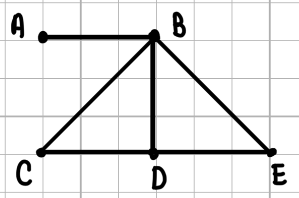
Lecture 30: Walks, path, trails



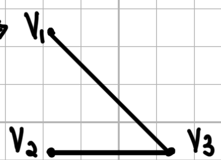
ABDEBD ⟶ walk (BD used twice)
CDEBA ⟶ path (trail + walk, X repeated vertices)
ADEBD ⟶ none (AD is not an edge)
ABCDBE ⟶ trail (repeats vertex but X edge)

Adjency matrix:

adjacent ⇒ 2 vertices joined by an edge

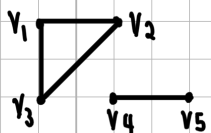
(i, j) ⟶ i = row
⟶ j = column

graph ⟶



} Adjacency matrix: $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

adjacency matrix: $\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ } graph: disconnected graph, V_4 and V_5 not joined to other vertices than between themselves

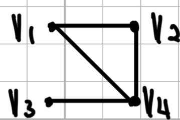


* WON'T BE ASKED TO MULTIPLY MATRICS

* (i,j) entry in the kth power of the adjacency matrix = number of walks of length k between V_i and V_j.
* length of walks = number of steps (edges)

Eg. Lecture slide 30 pg 12 – 14

graph G:



with m as adjacency matrix of G

(3,1) = entry of m^{k+1} the number of walks of length k+1 from V_3 to V_1?

$$m^{k+1} = m^k \times m = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

where by assumption a_{ij} is the number of walks length k from V_i to V_j

$$(3,1) \text{ entry} = (a_{31} \times 0) + (a_{32} \times 1) + (a_{33} \times 0) + (a_{34} \times 1)$$
$$= \underline{a_{32} + a_{34}}$$

↳ length of k walks from ($V_3 \to V_2$) & ($V_3 \to V_4$)
= length of k+1 walks from $V_3 \to V_1$

$(3,3)$ entry in $m^2 \Rightarrow \begin{pmatrix}1 & 1 & 0\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix} = (1\times1)+(1\times1)+(0\times0)$
$= 2$

so... continue the walks count :

$V_3 \to V_1 \to V_3$ & $V_3 \to V_2 \to V_3$

$m^2 \times m = m^3$

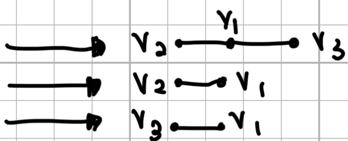
$$m^2 \times m = \begin{pmatrix}1 & 1 & 0\\1 & 1 & 0\\0 & 0 & 2\end{pmatrix} \times \begin{pmatrix}0 & 0 & 1\\0 & 0 & 1\\1 & 1 & 0\end{pmatrix}$$

has $(3,2)$ entry : $\begin{pmatrix}0 & 0 & 2\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} = (0\times0)+(0\times0)+(2\times1)$
$= 2$

number of walks length = 3 from V_3 and 2 from V_2

* Draw adjacency matrix, m graph:

$$m = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \end{array}\begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \begin{pmatrix}0 & 1 & 1\\1 & 0 & 0\\1 & 0 & 0\end{pmatrix}\end{array} \begin{array}{l} \to V_1 \text{ to } V_2 \text{ , } V_1 \text{ to } V_3 \\ \to V_2 \text{ to } V_1 \\ \to V_3 \text{ to } V_1 \end{array}$$



HENCE, graph =

* m^3 without calculation : $m^3 \begin{pmatrix}0 & 2 & 2\\2 & 0 & 0\\2 & 0 & 0\end{pmatrix}$

* m^3 with calculation :

$$m_2 = m \times m = \begin{pmatrix}0 & 1 & 1\\1 & 0 & 0\\1 & 0 & 0\end{pmatrix} \times \begin{pmatrix}0 & 1 & 1\\1 & 0 & 0\\1 & 0 & 0\end{pmatrix}$$

$$m_3 = m_2 \times m = \left[\begin{pmatrix}0 & 1 & 1\\1 & 0 & 0\\1 & 0 & 0\end{pmatrix} \times \begin{pmatrix}0 & 1 & 1\\1 & 0 & 0\\1 & 0 & 0\end{pmatrix}\right] \times \begin{pmatrix}0 & 1 & 1\\1 & 0 & 0\\1 & 0 & 0\end{pmatrix}$$