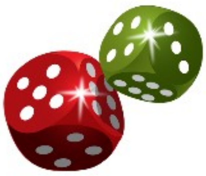
Mean and Variance for Discrete Random Variable



☺ Back to Discrete Probability Distributions
☺ Exercise for Mean and Variance

DRV → general discrete prob dist. $P(X=x)$ → table, function, piecewise; → cumulative prob dist → $P(X \le x)$; → tree diagram; → combination → prob → sum of prob = 1; → no (-ve); $0 \le P(X=x) \le 1$

DRV → uniform prob dist. $P(X=x) = \frac{1}{n}$

① Mean/average — Expected value = $E(X)$ = Mean = μ (miu) = $\sum x_i p_i$ → can be more than '1' → can be (-ve)

② Variance = $Var(X) = \sigma^2$ (sigma)^2 $= \sum \left[p_i (x_i - \mu)^2 \right]$ OR $E(X^2) - [E(X)]^2$ → can be more than '1' → always (+ve)

$\sigma^2 = \sum x_i^2 p_i - (\sum x_i p_i)^2$

③ Standard deviation = $SD(X) = |\sigma| = |\sqrt{Var(X)}|$ → can be more than 1 → always (+ve)

x	1	2
$P(X=x)$	0.2	0.8

① $E(X) = \mu = \sum x_i p_i = (1 \times 0.2) + (2 \times 0.8) = 1.8$

② $Var(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = [(1-1.8)^2 (0.2)] + [(2-1.8)^2 (0.8)] = 0.16$

$= E(X^2) - [E(X)]^2 = \sum x_i^2 p_i - (\sum x_i p_i)^2 = \underbrace{[(1^2 \times 0.2) + (2^2 \times 0.8)]}_{E(X^2)} - \underbrace{(1.8)^2}_{[E(X)]^2} = 0.16$

③ $SD(X) = |\sigma| = |\sqrt{0.16}| = 0.4$

Change of origin and scale

X – DRV → Y
$P(X=x)$ – discrete prob dist
$P(X \le x)$ – cumulative prob
$E(X) = \sum x_i p_i$
$Var(X) = \sum (x-\mu)^2 p$
$SD(X) = |\sigma| = |\sqrt{Var(X)}|$

If X is a random variable and $Y = aX + b$, where a and b are constants, then

'a' – scale (how big/small); 'b' – origin (moving)

$E(X)$, $Var(X)$, $SD(X)$

$E(Y) = aE(X) + b$ → both scale (a) & origin (b) will affect Expected value

$Var(Y) = a^2 Var(X)$ → only scale (a) will affect variance [(scale)^2]

$SD(Y) = |a| \cdot SD(X)$ → only scale (a) will affect std. deviation

Example 1

DRV → x	1	2	3	4
$P(X=x)$ → $P(X=x)$	0.1	0.2	0.4	0.3

1) with CAS
a) name column A → x, B → p
b) list down variable & prob: A → DRV, B → prob
c) column C → $E(X)$ → "= sum(x·p)"; $Var(X)$ → "= sum((x − μ)^2·p)"; $SD(X)$ → "= $\sqrt{Var}$"

(a) Find the expected value and variance by calculation and compare the answer using the statistical functions of the CAS calculator

2) without CAS / can still use scientific calculator → formula

$E(X) = \sum x \cdot p = (1 \times 0.1) + (2 \times 0.2) + (3 \times 0.4) + (4 \times 0.3) = 2.9$

$Var(X) = \sum (x-\mu)^2 \cdot p = [(1-2.9)^2 (0.1)] + [(2-2.9)^2 (0.2)] + [(3-2.9)^2 (0.4)] + [(4-2.9)^2 (0.3)] = 0.89$

or $\sum x^2 \cdot p - (\sum x \cdot p)^2 = [(1^2 \times 0.1) + (2^2 \times 0.2) + (3^2 \times 0.4) + (4^2 \times 0.3)] - (2.9)^2 = 0.89$

$SD(X) = |\sigma| = |\sqrt{0.89}| = 0.9434$

(b) If $Y = 3X$, find $E(Y)$ and $Var(Y)$

$E(X) = 2.9$
$Var(X) = 0.89$
$SD(X) = 0.9434$

scale

$E(Y) = 3 \cdot E(X) = 3 \times 2.9 = 8.7$
$Var(Y) = (3)^2 \cdot Var(X) = 9 \times 0.89 = 8.01$
$SD(Y) = |\sqrt{8.01}|$ or $= |(3) \cdot 0.9434| = 2.83$

(c) If $Z = 3X + 4$ find $E(Z)$ and $Var(Z)$

scale (a), origin (b)

$E(Z) = 3 \cdot E(X) + 4 = 3 \times (2.9) + 4 = 12.7$
$Var(Z) = (3)^2 \cdot Var(X) = 9 \times (0.89) = 8.01$
$SD(Z) = |\sqrt{8.01}| = |3 \times 0.9434| = 2.83$

Answers: (a) 2.9, 0.89 (b) 8.7, 8.01 (c) 12.7, 8.01

Example of exam like

Example 2

The expected value of the discrete probability distribution given below is 2.7. Find the values of p and q and hence determine *Var(X)*, the variance of X.

x	1	2	3	4	5
$P(X=x)$	0.3	p	0.2	q	0.1

$E(X) = 2.7$ → $(1 \times 0.3) + (2p) + (3 \times 0.2) + (4q) + (5 \times 0.1) = 2.7$
$2p + 4q = 2.7 - 0.3 - 0.6 - 0.5$
$2p + 4q = 1.3$ — ①

$E(X) = \sum x_i p_i$
$Var(X) = \sum (x_i - \mu)^2 p_i$
$SD(X) = |\sigma| = |\sqrt{Var(X)}|$

$0.3 + p + 0.2 + q + 0.1 = 1$
$p + q = 0.4$ — ②

$p = 0.15$, $q = 0.25$

CAS → menu → 3 → 1 → solve ({①②, p, q)

CAS

$Var(X) = \sum (x-\mu)^2 \cdot p$
$= [(1-2.7)^2 \cdot (0.3)] + [(2-2.7)^2 \cdot (0.15)]$
$+ [(3-2.7)^2 (0.2)] + [(4-2.7)^2 (0.25)]$
$+ [(5-2.7)^2 (0.1)]$
$= 1.91$

DRV
host/house – organise ... x → host

$p+q = 0.4$ —②

$p = 0.15,\ q = 0.25$

CAS → menu → 3 → 1 → solve ({ , p,q)

Answers: p=0.15, q= 0.25 Var(X) = 1.91

gambling
games
insurance company

host/house – organise
player/client – involved
? → X → host
client/player
DRV

Example 3 → application discrele prob dist

Sue and Bob are developing a gambling game. Each "play" of the game involves the rolling of two normal fair six sided dice and the two numbers on the uppermost faces being added together.

1. Any double, i.e. two 1s, two 2s, two 3s etc., pays $30 → player wins $30
2. A total of 7 pays $15. → player wins $15
3. Anything other than the above two outcomes pays nothing. → player wins nothing

If the discrete random variable, $X is the amount paid out on a single play, complete the following probability distribution table for X.

X – amount paid out

x ($ paid out)	0	15	30
$P(X = x)$	2/3	1/6	1/6

0.6667 0.1667 0.1667

In the long run, Sue and Bob want to make an average of $0.50 profit per game played. How much should they charge for each "play"?

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(\$30) = P(\text{any double}) = 6/36 = 1/6$

$P(\$15) = P(\text{total of } 7) = 6/36 = 1/6$

$P(\$0) = 1 - \frac{1}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

↳ E(X) – expected value that Sue & Bob have to pay

Answers: 2/3, 1/6, 1/6, Should charge $8 per day.

$E(X) = (0 \times 2/3) + (15 \times 1/6) + (30 \times 1/6) = \$7.50$ per game

average

charge ⇒ $7.50 + $0.50 → $8.00 per game

Basic of probability
experiment
outcome
sample space
union, intersection, complement
cond. prob
etc

discrete prob dist
DRV
discrete prob dist (general)
cumulative
uniform
mean / variance / SD
change in scale & origin