

6. For each Finite Automaton you found in the previous question, find the corresponding regular expression (using the GNFA approach).

Supplementary exercises

7. Construct a minimum state Finite Automaton that accepts only those strings defined by the regular expression: -?(N| N.| N.N| .N), where $N = [0\text{–}9]^+$.

8. We can prove that two regular expressions are equivalent by showing that their minimum state Finite Automaton are the same, except for state names. Using this technique, show that the following regular expressions are equivalent:

i. $(a \cup b)^*$

ii. $(a^* \cup b^*)^*$

iii. $((\epsilon \cup a)b^*)^*$