Guidelines for writing in mathematics

A correct answer to a mathematical problem is not just a string of equations, but also includes sentences explaining the method that was used and what the equations mean. Writing mathematics in a clear, precise and logical way is a communication skill. Like any skill, it takes practice and repetition to master.

The development of mathematical communication skills is one of the major outcomes of your studies in mathematics. Communication of mathematics is just as important (maybe even more important) than the actual mathematics. A significant fraction of marks are awarded for communication and presentation.

To get full marks your work

- should contain a clear description of the mathematics that justifies everything you do in full English sentences;
- should convince the marker that you understand the mathematical ideas as well as the calculations;
- should be neatly presented;
- should be understandable without too much effort by other typical students taking this course;
- should not contain any mistakes (mathematical, logical, spelling, etc.).

In essence, keep your answers clear, concise and to the point, and demonstrate to the reader/marker that you understand what you are doing.

Writing tips – short version

1. Explain what you are doing
2. Show all the important steps
3. Use correct mathematical notation
4. Present diagrams and graphs
5. Technically correct
6. Remember the little things

Final Advice

These are just a few hints to get you off the ground. The best examples of good mathematical writing you may have at hand are your text book, the lecture notes and solutions to problem sets and past exams. When you read these, pay particular attention to how the solutions to problems are presented. Watch the notation used by your teachers and always ask for clarification when you are not sure about its meaning.

Writing tips – in more detail

1. Explain what you are doing
    - It is important to explain what you are doing.
    - Write a clear description of the mathematics, in full sentences. Don't use abbreviated txt style.
    - Write your solutions in the same way that you would use if you were giving a running commentary of the process to one of your peers – include phrases where they assist.
    - Introduce your working with a simple sentence outlining what you intend to do (*Eg: I will use the method of cylindrical shells to find the volume of this solid*).
    - Keep in mind the assumed background knowledge for this unit. Trivial calculations and minor concepts need not be explained fully, but everything "new" should be.

2. Show all the important steps
    - Make sure that your sentences communicate clearly your ideas, and that these are organised in an orderly and logical manner.
    - Each new statement in a mathematical argument should follow logically from a previous one.
    - Give a reason of what has been done on each line, with enough detail so that others can follow what you are doing and you demonstrate to the marker that you know what you are doing.
    - Paragraphs can be used to separate important steps.

3. Use correct mathematical notation
    - Define all variables used and state any assumptions you made. For example: *Let x be the length and y the width of the box, measured in metres*.
    - Only use mathematical symbols that you are sure of their meaning. For example, different types of brackets have different meanings. Conventionally (x,y), [x,y] and {x,y} are the coordinates of a point (or an open interval), a closed interval, a set containing 2 elements, respectively.
    - When writing sentences do not use mathematics symbols as short hand. Symbols such as $\forall$, $\exists$,$\Rightarrow$, $\Leftrightarrow$, or abbreviations like "iff" and "s.t.". Instead write the symbols out in full "for all", "there exists", "implies' "if and only if", "such that".
    - Only use arrows or logic symbols in logic unit – where they have fixed meanings.
    - Starting a sentence with a symbol is considered poor form.

4. Present diagrams and graphs
    - Label the graph with a title and label the axes (include units if graphing data).
    - Label the important points. For example: turning points on graphs; vertices and line segments on diagrams.
    - Include any other graphing conventions, such as: direction arrows on vectors; tick-marks on congruent segments.
    - Include legends when more than one set of data is plotted on the same graph.
    - Is the scale sensible or appropriate? Graphs don't always need to start at zero, but be clear where they start.

- Numbering: In some cases you will have to include diagrams and figures; make sure you number these correctly (Eg: *Figure1, Figure 2,* etc) so you can refer to them anywhere in your assignment (Eg: … *as shown in Figure 1*).
- Formulas can also be labelled if you need to refer to them in some other parts of your report.

5. Technically correct

- Does not contain any mistakes (mathematical, typographical, grammatical, etc.).
- Is completely self-contained, including the statement of the different problems at the beginning and a summary of results at the end of each question.

6. Remember the little things

- Include the correct units wherever appropriate.
- Check that your final paragraph gives the answer to the question asked.
- Check that your answer makes sense.
- Are there any limitations on the results? Were any additional assumptions made?
- Check the spelling and grammar.
- Make a clean copy of your work. If your handwriting is illegible, whatever you are writing about will not be counted.

Commonly used words and phrases when writing in mathematics

- Therefore
- Hence
- It follows that
- Then
- In conclusion
- Let … be …
- This proves that
- Using … we have …
- After simplification, we find that …
- This follows from …
- Let … , where … is …
- Consequently
- As a result

Elements of a good mathematical/statistical/scientific report

Statistical and scientific reports also follow the same general outline as a solution to a mathematics question.

- State question/aim
- Define variables
- Outline techniques used
- State the result
- Check validity of solution and draw conclusions

Appendix A: A question with 5 example answers

Question:
Use the addition and double-angle formulae to derive an expression for $\sin(3x)$ *in terms of* $\sin x$*.*

For this problem there are three key steps in the solution process (see below), each of which might be awarded marks in a typical marking scheme. Additional marks might be awarded for stating clearly the correct final answer. In addition to those technical details, marks might typically be awarded for a clear explanation of the reasoning used and correct use of mathematical notation. (Note that the explanations and notation might attract more or fewer marks than shown here.)

Answer 1 - *Only the final answer is given*

$$\sin(3x) = 3\sin x - 4\sin^3 x$$

Comments: Although the answer is correct, you are expected to demonstrate that you understand how to solve a problem using mathematical reasoning, not just that you know the answer. This 'solution' might receive perhaps 1 mark because getting the final answer correct is usually worth a small amount in most marking schemes, but **most** of the marks for the question are awarded for showing the main steps and explaining clearly how to obtain the answer.

Answer 2 - *Steps partially shown, but no explanations and incorrect mathematical notation*

$$\sin(3x) \Rightarrow \sin x \cos(2x) + \cos x \sin(2x)$$
$$\Rightarrow \sin x \ (1 - 2\sin^2 x) + \cos x \ (2\sin x \cos x)$$
$$\therefore 3\sin x - 4\sin^3 x$$

Comments: There are no explanations or reasoning given or argument made. There is no English used. The expression on the final line is the correct final result but incorrect mathematical notation has been used. The 'implies' symbol ($\Rightarrow$) and 'therefore' symbol ($\therefore$) were used when an equals sign should have been used. Only partial marks would be awarded, with zero marks for the correct use of mathematical notation.

Answer 3 - *A mathematically correct but overly brief solution*

Use $\sin(x+y) = \sin x \cos y + \cos x \sin y$, with y=2x then $\sin 3x = \sin x \cos 2x + \cos x \ \sin 2x$.

Also $\sin(2x) = 2\sin x \cos x$ and $\cos(2x) = 1 - 2\sin^2 x \therefore \sin(3x) = 3\sin x \cos^2 x - \sin^3 x$.

Use $\cos^2 x = 1 - \sin^2 x$ then $\sin(3x) = 3\sin x - 4\sin^3 x$.

Comments: This answer includes a little more detail on the main steps but more words are needed to explain clearly what was being done and the reasoning behind each step. The sentences are grammatically poor. The answer do not include sufficient information for the method to be reproduced by an uninformed reader (most students would feel unsatisfied if this was provided in a textbook). The solution would receive some marks for the method and correct use of mathematical notation but zero marks for the explanation.

Answer 4 – *One example of a complete solution*

To obtain an expression for $\sin(3x)$, use 'summation formula' $\sin(x+y) = \sin x \cos y + \cos x \sin y$ and let $y = 2x$. This gives

$$\sin(3x) = \sin(x + 2x) = \sin x \cos(2x) + \cos x \sin(2x)\,.$$

Use the sine and cosine double-angle formulae to substitute for $\sin(2x) = 2\sin x \cos x$ and $\cos(2x) = \cos^2 x - \sin^2 x$. Then $\sin(3x)$can be written as

$$\sin(3x) = \sin x \left(\cos^2 x - \sin^2 x\right) + \cos x\,(2\sin x \cos x) = 3\sin x \cos^2 x - \sin^3 x.$$

The remaining $\cos^2 x$ term in this expression can be written in terms of $\sin x$ by using the Pythagorean identity written in the form $\cos^2 x = 1 - \sin^2 x$. Substituting it into the expression for $\sin(3x)$ above gives that

$$\sin(3x) = 3\sin x \left(1 - \sin^2 x\right) - \sin^3 x = 3\sin x - 4\sin^3 x$$

Therefore $\sin(3x)$ can be written in terms of $\sin x$ as: $\sin(3x) = 3\sin x - 4\sin^3 x$.

Comments: Notice how complete sentences are used to explain the reasoning behind each step, and intermediate mathematical steps are used to demonstrate the method. Also any additional results that are needed are stated so the reader knows their origin. The final answer is also stated in a full sentence to demonstrate that the requested problem has been solved as required. (If you were asked to repeat this problem with $\cos(3x)$ written in terms of $\cos x$ then it should be easy to follow the same approach.)

Answer 5 - *The final answer is correct but the method does not match the instructions*

Note that $e^{3ix} = (\cos x + i \sin x)^3$ from Euler's formula $e^{ix} = \cos x + i \sin x$ and the index laws. Expanding the cube and separating into real and imaginary parts gives that

$$(\cos x + i \sin x)^3 = \left(\cos^3 x - 3\sin^2 x \cos x\right) + i(3\cos^2 x\ \sin x - \sin^3 x)\,.$$

Taking the imaginary part of this implies that

$$\sin(3x) = \mathrm{Im}\left\{e^{3ix}\right\} = 3\cos^2 x\ \sin x - \sin^3 x\,.$$

Since $\cos^2 x = 1 - \sin^2 x$, this can be written in terms of $\sin x$ as

$$\sin(3x) = 3\left(1 - \sin^2 x\right)\sin x - \sin^3 x$$

and then collecting similar terms gives that

$$\sin(3x) = 3\sin x - 4\sin^3 x.$$

Comments: Again the answer is correct, but notice that the question asked you to 'Use the addition and double-angle formulae …' so you should follow those instructions – rather than find some other way to reach the answer. While the steps used were explained well, and it is technically correct, marks will be lost for not stating and using the formulae that were requested in the question. Note that **if** the question had included 'or otherwise', then the solution above would receive full marks. If you are interested in mathematics and want to show to the marker that you know another way to solve the problem then certainly do that, but make sure that you **also** complete it in the manner requested.

Appendix B: Common notation

Sets - let A and B be sets	
$x \in A$ $A \ni x$	x is a member of, is an element of, belongs to, or is in the set A
$A \subset B$	A is a subset of B, includes $A = B$
$A \cup B$	the union of sets A and B
$A \cap B$	the intersection of sets A and B
$A \backslash B$	the difference of sets A and B
$A \times B$	the Cartesian product of sets A and B; also written as A^2 if $A = B$
$\{x: P(x)\}$ $\{x \mid P(x)\}$	the set of x such that $P(x)$ is true For example, $\{x : 0 < x < 1\}$ is the set of all x which lie strictly between 0 and 1.
$\varnothing$	the empty set
N	the set of all natural numbers. **Caution:** sometimes 0 is included - check which convention is being used.
Z	the set of all integers
Q	the set of all rational numbers
R	the set of all real numbers
C	the set of all complex numbers

Logic – let P and Q be logical statements	
$P \Rightarrow Q$ $Q \Leftarrow P$	P implies Q ; P is sufficient for Q Q is necessary for P
$P \Leftrightarrow Q$	P holds if and only if Q holds ; P and Q are logically equivalent
$\neg P$	the negation of P
$P \wedge Q$	P and Q
$P \vee Q$	P or Q (or both)
$\exists$	the quantifier "exists"
$\forall x$	the quantifier "for all"

Functions – let A and B be sets, let $f : A \to B$ be a function	
$f : A \to B$	a function with domain A, range (co-domain) B and image $f(A) \subset B$
f^{-1}	the inverse of f (if it exists)
$f(a)$	the image of $a \in A$ under f
$f^{-1}(b)$	the preimage of $b \in B$ under f
$f \lceil g$	the composition of two functions (first g then f)
B^A	the set of all functions from A to B

Probability – let A and B be events, let X be a random variable	
P(A)	the probability of the event A
P(A ∩ B) P(A and B)	the probability of the joint event A and B
P(A ∪ B) P(A or B)	the probability of the event A or B (or both)
P(A | B)	the conditional probability of event A given that event B has occurred
$E(X)$	the expected value of X

Miscellaneous	
∞	infinity
$\sum_{i=1}^{n} x_i$	the sum $x_1 + x_2 + \ldots + x_n$
$\prod_{i=1}^{n} x_i$	the product $x_1 . x_2 . \ldots . x_n$
$n!$	n factorial (for a positive integer n)

A list of Greek letters used in mathematics, science & engineering can be found on Wikipedia.
http://en.wikipedia.org/wiki/Greek_letters_used_in_mathematics,_science,_and_engineering