

2×2 matrix

$$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\det = ad - bc$$

3×3 matrix

$$A^{-1} = (A|I) \rightarrow (V|I) \rightarrow (I|A^{-1})$$

$$A^{-1} = \frac{1}{\det A} [(-1)^{i+j} \det P_{ji}]$$

$$\det = \sum_{i=1}^n (-1)^{i+j} \dots$$

Determinants

- $n=1$
 $[a]$
 $\det = a$
- $n=2$
 $\det = ad - bc$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad |A| = 1(4) - 2(3) \\ = 4 - 6 \\ = -2$$

- $n > 2$
 - ↳ expansion along the first row (any row)

$$\det A = \sum_{j=i}^n (-1)^{i+j}$$

$$\cdot a_{ij} \cdot |S_{ij}|$$

$$A = \begin{pmatrix} 1 & 7 & 2 \\ 3 & 4 & 5 \\ 6 & 0 & 9 \end{pmatrix}$$

$$\begin{pmatrix} + & + & - \\ - & + & + \\ + & - & + \\ - & + & - \end{pmatrix}$$

$$|A| = (-1)^{1+1} (1) \begin{pmatrix} 4 & 5 \\ 0 & 9 \end{pmatrix} + (-1)^{1+2} (7) \begin{pmatrix} 3 & 5 \\ 6 & 9 \end{pmatrix} + (-1)^{1+3} (2) \begin{pmatrix} 3 & 4 \\ 6 & 0 \end{pmatrix} \\ = (36 - 0) - 7(27 - 30) + 2(0 - 24) \\ = 36 - 7(-3) - 48 \\ = 9$$

Cramer's rules

$$\rightarrow A^{-1}B = x$$

$$n = \frac{\det A}{\det A}$$

$$Ax = B$$

A_i = A matrix where i th column
is replaced by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Fastest way = Gaussian elimination

$$Ax = b$$

x

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$$

$$\begin{aligned} \det A &= 2(-1-6) - 3(-1-3) - 1(2-1) \\ &= 2(-7) - 3(-4) - (1) \\ &= -14 + 12 + (-1) \\ &= -3 \end{aligned}$$

$$x_1 = \frac{\det A_1}{\det A}, \quad = \frac{-3}{-3} = 1$$

$$x_1 : \begin{pmatrix} 4 & 3 & -1 \\ 1 & 1 & 3 \\ 3 & 0 & -1 \end{pmatrix} = 4(-7) - 3(-1-9) + (-1)(3-2) \\ = -28 + 30 - 1 \\ = 3$$

$$x_2 : \begin{pmatrix} 2 & 4 & -1 \\ 1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix}$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{-6}{-3} = 2$$

$$\begin{aligned} \det A_2 &= 2(-1-9) - 4(-1-3) + (-1)(3-1) \\ &= -20 + 16 - 1 \\ &= -6 \end{aligned}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$$

INVERSE WITH GAUSS-JORDAN ELIMINATION

↳ $[A|I] \rightarrow [U|I] \rightarrow [I|B]$

↳ tutorial 9 :

$$A = \left(\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 & 1 & 0 \\ 1 & 2 & -1 & 0 & 0 & 1 \end{array} \right)$$

elimination

$$R_2 = R_1 - 2R_2$$

$$R_2 = R_2 - R_3$$

$$= \left(\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & 1 & -2 & 0 \\ 0 & 1 & 4 & 0 & 1 & -1 \end{array} \right)$$

$$R_3 = R_3 + R_2$$

$$= \left(\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & 1 & -2 & 0 \\ 0 & 0 & -3 & 1 & -1 & -1 \end{array} \right)$$

$$R_3 = R_3 / -3$$

$$= \left(\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & 1 & -2 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array} \right)$$

$$R_1 = R_1 + R_3$$

$$R_3 = R_2 + 7R_3$$

$$= \left(\begin{array}{ccc|ccc} 2 & 3 & 0 & \frac{2}{3} & \frac{4}{3} & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{4}{3} & \frac{1}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{4}{3} & \frac{1}{3} \end{array} \right)$$

$$R_1 = R_1 - 3R_2$$

$$= \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{14}{3} & -\frac{2}{3} & -\frac{20}{3} \\ 0 & 1 & 0 & -\frac{4}{3} & \frac{1}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array} \right)$$

$$R_1 = R_1 / 2$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{7}{3} & -\frac{1}{3} & -\frac{10}{3} \\ 0 & 1 & 0 & -\frac{4}{3} & \frac{1}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{4}{3} & \frac{1}{3} \end{array} \right)$$

$$\therefore \left(\begin{array}{ccc} -\frac{7}{3} & -\frac{1}{3} & -\frac{10}{3} \\ -\frac{4}{3} & \frac{1}{3} & \frac{7}{3} \\ -\frac{1}{3} & \frac{4}{3} & \frac{1}{3} \end{array} \right) = A^{-1}$$

$$R_2 = R_1 - 2R_2$$

$$R_2 = R_2 - R_3$$

$$= \left(\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & 1 & -2 & 0 \\ 0 & -1 & 4 & 0 & 1 & -1 \end{array} \right)$$

$$R_2 = R_1 - 2R_2$$

$$\begin{aligned} 2 - 2(0) &= 2 - 0 \\ &= 2 \end{aligned}$$

$$R_3 = R_3 + R_2$$

$$= \left(\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & 1 & -2 & 0 \\ 0 & 0 & -3 & 1 & -1 & -1 \end{array} \right)$$

$$3 - 2(1) = 3 - 2$$

$$= 1$$

$$\begin{aligned} -1 - 2(-7) &= -1 + 14 \\ &= -13 \end{aligned}$$

① Find direction

② Find point

③ FOLLOW 1.38

1)(c) 2 lines, min distance

5)(b) LK 4 pass **(Q6)**