MONASH University

2020 Semester Two (November-December 2020) Examination Period

Faculty of Science

EXAM CODES:	MAT1841
TITLE OF PAPER:	Continuous Math. for Comp. Sci.
EXAM DURATION:	3 hours 10 mins

Rules

During an exam, you must not have in your possession any item/material that has not been authorised for your exam. This includes books, notes, paper, electronic device/s, mobile phone, smart watch/device, calculator, pencil case, or writing on any part of your body. Any authorised items are listed above. Items/materials on your desk, chair, in your clothing or otherwise on your person will be deemed to be in your possession.

You must not retain, copy, memorise or note down any exam content for personal use or to share with any other person by any means following your exam.

You must comply with any instructions given to you by an exam supervisor.

As a student, and under Monash University's Student Academic Integrity procedure, you must undertake your in-semester tasks, and end-of-semester tasks, including exams, with honesty and integrity. In exams, you must not allow anyone else to do work for you and you must not do any work for others. You must not contact, or attempt to contact, another person in an attempt to gain unfair advantage during your exam session. Assessors may take reasonable steps to check that your work displays the expected standards of academic integrity.

Failure to comply with the above instructions, or attempting to cheat or cheating in an exam may constitute a breach of instructions under regulation 23 of the Monash University (Academic Board) Regulations or may constitute an act of academic misconduct under Part 7 of the Monash University (Council) Regulations.

<u>Authorised Materials</u>

CALCULATORS	☑ YES	☐ NO	Calculator
DICTIONARIES	☐ YES	☑ NO	
NOTES	☐ YES	☑ NO	
PERMITTED ITEM	☐ YES	☑ NO	

if yes, items permitted are:

Instructions

ALL answers in this exam that are not multiple choice are integers or fractions.

INTEGERS: You must enter integers in their simplest form.

- For example, enter the number twenty-three as 23.
- Extra spaces before, in the middle or following the integer, or inputs such as 22.999... or 23.000 or 24-1, etc. will ALL result in an answer to be counted as incorrect.

FRACTIONS: Similarly, you must enter non-integer fractions in lowest terms and in their simplest form possible.

- So, enter three halves as 3/2.
- Extra spaces before, in the middle or following a fraction, or input as 6/4, 1.5, etc. will ALL result in an answer to counted as incorrect.

NEGATIVE SIGN: If you want to enter a negative integer or fraction you must enter the negative sign immediately before the number, e.g. -23 or -3/2. Again, extra spaces in between the minus sign and the number or anywhere else will result in an answer to be counted as incorrect.

ATTEMPT ALL QUESTIONS: There are no negative marks for incorrect answers.

Instructions

Information

ALL answers in this exam that are not multiple choice areintegers or fractions.

INTEGERS: You must enter integers in their simplest form.

- For example, enter the number twenty-three as 23.
- Extra spaces before, in the middle or following the integer, or inputs such as 22.999... or 23.000 or 24-1, etc. will ALL result in an answer to be counted as incorrect.

FRACTIONS: Similarly, you must enter non-integer fractions in lowest terms and in their simplest form possible.

- So, enter three halves as 3/2.
- Extra spaces before, in the middle or following a fraction, or input as 6/4, 1.5, etc. will ALL result in an answer to counted as incorrect.

NEGATIVE SIGN: If you want to enter a negative integer or fraction you must enter the negative sign immediately before the number, e.g. -23 or -3/2. Again, extra spaces in between the minus sign and the number or anywhere else will result in an answer to be counted as incorrect.

ATTEMPT ALL QUESTIONS: There are no negative marks for incorrect answers.

Formula Sheets

Information

Please Note: In some questions, the first line has been added to serve as a header to appear on the Summary page, and should be ignored if it's an incomplete sentence.

Information

Formulae:

Scalar and vector projections:
The **scalar projection**, v_w, of $\mathbf{v}$ in the direction of $\mathbf{w}$ is given by

$$v_w = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|}$$

The **vector projection**, $\mathbf{v}_w$, of $\mathbf{v}$ in the direction of $\mathbf{w}$ is given by

$$\mathbf{v}_w = \left(\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|^2}\right)\mathbf{w}$$

Vector cross product:

The vector cross product of vectors $\mathbf{v} = (v_x, v_y, v_z)$ and $\mathbf{w} = (w_x, w_y, w_z)$ is

$$\mathbf{v} \times \mathbf{w} = (v_y w_z - v_z w_y, v_z w_x - v_x w_z, v_x w_y - v_y w_x)$$

Vector equation of a plane:

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{d}) = 0$$

Matrix inverse (2×2):

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

for $ad - bc \neq 0$.

Schematic of Gauss-Jordan algorithm:

$$[A|I] \xrightarrow[G.A]{} [U|*] \xrightarrow[J.A]{} [I|B] \quad \text{where } B = A^{-1}.$$

Derivative definition:
The **derivative** of $f(x)$ at the point x is defined as

$$\frac{\mathrm{d}f}{\mathrm{d}x} = f'(x) = \lim_{\Delta x \to 0}\left(\frac{\Delta f}{\Delta x}\right) = \lim_{\Delta x \to 0}\left(\frac{f(x + \Delta x) - f(x)}{\Delta x}\right).$$

Information

Some rules for finding derivatives:

Description	Function	Derivative
Sum (or difference) of functions	$f(x) \pm g(x)$	$f'(x) \pm g'(x)$
Product of functions	$f(x)g(x)$	$f(x)g'(x) + g(x)f'(x)$
Quotient of functions	$\dfrac{f(x)}{g(x)}$	$\dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

Chain rule for composite functions

If $u = g(x)$ and $y = f(u)$ so that $y = f\,(g(x))$ then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} = f'(u)g'(x)$$

Derivative rule for inverse functions

$$\text{If } y = f^{-1}(x) \Leftrightarrow x = f(y), \text{ then } \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\mathrm{d}x/\mathrm{d}y} = \frac{1}{f'(y)}$$

Parametric differentiation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\Big/\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{g'(t)}{f'(t)} \quad \text{where } f'(t) = \frac{\mathrm{d}f}{\mathrm{d}t} \text{ and } g'(t) = \frac{\mathrm{d}g}{\mathrm{d}t}.$$

Taylor series at $x = 0$:

$$T_n(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n.$$

Integration by substitution:

$$I = \int f(x)dx = \int f(x(u))\frac{dx}{du}du$$

Integration by parts.

$$\int f\frac{dg}{dx}dx = fg - \int g\frac{df}{dx}dx$$

Fundamental Theorem of Calculus:

If $f(x)$ is a continuous function on the interval $[a, b]$ and there is a function $F(x)$ such that $F'(x) = f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Area between two curves. Given two continuous functions $f(x)$ and $g(x)$ where $f(x) \geq g(x)$ for all x in the interval $[a, b]$, the area of the region bounded by the curves $y = f(x)$ and $y = g(x)$, and the lines $x = a$ and $x = b$ is given by the definite integral

$$\int_a^b [f(x) - g(x)]dx$$

Trapezoidal rule:

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left(f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_i)\right).$$

Tangent plane to surface:

$$z = f(a,b) + f_x(a,b)\cdot(x-a) + f_y(a,b)\cdot(y-b)$$

Multivariate chain-rule:

$$\frac{df}{ds} = \frac{\partial f}{\partial x}\frac{dx}{ds} + \frac{\partial f}{\partial y}\frac{dy}{ds}$$

Directional derivative:
The *directional derivative* df/ds of a function f in the direction $\underset{\sim}{t}$ is given by

$$\frac{df}{ds} = \underset{\sim}{t}\cdot\nabla f = \nabla_{\underset{\sim}{t}} f$$

where the *gradient* ∇f is defined by

$$\nabla f = \frac{\partial f}{\partial x}\underset{\sim}{i} + \frac{\partial f}{\partial y}\underset{\sim}{j}$$

and $\underset{\sim}{t}$ is a unit vector, $\underset{\sim}{t}\cdot\underset{\sim}{t} = 1$.

Quadratic approximation to surface:

$$T_2(x,y) = f(a,b) + f_x(a,b)\cdot(x-a) + f_y(a,b)\cdot(y-b) + \frac{1}{2!}\left[f_{xx}(a,b)(x-a)^2 + 2f_{xy}(a,b)(x-a)(y-b) + f_{yy}(a,b)(y-b)^2\right]$$

Information

Table of the derivatives of the basic functions of calculus	
Original function f	Derivative function f'
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x \equiv 1 + \tan^2 x$
$\operatorname{cosec} x \equiv 1/\sin x$	$-\operatorname{cosec} x \cdot \cot x$
$\sec x \equiv 1/\cos x$	$\sec x \cdot \tan x$
$\cot x \equiv 1/\tan x$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x$ domain: $-1 \leq x \leq 1$ (ie $\|x\| \leq 1$)	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$ domain: $-1 \leq x \leq 1$ (ie $\|x\| \leq 1$)	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$ domain: $-\infty < x < \infty$	$\frac{1}{1+x^2}$
e^x	e^x
$\ln x$ domain: $x > 0$	$\frac{1}{x}$

Table of Useful Power Series

Series	Domain
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$	$-1 < x < 1$
$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots + (-1)^n x^n + \ldots$	$-1 < x < 1$
$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots + \frac{1}{n!}x^n + \ldots$	$-\infty < x < \infty$
$\ln(1+x) \equiv \log_e(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$ $+(-1)^n \frac{x^{n+1}}{n+1} + \ldots$	$-1 < x \leq 1$
$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots$	$-\infty < x < \infty$
$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots$	$-\infty < x < \infty$

Vectors, Lines and Planes

Question 1

What is the first component of the cross product of the vectors (5, -2, 0) and (2, 1, 2)?

[Note: the first component of the vector (5, -2, 0) is 5.]

1 Mark

Question 3

4 Marks

The plane through the points (1, 5, 2), (5, 0, -4) and (-3, -2, 0) can be written in the form $ax + by + cz = d$,

where a = , b = , c = , d = .

[This equation must be reduced to the lowest possible integer values.]

Question 5

2 Marks

The line (-5, -8, 10) + t(2, 2, -3) intersects the plane $3x - y - 4z = 1$ when the parametric variable $t =$.

The x coordinate (i.e. the first component) of this point of intersection is $x =$.

Question 7

What is the minimum distance between the following two lines?

$(x(t), y(t), z(t)) = (-1, 4, 1) + t(1, 1, 0)$ and
$(x(s), y(s), z(s)) = (2, -1, -1) + s(0, 1, -2)$?

4
Marks

Select one:

- a.
  6
- b.
  1
- c.
  3
- d.
  2√8

Question 9

What is first entry (1st row, 1st column) of the matrix $(BA)^{-1}$ for the following matrices?

3
Marks

$$A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & -1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$

Select one:

- a. -1/13
- b. 1/13
- c. -6/13
- d. -7/13

Question 11

Consider a linear system of m equations and n variables such that the coefficients form an $m \times n$ matrix, A. Let A have rank r. Which of the following statements is FALSE?

1 Mark

Select one:

- a. r must be less than or equal to n.
- b. The number of free variables is equal to $n - r$.
- c. m must be less than or equal to r.
- d. The number of pivot variables is equal to r.

Question 13

Consider the system of 4 equations and 3 unknowns:

6 Marks

$x - 2y + 2z = -4$

$2x + y - 3z = 7$

$-x + y + z = 1$

$2x + 2y - z = c$

The rank of the system is .

The system will have one unique solution if the constant c is .

For this value of c, the resulting solution has a value of for z.

Question 15

Which of the following is an expression for the tangent line to the curve defined by the parameterisation

3 Marks

$x(t) = t^3 - 4t^2$ and

$y(t) = 2t^3 - 4$

at the value $t = 1$?

Select one:

- a. $y = -3x/5 - 19/5$
- b. $y = -3x/5 + 1/5$
- c. $y = -6x/5 - 28/5$
- d. $y = -3x/5 + 2$

Question 17

The power series $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$ is the Maclaurin series of which function?

2
Marks

Select one:

- a. $\sin(-x)$
- b. $\cos(x)$
- c. e^{-x}
- d. $\ln(x)$
- e. $\arctan(-x)$

Question 19

3
Marks

Given the data set

x_1	x_2	x_3	x_4
y_1	y_2	y_3	y_4

and the polynomials $p_1(x), p_2(x), p_3(x)$ comprising the natural cubic spline interpolating the data set. Which of the following is/are true?

I) $p_i(x_i) = y_{i-1}$ for all $i = 1, 2, 3.$

II) $p_i(x_i) = y_i$ for all $i = 1, 2, 3.$

III) $p_{i-1}(x_i) = p_i(x_{i-1})$ for all $i = 2, 3.$

IV) $p_{i-1}(x_i) = p_i(x_i)$ for all $i = 2, 3.$

V) $p'_{i-1}(x_i) = p'_i(x_{i-1})$ for all $i = 2, 3.$

VI) $p'_{i-1}(x_i) = p'_i(x_i)$ for all $i = 2, 3.$

VII) The fourth derivative of $p_i(x)$ is positive for all $i.$

VIII) The fourth derivative of $p_i(x)$ is negative for all $i.$

Select one:

- a. I), III), and V).
- b. I), V), and VII).
- c. II), IV), and VI).
- d. III), V), and VIII).
- e. All of the above
- f. None of the above

Integration

Question 21

Which one of the following is a valid expression of the integral

$\int x\sqrt{x+3}\ dx$?

2
Marks

Select one:

- a. $\frac{2}{5}\left(x+3\right)^{\frac{5}{2}}-2\left(x+3\right)^{\frac{3}{2}}+c$
- b. $\frac{2}{3}\left(x+3\right)^{\frac{3}{2}}-2\left(x+3\right)^{\frac{5}{2}}+c$
- c. $\frac{2}{5}\left(x+3\right)^{\frac{1}{2}}-2\left(x+3\right)^{\frac{5}{2}}+c$
- d. $\frac{2}{5}\left(x+3\right)^{\frac{3}{2}}-2\left(x+3\right)^{\frac{1}{2}}+c$

Question 23

Given the functions $f(x) = 3 - x^2$ and $g(x) = 1 - x$, use the Fundamental Theorem of Calculus to calculate the area bounded by the two curves over the interval $0 \le x \le 2$. Please write the answer as either an integer or a simple, reduced fraction.

4
Marks

Multivariable Calculus

Question 25

The equation of the tangent plane to the ellipsoid $f(x, y, z) = x^2/9 + y^2/25 + z^2 = 1$ at the point (0, 4, 3/5) is:

4 Marks

Select one:

- a. $4y + 15z = 25$
- b. $4x + 4y + 15z = 25$
- c. $4x - 10z = 25$
- d. $4y - 15z = -2$

Question 27

Working in R^3, what is the shape of the surface defined by the equation $\frac{z^2}{25}+\frac{y^2}{4}=(x-1)^2$?

2
Marks

Select one:

- a.
  a paraboloid
- b.
  a hyperboloid of one sheet
- c.
  an elliptic cone
- d.
  an ellipsoid

Question 29

A rectangular box of dimensions *x*, *y* and *z* is required to have a fixed volume of 20 cm^3. The volume of the box is defined as *V*(*x*, *y*, *z*) = *xyz*. Let the cost of constructing the box be defined as *C*(*x*, *y*, *z*) = 6*xz* + 6*yz* + 4*xy*.

2
Marks

The *constrained* cost of constructing the box may be represented as which one of the following?

Select one:

- a.

$$C(x,y,z)=\frac{120}{y}+\frac{120}{x}+4xy$$

- b.

$$C(x,y,z)=120x^2z^2y+120y^2z^2x+80x^2yz$$

- c.

$$C(x,y,z)=\frac{6}{20y}+\frac{6}{20x}+\frac{1}{5z}$$

- d.

$$C(x,y,z)=\frac{6}{x}+\frac{6}{y}+\frac{4}{yz}$$