Bernoulli Distributions

- Back to Discrete Probability Distributions
- Exercise for Bernoulli Distributions

eg; flip a coin once — 0.5 H (win) → success; 0.5 T (lose) → failure

DRV → general discrete prob [P(X=x)]
↳ cumulative prob distri [P(X≤x)]
↳ uniform prob distribution [P(X=x) = 1/n] ↳ no of outcome
↳ mean/variance/SD → general (a) & uniform (b)
↳ change in scale & origin
↳ Bernoulli; Binomial

* Bernoulli → 1 trial, 2 mutually exclusive outcomes
success (x=1) probability ↳ P(X=1) = p (success)
failure (x=0) ↳ P(X=0) = 1-p (failure)

A trial which can be considered to have just two mutually exclusive and exhaustive outcomes, referred to as success and failure, is called a Bernoulli trial.

The associated random variable, with its two possible values, 0 and 1, is called a Bernoulli random variable.

The Bernoulli random variable has parameter p, the probability of obtaining a 1.
If the probability of success is p, then the probability of failure will be (1-p).

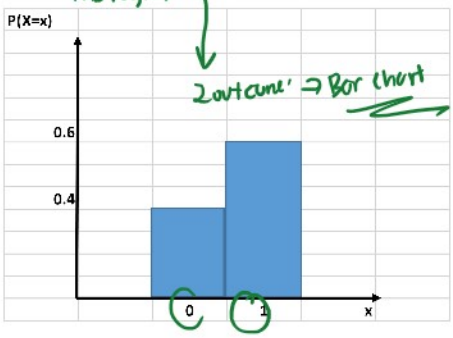
	Failure	Success
x	0	1
$P(X=x)$	1-p	p

} discrete prob distribution Table for Bernoulli

Example 1

	Failure	Success
x	0	1
$P(X=x)$	0.4	0.6

Histogram → 2 outcome → Bar chart



Example 2

X - number on the uppermost of the die

If a six-sided die is rolled then, a uniform discrete random variable with probability distribution:

x	1 ✓	2 ✓	3 ✓	4 ✓	5 ✓	6 ✓
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

uniform

But if the concern is only on "getting a six", it is success and "not getting a six" is failure, then it is Bernoulli random variable with probability distribution:

Bernoulli

	Failure	Success
x	0	1

X - getting a six

Bernoulli random variable with probability distribution:

Bernoulli

	Failure	Success
x	0	1
$P(X=x)$	$\frac{5}{6}$	$\frac{1}{6}$

X - getting a six

P(getting a six) = 1/6 (success)

P(not getting a six) = 5/6 (failure)

Example 3

Flipping a coin with obtaining a head being considered a success

	Failure	Success
x	0	1
$P(X=x)$	0.5	0.5
	1-p	p

0.5 H (success) → P(X=1) = 0.5

0.5 T (failure) → P(X=0) = 0.5

Answers: 0.5,0.5

Example 4

Guessing the answer to a multiple choice in which there are four answers to choose from, only one of which is correct.

	Failure	Success
x	0	1
$P(X=x)$	3/4	1/4
	(1-p)	p

10 → 1/4 correct → P(X=1) = 1/4 (success)

3/4 incorrect → P(X=0) = 3/4 (failure)

A

B

C

D

Answers: 3/4,1/4

$E(X) = \sum x_i p_i$ $\quad SD(X) = |\sigma| = |\sqrt{Var(X)}|$

$Var(X) = \sum (x_i - \mu)^2 . p_i$

Mean and variance for Bernoulli distribution with parameter p

	Failure	Success
x	0	1
$P(X=x)$	1-p	p

Expected value = $E(X)$ = Mean = $\mu = \sum x_i p_i$

= 0 x (1-p) + 1 x p

= p

} no need to remember derivative

Variance = $\sigma^2 = \sum [p_i (x_i - \mu)^2]$

= (1-p) x (0 − p)² + p x (1 − p)²

= p (1 − p) (p + 1 − p)

= p (1 − p)

given in formula sheet

① Bernoulli dist

a) 1 trial

b) 2 outcomes (mutually exclusive)

c) success (X = 1) → P(X = 1) = p

d) failure (X = 0) → P(X = 0) = (1-p) / q

e) E(X) = p

f) Var(X) = p(1-p)

g) SD(X) = $|\sqrt{Var(X)}| = |\sqrt{p(1-p)}|$

Example 1

Flipping a coin with obtaining a head being considered a success

	Failure	Success
x	0	1
$P(X=x)$	0.5	0.5
	(1-p)	p

H

T

Find the mean and standard deviation for the event above.

Answers: 0.5, 0.5

E(X) = p = 0.5

SD(X) = $|\sqrt{p(1-p)}| = |\sqrt{0.25}| = 0.5$

Example 2

Guessing the answer to a multiple choice in which there are four answers to choose from, only one of which is correct.

	Failure	Success
x	0	1
$P(X=x)$	3/4	1/4
	(1-p)	p

E(X) = p = 1/4

SD(X) = $|\sqrt{\frac{1}{4} \times \frac{3}{4}}| = \frac{\sqrt{3}}{4} = 0.4330$

	Failure	Success
x	0	1
$P(X = x)$	3/4	1/4

(1-p) → p

$E(X) = p = \frac{1}{4}$

$SD(X) = \sqrt{\frac{1}{4} \times \frac{3}{4}} = \frac{\sqrt{3}}{4} = 0.4330$

Find the mean and standard deviation for the event above.

Answers: 1/4, 0.4330