CALCULUS I

Practice Problems

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Table of Contents

Preface

Here are a set of practice problems for the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

1. If you'd like a pdf document containing the solutions the download tab above contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.

2. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Outline

Here is a listing of sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Review - In this chapter we give a brief review of selected topics from Algebra and Trig that are vital to surviving a Calculus course. Included are Functions, Trig Functions, Solving Trig Equations and Equations, Exponential/Logarithm Functions and Solving Exponential/Logarithm Equations.

Functions – In this section we will cover function notation/evaluation, determining the domain and range of a function and function composition.

Inverse Functions – In this section we will define an inverse function and the notation used for inverse functions. We will also discuss the process for finding an inverse function.

Trig Functions – In this section we will give a quick review of trig functions. We will cover the basic notation, relationship between the trig functions, the right triangle definition of the trig functions. We will also cover evaluation of trig functions as well as the unit circle (one of the most important ideas from a trig class!) and how it can be used to evaluate trig functions.

Solving Trig Equations – In this section we will discuss how to solve trig equations. The answers to the equations in this section will all be one of the “standard” angles that most students have memorized after a trig class. However, the process used here can be used for any answer regardless of it being one of the standard angles or not.

Solving Trig Equations with Calculators, Part I – In this section we will discuss solving trig equations when the answer will (generally) require the use of a calculator (i.e. they aren’t one of the standard angles). Note however, the process used here is identical to that for when the answer is one of the standard angles. The only difference is that the answers in here can be a little messy due to the need of a calculator. Included is a brief discussion of inverse trig functions.

Solving Trig Equations with Calculators, Part II – In this section we will continue our discussion of solving trig equations when a calculator is needed to get the answer. The equations in this section tend to be a little trickier than the "normal" trig equation and are not always covered in a trig class.

Exponential Functions – In this section we will discuss exponential functions. We will cover the basic definition of an exponential function, the natural exponential function, i.e. $\mathbf{e}^x$, as well as the properties and graphs of exponential functions.

Logarithm Functions – In this section we will discuss logarithm functions, evaluation of logarithms and their properties. We will discuss many of the basic manipulations of logarithms that commonly occur in Calculus (and higher) classes. Included is a discussion of the natural (\(\ln(x)\)) and common logarithm (\(\log(x)\)) as well as the change of base formula.

Exponential and Logarithm Equations – In this section we will discuss various methods for solving equations that involve exponential functions or logarithm functions.

Common Graphs – In this section we will do a very quick review of many of the most common functions and their graphs that typically show up in a Calculus class.

Limits - In this chapter we introduce the concept of limits. We will discuss the interpretation/meaning of a limit, how to evaluate limits, the definition and evaluation of one-sided limits, evaluation of infinite limits, evaluation of limits at infinity, continuity and the Intermediate Value Theorem. We will also give a brief introduction to a precise definition of the limit and how to use it to evaluate limits.

Tangent Lines and Rates of Change – In this section we will introduce two problems that we will see time and again in this course : Rate of Change of a function and Tangent Lines to functions. Both of these problems will be used to introduce the concept of limits, although we won't formally give the definition or notation until the next section.
The Limit – In this section we will introduce the notation of the limit. We will also take a conceptual look at limits and try to get a grasp on just what they are and what they can tell us. We will be estimating the value of limits in this section to help us understand what they tell us. We will actually start computing limits in a couple of sections.
One-Sided Limits – In this section we will introduce the concept of one-sided limits. We will discuss the differences between one-sided limits and limits as well as how they are related to each other.
Limit Properties – In this section we will discuss the properties of limits that we'll need to use in computing limits (as opposed to estimating them as we've done to this point). We will also compute a couple of basic limits in this section.
Computing Limits – In this section we will looks at several types of limits that require some work before we can use the limit properties to compute them. We will also look at computing limits of piecewise functions and use of the Squeeze Theorem to compute some limits.
Infinite Limits – In this section we will look at limits that have a value of infinity or negative infinity. We'll also take a brief look at vertical asymptotes.
Limits At Infinity, Part I – In this section we will start looking at limits at infinity, i.e. limits in which the variable gets very large in either the positive or negative sense. We will concentrate on polynomials and rational expressions in this section. We'll also take a brief look at horizontal asymptotes.
Limits At Infinity, Part II – In this section we will continue covering limits at infinity. We'll be looking at exponentials, logarithms and inverse tangents in this section.
Continuity – In this section we will introduce the concept of continuity and how it relates to limits. We will also see the Intermediate Value Theorem in this section and how it can be used to determine if functions have solutions in a given interval.
The Definition of the Limit – In this section we will give a precise definition of several of the limits covered in this section. We will work several basic examples illustrating how to use this precise definition to compute a limit. We'll also give a precise definition of continuity.

Derivatives – In this chapter we introduce Derivatives. We cover the standard derivatives formulas including the product rule, quotient rule and chain rule as well as derivatives of polynomials, roots, trig functions, inverse trig functions, hyperbolic functions, exponential functions and logarithm functions. We also cover implicit differentiation, related rates, higher order derivatives and logarithmic differentiation.
The Definition of the Derivative – In this section we define the derivative, give various notations for the derivative and work a few problems illustrating how to use the definition of the derivative to actually compute the derivative of a function.
Interpretation of the Derivative – In this section we give several of the more important interpretations of the derivative. We discuss the rate of change of a function, the velocity of a moving object and the slope of the tangent line to a graph of a function.
Differentiation Formulas – In this section we give most of the general derivative formulas and properties used when taking the derivative of a function. Examples in this section concentrate mostly on polynomials, roots and more generally variables raised to powers.

Product and Quotient Rule – In this section we will give two of the more important formulas for differentiating functions. We will discuss the Product Rule and the Quotient Rule allowing us to differentiate functions that, up to this point, we were unable to differentiate.
Derivatives of Trig Functions – In this section we will discuss differentiating trig functions. Derivatives of all six trig functions are given and we show the derivation of the derivative of sin(x) and tan(x).
Derivatives of Exponential and Logarithm Functions – In this section we derive the formulas for the derivatives of the exponential and logarithm functions.
Derivatives of Inverse Trig Functions – In this section we give the derivatives of all six inverse trig functions. We show the derivation of the formulas for inverse sine, inverse cosine and inverse tangent.
Derivatives of Hyperbolic Functions – In this section we define the hyperbolic functions, give the relationships between them and some of the basic facts involving hyperbolic functions. We also give the derivatives of each of the six hyperbolic functions and show the derivation of the formula for hyperbolic sine.
Chain Rule – In this section we discuss one of the more useful and important differentiation formulas, The Chain Rule. With the chain rule in hand we will be able to differentiate a much wider variety of functions. As you will see throughout the rest of your Calculus courses a great many of derivatives you take will involve the chain rule!
Implicit Differentiation – In this section we will discuss implicit differentiation. Not every function can be explicitly written in terms of the independent variable, e.g. y = f(x) and yet we will still need to know what f'(x) is. Implicit differentiation will allow us to find the derivative in these cases. Knowing implicit differentiation will allow us to do one of the more important applications of derivatives, Related Rates (the next section).
Related Rates – In this section we will discuss the only application of derivatives in this section, Related Rates. In related rates problems we are give the rate of change of one quantity in a problem and asked to determine the rate of one (or more) quantities in the problem. This is often one of the more difficult sections for students. We work quite a few problems in this section so hopefully by the end of this section you will get a decent understanding on how these problems work
Higher Order Derivatives – In this section we define the concept of higher order derivatives and give a quick application of the second order derivative and show how implicit differentiation works for higher order derivatives.
Logarithmic Differentiation – In this section we will discuss logarithmic differentiation. Logarithmic differentiation gives an alternative method for differentiating products and quotients (sometimes easier than using product and quotient rule). More importantly, however, is the fact that logarithm differentiation allows us to differentiate functions that are in the form of one function raised to another function, i.e. there are variables in both the base and exponent of the function.

Applications of Derivatives – In this chapter we will cover many of the major applications of derivatives. Applications included are determining absolute and relative minimum and maximum function values (both with and without constraints), sketching the graph of a function without using a computational aid, determining the Linear Approximation of a function, L'Hospital's Rule (allowing us to compute some limits we could not prior to this), Newton's Method (allowing us to approximate solutions to equations) as well as a few basic Business applications.

Rates of Change – In this section we review the main application/interpretation of derivatives from the previous chapter (i.e. rates of change) that we will be using in many of the applications in this chapter.
Critical Points – In this section we give the definition of critical points. Critical points will show up in most of the sections in this chapter, so it will be important to understand them and how to find them. We will work a number of examples illustrating how to find them for a wide variety of functions.
Minimum and Maximum Values –In this section we define absolute (or global) minimum and maximum values of a function and relative (or local) minimum and maximum values of a function. It is important to understand the difference between the two types of minimum/maximum (collectively called extrema) values for many of the applications in this chapter and so we use a variety of examples to help with this. We also give the Extreme Value Theorem and Fermat's Theorem, both of which are very important in the many of the applications we'll see in this chapter.
Finding Absolute Extrema – In this section we discuss how to find the absolute (or global) minimum and maximum values of a function. In other words, we will be finding the largest and smallest values that a function will have.
The Shape of a Graph, Part I – In this section we will discuss what the first derivative of a function can tell us about the graph of a function. The first derivative will allow us to identify the relative (or local) minimum and maximum values of a function and where a function will be increasing and decreasing. We will also give the First Derivative test which will allow us to classify critical points as relative minimums, relative maximums or neither a minimum or a maximum.
The Shape of a Graph, Part II – In this section we will discuss what the second derivative of a function can tell us about the graph of a function. The second derivative will allow us to determine where the graph of a function is concave up and concave down. The second derivative will also allow us to identify any inflection points (i.e. where concavity changes) that a function may have. We will also give the Second Derivative Test that will give an alternative method for identifying some critical points (but not all) as relative minimums or relative maximums.
The Mean Value Theorem – In this section we will give Rolle's Theorem and the Mean Value Theorem. With the Mean Value Theorem we will prove a couple of very nice facts, one of which will be very useful in the next chapter.
Optimization Problems – In this section we will be determining the absolute minimum and/or maximum of a function that depends on two variables given some constraint, or relationship, that the two variables must always satisfy. We will discuss several methods for determining the absolute minimum or maximum of the function. Examples in this section tend to center around geometric objects such as squares, boxes, cylinders, etc.
More Optimization Problems – In this section we will continue working optimization problems. The examples in this section tend to be a little more involved and will often involve situations that will be more easily described with a sketch as opposed to the 'simple' geometric objects we looked at in the previous section.
L'Hospital's Rule and Indeterminate Forms – In this section we will revisit indeterminate forms and limits and take a look at L'Hospital's Rule. L'Hospital's Rule will allow us to evaluate some limits we were not able to previously.
Linear Approximations – In this section we discuss using the derivative to compute a linear approximation to a function. We can use the linear approximation to a function to approximate values of the function at certain points. While it might not seem like a useful thing to do with

when we have the function there really are reasons that one might want to do this. We give two ways this can be useful in the examples.
Differentials – In this section we will compute the differential for a function. We will give an application of differentials in this section. However, one of the more important uses of differentials will come in the next chapter and unfortunately we will not be able to discuss it until then.
Newton's Method – In this section we will discuss Newton's Method. Newton's Method is an application of derivatives that will allow us to approximate solutions to an equation. There are many equations that cannot be solved directly and with this method we can get approximations to the solutions to many of those equations.
Business Applications – In this section we will give a cursory discussion of some basic applications of derivatives to the business field. We will revisit finding the maximum and/or minimum function value and we will define the marginal cost function, the average cost, the revenue function, the marginal revenue function and the marginal profit function. Note that this section is only intended to introduce these concepts and not teach you everything about them.

Integrals – In this chapter we will give an introduction to definite and indefinite integrals. We will discuss the definition and properties of each type of integral as well as how to compute them including the Substitution Rule. We will give the Fundamental Theorem of Calculus showing the relationship between derivatives and integrals. We will also discuss the Area Problem, an important interpretation of the definite integral.
Indefinite Integrals – In this section we will start off the chapter with the definition and properties of indefinite integrals. We will not be computing many indefinite integrals in this section. This section is devoted to simply defining what an indefinite integral is and to give many of the properties of the indefinite integral. Actually computing indefinite integrals will start in the next section.
Computing Indefinite Integrals – In this section we will compute some indefinite integrals. The integrals in this section will tend to be those that do not require a lot of manipulation of the function we are integrating in order to actually compute the integral. As we will see starting in the next section many integrals do require some manipulation of the function before we can actually do the integral. We will also take a quick look at an application of indefinite integrals.
Substitution Rule for Indefinite Integrals – In this section we will start using one of the more common and useful integration techniques – The Substitution Rule. With the substitution rule we will be able integrate a wider variety of functions. The integrals in this section will all require some manipulation of the function prior to integrating unlike most of the integrals from the previous section where all we really needed were the basic integration formulas.
More Substitution Rule – In this section we will continue to look at the substitution rule. The problems in this section will tend to be a little more involved than those in the previous section.
Area Problem – In this section we start off with the motivation for definite integrals and give one of the interpretations of definite integrals. We will be approximating the amount of area that lies between a function and the \(x\)-axis. As we will see in the next section this problem will lead us to the definition of the definite integral and will be one of the main interpretations of the definite integral that we'll be looking at in this material.
Definition of the Definite Integral – In this section we will formally define the definite integral, give many of its properties and discuss a couple of interpretations of the definite integral. We will also look at the first part of the Fundamental Theorem of Calculus which shows the very close relationship between derivatives and integrals.

Computing Definite Integrals – In this section we will take a look at the second part of the Fundamental Theorem of Calculus. This will show us how we compute definite integrals without using (the often very unpleasant) definition. The examples in this section can all be done with a basic knowledge of indefinite integrals and will not require the use of the substitution rule. Included in the examples in this section are computing definite integrals of piecewise and absolute value functions.

Substitution Rule for Definite Integrals – In this section we will revisit the substitution rule as it applies to definite integrals. The only real requirements to being able to do the examples in this section are being able to do the substitution rule for indefinite integrals and understanding how to compute definite integrals in general.

Applications of Integrals – In this chapter we will take a look at some applications of integrals. We will look at Average Function Value, Area Between Curves, Volume (both solids of revolution and other solids) and Work.

Average Function Value – In this section we will look at using definite integrals to determine the average value of a function on an interval. We will also give the Mean Value Theorem for Integrals.

Area Between Curves – In this section we'll take a look at one of the main applications of definite integrals in this chapter. We will determine the area of the region bounded by two curves.

Volumes of Solids of Revolution / Method of Rings – In this section, the first of two sections devoted to finding the volume of a solid of revolution, we will look at the method of rings/disks to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the *x* or *y*-axis) around a vertical or horizontal axis of rotation.

Volumes of Solids of Revolution / Method of Cylinders – In this section, the second of two sections devoted to finding the volume of a solid of revolution, we will look at the method of cylinders/shells to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the *x* or *y*-axis) around a vertical or horizontal axis of rotation.

More Volume Problems – In the previous two sections we looked at solids that could be found by treating them as a solid of revolution. Not all solids can be thought of as solids of revolution and, in fact, not all solids of revolution can be easily dealt with using the methods from the previous two sections. So, in this section we'll take a look at finding the volume of some solids that are either not solids of revolutions or are not easy to do as a solid of revolution.

Work – In this section we will look at is determining the amount of work required to move an object subject to a force over a given distance.

Chapter 1 : Review

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Section 1-1 : Functions

For problems 1 – 4 the given functions perform the indicated function evaluations.

1. $f(x) = 3 - 5x - 2x^2$

(a) $f(4)$ (b) $f(0)$ (c) $f(-3)$

(d) $f(6-t)$ (e) $f(7-4x)$ (f) $f(x+h)$

2. $g(t) = \dfrac{t}{2t+6}$

(a) $g(0)$ (b) $g(-3)$ (c) $g(10)$

(d) $g(x^2)$ (e) $g(t+h)$ (f) $g(t^2 - 3t + 1)$

3. $h(z) = \sqrt{1-z^2}$

(a) $h(0)$ (b) $h\left(-\frac{1}{2}\right)$ (c) $h\left(\frac{1}{2}\right)$

(d) $h(9z)$ (e) $h(z^2 - 2z)$ (f) $h(z+k)$

4. $R(x) = \sqrt{3+x} - \dfrac{4}{x+1}$

(a) $R(0)$ (b) $R(6)$ (c) $R(-9)$

(d) $R(x+1)$ (e) $R(x^4 - 3)$ (f) $R\left(\frac{1}{x} - 1\right)$

The difference quotient of a function $f(x)$ is defined to be,

$$\frac{f(x+h) - f(x)}{h}$$

For problems 5 – 9 compute the difference quotient of the given function.

5. $f(x) = 4x - 9$

6. $g(x) = 6 - x^2$

7. $f(t) = 2t^2 - 3t + 9$

8. $y(z) = \dfrac{1}{z+2}$

9. $A(t) = \dfrac{2t}{3-t}$

For problems 10 – 17 determine all the roots of the given function.

10. $f(x) = x^5 - 4x^4 - 32x^3$

11. $R(y) = 12y^2 + 11y - 5$

12. $h(t) = 18 - 3t - 2t^2$

13. $g(x) = x^3 + 7x^2 - x$

14. $W(x) = x^4 + 6x^2 - 27$

15. $f(t) = t^{\frac{5}{3}} - 7t^{\frac{4}{3}} - 8t$

16. $h(z) = \dfrac{z}{z-5} - \dfrac{4}{z-8}$

17. $g(w) = \dfrac{2w}{w+1} + \dfrac{w-4}{2w-3}$

For problems 18 – 22 find the domain and range of the given function.

18. $Y(t) = 3t^2 - 2t + 1$

19. $g(z) = -z^2 - 4z + 7$

20. $f(z) = 2 + \sqrt{z^2 + 1}$

21. $h(y) = -3\sqrt{14 + 3y}$

22. $M(x) = 5 - |x + 8|$

For problems 23 – 32 find the domain of the given function.

23. $f(w) = \dfrac{w^3 - 3w + 1}{12w - 7}$

24. $R(z) = \dfrac{5}{z^3 + 10z^2 + 9z}$

25. $g(t) = \dfrac{6t - t^3}{7 - t - 4t^2}$

26. $g(x) = \sqrt{25 - x^2}$

27. $h(x) = \sqrt{x^4 - x^3 - 20x^2}$

28. $P(t) = \dfrac{5t + 1}{\sqrt{t^3 - t^2 - 8t}}$

29. $f(z) = \sqrt{z - 1} + \sqrt{z + 6}$

30. $h(y) = \sqrt{2y + 9} - \dfrac{1}{\sqrt{2 - y}}$

31. $A(x) = \dfrac{4}{x - 9} - \sqrt{x^2 - 36}$

32. $Q(y) = \sqrt{y^2 + 1} - \sqrt[3]{1 - y}$

For problems 33 – 36 compute $(f \circ g)(x)$ and $(g \circ f)(x)$ for each of the given pair of functions.

33. $f(x) = 4x - 1$, $g(x) = \sqrt{6 + 7x}$

34. $f(x) = 5x + 2$, $g(x) = x^2 - 14x$

35. $f(x) = x^2 - 2x + 1$, $g(x) = 8 - 3x^2$

36. $f(x) = x^2 + 3$, $g(x) = \sqrt{5 + x^2}$

Section 1-2 : Inverse Functions

For each of the following functions find the inverse of the function. Verify your inverse by computing one or both of the composition as discussed in this section.

1. $f(x) = 6x + 15$

2. $h(x) = 3 - 29x$

3. $R(x) = x^3 + 6$

4. $g(x) = 4(x-3)^5 + 21$

5. $W(x) = \sqrt[5]{9 - 11x}$

6. $f(x) = \sqrt[7]{5x + 8}$

7. $h(x) = \dfrac{1 + 9x}{4 - x}$

8. $f(x) = \dfrac{6 - 10x}{8x + 7}$

Section 1-3 : Trig Functions

Determine the exact value of each of the following without using a calculator.

Note that the point of these problems is not really to learn how to find the value of trig functions but instead to get you comfortable with the unit circle since that is a very important skill that will be needed in solving trig equations.

1. $\cos\left(\frac{5\pi}{6}\right)$

2. $\sin\left(-\frac{4\pi}{3}\right)$

3. $\sin\left(\frac{7\pi}{4}\right)$

4. $\cos\left(-\frac{2\pi}{3}\right)$

5. $\tan\left(\frac{3\pi}{4}\right)$

6. $\sec\left(-\frac{11\pi}{6}\right)$

7. $\cos\left(\frac{8\pi}{3}\right)$

8. $\tan\left(-\frac{\pi}{3}\right)$

9. $\tan\left(\frac{15\pi}{4}\right)$

10. $\sin\left(-\frac{11\pi}{3}\right)$

11. $\sec\left(\frac{29\pi}{4}\right)$

Section 1-4 : Solving Trig Equations

Without using a calculator find the solution(s) to the following equations. If an interval is given find only those solutions that are in the interval. If no interval is given find all solutions to the equation.

1. $4\sin\left(3t\right)=2$

2. $4\sin\left(3t\right)=2$ in $\left[0,\frac{4\pi}{3}\right]$

3. $2\cos\left(\frac{x}{3}\right)+\sqrt{2}=0$

4. $2\cos\left(\frac{x}{3}\right)+\sqrt{2}=0$ in $\left[-7\pi,7\pi\right]$

5. $4\cos\left(6z\right)=\sqrt{12}$ in $\left[0,\frac{\pi}{2}\right]$

6. $2\sin\left(\frac{3y}{2}\right)+\sqrt{3}=0$ in $\left[-\frac{7\pi}{3},0\right]$

7. $8\tan\left(2x\right)-5=3$ in $\left[-\frac{\pi}{2},\frac{3\pi}{2}\right]$

8. $16=-9\sin\left(7x\right)-4$ in $\left[-2\pi,\frac{9\pi}{4}\right]$

9. $\sqrt{3}\tan\left(\frac{t}{4}\right)+5=4$ in $\left[0,4\pi\right]$

10. $\sqrt{3}\csc\left(9z\right)-7=-5$ in $\left[-\frac{\pi}{3},\frac{4\pi}{9}\right]$

11. $1-14\cos\left(\frac{2x}{5}\right)=-6$ in $\left[5\pi,\frac{40\pi}{3}\right]$

12. $15=17+4\cos\left(\frac{y}{7}\right)$ in $\left[10\pi,15\pi\right]$

Section 1-5 : Solving Trig Equations with Calculators, Part I

Find the solution(s) to the following equations. If an interval is given find only those solutions that are in the interval. If no interval is given find all solutions to the equation. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $7\cos\left(4x\right)+11=10$

2. $6+5\cos\left(\frac{x}{3}\right)=10$ in $\left[0,38\right]$

3. $3=6-11\sin\left(\frac{t}{8}\right)$

4. $4\sin\left(6z\right)+\frac{13}{10}=-\frac{3}{10}$ in $\left[0,2\right]$

5. $9\cos\left(\frac{4z}{9}\right)+21\sin\left(\frac{4z}{9}\right)=0$ in $\left[-10,10\right]$

6. $3\tan\left(\frac{w}{4}\right)-1=11-2\tan\left(\frac{w}{4}\right)$ in $\left[-50,0\right]$

7. $17-3\sec\left(\frac{z}{2}\right)=2$ in $\left[20,45\right]$

8. $12\sin\left(7y\right)+11=3+4\sin\left(7y\right)$ in $\left[-2,-\frac{1}{2}\right]$

9. $5-14\tan\left(8x\right)=30$ in $\left[-1,1\right]$

10. $0=18+2\csc\left(\frac{t}{3}\right)$ in $\left[0,5\right]$

11. $\frac{1}{2}\cos\left(\frac{x}{8}\right)+\frac{1}{4}=\frac{2}{3}$ in $\left[0,100\right]$

12. $\frac{4}{3}=1+3\sec\left(2t\right)$ in $\left[-4,6\right]$

Section 1-6 : Solving Trig Equations with Calculators, Part II

Find all the solution(s) to the following equations. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $3-14\sin\left(12t+7\right)=13$

2. $3\sec\left(4-9z\right)-24=0$

3. $4\sin\left(x+2\right)-15\sin\left(x+2\right)\tan\left(4x\right)=0$

4. $3\cos\left(\frac{3y}{7}\right)\sin\left(\frac{y}{2}\right)+14\cos\left(\frac{3y}{7}\right)=0$

5. $7\cos^2\left(3x\right)-\cos\left(3x\right)=0$

6. $\tan^2\left(\frac{w}{4}\right)=\tan\left(\frac{w}{4}\right)+12$

7. $4\csc^2\left(1-t\right)+6=25\csc\left(1-t\right)$

8. $4y\sec\left(7y\right)=-21y$

9. $10x^2\sin\left(3x+2\right)=7x\sin\left(3x+2\right)$

10. $\left(2t-3\right)\tan\left(\frac{6t}{11}\right)=15-10t$

Section 1-7 : Exponential Functions

Sketch the graphs of each of the following functions.

1. $f\left(x\right)=3^{1+2x}$

2. $h\left(x\right)=2^{3-\frac{x}{4}}-7$

3. $h\left(t\right)=8+3\mathbf{e}^{2t-4}$

4. $g\left(z\right)=10-\frac{1}{4}\mathbf{e}^{-2-3z}$

Section 1-8 : Logarithm Functions

Without using a calculator determine the exact value of each of the following.

1. $\log_3 81$

2. $\log_5 125$

3. $\log_2 \dfrac{1}{8}$

4. $\log_{\frac{1}{4}} 16$

5. $\ln \mathbf{e}^4$

6. $\log \dfrac{1}{100}$

Write each of the following in terms of simpler logarithms

7. $\log\left(3x^4 y^{-7}\right)$

8. $\ln\left(x\sqrt{y^2+z^2}\right)$

9. $\log_4\left(\dfrac{x-4}{y^2\sqrt[5]{z}}\right)$

Combine each of the following into a single logarithm with a coefficient of one.

10. $2\log_4 x + 5\log_4 y - \dfrac{1}{2}\log_4 z$

11. $3\ln(t+5) - 4\ln t - 2\ln(s-1)$

12. $\dfrac{1}{3}\log a - 6\log b + 2$

Use the change of base formula and a calculator to find the value of each of the following.

13. $\log_{12} 35$

14. $\log_{\frac{2}{3}} 53$

Section 1-9 : Exponential and Logarithm Equations

For problems 1 – 12 find all the solutions to the given equation. If there is no solution to the equation clearly explain why.

1. $12 - 4\mathbf{e}^{7+3x} = 7$

2. $1 = 10 - 3\mathbf{e}^{z^2 - 2z}$

3. $2t - t\mathbf{e}^{6t-1} = 0$

4. $4x + 1 = (12x + 3)\mathbf{e}^{x^2 - 2}$

5. $2\mathbf{e}^{3y+8} - 11\mathbf{e}^{5-10y} = 0$

6. $14\mathbf{e}^{6-x} + \mathbf{e}^{12x-7} = 0$

7. $1 - 8\ln\left(\dfrac{2x-1}{7}\right) = 14$

8. $\ln(y-1) = 1 + \ln(3y+2)$

9. $\log(w) + \log(w-21) = 2$

10. $2\log(z) - \log(7z-1) = 0$

11. $16 = 17^{t-2} + 11$

12. $2^{3-8w} - 7 = 11$

Compound Interest. If we put *P* dollars into an account that earns interest at a rate of *r* (written as a decimal as opposed to the standard percent) for *t* years then,

a. if interest is compounded *m* times per year we will have,
$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$
dollars after *t* years.

b. if interest is compounded continuously we will have,
$$A = P\mathbf{e}^{rt}$$
dollars after *t* years.

13. We have \$10,000 to invest for 44 months. How much money will we have if we put the money into an account that has an annual interest rate of 5.5% and interest is compounded
(a) quarterly (b) monthly (c) continuously

14. We are starting with \$5000 and we're going to put it into an account that earns an annual interest rate of 12%. How long should we leave the money in the account in order to double our money if interest is compounded
(a) quarterly (b) monthly (c) continuously

Exponential Growth/Decay. Many quantities in the world can be modeled (at least for a short time) by the exponential growth/decay equation.

$$Q = Q_0 \mathbf{e}^{kt}$$

If k is positive we will get exponential growth and if k is negative we will get exponential decay.

15. A population of bacteria initially has 250 present and in 5 days there will be 1600 bacteria present.
(a) Determine the exponential growth equation for this population.
(b) How long will it take for the population to grow from its initial population of 250 to a population of 2000?

16. We initially have 100 grams of a radioactive element and in 1250 years there will be 80 grams left.
(a) Determine the exponential decay equation for this element.
(b) How long will it take for half of the element to decay?
(c) How long will it take until there is only 1 gram of the element left?

Section 1-10 : Common Graphs

Without using a graphing calculator sketch the graph of each of the following.

1. $y = \frac{4}{3}x - 2$

2. $f(x) = |x - 3|$

3. $g(x) = \sin(x) + 6$

4. $f(x) = \ln(x) - 5$

5. $h(x) = \cos\left(x + \frac{\pi}{2}\right)$

6. $h(x) = (x - 3)^2 + 4$

7. $W(x) = \mathbf{e}^{x+2} - 3$

8. $f(y) = (y - 1)^2 + 2$

9. $R(x) = -\sqrt{x}$

10. $g(x) = \sqrt{-x}$

11. $h(x) = 2x^2 - 3x + 4$

12. $f(y) = -4y^2 + 8y + 3$

13. $(x + 1)^2 + (y - 5)^2 = 9$

14. $x^2 - 4x + y^2 - 6y - 87 = 0$

15. $25(x + 2)^2 + \frac{y^2}{4} = 1$

16. $x^2 + \frac{(y-6)^2}{9} = 1$

17. $\frac{x^2}{36} - \frac{y^2}{49} = 1$

18. $(y+2)^2 - \frac{(x+4)^2}{16} = 1$

Chapter 2 : Limits

Here are a set of practice problems for the Limits chapter of the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

5. If you'd like a pdf document containing the solutions the download tab on the website contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.

6. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Tangent Lines and Rates of Change – In this section we will introduce two problems that we will see time and again in this course : Rate of Change of a function and Tangent Lines to functions. Both of these problems will be used to introduce the concept of limits, although we won't formally give the definition or notation until the next section.

The Limit – In this section we will introduce the notation of the limit. We will also take a conceptual look at limits and try to get a grasp on just what they are and what they can tell us. We will be estimating the value of limits in this section to help us understand what they tell us. We will actually start computing limits in a couple of sections.

One-Sided Limits – In this section we will introduce the concept of one-sided limits. We will discuss the differences between one-sided limits and limits as well as how they are related to each other.

Limit Properties – In this section we will discuss the properties of limits that we'll need to use in computing limits (as opposed to estimating them as we've done to this point). We will also compute a couple of basic limits in this section.

Computing Limits – In this section we will looks at several types of limits that require some work before we can use the limit properties to compute them. We will also look at computing limits of piecewise functions and use of the Squeeze Theorem to compute some limits.

Infinite Limits – In this section we will look at limits that have a value of infinity or negative infinity. We'll also take a brief look at vertical asymptotes.

Limits At Infinity, Part I – In this section we will start looking at limits at infinity, i.e. limits in which the variable gets very large in either the positive or negative sense. We will concentrate on polynomials and rational expressions in this section. We'll also take a brief look at horizontal asymptotes.

Limits At Infinity, Part II – In this section we will continue covering limits at infinity. We'll be looking at exponentials, logarithms and inverse tangents in this section.

Continuity – In this section we will introduce the concept of continuity and how it relates to limits. We will also see the Intermediate Value Theorem in this section and how it can be used to determine if functions have solutions in a given interval.

The Definition of the Limit – In this section we will give a precise definition of several of the limits covered in this section. We will work several basic examples illustrating how to use this precise definition to compute a limit. We'll also give a precise definition of continuity.

Section 2-1 : Tangent Lines and Rates of Change

1. For the function $f(x) = 3(x+2)^2$ and the point *P* given by $x = -3$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) -3.5	(ii) -3.1	(iii) -3.01	(iv) -3.001	(v) -3.0001
(vi) -2.5	(vii) -2.9	(viii) -2.99	(ix) -2.999	(x) -2.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $f(x)$ at $x = -3$ and write down the equation of the tangent line.

2. For the function $g(x) = \sqrt{4x+8}$ and the point *P* given by $x = 2$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) 2.5	(ii) 2.1	(iii) 2.01	(iv) 2.001	(v) 2.0001
(vi) 1.5	(vii) 1.9	(viii) 1.99	(ix) 1.999	(x) 1.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $g(x)$ at $x = 2$ and write down the equation of the tangent line.

3. For the function $W(x) = \ln(1+x^4)$ and the point *P* given by $x = 1$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) 1.5	(ii) 1.1	(iii) 1.01	(iv) 1.001	(v) 1.0001
(vi) 0.5	(vii) 0.9	(viii) 0.99	(ix) 0.999	(x) 0.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $W(x)$ at $x = 1$ and write down the equation of the tangent line.

4. The volume of air in a balloon is given by $V\left(t\right)=\dfrac{6}{4t+1}$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the volume of air in the balloon between $t = 0.25$ and the following values of *t*.

(i) 1 (ii) 0.5 (iii) 0.251 (iv) 0.2501 (v) 0.25001
(vi) 0 (vii) 0.1 (viii) 0.249 (ix) 0.2499 (x) 0.24999

(b) Use the information from (a) to estimate the instantaneous rate of change of the volume of air in the balloon at $t = 0.25$.

5. The population (in hundreds) of fish in a pond is given by $P\left(t\right)=2t+\sin\left(2t-10\right)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the population of fish between $t = 5$ and the following values of *t*. Make sure your calculator is set to radians for the computations.

(i) 5.5 (ii) 5.1 (iii) 5.01 (iv) 5.001 (v) 5.0001
(vi) 4.5 (vii) 4.9 (viii) 4.99 (ix) 4.999 (x) 4.9999

(b) Use the information from (a) to estimate the instantaneous rate of change of the population of the fish at $t = 5$.

6. The position of an object is given by $s\left(t\right)=\cos^2\left(\dfrac{3t-6}{2}\right)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 2$ and the following values of *t*. Make sure your calculator is set to radians for the computations.

(i) 2.5 (ii) 2.1 (iii) 2.01 (iv) 2.001 (v) 2.0001
(vi) 1.5 (vii) 1.9 (viii) 1.99 (ix) 1.999 (x) 1.9999

(b) Use the information from (a) to estimate the instantaneous velocity of the object at $t = 2$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

7. The position of an object is given by $s\left(t\right)=\left(8-t\right)\left(t+6\right)^{\frac{3}{2}}$. Note that a negative position here simply means that the position is to the left of the "zero position" and is perfectly acceptable. Answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t=10$ and the following values of *t*.

(i) 10.5	(ii) 10.1	(iii) 10.01	(iv) 10.001	(v) 10.0001
(vi) 9.5	(vii) 9.9	(viii) 9.99	(ix) 9.999	(x) 9.9999

(b) Use the information from (a) to estimate the instantaneous velocity of the object at $t=10$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

Section 2-2 : The Limit

1. For the function $f(x) = \dfrac{8 - x^3}{x^2 - 4}$ answer each of the following questions.

(a) Evaluate the function at the following values of x compute (accurate to at least 8 decimal places).

(i) 2.5	(ii) 2.1	(iii) 2.01	(iv) 2.001	(v) 2.0001
(vi) 1.5	(vii) 1.9	(viii) 1.99	(ix) 1.999	(x) 1.9999

(b) Use the information from (a) to estimate the value of $\displaystyle\lim_{x \to 2} \frac{8 - x^3}{x^2 - 4}$.

2. For the function $R(t) = \dfrac{2 - \sqrt{t^2 + 3}}{t + 1}$ answer each of the following questions.

(a) Evaluate the function at the following values of t compute (accurate to at least 8 decimal places).

(i) -0.5	(ii) -0.9	(iii) -0.99	(iv) -0.999	(v) -0.9999
(vi) -1.5	(vii) -1.1	(viii) -1.01	(ix) -1.001	(x) -1.0001

(b) Use the information from (a) to estimate the value of $\displaystyle\lim_{t \to -1} \frac{2 - \sqrt{t^2 + 3}}{t + 1}$.

3. For the function $g(\theta) = \dfrac{\sin(7\theta)}{\theta}$ answer each of the following questions.

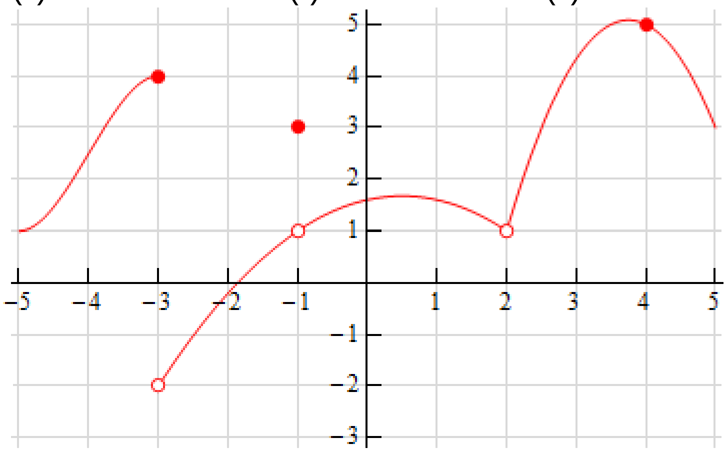
(a) Evaluate the function at the following values of θ compute (accurate to at least 8 decimal places). Make sure your calculator is set to radians for the computations.

(i) 0.5	(ii) 0.1	(iii) 0.01	(iv) 0.001	(v) 0.0001
(vi) -0.5	(vii) -0.1	(viii) -0.01	(ix) -0.001	(x) -0.0001

(b) Use the information from (a) to estimate the value of $\displaystyle\lim_{\theta \to 0} \frac{\sin(7\theta)}{\theta}$.

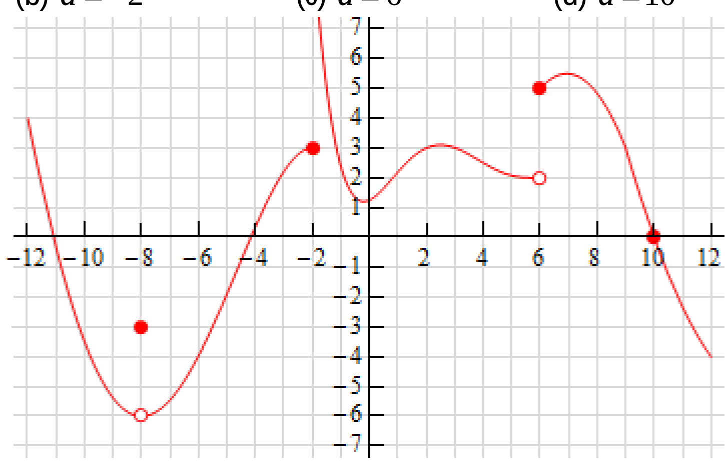
4. Below is the graph of $f\left(x\right)$. For each of the given points determine the value of $f\left(a\right)$ and $\lim\limits_{x\to a} f\left(x\right)$. If any of the quantities do not exist clearly explain why.

(a) $a=-3$ (b) $a=-1$ (c) $a=2$ (d) $a=4$



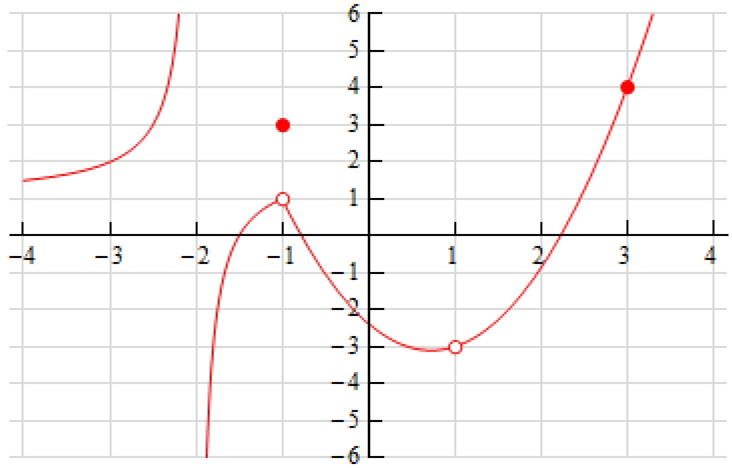
5. Below is the graph of $f\left(x\right)$. For each of the given points determine the value of $f\left(a\right)$ and $\lim\limits_{x\to a} f\left(x\right)$. If any of the quantities do not exist clearly explain why.

(a) $a=-8$ (b) $a=-2$ (c) $a=6$ (d) $a=10$



6. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim\limits_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

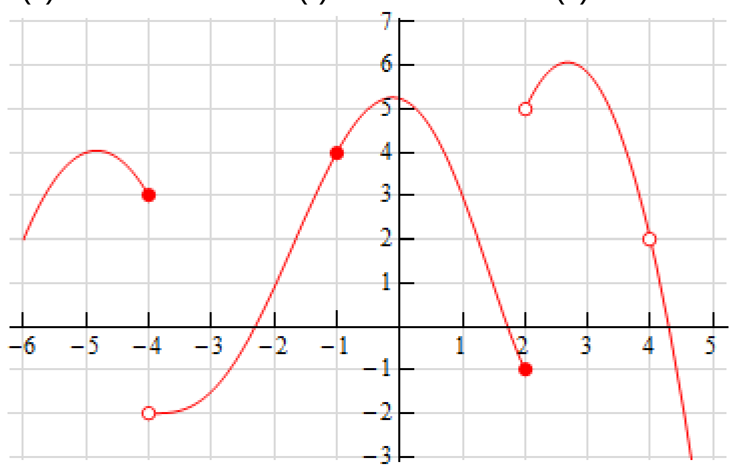
(a) $a = -2$ (b) $a = -1$ (c) $a = 1$ (d) $a = 3$



Section 2-3 : One–Sided Limits

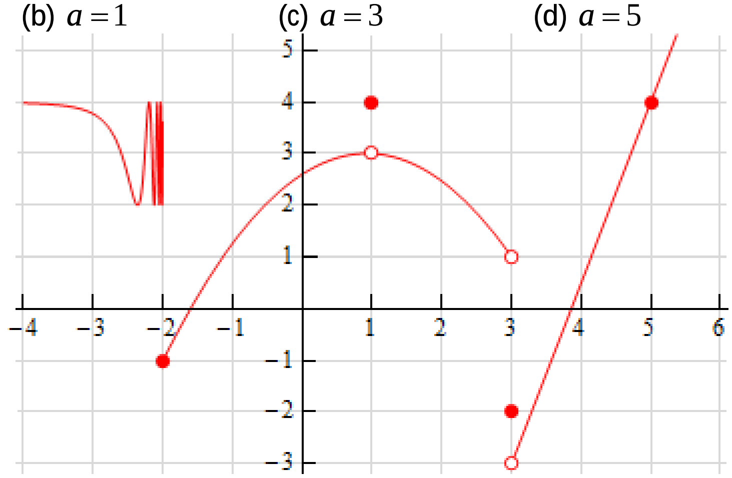
1. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim\limits_{x\to a^-} f(x)$, $\lim\limits_{x\to a^+} f(x)$, and $\lim\limits_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a=-4$ (b) $a=-1$ (c) $a=2$ (d) $a=4$



2. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim\limits_{x\to a^-} f(x)$, $\lim\limits_{x\to a^+} f(x)$, and $\lim\limits_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a=-2$ (b) $a=1$ (c) $a=3$ (d) $a=5$



3. Sketch a graph of a function that satisfies each of the following conditions.

$$\lim_{x\to 2^-} f(x)=1 \qquad \lim_{x\to 2^+} f(x)=-4 \qquad f(2)=1$$

4. Sketch a graph of a function that satisfies each of the following conditions.

$\lim\limits_{x \to 3^-} f(x) = 0$ $\lim\limits_{x \to 3^+} f(x) = 4$ $f(3)$ does not exist

$\lim\limits_{x \to -1} f(x) = -3$ $f(-1) = 2$

Section 2-4 : Limit Properties

1. Given $\lim\limits_{x \to 8} f(x) = -9$, $\lim\limits_{x \to 8} g(x) = 2$ and $\lim\limits_{x \to 8} h(x) = 4$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim\limits_{x \to 8} \left[2f(x) - 12h(x)\right]$

(b) $\lim\limits_{x \to 8} \left[3h(x) - 6\right]$

(c) $\lim\limits_{x \to 8} \left[g(x)h(x) - f(x)\right]$

(d) $\lim\limits_{x \to 8} \left[f(x) - g(x) + h(x)\right]$

2. Given $\lim\limits_{x \to -4} f(x) = 1$, $\lim\limits_{x \to -4} g(x) = 10$ and $\lim\limits_{x \to -4} h(x) = -7$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim\limits_{x \to -4} \left[\dfrac{f(x)}{g(x)} - \dfrac{h(x)}{f(x)}\right]$

(b) $\lim\limits_{x \to -4} \left[f(x)g(x)h(x)\right]$

(c) $\lim\limits_{x \to -4} \left[\dfrac{1}{h(x)} + \dfrac{3 - f(x)}{g(x) + h(x)}\right]$

(d) $\lim\limits_{x \to -4} \left[2h(x) - \dfrac{1}{h(x) + 7f(x)}\right]$

3. Given $\lim\limits_{x \to 0} f(x) = 6$, $\lim\limits_{x \to 0} g(x) = -4$ and $\lim\limits_{x \to 0} h(x) = -1$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim\limits_{x \to 0} \left[f(x) + h(x)\right]^3$

(b) $\lim\limits_{x \to 0} \sqrt{g(x)h(x)}$

(c) $\lim\limits_{x \to 0} \sqrt[3]{11 + \left[g(x)\right]^2}$

(d) $\lim\limits_{x \to 0} \sqrt{\dfrac{f(x)}{h(x) - g(x)}}$

For each of the following limits use the limit properties given in this section to compute the limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

4. $\lim\limits_{t \to -2} \left(14 - 6t + t^3\right)$

5. $\lim\limits_{x \to 6} \left(3x^2 + 7x - 16\right)$

6. $\displaystyle\lim_{w \to 3} \frac{w^2 - 8w}{4 - 7w}$

7. $\displaystyle\lim_{x \to -5} \frac{x + 7}{x^2 + 3x - 10}$

8. $\displaystyle\lim_{z \to 0} \sqrt{z^2 + 6}$

9. $\displaystyle\lim_{x \to 10} \left(4x + \sqrt[3]{x - 2}\right)$

Section 2-5 : Computing Limits

For problems 1 – 9 evaluate the limit, if it exists.

1. $\lim\limits_{x \to 2} \left(8 - 3x + 12x^2\right)$

2. $\lim\limits_{t \to -3} \dfrac{6 + 4t}{t^2 + 1}$

3. $\lim\limits_{x \to -5} \dfrac{x^2 - 25}{x^2 + 2x - 15}$

4. $\lim\limits_{z \to 8} \dfrac{2z^2 - 17z + 8}{8 - z}$

5. $\lim\limits_{y \to 7} \dfrac{y^2 - 4y - 21}{3y^2 - 17y - 28}$

6. $\lim\limits_{h \to 0} \dfrac{(6 + h)^2 - 36}{h}$

7. $\lim\limits_{z \to 4} \dfrac{\sqrt{z} - 2}{z - 4}$

8. $\lim\limits_{x \to -3} \dfrac{\sqrt{2x + 22} - 4}{x + 3}$

9. $\lim\limits_{x \to 0} \dfrac{x}{3 - \sqrt{x + 9}}$

10. Given the function

$$f(x) = \begin{cases} 7 - 4x & x < 1 \\ x^2 + 2 & x \geq 1 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim\limits_{x \to -6} f(x)$ (b) $\lim\limits_{x \to 1} f(x)$

11. Given

$$h(z) = \begin{cases} 6z & z \le -4 \\ 1-9z & z > -4 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim\limits_{z \to 7} h(z)$ (b) $\lim\limits_{z \to -4} h(z)$

For problems 12 & 13 evaluate the limit, if it exists.

12. $\lim\limits_{x \to 5} \left(10 + |x-5|\right)$

13. $\lim\limits_{t \to -1} \dfrac{t+1}{|t+1|}$

14. Given that $7x \le f(x) \le 3x^2 + 2$ for all x determine the value of $\lim\limits_{x \to 2} f(x)$.

15. Use the Squeeze Theorem to determine the value of $\lim\limits_{x \to 0} x^4 \sin\left(\dfrac{\pi}{x}\right)$.

Section 2-6 : Infinite Limits

For problems 1 – 6 evaluate the indicated limits, if they exist.

1. For $f\left(x\right)=\dfrac{9}{\left(x-3\right)^5}$ evaluate,

(a) $\lim\limits_{x\to 3^-} f\left(x\right)$ (b) $\lim\limits_{x\to 3^+} f\left(x\right)$ (c) $\lim\limits_{x\to 3} f\left(x\right)$

2. For $h\left(t\right)=\dfrac{2t}{6+t}$ evaluate,

(a) $\lim\limits_{t\to -6^-} h\left(t\right)$ (b) $\lim\limits_{t\to -6^+} h\left(t\right)$ (c) $\lim\limits_{t\to -6} h\left(t\right)$

3. For $g\left(z\right)=\dfrac{z+3}{\left(z+1\right)^2}$ evaluate,

(a) $\lim\limits_{z\to -1^-} g\left(z\right)$ (b) $\lim\limits_{z\to -1^+} g\left(z\right)$ (c) $\lim\limits_{z\to -1} g\left(z\right)$

4. For $g\left(x\right)=\dfrac{x+7}{x^2-4}$ evaluate,

(a) $\lim\limits_{x\to 2^-} g\left(x\right)$ (b) $\lim\limits_{x\to 2^+} g\left(x\right)$ (c) $\lim\limits_{x\to 2} g\left(x\right)$

5. For $h\left(x\right)=\ln\left(-x\right)$ evaluate,

(a) $\lim\limits_{x\to 0^-} h\left(x\right)$ (b) $\lim\limits_{x\to 0^+} h\left(x\right)$ (c) $\lim\limits_{x\to 0} h\left(x\right)$

6. For $R\left(y\right)=\tan\left(y\right)$ evaluate,

(a) $\lim\limits_{y\to \frac{3\pi}{2}^-} R\left(y\right)$ (b) $\lim\limits_{y\to \frac{3\pi}{2}^+} R\left(y\right)$ (c) $\lim\limits_{y\to \frac{3\pi}{2}} R\left(y\right)$

For problems 7 & 8 find all the vertical asymptotes of the given function.

7. $f\left(x\right)=\dfrac{7x}{\left(10-3x\right)^4}$

8. $g\left(x\right)=\dfrac{-8}{\left(x+5\right)\left(x-9\right)}$

Section 2-7 : Limits at Infinity, Part I

1. For $f\left(x\right)=4x^7-18x^3+9$ evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to-\infty} f\left(x\right)$ (b) $\displaystyle\lim_{x\to\infty} f\left(x\right)$

2. For $h\left(t\right)=\sqrt[3]{t}+12t-2t^2$ evaluate each of the following limits.

(a) $\displaystyle\lim_{t\to-\infty} h\left(t\right)$ (b) $\displaystyle\lim_{t\to\infty} h\left(t\right)$

For problems 3 – 10 answer each of the following questions.

(a) Evaluate $\displaystyle\lim_{x\to-\infty} f\left(x\right)$.

(b) Evaluate $\displaystyle\lim_{x\to\infty} f\left(x\right)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

3. $f\left(x\right)=\dfrac{8-4x^2}{9x^2+5x}$

4. $f\left(x\right)=\dfrac{3x^7-4x^2+1}{5-10x^2}$

5. $f\left(x\right)=\dfrac{20x^4-7x^3}{2x+9x^2+5x^4}$

6. $f\left(x\right)=\dfrac{x^3-2x+11}{3-6x^5}$

7. $f\left(x\right)=\dfrac{x^6-x^4+x^2-1}{7x^6+4x^3+10}$

8. $f\left(x\right)=\dfrac{\sqrt{7+9x^2}}{1-2x}$

9. $f\left(x\right)=\dfrac{x+8}{\sqrt{2x^2+3}}$

10. $f\left(x\right)=\dfrac{8+x-4x^2}{\sqrt{6+x^2+7x^4}}$

Section 2-8 : Limits At Infinity, Part II

For problems 1 – 6 evaluate (a) $\lim\limits_{x \to -\infty} f\left(x\right)$ and (b) $\lim\limits_{x \to \infty} f\left(x\right)$.

1. $f\left(x\right) = \mathbf{e}^{8+2x-x^3}$

2. $f\left(x\right) = \mathbf{e}^{\frac{6x^2+x}{5+3x}}$

3. $f\left(x\right) = 2\mathbf{e}^{6x} - \mathbf{e}^{-7x} - 10\mathbf{e}^{4x}$

4. $f\left(x\right) = 3\mathbf{e}^{-x} - 8\mathbf{e}^{-5x} - \mathbf{e}^{10x}$

5. $f\left(x\right) = \dfrac{\mathbf{e}^{-3x} - 2\mathbf{e}^{8x}}{9\mathbf{e}^{8x} - 7\mathbf{e}^{-3x}}$

6. $f\left(x\right) = \dfrac{\mathbf{e}^{-7x} - 2\mathbf{e}^{3x} - \mathbf{e}^{x}}{\mathbf{e}^{-x} + 16\mathbf{e}^{10x} + 2\mathbf{e}^{-4x}}$

For problems 7 – 12 evaluate the given limit.

7. $\lim\limits_{t \to -\infty} \ln\left(4 - 9t - t^3\right)$

8. $\lim\limits_{z \to -\infty} \ln\left(\dfrac{3z^4 - 8}{2 + z^2}\right)$

9. $\lim\limits_{x \to \infty} \ln\left(\dfrac{11 + 8x}{x^3 + 7x}\right)$

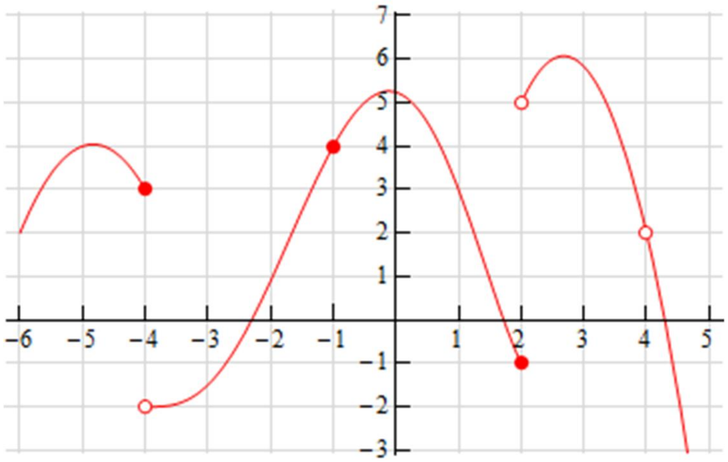
10. $\lim\limits_{x \to -\infty} \tan^{-1}\left(7 - x + 3x^5\right)$

11. $\lim\limits_{t \to \infty} \tan^{-1}\left(\dfrac{4 + 7t}{2 - t}\right)$

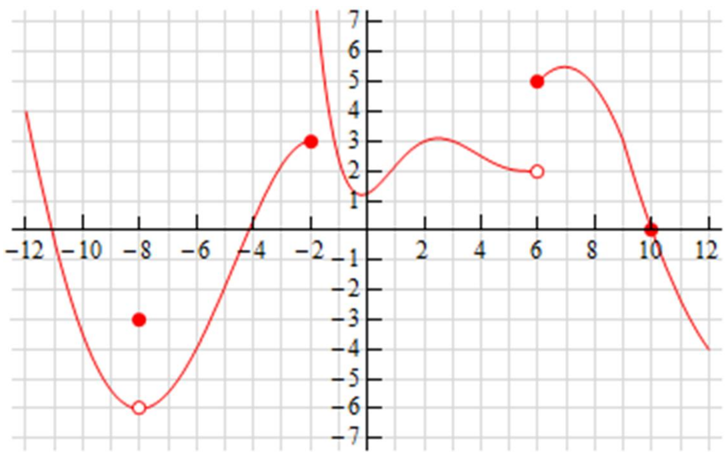
12. $\lim\limits_{w \to \infty} \tan^{-1}\left(\dfrac{3w^2 - 9w^4}{4w - w^3}\right)$

Section 2-9 : Continuity

1. The graph of $f\left(x\right)$ is given below. Based on this graph determine where the function is discontinuous.



2. The graph of $f\left(x\right)$ is given below. Based on this graph determine where the function is discontinuous.



For problems 3 – 7 using only Properties 1 – 9 from the Limit Properties section, one-sided limit properties (if needed) and the definition of continuity determine if the given function is continuous or discontinuous at the indicated points.

3. $f\left(x\right)=\dfrac{4x+5}{9-3x}$

(a) $x=-1$, (b) $x=0$, (c) $x=3$

4. $g\left(z\right)=\dfrac{6}{z^2-3z-10}$

(a) $z=-2$, (b) $z=0$, (c) $z=5$

5. $g\left(x\right)=\begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$
   (a) $x=4$, (b) $x=6$

6. $h\left(t\right)=\begin{cases} t^2 & t<-2 \\ t+6 & t\geq -2 \end{cases}$
   (a) $t=-2$, (b) $t=10$

7. $g\left(x\right)=\begin{cases} 1-3x & x<-6 \\ 7 & x=-6 \\ x^3 & -6<x<1 \\ 1 & x=1 \\ 2-x & x>1 \end{cases}$
   (a) $x=-6$, (b) $x=1$

For problems 8 – 12 determine where the given function is discontinuous.

8. $f\left(x\right)=\dfrac{x^2-9}{3x^2+2x-8}$

9. $R\left(t\right)=\dfrac{8t}{t^2-9t-1}$

10. $h\left(z\right)=\dfrac{1}{2-4\cos\left(3z\right)}$

11. $y\left(x\right)=\dfrac{x}{7-\mathbf{e}^{2x+3}}$

12. $g\left(x\right)=\tan\left(2x\right)$

For problems 13 – 15 use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.

13. $25-8x^2-x^3=0$ on $\left[-2,4\right]$

14. $w^2-4\ln\left(5w+2\right)=0$ on $\left[0,4\right]$

15. $4t+10\mathbf{e}^t-\mathbf{e}^{2t}=0$ on $\left[1,3\right]$

Section 2-10 : The Definition of the Limit

Use the definition of the limit to prove the following limits.

1. $\lim\limits_{x \to 3} x = 3$

2. $\lim\limits_{x \to -1} \left(x + 7 \right) = 6$

3. $\lim\limits_{x \to 2} x^2 = 4$

4. $\lim\limits_{x \to -3} \left(x^2 + 4x + 1 \right) = -2$

5. $\lim\limits_{x \to 1} \dfrac{1}{\left(x - 1 \right)^2} = \infty$

6. $\lim\limits_{x \to 0^-} \dfrac{1}{x} = -\infty$

7. $\lim\limits_{x \to \infty} \dfrac{1}{x^2} = 0$

Chapter 3 : Derivatives

Here are a set of practice problems for the Derivatives chapter of the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

7. If you'd like a pdf document containing the solutions the download tab on the website contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.

8. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

The Definition of the Derivative – In this section we define the derivative, give various notations for the derivative and work a few problems illustrating how to use the definition of the derivative to actually compute the derivative of a function.

Interpretation of the Derivative – In this section we give several of the more important interpretations of the derivative. We discuss the rate of change of a function, the velocity of a moving object and the slope of the tangent line to a graph of a function.

Differentiation Formulas – In this section we give most of the general derivative formulas and properties used when taking the derivative of a function. Examples in this section concentrate mostly on polynomials, roots and more generally variables raised to powers.

Product and Quotient Rule – In this section we will give two of the more important formulas for differentiating functions. We will discuss the Product Rule and the Quotient Rule allowing us to differentiate functions that, up to this point, we were unable to differentiate.

Derivatives of Trig Functions – In this section we will discuss differentiating trig functions. Derivatives of all six trig functions are given and we show the derivation of the derivative of sin(x) and tan(x).

Derivatives of Exponential and Logarithm Functions – In this section we derive the formulas for the derivatives of the exponential and logarithm functions.

Derivatives of Inverse Trig Functions – In this section we give the derivatives of all six inverse trig functions. We show the derivation of the formulas for inverse sine, inverse cosine and inverse tangent.

Derivatives of Hyperbolic Functions – In this section we define the hyperbolic functions, give the relationships between them and some of the basic facts involving hyperbolic functions. We also give the derivatives of each of the six hyperbolic functions and show the derivation of the formula for hyperbolic sine.

Chain Rule – In this section we discuss one of the more useful and important differentiation formulas, The Chain Rule. With the chain rule in hand we will be able to differentiate a much wider variety of functions. As you will see throughout the rest of your Calculus courses a great many of derivatives you take will involve the chain rule!

Implicit Differentiation – In this section we will discuss implicit differentiation. Not every function can be explicitly written in terms of the independent variable, e.g. y = f(x) and yet we will still need to know what f'(x) is. Implicit differentiation will allow us to find the derivative in these cases. Knowing implicit differentiation will allow us to do one of the more important applications of derivatives, Related Rates (the next section).

Related Rates – In this section we will discuss the only application of derivatives in this section, Related Rates. In related rates problems we are give the rate of change of one quantity in a problem and asked to determine the rate of one (or more) quantities in the problem. This is often one of the more difficult sections for students. We work quite a few problems in this section so hopefully by the end of this section you will get a decent understanding on how these problems work.

Higher Order Derivatives – In this section we define the concept of higher order derivatives and give a quick application of the second order derivative and show how implicit differentiation works for higher order derivatives.

Logarithmic Differentiation – In this section we will discuss logarithmic differentiation. Logarithmic differentiation gives an alternative method for differentiating products and quotients (sometimes easier than using product and quotient rule). More importantly, however, is the fact that logarithm differentiation allows us to differentiate functions that are in the form of one function raised to another function, i.e. there are variables in both the base and exponent of the function.

Section 3-1 : The Definition of the Derivative

Use the definition of the derivative to find the derivative of the following functions.

1. $f(x) = 6$

2. $V(t) = 3 - 14t$

3. $g(x) = x^2$

4. $Q(t) = 10 + 5t - t^2$

5. $W(z) = 4z^2 - 9z$

6. $f(x) = 2x^3 - 1$

7. $g(x) = x^3 - 2x^2 + x - 1$

8. $R(z) = \dfrac{5}{z}$

9. $V(t) = \dfrac{t+1}{t+4}$

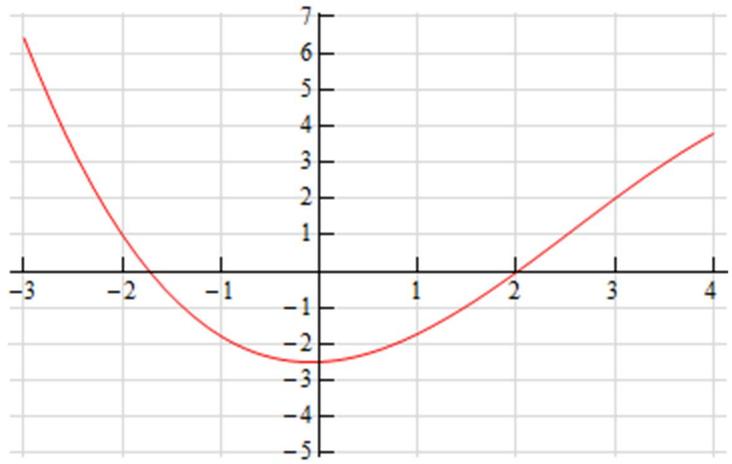
10. $Z(t) = \sqrt{3t - 4}$

11. $f(x) = \sqrt{1 - 9x}$

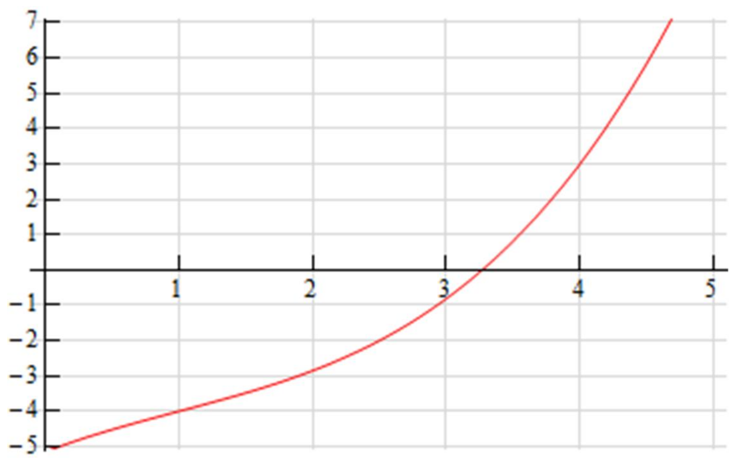
Section 3-2 : Interpretation of the Derivative

For problems 1 and 2 use the graph of the function, $f\left(x\right)$, estimate the value of $f'\left(a\right)$ for the given values of *a*.

1. (a) $a = -2$ (b) $a = 3$



2. (a) $a = 1$ (b) $a = 4$



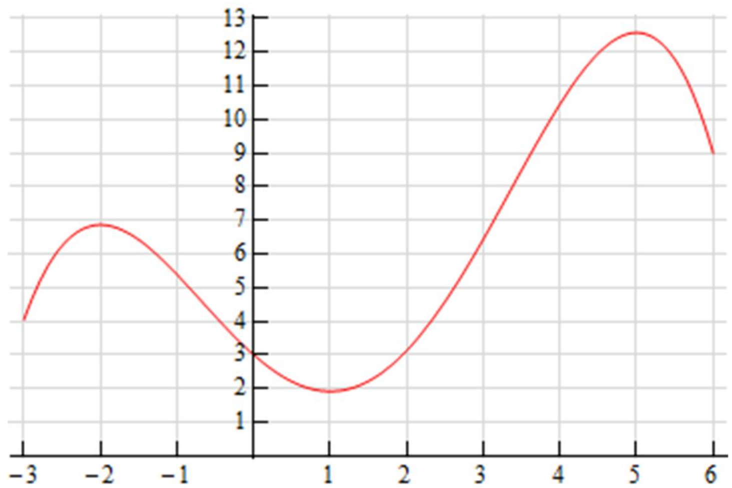
For problems 3 and 4 sketch the graph of a function that satisfies the given conditions.

3. $f\left(1\right) = 3$, $f'\left(1\right) = 1$, $f\left(4\right) = 5$, $f'\left(4\right) = -2$

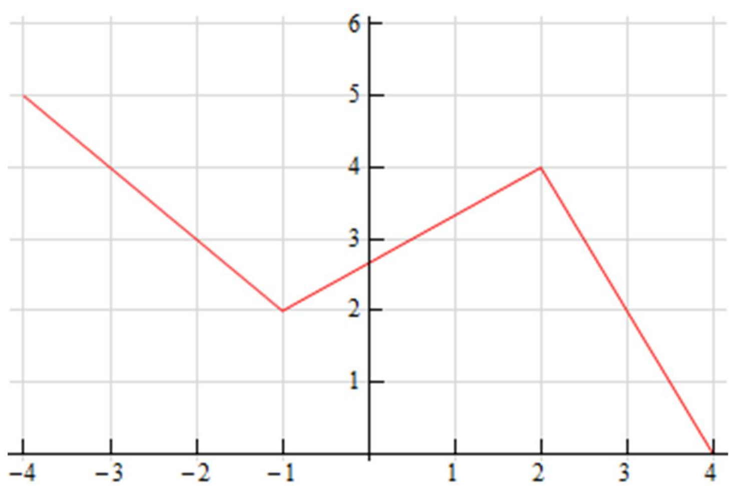
4. $f\left(-3\right) = 5$, $f'\left(-3\right) = -2$, $f\left(1\right) = 2$, $f'\left(1\right) = 0$, $f\left(4\right) = -2$, $f'\left(4\right) = -3$

For problems 5 and 6 the graph of a function, $f(x)$, is given. Use this to sketch the graph of the derivative, $f'(x)$.

5.



6.



7. Answer the following questions about the function $W(z) = 4z^2 - 9z$.

(a) Is the function increasing or decreasing at $z = -1$?

(b) Is the function increasing or decreasing at $z = 2$?

(c) Does the function ever stop changing? If yes, at what value(s) of *z* does the function stop changing?

8. What is the equation of the tangent line to $f(x) = 3 - 14x$ at $x = 8$.

9. The position of an object at any time *t* is given by $s\left(t\right)=\frac{t+1}{t+4}$.

(a) Determine the velocity of the object at any time *t*.
(b) Does the object ever stop moving? If yes, at what time(s) does the object stop moving?

10. What is the equation of the tangent line to $f\left(x\right)=\frac{5}{x}$ at $x=\frac{1}{2}$?

11. Determine where, if anywhere, the function $g\left(x\right)=x^3-2x^2+x-1$ stops changing.

12. Determine if the function $Z\left(t\right)=\sqrt{3t-4}$ increasing or decreasing at the given points.

(a) $t=5$
(b) $t=10$
(c) $t=300$

13. Suppose that the volume of water in a tank for $0\le t\le 6$ is given by $Q\left(t\right)=10+5t-t^2$.

(a) Is the volume of water increasing or decreasing at $t=0$?
(b) Is the volume of water increasing or decreasing at $t=6$?
(c) Does the volume of water ever stop changing? If yes, at what times(s) does the volume stop changing?

Section 3-3 : Differentiation Formulas

For problems 1 – 12 find the derivative of the given function.

1. $f(x) = 6x^3 - 9x + 4$

2. $y = 2t^4 - 10t^2 + 13t$

3. $g(z) = 4z^7 - 3z^{-7} + 9z$

4. $h(y) = y^{-4} - 9y^{-3} + 8y^{-2} + 12$

5. $y = \sqrt{x} + 8\sqrt[3]{x} - 2\sqrt[4]{x}$

6. $f(x) = 10\sqrt[5]{x^3} - \sqrt{x^7} + 6\sqrt[3]{x^8} - 3$

7. $f(t) = \dfrac{4}{t} - \dfrac{1}{6t^3} + \dfrac{8}{t^5}$

8. $R(z) = \dfrac{6}{\sqrt{z^3}} + \dfrac{1}{8z^4} - \dfrac{1}{3z^{10}}$

9. $z = x(3x^2 - 9)$

10. $g(y) = (y-4)(2y + y^2)$

11. $h(x) = \dfrac{4x^3 - 7x + 8}{x}$

12. $f(y) = \dfrac{y^5 - 5y^3 + 2y}{y^3}$

13. Determine where, if anywhere, the function $f(x) = x^3 + 9x^2 - 48x + 2$ is not changing.

14. Determine where, if anywhere, the function $y = 2z^4 - z^3 - 3z^2$ is not changing.

15. Find the tangent line to $g(x) = \dfrac{16}{x} - 4\sqrt{x}$ at $x = 4$.

16. Find the tangent line to $f\left(x\right)=7x^4+8x^{-6}+2x$ at $x=-1$.

17. The position of an object at any time t is given by $s\left(t\right)=3t^4-40t^3+126t^2-9$.

(a) Determine the velocity of the object at any time t.
(b) Does the object ever stop changing?
(c) When is the object moving to the right and when is the object moving to the left?

18. Determine where the function $h\left(z\right)=6+40z^3-5z^4-4z^5$ is increasing and decreasing.

19. Determine where the function $R\left(x\right)=\left(x+1\right)\left(x-2\right)^2$ is increasing and decreasing.

20. Determine where, if anywhere, the tangent line to $f\left(x\right)=x^3-5x^2+x$ is parallel to the line $y=4x+23$.

Section 3-4 : Product and Quotient Rule

For problems 1 – 6 use the Product Rule or the Quotient Rule to find the derivative of the given function.

1. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

2. $y = \left(1 + \sqrt{x^3}\right)\left(x^{-3} - 2\sqrt[3]{x}\right)$

3. $h(z) = (1 + 2z + 3z^2)(5z + 8z^2 - z^3)$

4. $g(x) = \dfrac{6x^2}{2 - x}$

5. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

6. $f(x) = \dfrac{\sqrt{x} + 2x}{7x - 4x^2}$

7. If $f(2) = -8$, $f'(2) = 3$, $g(2) = 17$ and $g'(2) = -4$ determine the value of $(f\,g)'(2)$.

8. If $f(x) = x^3 g(x)$, $g(-7) = 2$, $g'(-7) = -9$ determine the value of $f'(-7)$.

9. Find the equation of the tangent line to $f(x) = \left(1 + 12\sqrt{x}\right)(4 - x^2)$ at $x = 9$.

10. Determine where $f(x) = \dfrac{x - x^2}{1 + 8x^2}$ is increasing and decreasing.

11. Determine where $V(t) = (4 - t^2)(1 + 5t^2)$ is increasing and decreasing.

Section 3-5 : Derivatives of Trig Functions

For problems 1 – 3 evaluate the given limit.

1. $\displaystyle\lim_{z \to 0} \frac{\sin(10z)}{z}$

2. $\displaystyle\lim_{\alpha \to 0} \frac{\sin(12\alpha)}{\sin(5\alpha)}$

3. $\displaystyle\lim_{x \to 0} \frac{\cos(4x) - 1}{x}$

For problems 4 – 10 differentiate the given function.

4. $f(x) = 2\cos(x) - 6\sec(x) + 3$

5. $g(z) = 10\tan(z) - 2\cot(z)$

6. $f(w) = \tan(w)\sec(w)$

7. $h(t) = t^3 - t^2 \sin(t)$

8. $y = 6 + 4\sqrt{x}\csc(x)$

9. $R(t) = \dfrac{1}{2\sin(t) - 4\cos(t)}$

10. $Z(v) = \dfrac{v + \tan(v)}{1 + \csc(v)}$

11. Find the tangent line to $f(x) = \tan(x) + 9\cos(x)$ at $x = \pi$.

12. The position of an object is given by $s(t) = 2 + 7\cos(t)$ determine all the points where the object is not moving.

13. Where in the range $[-2, 7]$ is the function $f(x) = 4\cos(x) - x$ is increasing and decreasing.

Section 3-6 : Derivatives of Exponential and Logarithm Functions

For problems 1 – 6 differentiate the given function.

1. $f(x) = 2\mathbf{e}^x - 8^x$

2. $g(t) = 4\log_3(t) - \ln(t)$

3. $R(w) = 3^w \log(w)$

4. $y = z^5 - \mathbf{e}^z \ln(z)$

5. $h(y) = \dfrac{y}{1-\mathbf{e}^y}$

6. $f(t) = \dfrac{1+5t}{\ln(t)}$

7. Find the tangent line to $f(x) = 7^x + 4\mathbf{e}^x$ at $x = 0$.

8. Find the tangent line to $f(x) = \ln(x)\log_2(x)$ at $x = 2$.

9. Determine if $V(t) = \dfrac{t}{\mathbf{e}^t}$ is increasing or decreasing at the following points.

(a) $t = -4$ (b) $t = 0$ (c) $t = 10$

10. Determine if $G(z) = (z-6)\ln(z)$ is increasing or decreasing at the following points.

(a) $z = 1$ (b) $z = 5$ (c) $z = 20$

Section 3-7 : Derivatives of Inverse Trig Functions

For each of the following problems differentiate the given function.

1. $T\left(z\right) = 2\cos\left(z\right) + 6\cos^{-1}\left(z\right)$

2. $g\left(t\right) = \csc^{-1}\left(t\right) - 4\cot^{-1}\left(t\right)$

3. $y = 5x^6 - \sec^{-1}\left(x\right)$

4. $f\left(w\right) = \sin\left(w\right) + w^2\tan^{-1}\left(w\right)$

5. $h\left(x\right) = \dfrac{\sin^{-1}\left(x\right)}{1+x}$

Section 3-8 : Derivatives of Hyperbolic Functions

For each of the following problems differentiate the given function.

1. $f\left(x\right)=\sinh\left(x\right)+2\cosh\left(x\right)-\operatorname{sech}\left(x\right)$

2. $R\left(t\right)=\tan\left(t\right)+t^{2}\operatorname{csch}\left(t\right)$

3. $g\left(z\right)=\dfrac{z+1}{\tanh\left(z\right)}$

Section 3-9 : Chain Rule

For problems 1 – 27 differentiate the given function.

1. $f(x) = \left(6x^2 + 7x\right)^4$

2. $g(t) = \left(4t^2 - 3t + 2\right)^{-2}$

3. $y = \sqrt[3]{1 - 8z}$

4. $R(w) = \csc(7w)$

5. $G(x) = 2\sin\left(3x + \tan(x)\right)$

6. $h(u) = \tan(4 + 10u)$

7. $f(t) = 5 + \mathbf{e}^{4t + t^7}$

8. $g(x) = \mathbf{e}^{1 - \cos(x)}$

9. $H(z) = 2^{1 - 6z}$

10. $u(t) = \tan^{-1}(3t - 1)$

11. $F(y) = \ln\left(1 - 5y^2 + y^3\right)$

12. $V(x) = \ln\left(\sin(x) - \cot(x)\right)$

13. $h(z) = \sin\left(z^6\right) + \sin^6(z)$

14. $S(w) = \sqrt{7w} + \mathbf{e}^{-w}$

15. $g(z) = 3z^7 - \sin\left(z^2 + 6\right)$

16. $f(x) = \ln\left(\sin(x)\right) - \left(x^4 - 3x\right)^{10}$

17. $h(t) = t^6\sqrt{5t^2 - t}$

18. $q(t) = t^2 \ln\left(t^5\right)$

19. $g(w) = \cos(3w)\sec(1-w)$

20. $y = \dfrac{\sin(3t)}{1+t^2}$

21. $K(x) = \dfrac{1+\mathbf{e}^{-2x}}{x+\tan(12x)}$

22. $f(x) = \cos\left(x^2\mathbf{e}^x\right)$

23. $z = \sqrt{5x+\tan(4x)}$

24. $f(t) = \left(\mathbf{e}^{-6t}+\sin(2-t)\right)^3$

25. $g(x) = \left(\ln\left(x^2+1\right)-\tan^{-1}(6x)\right)^{10}$

26. $h(z) = \tan^4\left(z^2+1\right)$

27. $f(x) = \left(\sqrt[3]{12x}+\sin^2(3x)\right)^{-1}$

28. Find the tangent line to $f(x) = 4\sqrt{2x}-6\mathbf{e}^{2-x}$ at $x=2$.

29. Determine where $V(z) = z^4(2z-8)^3$ is increasing and decreasing.

30. The position of an object is given by $s(t) = \sin(3t)-2t+4$. Determine where in the interval $[0,3]$ the object is moving to the right and moving to the left.

31. Determine where $A(t) = t^2\mathbf{e}^{5-t}$ is increasing and decreasing.

32. Determine where in the interval $[-1,20]$ the function $f(x) = \ln\left(x^4+20x^3+100\right)$ is increasing and decreasing.

Section 3-10 : Implicit Differentiation

For problems 1 – 3 do each of the following.

(a) Find y' by solving the equation for *y* and differentiating directly.
(b) Find y' by implicit differentiation.
(c) Check that the derivatives in (a) and (b) are the same.

1. $\dfrac{x}{y^3} = 1$

2. $x^2 + y^3 = 4$

3. $x^2 + y^2 = 2$

For problems 4 – 9 find y' by implicit differentiation.

4. $2y^3 + 4x^2 - y = x^6$

5. $7y^2 + \sin\left(3x\right) = 12 - y^4$

6. $\mathbf{e}^x - \sin\left(y\right) = x$

7. $4x^2y^7 - 2x = x^5 + 4y^3$

8. $\cos\left(x^2 + 2y\right) + x\,\mathbf{e}^{y^2} = 1$

9. $\tan\left(x^2y^4\right) = 3x + y^2$

For problems 10 & 11 find the equation of the tangent line at the given point.

10. $x^4 + y^2 = 3$ at $\left(1, -\sqrt{2}\right)$.

11. $y^2\mathbf{e}^{2x} = 3y + x^2$ at $\left(0, 3\right)$.

For problems 12 & 13 assume that $x = x\left(t\right)$, $y = y\left(t\right)$ and $z = z\left(t\right)$ then differentiate the given equation with respect to *t*.

12. $x^2 - y^3 + z^4 = 1$

13. $x^2\cos(y) = \sin(y^3 + 4z)$

Section 3-11 : Related Rates

1. In the following assume that *x* and *y* are both functions of *t*. Given $x = -2$, $y = 1$ and $x' = -4$ determine y' for the following equation.

$$6y^2 + x^2 = 2 - x^3\mathbf{e}^{4-4y}$$

2. In the following assume that *x*, *y* and *z* are all functions of *t*. Given $x = 4$, $y = -2$, $z = 1$, $x' = 9$ and $y' = -3$ determine z' for the following equation.

$$x(1-y) + 5z^3 = y^2z^2 + x^2 - 3$$

3. For a certain rectangle the length of one side is always three times the length of the other side.

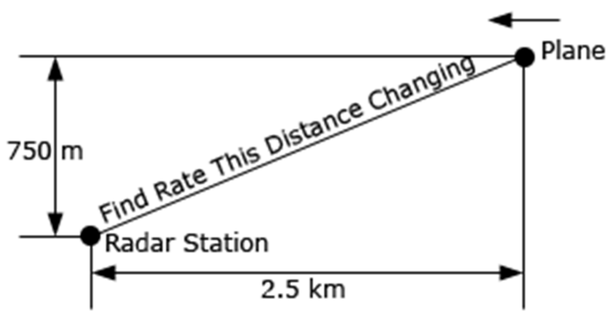
(a) If the shorter side is decreasing at a rate of 2 inches/minute at what rate is the longer side decreasing?

(b) At what rate is the enclosed area decreasing when the shorter side is 6 inches long and is decreasing at a rate of 2 inches/minute?

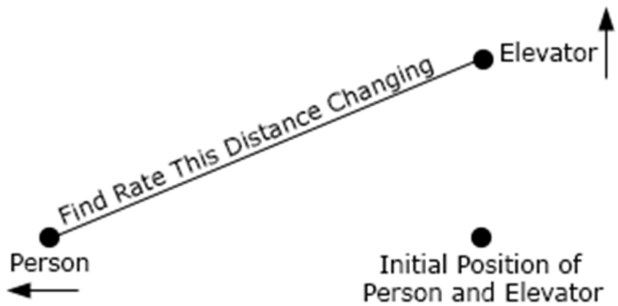
4. A thin sheet of ice is in the form of a circle. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of 0.5 m^2/sec at what rate is the radius decreasing when the area of the sheet is 12 m^2 ?

5. A person is standing 350 feet away from a model rocket that is fired straight up into the air at a rate of 15 ft/sec. At what rate is the distance between the person and the rocket increasing (a) 20 seconds after liftoff? (b) 1 minute after liftoff?

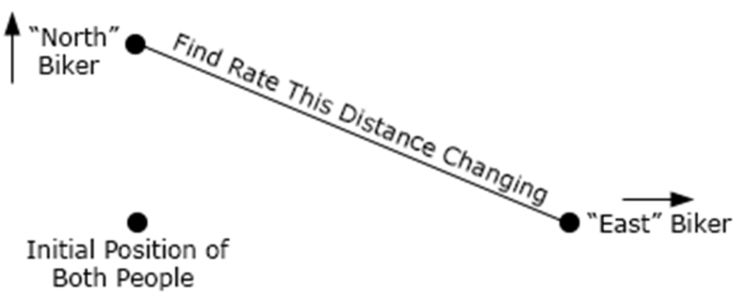
6. A plane is 750 meters in the air flying parallel to the ground at a speed of 100 m/s and is initially 2.5 kilometers away from a radar station. At what rate is the distance between the plane and the radar station changing (a) initially and (b) 30 seconds after it passes over the radar station?



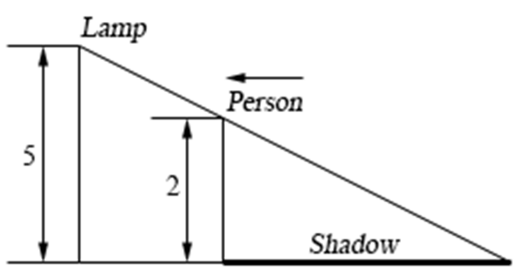
7. Two people are at an elevator. At the same time one person starts to walk away from the elevator at a rate of 2 ft/sec and the other person starts going up in the elevator at a rate of 7 ft/sec. What rate is the distance between the two people changing 15 seconds later?



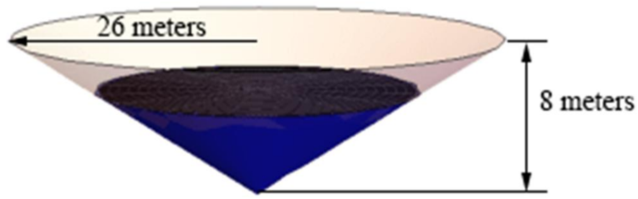
8. Two people on bikes are at the same place. One of the bikers starts riding directly north at a rate of 8 m/sec. Five seconds after the first biker started riding north the second starts to ride directly east at a rate of 5 m/sec. At what rate is the distance between the two riders increasing 20 seconds after the second person started riding?



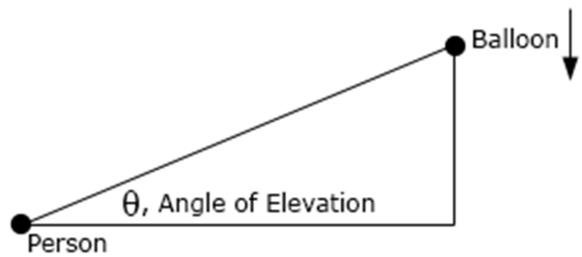
9. A light is mounted on a wall 5 meters above the ground. A 2 meter tall person is initially 10 meters from the wall and is moving towards the wall at a rate of 0.5 m/sec. After 4 seconds of moving is the tip of the shadow moving (a) towards or away from the person and (b) towards or away from the wall?



10. A tank of water in the shape of a cone is being filled with water at a rate of 12 m^3/sec. The base radius of the tank is 26 meters and the height of the tank is 8 meters. At what rate is the depth of the water in the tank changing when the radius of the top of the water is 10 meters? Note the image below is not completely to scale….



11. The angle of elevation is the angle formed by a horizontal line and a line joining the observer's eye to an object above the horizontal line. A person is 500 feet way from the launch point of a hot air balloon. The hot air balloon is starting to come back down at a rate of 15 ft/sec. At what rate is the angle of elevation, θ, changing when the hot air balloon is 200 feet above the ground.



Section 3-12 : Higher Order Derivatives

For problems 1 – 5 determine the fourth derivative of the given function.

1. $h(t) = 3t^7 - 6t^4 + 8t^3 - 12t + 18$

2. $V(x) = x^3 - x^2 + x - 1$

3. $f(x) = 4\sqrt[5]{x^3} - \dfrac{1}{8x^2} - \sqrt{x}$

4. $f(w) = 7\sin\left(\frac{w}{3}\right) + \cos(1 - 2w)$

5. $y = \mathbf{e}^{-5z} + 8\ln\left(2z^4\right)$

For problems 6 – 9 determine the second derivative of the given function.

6. $g(x) = \sin\left(2x^3 - 9x\right)$

7. $z = \ln\left(7 - x^3\right)$

8. $Q(v) = \dfrac{2}{\left(6 + 2v - v^2\right)^4}$

9. $H(t) = \cos^2(7t)$

For problems 10 & 11 determine the second derivative of the given function.

10. $2x^3 + y^2 = 1 - 4y$

11. $6y - xy^2 = 1$

Section 3-13 : Logarithmic Differentiation

For problems 1 – 3 use logarithmic differentiation to find the first derivative of the given function.

1. $f(x) = (5-3x^2)^7 \sqrt{6x^2+8x-12}$

2. $y = \dfrac{\sin(3z+z^2)}{(6-z^4)^3}$

3. $h(t) = \dfrac{\sqrt{5t+8}\ \sqrt[3]{1-9\cos(4t)}}{\sqrt[4]{t^2+10t}}$

For problems 4 & 5 find the first derivative of the given function.

4. $g(w) = (3w-7)^{4w}$

5. $f(x) = (2x-\mathbf{e}^{8x})^{\sin(2x)}$

Chapter 4 : Applications of Derivatives

Here are a set of practice problems for the Applications of Derivatives chapter of the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

9. If you'd like a pdf document containing the solutions the download tab on the website contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.

10. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Rates of Change – In this section we review the main application/interpretation of derivatives from the previous chapter (i.e. rates of change) that we will be using in many of the applications in this chapter.

Critical Points – In this section we give the definition of critical points. Critical points will show up in most of the sections in this chapter, so it will be important to understand them and how to find them. We will work a number of examples illustrating how to find them for a wide variety of functions.

Minimum and Maximum Values – In this section we define absolute (or global) minimum and maximum values of a function and relative (or local) minimum and maximum values of a function. It is important to understand the difference between the two types of minimum/maximum (collectively called extrema) values for many of the applications in this chapter and so we use a variety of examples to help with this. We also give the Extreme Value Theorem and Fermat's Theorem, both of which are very important in the many of the applications we'll see in this chapter.

Finding Absolute Extrema – In this section we discuss how to find the absolute (or global) minimum and maximum values of a function. In other words, we will be finding the largest and smallest values that a function will have.

The Shape of a Graph, Part I – In this section we will discuss what the first derivative of a function can tell us about the graph of a function. The first derivative will allow us to identify the relative (or local) minimum and maximum values of a function and where a function will be increasing and decreasing. We will also give the First Derivative test which will allow us to classify critical points as relative minimums, relative maximums or neither a minimum or a maximum.

The Shape of a Graph, Part II – In this section we will discuss what the second derivative of a function can tell us about the graph of a function. The second derivative will allow us to determine where the graph of a function is concave up and concave down. The second derivative will also allow us to identify any inflection points (i.e. where concavity changes) that a function may have. We will also give the Second Derivative Test that will give an alternative method for identifying some critical points (but not all) as relative minimums or relative maximums.

The Mean Value Theorem – In this section we will give Rolle's Theorem and the Mean Value Theorem. With the Mean Value Theorem we will prove a couple of very nice facts, one of which will be very useful in the next chapter.

Optimization Problems – In this section we will be determining the absolute minimum and/or maximum of a function that depends on two variables given some constraint, or relationship, that the two variables must always satisfy. We will discuss several methods for determining the absolute minimum or maximum of the function. Examples in this section tend to center around geometric objects such as squares, boxes, cylinders, etc.

More Optimization Problems – In this section we will continue working optimization problems. The examples in this section tend to be a little more involved and will often involve situations that will be more easily described with a sketch as opposed to the 'simple' geometric objects we looked at in the previous section.

L'Hospital's Rule and Indeterminate Forms – In this section we will revisit indeterminate forms and limits and take a look at L'Hospital's Rule. L'Hospital's Rule will allow us to evaluate some limits we were not able to previously.

Linear Approximations – In this section we discuss using the derivative to compute a linear approximation to a function. We can use the linear approximation to a function to approximate values of the function at certain points. While it might not seem like a useful thing to do with when we have the function there really are reasons that one might want to do this. We give two ways this can be useful in the examples.

Differentials – In this section we will compute the differential for a function. We will give an application of differentials in this section. However, one of the more important uses of differentials will come in the next chapter and unfortunately we will not be able to discuss it until then.

Newton's Method – In this section we will discuss Newton's Method. Newton's Method is an application of derivatives that will allow us to approximate solutions to an equation. There are many equations that cannot be solved directly and with this method we can get approximations to the solutions to many of those equations.

Business Applications – In this section we will give a cursory discussion of some basic applications of derivatives to the business field. We will revisit finding the maximum and/or minimum function value and we will define the marginal cost function, the average cost, the revenue function, the marginal revenue function and the marginal profit function. Note that this section is only intended to introduce these concepts and not teach you everything about them.

Section 4-1 : Rates of Change

As noted in the text for this section the purpose of this section is only to remind you of certain types of applications that were discussed in the previous chapter. As such there aren't any problems written for this section. Instead here is a list of links (note that these will only be active links in the web version and not the pdf version) to problems from the relevant sections from the previous chapter.

Each of the following sections has a selection of increasing/decreasing problems towards the bottom of the problem set.

Differentiation Formulas
Product & Quotient Rules
Derivatives of Trig Functions
Derivatives of Exponential and Logarithm Functions
Chain Rule

Related Rates problems are in the Related Rates section.

Section 4-2 : Critical Points

Determine the critical points of each of the following functions.

1. $f(x) = 8x^3 + 81x^2 - 42x - 8$

2. $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$

3. $g(w) = 2w^3 - 7w^2 - 3w - 2$

4. $g(x) = x^6 - 2x^5 + 8x^4$

5. $h(z) = 4z^3 - 3z^2 + 9z + 12$

6. $Q(x) = (2 - 8x)^4 (x^2 - 9)^3$

7. $f(z) = \dfrac{z+4}{2z^2 + z + 8}$

8. $R(x) = \dfrac{1-x}{x^2 + 2x - 15}$

9. $r(y) = \sqrt[5]{y^2 - 6y}$

10. $h(t) = 15 - (3-t)\left[t^2 - 8t + 7\right]^{\frac{1}{3}}$

11. $s(z) = 4\cos(z) - z$

12. $f(y) = \sin\left(\frac{y}{3}\right) + \frac{2y}{9}$

13. $V(t) = \sin^2(3t) + 1$

14. $f(x) = 5x\,\mathbf{e}^{9-2x}$

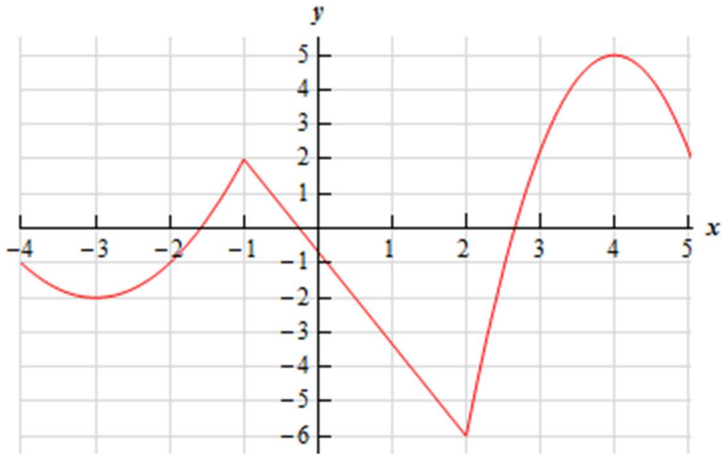
15. $g(w) = \mathbf{e}^{w^3 - 2w^2 - 7w}$

16. $R(x) = \ln(x^2 + 4x + 14)$

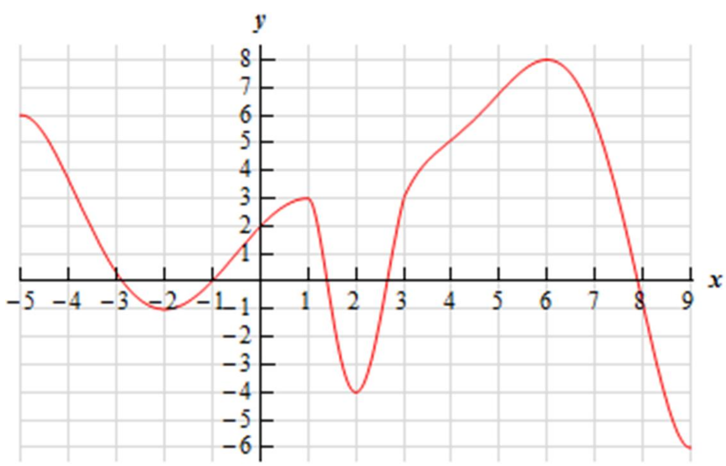
17. $A(t) = 3t - 7\ln(8t + 2)$

Section 4-3 : Minimum and Maximum Values

1. Below is the graph of some function, $f(x)$. Identify all of the relative extrema and absolute extrema of the function.



2. Below is the graph of some function, $f(x)$. Identify all of the relative extrema and absolute extrema of the function.



3. Sketch the graph of $g(x) = x^2 - 4x$ and identify all the relative extrema and absolute extrema of the function on each of the following intervals.

(a) $(-\infty, \infty)$

(b) $[-1, 4]$

(c) $[1, 3]$

(d) $[3, 5]$

(e) $(-1, 5]$

4. Sketch the graph of $h(x) = -(x+4)^3$ and identify all the relative extrema and absolute extrema of the function on each of the following intervals.

(a) $(-\infty, \infty)$

(b) $[-5.5, -2]$

(c) $[-4, -3)$

(d) $[-4, -3]$

5. Sketch the graph of some function on the interval $[1, 6]$ that has an absolute maximum at $x = 6$ and an absolute minimum at $x = 3$.

6. Sketch the graph of some function on the interval $[-4, 3]$ that has an absolute maximum at $x = -3$ and an absolute minimum at $x = 2$.

7. Sketch the graph of some function that meets the following conditions :

(a) The function is continuous.
(b) Has two relative minimums.
(c) One of relative minimums is also an absolute minimum and the other relative minimum is not an absolute minimum.
(d) Has one relative maximum.
(e) Has no absolute maximum.

Section 4-4 : Finding Absolute Extrema

For each of the following problems determine the absolute extrema of the given function on the specified interval.

1. $f(x) = 8x^3 + 81x^2 - 42x - 8$ on $[-8, 2]$

2. $f(x) = 8x^3 + 81x^2 - 42x - 8$ on $[-4, 2]$

3. $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$ on $[-4.5,\ 4]$

4. $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$ on $[0, 7]$

5. $h(z) = 4z^3 - 3z^2 + 9z + 12$ on $[-2, 1]$

6. $g(x) = 3x^4 - 26x^3 + 60x^2 - 11$ on $[1, 5]$

7. $Q(x) = (2 - 8x)^4 (x^2 - 9)^3$ on $[-3, 3]$

8. $h(w) = 2w^3 (w + 2)^5$ on $\left[-\frac{5}{2}, \frac{1}{2}\right]$

9. $f(z) = \dfrac{z + 4}{2z^2 + z + 8}$ on $[-10, 0]$

10. $A(t) = t^2 (10 - t)^{\frac{2}{3}}$ on $[2,\ 10.5]$

11. $f(y) = \sin\left(\frac{y}{3}\right) + \frac{2y}{9}$ on $[-10, 15]$

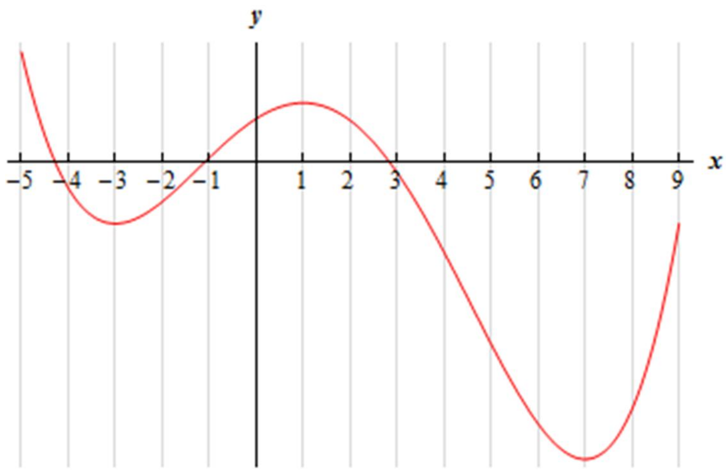
12. $g(w) = \mathbf{e}^{w^3 - 2w^2 - 7w}$ on $\left[-\frac{1}{2},\ \frac{5}{2}\right]$

13. $R(x) = \ln\left(x^2 + 4x + 14\right)$ on $[-4, 2]$

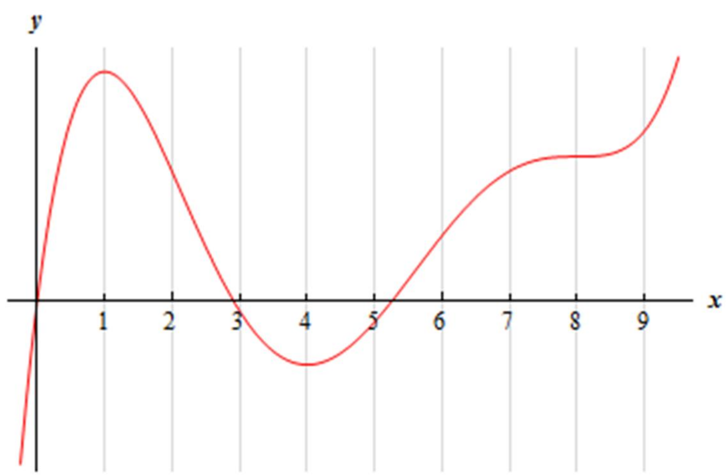
Section 4-5 : The Shape of a Graph, Part I

For problems 1 & 2 the graph of a function is given. Determine the intervals on which the function increases and decreases.

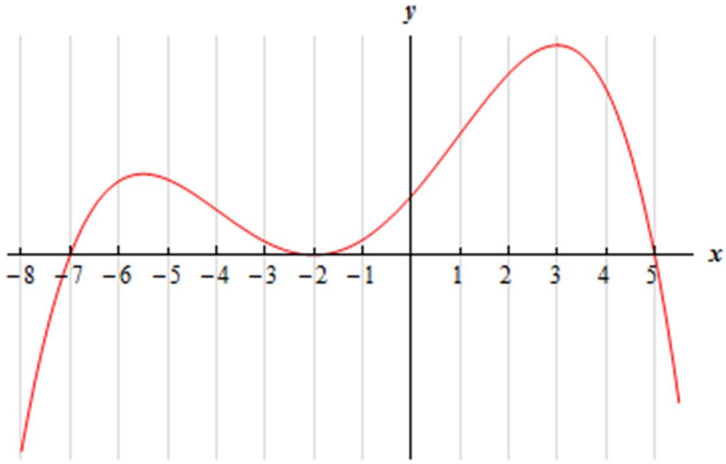
1.



2.



3. Below is the graph of the derivative of a function. From this graph determine the intervals in which the function increases and decreases.



4. This problem is about some function. All we know about the function is that it exists everywhere and we also know the information given below about the derivative of the function. Answer each of the following questions about this function.

(a) Identify the critical points of the function.
(b) Determine the intervals on which the function increases and decreases.
(c) Classify the critical points as relative maximums, relative minimums or neither.

$$f'(-5)=0 \qquad f'(-2)=0 \qquad f'(4)=0 \qquad f'(8)=0$$

$$f'(x)<0 \quad \text{on} \quad (-5,-2),\ (-2,4),\ (8,\infty) \qquad\qquad f'(x)>0 \quad \text{on} \quad (-\infty,-5),\ (4,8)$$

For problems 5 – 12 answer each of the following.

(a) Identify the critical points of the function.
(b) Determine the intervals on which the function increases and decreases.
(c) Classify the critical points as relative maximums, relative minimums or neither.

5. $f(x)=2x^3-9x^2-60x$

6. $h(t)=50+40t^3-5t^4-4t^5$

7. $y=2x^3-10x^2+12x-12$

8. $p(x)=\cos(3x)+2x$ on $\left[-\frac{3}{2},\, 2\right]$

9. $R(z)=2-5z-14\sin\left(\frac{z}{2}\right)$ on $[-10,7]$

10. $h(t)=t^2\sqrt[3]{t-7}$

11. $f(w) = w\mathbf{e}^{2-\frac{1}{2}w^2}$

12. $g(x) = x - 2\ln(1+x^2)$

13. For some function, $f(x)$, it is known that there is a relative maximum at $x = 4$. Answer each of the following questions about this function.

(a) What is the simplest form for the derivative of this function? Note : There really are many possible forms of the derivative so to make the rest of this problem as simple as possible you will want to use the simplest form of the derivative that you can come up with.

(b) Using your answer from (a) determine the most general form of the function.

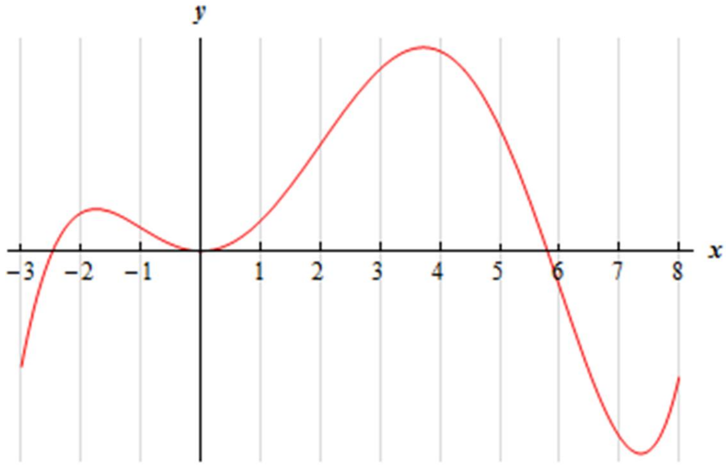
(c) Given that $f(4) = 1$ find a function that will have a relative maximum at $x = 4$. Note : You should be able to use your answer from (b) to determine an answer to this part.

14. Given that $f(x)$ and $g(x)$ are increasing functions. If we define $h(x) = f(x) + g(x)$ show that $h(x)$ is an increasing function.

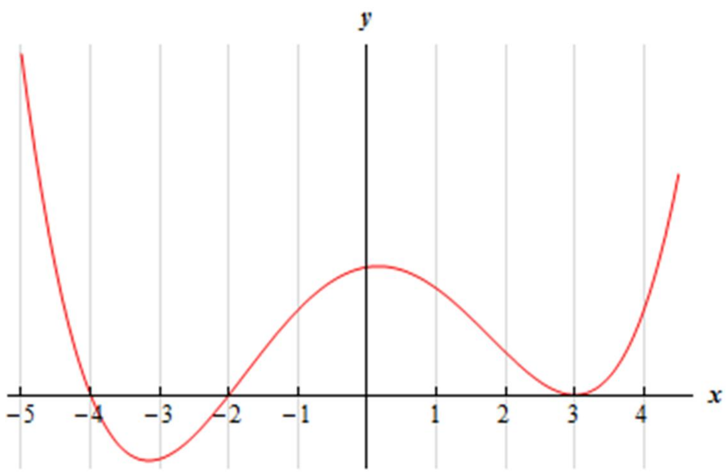
15. Given that $f(x)$ is an increasing function and define $h(x) = \left[f(x)\right]^2$. Will $h(x)$ be an increasing function? If yes, prove that $h(x)$ is an increasing function. If not, can you determine any other conditions needed on the function $f(x)$ that will guarantee that $h(x)$ will also increase?

Section 4-6 : The Shape of a Graph, Part I

1. The graph of a function is given below. Determine the intervals on which the function is concave up and concave down.



2. Below is the graph the 2nd derivative of a function. From this graph determine the intervals in which the function is concave up and concave down.



For problems 3 – 8 answer each of the following.

(a) Determine a list of possible inflection points for the function.
(b) Determine the intervals on which the function is concave up and concave down.
(c) Determine the inflection points of the function.

3. $f(x) = 12 + 6x^2 - x^3$

4. $g(z) = z^4 - 12z^3 + 84z + 4$

5. $h(t) = t^4 + 12t^3 + 6t^2 - 36t + 2$

6. $h(w) = 8 - 5w + 2w^2 - \cos(3w)$ on $[-1, 2]$

7. $R(z) = z(z+4)^{\frac{2}{3}}$

8. $h(x) = \mathbf{e}^{4-x^2}$

For problems 9 – 14 answer each of the following.

(a) Identify the critical points of the function.
(b) Determine the intervals on which the function increases and decreases.
(c) Classify the critical points as relative maximums, relative minimums or neither.
(d) Determine the intervals on which the function is concave up and concave down.
(e) Determine the inflection points of the function.
(f) Use the information from steps (a) – (e) to sketch the graph of the function.

9. $g(t) = t^5 - 5t^4 + 8$

10. $f(x) = 5 - 8x^3 - x^4$

11. $h(z) = z^4 - 2z^3 - 12z^2$

12. $Q(t) = 3t - 8\sin\left(\frac{t}{2}\right)$ on $[-7, 4]$

13. $f(x) = x^{\frac{4}{3}}(x-2)$

14. $P(w) = w\mathbf{e}^{4w}$

15. Determine the minimum degree of a polynomial that has exactly one inflection point.

16. Suppose that we know that $f(x)$ is a polynomial with critical points $x = -1$, $x = 2$ and $x = 6$. If we also know that the 2nd derivative is $f''(x) = -3x^2 + 14x - 4$. If possible, classify each of the critical points as relative minimums, relative maximums. If it is not possible to classify the critical points clearly explain why they cannot be classified.

Section 4-7 : The Mean Value Theorem

For problems 1 & 2 determine all the number(s) *c* which satisfy the conclusion of Rolle's Theorem for the given function and interval.

1. $f(x) = x^2 - 2x - 8$ on $[-1,3]$

2. $g(t) = 2t - t^2 - t^3$ on $[-2,1]$

For problems 3 & 4 determine all the number(s) *c* which satisfy the conclusion of the Mean Value Theorem for the given function and interval.

3. $h(z) = 4z^3 - 8z^2 + 7z - 2$ on $[2,5]$

4. $A(t) = 8t + \mathbf{e}^{-3t}$ on $[-2,3]$

5. Suppose we know that $f(x)$ is continuous and differentiable on the interval $[-7,0]$, that $f(-7) = -3$ and that $f'(x) \le 2$. What is the largest possible value for $f(0)$?

6. Show that $f(x) = x^3 - 7x^2 + 25x + 8$ has exactly one real root.

Section 4-8 : Optimization

1. Find two positive numbers whose sum is 300 and whose product is a maximum.

2. Find two positive numbers whose product is 750 and for which the sum of one and 10 times the other is a minimum.

3. Let x and y be two positive numbers such that $x + 2y = 50$ and $\left(x+1\right)\left(y+2\right)$ is a maximum.

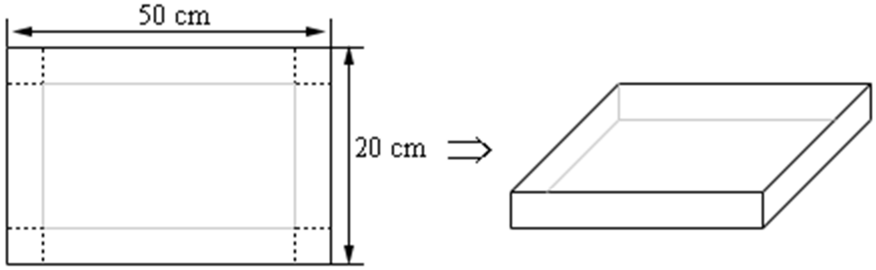
4. We are going to fence in a rectangular field. If we look at the field from above the cost of the vertical sides are \$10/ft, the cost of the bottom is \$2/ft and the cost of the top is \$7/ft. If we have \$700 determine the dimensions of the field that will maximize the enclosed area.

5. We have 45 m^2 of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume.

6. We want to build a box whose base length is 6 times the base width and the box will enclose 20 in^3. The cost of the material of the sides is \$3/in^2 and the cost of the top and bottom is \$15/in^2. Determine the dimensions of the box that will minimize the cost.

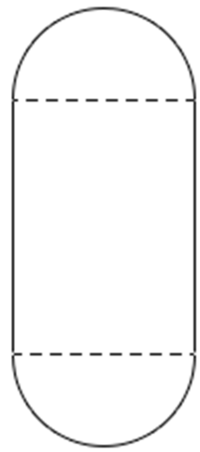
7. We want to construct a cylindrical can with a bottom but no top that will have a volume of 30 cm^3. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.

8. We have a piece of cardboard that is 50 cm by 20 cm and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.

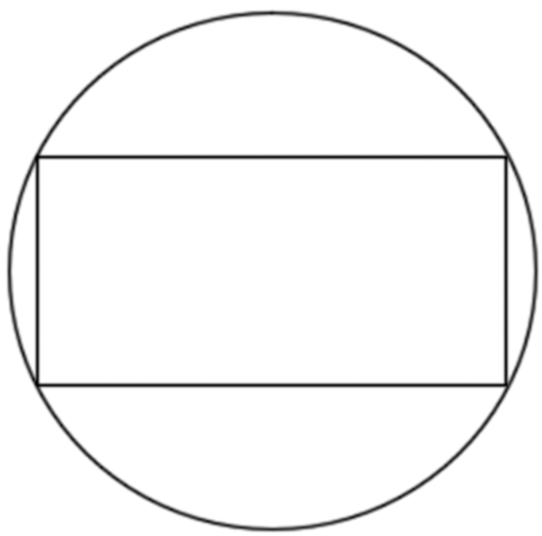


Section 4-9 : More Optimization

1. We want to construct a window whose middle is a rectangle and the top and bottom of the window are semi-circles. If we have 50 meters of framing material what are the dimensions of the window that will let in the most light?



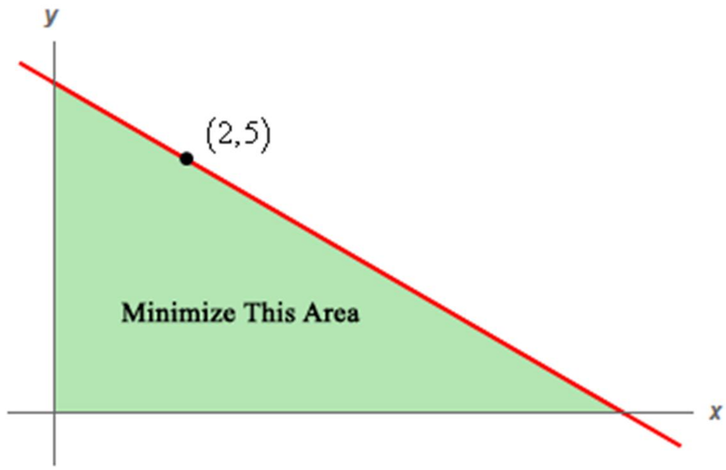
2. Determine the area of the largest rectangle that can be inscribed in a circle of radius 1.



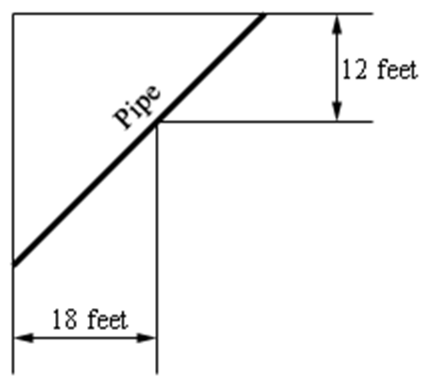
3. Find the point(s) on $x = 3 - 2y^2$ that are closest to $\left(-4, 0\right)$.

4. An 80 cm piece of wire is cut into two pieces. One piece is bent into an equilateral triangle and the other will be bent into a rectangle with one side 4 times the length of the other side. Determine where, if anywhere, the wire should be cut to maximize the area enclosed by the two figures.

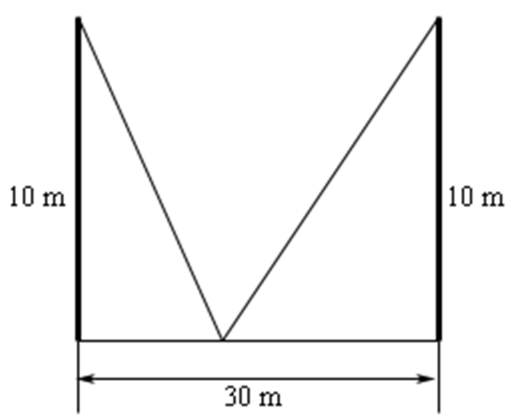
5. A line through the point $\left(2,5\right)$ forms a right triangle with the *x*-axis and *y*-axis in the 1st quadrant. Determine the equation of the line that will minimize the area of this triangle.



6. A piece of pipe is being carried down a hallway that is 18 feet wide. At the end of the hallway there is a right-angled turn and the hallway narrows down to 12 feet wide. What is the longest pipe (always keeping it horizontal) that can be carried around the turn in the hallway?



7. Two 10 meter tall poles are 30 meters apart. A length of wire is attached to the top of each pole and it is staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?



Section 4-10 : L'Hospital's Rule and Indeterminate Forms

Use L'Hospital's Rule to evaluate each of the following limits.

1. $\displaystyle\lim_{x\to 2}\frac{x^3-7x^2+10x}{x^2+x-6}$

2. $\displaystyle\lim_{w\to -4}\frac{\sin(\pi w)}{w^2-16}$

3. $\displaystyle\lim_{t\to\infty}\frac{\ln(3t)}{t^2}$

4. $\displaystyle\lim_{z\to 0}\frac{\sin(2z)+7z^2-2z}{z^2(z+1)^2}$

5. $\displaystyle\lim_{x\to -\infty}\frac{x^2}{\mathbf{e}^{1-x}}$

6. $\displaystyle\lim_{z\to\infty}\frac{z^2+\mathbf{e}^{4z}}{2z-\mathbf{e}^{z}}$

7. $\displaystyle\lim_{t\to\infty}\left[t\ln\left(1+\frac{3}{t}\right)\right]$

8. $\displaystyle\lim_{w\to 0^+}\left[w^2\ln\left(4w^2\right)\right]$

9. $\displaystyle\lim_{x\to 1^+}\left[(x-1)\tan\left(\tfrac{\pi}{2}x\right)\right]$

10. $\displaystyle\lim_{y\to 0^+}\left[\cos(2y)\right]^{1/y^2}$

11. $\displaystyle\lim_{x\to\infty}\left[\mathbf{e}^x+x\right]^{1/x}$

Section 4-11 : Linear Approximations

For problems 1 & 2 find a linear approximation to the function at the given point.

1. $f\left(x\right)=3x\,\mathbf{e}^{2x-10}$ at $x=5$

2. $h\left(t\right)=t^4-6t^3+3t-7$ at $t=-3$

3. Find the linear approximation to $g\left(z\right)=\sqrt[4]{z}$ at $z=2$. Use the linear approximation to approximate the value of $\sqrt[4]{3}$ and $\sqrt[4]{10}$. Compare the approximated values to the exact values.

4. Find the linear approximation to $f\left(t\right)=\cos\left(2t\right)$ at $t=\frac{1}{2}$. Use the linear approximation to approximate the value of $\cos\left(2\right)$ and $\cos\left(18\right)$. Compare the approximated values to the exact values.

5. Without using any kind of computational aid use a linear approximation to estimate the value of $\mathbf{e}^{0.1}$.

Section 4-12 : Differentials

For problems 1 – 3 compute the differential of the given function.

1. $f(x) = x^2 - \sec(x)$

2. $w = \mathbf{e}^{x^4 - x^2 + 4x}$

3. $h(z) = \ln(2z)\sin(2z)$

4. Compute dy and Δy for $y = \mathbf{e}^{x^2}$ as *x* changes from 3 to 3.01.

5. Compute dy and Δy for $y = x^5 - 2x^3 + 7x$ as *x* changes from 6 to 5.9.

6. The sides of a cube are found to be 6 feet in length with a possible error of no more than 1.5 inches. What is the maximum possible error in the volume of the cube if we use this value of the length of the side to compute the volume?

Section 4-13 : Newton's Method

For problems 1 & 2 use Newton's Method to determine x_2 for the given function and given value of x_0.

1. $f\left(x\right)=x^3-7x^2+8x-3$, $x_0=5$

2. $f\left(x\right)=x\cos\left(x\right)-x^2$, $x_0=1$

For problems 3 & 4 use Newton's Method to find the root of the given equation, accurate to six decimal places, that lies in the given interval.

3. $x^4-5x^3+9x+3=0$ in $\left[4,6\right]$

4. $2x^2+5=\mathbf{e}^x$ in $\left[3,4\right]$

For problems 5 & 6 use Newton's Method to find all the roots of the given equation accurate to six decimal places.

5. $x^3-x^2-15x+1=0$

6. $2-x^2=\sin\left(x\right)$

Section 4-14 : Business Applications

1. A company can produce a maximum of 1500 widgets in a year. If they sell x widgets during the year then their profit, in dollars, is given by,

$$P(x) = 30,000,000 - 360,000x + 750x^2 - \tfrac{1}{3}x^3$$

How many widgets should they try to sell in order to maximize their profit?

2. A management company is going to build a new apartment complex. They know that if the complex contains x apartments the maintenance costs for the building, landscaping etc. will be,

$$C(x) = 4000 + 14x - 0.04x^2$$

The land they have purchased can hold a complex of at most 500 apartments. How many apartments should the complex have in order to minimize the maintenance costs?

3. The production costs, in dollars, per day of producing x widgets is given by,

$$C(x) = 1750 + 6x - 0.04x^2 + 0.0003x^3$$

What is the marginal cost when $x = 175$ and $x = 300$? What do your answers tell you about the production costs?

4. The production costs, in dollars, per month of producing x widgets is given by,

$$C(x) = 200 + 0.5x + \frac{10000}{x}$$

What is the marginal cost when $x = 200$ and $x = 500$? What do your answers tell you about the production costs?

5. The production costs, in dollars, per week of producing x widgets is given by,

$$C(x) = 4000 - 32x + 0.08x^2 + 0.00006x^3$$

and the demand function for the widgets is given by,

$$p(x) = 250 + 0.02x - 0.001x^2$$

What is the marginal cost, marginal revenue and marginal profit when $x = 200$ and $x = 400$? What do these numbers tell you about the cost, revenue and profit?

Chapter 5 : Integrals

Here are a set of practice problems for the Integrals chapter of the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

11. If you'd like a pdf document containing the solutions the download tab on the website contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.

12. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Indefinite Integrals – In this section we will start off the chapter with the definition and properties of indefinite integrals. We will not be computing many indefinite integrals in this section. This section is devoted to simply defining what an indefinite integral is and to give many of the properties of the indefinite integral. Actually computing indefinite integrals will start in the next section.

Computing Indefinite Integrals – In this section we will compute some indefinite integrals. The integrals in this section will tend to be those that do not require a lot of manipulation of the function we are integrating in order to actually compute the integral. As we will see starting in the next section many integrals do require some manipulation of the function before we can actually do the integral. We will also take a quick look at an application of indefinite integrals.

Substitution Rule for Indefinite Integrals – In this section we will start using one of the more common and useful integration techniques – The Substitution Rule. With the substitution rule we will be able integrate a wider variety of functions. The integrals in this section will all require some manipulation of the function prior to integrating unlike most of the integrals from the previous section where all we really needed were the basic integration formulas.

More Substitution Rule – In this section we will continue to look at the substitution rule. The problems in this section will tend to be a little more involved than those in the previous section.

Area Problem – In this section we start off with the motivation for definite integrals and give one of the interpretations of definite integrals. We will be approximating the amount of area that lies between a function and the \(x\)-axis. As we will see in the next section this problem will lead us to the definition of the definite integral and will be one of the main interpretations of the definite integral that we'll be looking at in this material.

Definition of the Definite Integral – In this section we will formally define the definite integral, give many of its properties and discuss a couple of interpretations of the definite integral. We will also look at the first part of the Fundamental Theorem of Calculus which shows the very close relationship between derivatives and integrals.

Computing Definite Integrals – In this section we will take a look at the second part of the Fundamental Theorem of Calculus. This will show us how we compute definite integrals without using (the often very unpleasant) definition. The examples in this section can all be done with a basic knowledge of indefinite integrals and will not require the use of the substitution rule. Included in the examples in this section are computing definite integrals of piecewise and absolute value functions.

Substitution Rule for Definite Integrals – In this section we will revisit the substitution rule as it applies to definite integrals. The only real requirements to being able to do the examples in this section are being able to do the substitution rule for indefinite integrals and understanding how to compute definite integrals in general.

Section 5-1 : Indefinite Integrals

1. Evaluate each of the following indefinite integrals.
   (a) $\int 6x^5 - 18x^2 + 7\,dx$
   (b) $\int 6x^5\,dx - 18x^2 + 7$

2. Evaluate each of the following indefinite integrals.
   (a) $\int 40x^3 + 12x^2 - 9x + 14\,dx$
   (b) $\int 40x^3 + 12x^2 - 9x\,dx + 14$
   (c) $\int 40x^3 + 12x^2\,dx - 9x + 14$

For problems 3 – 5 evaluate the indefinite integral.

3. $\int 12t^7 - t^2 - t + 3\,dt$

4. $\int 10w^4 + 9w^3 + 7w\;dw$

5. $\int z^6 + 4z^4 - z^2\,dz$

6. Determine $f(x)$ given that $f'(x) = 6x^8 - 20x^4 + x^2 + 9$.

7. Determine $h(t)$ given that $h'(t) = t^4 - t^3 + t^2 + t - 1$.

Section 5-2 : Computing Indefinite Integrals

For problems 1 – 21 evaluate the given integral.

1. $\int 4x^6 - 2x^3 + 7x - 4\,dx$

2. $\int z^7 - 48z^{11} - 5z^{16}\,dz$

3. $\int 10t^{-3} + 12t^{-9} + 4t^3\,dt$

4. $\int w^{-2} + 10w^{-5} - 8\,dw$

5. $\int 12\,dy$

6. $\int \sqrt[3]{w} + 10\,\sqrt[5]{w^3}\,dw$

7. $\int \sqrt{x^7} - 7\sqrt[6]{x^5} + 17\,\sqrt[3]{x^{10}}\,dx$

8. $\displaystyle\int \frac{4}{x^2} + 2 - \frac{1}{8x^3}\,dx$

9. $\displaystyle\int \frac{7}{3y^6} + \frac{1}{y^{10}} - \frac{2}{\sqrt[3]{y^4}}\,dy$

10. $\int \left(t^2 - 1\right)\left(4 + 3t\right)dt$

11. $\displaystyle\int \sqrt{z}\left(z^2 - \frac{1}{4z}\right)dz$

12. $\displaystyle\int \frac{z^8 - 6z^5 + 4z^3 - 2}{z^4}\,dz$

13. $\displaystyle\int \frac{x^4 - \sqrt[3]{x}}{6\sqrt{x}}\,dx$

14. $\int \sin\left(x\right) + 10\csc^2\left(x\right)dx$

15. $\int 2\cos(w) - \sec(w)\tan(w)\,dw$

16. $\int 12 + \csc(\theta)\left[\sin(\theta) + \csc(\theta)\right]d\theta$

17. $\int 4\mathbf{e}^{z} + 15 - \frac{1}{6z}\,dz$

18. $\int t^{3} - \frac{\mathbf{e}^{-t} - 4}{\mathbf{e}^{-t}}\,dt$

19. $\int \frac{6}{w^{3}} - \frac{2}{w}\,dw$

20. $\int \frac{1}{1+x^{2}} + \frac{12}{\sqrt{1-x^{2}}}\,dx$

21. $\int 6\cos(z) + \frac{4}{\sqrt{1-z^{2}}}\,dz$

22. Determine $f(x)$ given that $f'(x) = 12x^{2} - 4x$ and $f(-3) = 17$.

23. Determine $g(z)$ given that $g'(z) = 3z^{3} + \frac{7}{2\sqrt{z}} - \mathbf{e}^{z}$ and $g(1) = 15 - \mathbf{e}$.

24. Determine $h(t)$ given that $h''(t) = 24t^{2} - 48t + 2$, $h(1) = -9$ and $h(-2) = -4$.

Section 5-3 : Substitution Rule for Indefinite Integrals

For problems 1 – 16 evaluate the given integral.

1. $\int (8x-12)(4x^2-12x)^4\,dx$

2. $\int 3t^{-4}(2+4t^{-3})^{-7}\,dt$

3. $\int (3-4w)(4w^2-6w+7)^{10}\,dw$

4. $\int 5(z-4)\,\sqrt[3]{z^2-8z}\,dz$

5. $\int 90x^2\sin(2+6x^3)\,dx$

6. $\int \sec(1-z)\tan(1-z)\,dz$

7. $\int (15t^{-2}-5t)\cos(6t^{-1}+t^2)\,dt$

8. $\int (7y-2y^3)\mathbf{e}^{y^4-7y^2}\,dy$

9. $\int \dfrac{4w+3}{4w^2+6w-1}\,dw$

10. $\int (\cos(3t)-t^2)(\sin(3t)-t^3)^5\,dt$

11. $\int 4\left(\dfrac{1}{z}-\mathbf{e}^{-z}\right)\cos(\mathbf{e}^{-z}+\ln z)\,dz$

12. $\int \sec^2(v)\,\mathbf{e}^{1+\tan(v)}\,dv$

13. $\int 10\sin(2x)\cos(2x)\sqrt{\cos^2(2x)-5}\,dx$

14. $\int \dfrac{\csc(x)\cot(x)}{2-\csc(x)}\,dx$

15. $\int \dfrac{6}{7+y^2}\,dy$

16. $\displaystyle\int \frac{1}{\sqrt{4-9w^2}}\,dw$

17. Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{3x}{1+9x^2}\,dx$

(b) $\displaystyle\int \frac{3x}{\left(1+9x^2\right)^4}\,dx$

(c) $\displaystyle\int \frac{3}{1+9x^2}\,dx$

Section 5-4 : More Substitution Rule

Evaluate each of the following integrals.

1. $\int 4\sqrt{5+9t} + 12(5+9t)^7 \, dt$

2. $\int 7x^3 \cos\left(2+x^4\right) - 8x^3 \mathbf{e}^{2+x^4} \, dx$

3. $\displaystyle\int \frac{6\mathbf{e}^{7w}}{\left(1-8\mathbf{e}^{7w}\right)^3} + \frac{14\mathbf{e}^{7w}}{1-8\mathbf{e}^{7w}} \, dw$

4. $\int x^4 - 7x^5 \cos\left(2x^6+3\right) dx$

5. $\displaystyle\int \mathbf{e}^z + \frac{4\sin(8z)}{1+9\cos(8z)} \, dz$

6. $\int 20\mathbf{e}^{2-8w}\sqrt{1+\mathbf{e}^{2-8w}} + 7w^3 - 6\sqrt[3]{w} \, dw$

7. $\int (4+7t)^3 - 9t\sqrt[4]{5t^2+3} \, dt$

8. $\displaystyle\int \frac{6x-x^2}{x^3-9x^2+8} - \csc^2\left(\frac{3x}{2}\right) dx$

9. $\int 7(3y+2)\left(4y+3y^2\right)^3 + \sin(3+8y) \, dy$

10. $\int \sec^2(2t)\left[9+7\tan(2t) - \tan^2(2t)\right] dt$

11. $\displaystyle\int \frac{8-w}{4w^2+9} \, dw$

12. $\displaystyle\int \frac{7x+2}{\sqrt{1-25x^2}} \, dx$

13. $\int z^7\left(8+3z^4\right)^8 \, dz$

Section 5-5 : Area Problem

For problems 1 – 3 estimate the area of the region between the function and the *x*-axis on the given interval using $n = 6$ and using,

(a) the right end points of the subintervals for the height of the rectangles,
(b) the left end points of the subintervals for the height of the rectangles and,
(c) the midpoints of the subintervals for the height of the rectangles.

1. $f\left(x\right) = x^3 - 2x^2 + 4$ on $\left[1,4\right]$

2. $g\left(x\right) = 4 - \sqrt{x^2 + 2}$ on $\left[-1,3\right]$

3. $h\left(x\right) = -x\cos\left(\frac{x}{3}\right)$ on $\left[0,3\right]$

4. Estimate the net area between $f\left(x\right) = 8x^2 - x^5 - 12$ and the *x*-axis on $\left[-2,2\right]$ using $n = 8$ and the midpoints of the subintervals for the height of the rectangles. Without looking at a graph of the function on the interval does it appear that more of the area is above or below the *x*-axis?

Section 5-6 : Definition of the Definite Integral

For problems 1 & 2 use the definition of the definite integral to evaluate the integral. Use the right end point of each interval for x_i^*.

1. $\int_1^4 2x+3\,dx$

2. $\int_0^1 6x\left(x-1\right)dx$

3. Evaluate : $\displaystyle\int_4^4 \frac{\cos\left(\mathbf{e}^{3x}+x^2\right)}{x^4+1}\,dx$

For problems 4 & 5 determine the value of the given integral given that $\int_6^{11} f\left(x\right)dx=-7$ and $\int_6^{11} g\left(x\right)dx=24$.

4. $\int_{11}^6 9f\left(x\right)dx$

5. $\int_6^{11} 6g\left(x\right)-10f\left(x\right)dx$

6. Determine the value of $\int_2^9 f\left(x\right)dx$ given that $\int_5^2 f\left(x\right)dx=3$ and $\int_5^9 f\left(x\right)dx=8$.

7. Determine the value of $\int_{-4}^{20} f\left(x\right)dx$ given that $\int_{-4}^0 f\left(x\right)dx=-2$, $\int_{31}^0 f\left(x\right)dx=19$ and $\int_{20}^{31} f\left(x\right)dx=-21$.

For problems 8 & 9 sketch the graph of the integrand and use the area interpretation of the definite integral to determine the value of the integral.

8. $\int_1^4 3x-2\,dx$

9. $\int_0^5 -4x\,dx$

For problems 10 – 12 differentiate each of the following integrals with respect to *x*.

10. $\int_4^x 9\cos^2\left(t^2-6t+1\right)dt$

11. $\displaystyle\int_{7}^{\sin(6x)} \sqrt{t^2+4}\,dt$

12. $\displaystyle\int_{3x^2}^{-1} \frac{\mathbf{e}^t - 1}{t}\,dt$

Section 5-7 : Computing Definite Integrals

1. Evaluate each of the following integrals.

   a. $\displaystyle\int \cos\left(x\right) - \frac{3}{x^5}\,dx$

   b. $\displaystyle\int_{-3}^{4} \cos\left(x\right) - \frac{3}{x^5}\,dx$

   c. $\displaystyle\int_{1}^{4} \cos\left(x\right) - \frac{3}{x^5}\,dx$

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

2. $\int_{1}^{6} 12x^3 - 9x^2 + 2\,dx$

3. $\int_{-2}^{1} 5z^2 - 7z + 3\,dz$

4. $\int_{3}^{0} 15w^4 - 13w^2 + w\,dw$

5. $\displaystyle\int_{1}^{4} \frac{8}{\sqrt{t}} - 12\sqrt{t^3}\,dt$

6. $\displaystyle\int_{1}^{2} \frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3}\,dz$

7. $\displaystyle\int_{-2}^{4} x^6 - x^4 + \frac{1}{x^2}\,dx$

8. $\int_{-4}^{-1} x^2\left(3 - 4x\right)dx$

9. $\displaystyle\int_{2}^{1} \frac{2y^3 - 6y^2}{y^2}\,dy$

10. $\int_{0}^{\frac{\pi}{2}} 7\sin\left(t\right) - 2\cos\left(t\right)dt$

11. $\int_{0}^{\pi} \sec\left(z\right)\tan\left(z\right) - 1\,dz$

12. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2(w) - 8\csc(w)\cot(w)\,dw$

13. $\displaystyle\int_0^2 \mathbf{e}^x + \frac{1}{x^2+1}\,dx$

14. $\displaystyle\int_{-5}^{-2} 7\mathbf{e}^y + \frac{2}{y}\,dy$

15. $\int_0^4 f(t)\,dt$ where $f(t) = \begin{cases} 2t & t > 1 \\ 1-3t^2 & t \le 1 \end{cases}$

16. $\int_{-6}^{1} g(z)\,dz$ where $g(z) = \begin{cases} 2-z & z > -2 \\ 4\mathbf{e}^z & z \le -2 \end{cases}$

17. $\int_3^6 |2x-10|\,dx$

18. $\int_{-1}^0 |4w+3|\,dw$

Section 5-8 : Substitution Rule for Definite Integrals

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

1. $\displaystyle\int_0^1 3\left(4x+x^4\right)\left(10x^2+x^5-2\right)^6\,dx$

2. $\displaystyle\int_0^{\frac{\pi}{4}} \frac{8\cos\left(2t\right)}{\sqrt{9-5\sin\left(2t\right)}}\,dt$

3. $\displaystyle\int_\pi^0 \sin\left(z\right)\cos^3\left(z\right)dz$

4. $\displaystyle\int_1^4 \sqrt{w}\,\mathbf{e}^{1-\sqrt{w^3}}\,dw$

5. $\displaystyle\int_{-4}^{-1} \sqrt[3]{5-2y}+\frac{7}{5-2y}\,dy$

6. $\displaystyle\int_{-1}^2 x^3+\mathbf{e}^{\frac{1}{4}x}\,dx$

7. $\displaystyle\int_\pi^{\frac{3\pi}{2}} 6\sin\left(2w\right)-7\cos\left(w\right)dw$

8. $\displaystyle\int_1^5 \frac{2x^3+x}{x^4+x^2+1}-\frac{x}{x^2-4}\,dx$

9. $\displaystyle\int_{-2}^0 t\sqrt{3+t^2}+\frac{3}{\left(6t-1\right)^2}\,dt$

10. $\displaystyle\int_{-2}^1 \left(2-z\right)^3+\sin\left(\pi z\right)\left[3+2\cos\left(\pi z\right)\right]^3\,dz$

Chapter 6 : Applications of Integrals

Here are a set of practice problems for the Applications of Integrals chapter of the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

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Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Average Function Value – In this section we will look at using definite integrals to determine the average value of a function on an interval. We will also give the Mean Value Theorem for Integrals.

Area Between Curves – In this section we'll take a look at one of the main applications of definite integrals in this chapter. We will determine the area of the region bounded by two curves.

Volumes of Solids of Revolution / Method of Rings – In this section, the first of two sections devoted to finding the volume of a solid of revolution, we will look at the method of rings/disks to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the x or y-axis) around a vertical or horizontal axis of rotation.

Volumes of Solids of Revolution / Method of Cylinders – In this section, the second of two sections devoted to finding the volume of a solid of revolution, we will look at the method of cylinders/shells to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the x or y-axis) around a vertical or horizontal axis of rotation.

More Volume Problems – In the previous two sections we looked at solids that could be found by treating them as a solid of revolution. Not all solids can be thought of as solids of revolution and, in fact, not all solids of revolution can be easily dealt with using the methods from the previous two sections. So, in this section we'll take a look at finding the volume of some solids that are either not solids of revolutions or are not easy to do as a solid of revolution.

Work – In this section we will look at is determining the amount of work required to move an object subject to a force over a given distance.

Section 6-1 : Average Function Value

For problems 1 & 2 determine f_{avg} for the function on the given interval.

1. $f\left(x\right)=8x-3+5\mathbf{e}^{2-x}$ on $\left[0,2\right]$

2. $f\left(x\right)=\cos\left(2x\right)-\sin\left(\frac{x}{2}\right)$ on $\left[-\frac{\pi}{2},\pi\right]$

3. Find f_{avg} for $f\left(x\right)=4x^2-x+5$ on $\left[-2,3\right]$ and determine the value(s) of *c* in $\left[-2,3\right]$ for which $f\left(c\right)=f_{\text{avg}}$.

Section 6-2 : Area Between Curves

1. Determine the area below $f(x) = 3 + 2x - x^2$ and above the x-axis.

2. Determine the area to the left of $g(y) = 3 - y^2$ and to the right of $x = -1$.

For problems 3 – 11 determine the area of the region bounded by the given set of curves.

3. $y = x^2 + 2$, $y = \sin(x)$, $x = -1$ and $x = 2$

4. $y = \dfrac{8}{x}$, $y = 2x$ and $x = 4$

5. $x = 3 + y^2$, $x = 2 - y^2$, $y = 1$ and $y = -2$

6. $x = y^2 - y - 6$ and $x = 2y + 4$

7. $y = x\sqrt{x^2 + 1}$, $y = \mathbf{e}^{-\frac{1}{2}x}$, $x = -3$ and the y-axis

8. $y = 4x + 3$, $y = 6 - x - 2x^2$, $x = -4$ and $x = 2$

9. $y = \dfrac{1}{x+2}$, $y = (x+2)^2$, $x = -\dfrac{3}{2}$, $x = 1$

10. $x = y^2 + 1$, $x = 5$, $y = -3$ and $y = 3$

11. $x = \mathbf{e}^{1+2y}$, $x = \mathbf{e}^{1-y}$, $y = -2$ and $y = 1$

Section 6-3 : Volumes of Solids of Revolution / Method of Rings

For each of the following problems use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $y = \sqrt{x}$, $y = 3$ and the *y*-axis about the *y*-axis.

2. Rotate the region bounded by $y = 7 - x^2$, $x = -2$, $x = 2$ and the *x*-axis about the *x*-axis.

3. Rotate the region bounded by $x = y^2 - 6y + 10$ and $x = 5$ about the *y*-axis.

4. Rotate the region bounded by $y = 2x^2$ and $y = x^3$ about the *x*-axis.

5. Rotate the region bounded by $y = 6\mathbf{e}^{-2x}$ and $y = 6 + 4x - 2x^2$ between $x = 0$ and $x = 1$ about the line $y = -2$.

6. Rotate the region bounded by $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$ and $x = 5$ about the line $y = 8$.

7. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $x = 24$.

8. Rotate the region bounded by $y = 2x + 1$, $x = 4$ and $y = 3$ about the line $x = -4$.

Section 6-4 : Volumes of Solids of Revolution / Method of Cylinders

For each of the following problems use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $x = (y-2)^2$, the *x*-axis and the *y*-axis about the *x*-axis.

2. Rotate the region bounded by $y = \dfrac{1}{x}$, $x = \dfrac{1}{2}$, $x = 4$ and the *x*-axis about the *y*-axis.

3. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the *y*-axis. For this problem assume that $x \geq 0$.

4. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the *x*-axis. For this problem assume that $x \geq 0$.

5. Rotate the region bounded by $y = 2x+1$, $y = 3$ and $x = 4$ about the line $y = 10$.

6. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $y = -8$.

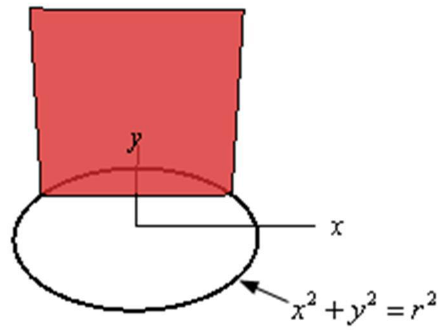
7. Rotate the region bounded by $y = x^2 - 6x + 9$ and $y = -x^2 + 6x - 1$ about the line $x = 8$.

8. Rotate the region bounded by $y = \dfrac{\mathbf{e}^{\frac{1}{2}x}}{x+2}$, $y = 5 - \frac{1}{4}x$, $x = -1$ and $x = 6$ about the line $x = -2$.

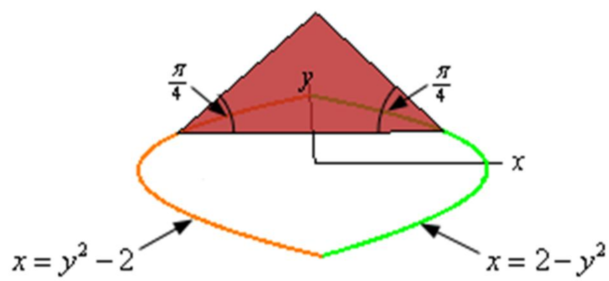
Section 6-5 : More Volume Problems

1. Find the volume of a pyramid of height *h* whose base is an equilateral triangle of length *L*.

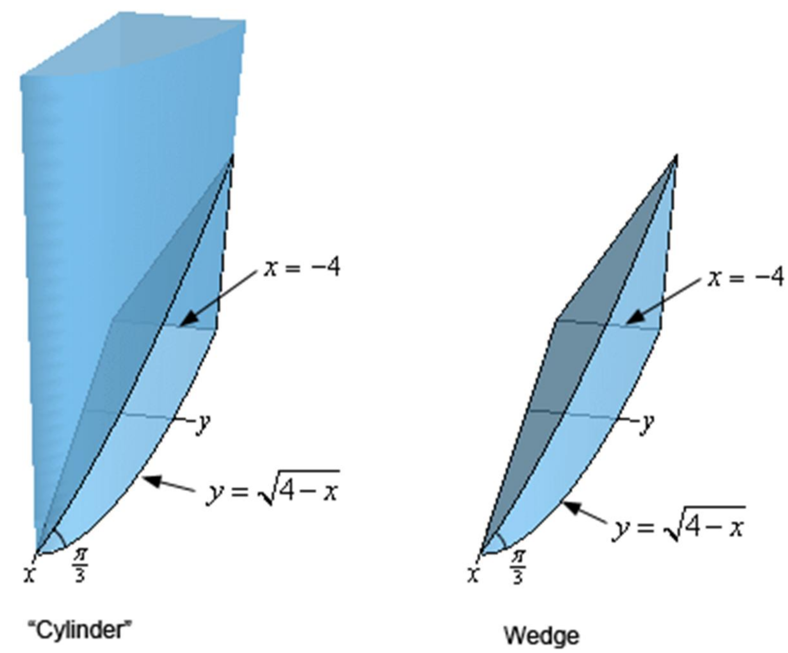
2. Find the volume of the solid whose base is a disk of radius *r* and whose cross-sections are squares. See figure below to see a sketch of the cross-sections.



3. Find the volume of the solid whose base is the region bounded by $x=2-y^2$ and $x=y^2-2$ and whose cross-sections are isosceles triangles with the base perpendicular to the *y*-axis and the angle between the base and the two sides of equal length is $\frac{\pi}{4}$. See figure below to see a sketch of the cross-sections.



4. Find the volume of a wedge cut out of a "cylinder" whose base is the region bounded by $y = \sqrt{4-x}$, $x = -4$ and the *x*-axis. The angle between the top and bottom of the wedge is $\frac{\pi}{3}$. See the figure below for a sketch of the "cylinder" and the wedge (the positive *x*-axis and positive *y*-axis are shown in the sketch – they are just in a different orientation).



Section 6-6 : Work

1. A force of $F(x) = x^2 - \cos(3x) + 2$, x is in meters, acts on an object. What is the work required to move the object from $x = 3$ to $x = 7$?

2. A spring has a natural length of 18 inches and a force of 20 lbs is required to stretch and hold the spring to a length of 24 inches. What is the work required to stretch the spring from a length of 21 inches to a length of 26 inches?

3. A cable with mass ½ kg/meter is lifting a load of 150 kg that is initially at the bottom of a 50 meter shaft. How much work is required to lift the load ¼ of the way up the shaft?

4. A tank of water is 15 feet long and has a cross section in the shape of an equilateral triangle with sides 2 feet long (point of the triangle points directly down). The tank is filled with water to a depth of 9 inches. Determine the amount of work needed to pump all of the water to the top of the tank. Assume that the weight of the water is 62 lb/ft^3.

