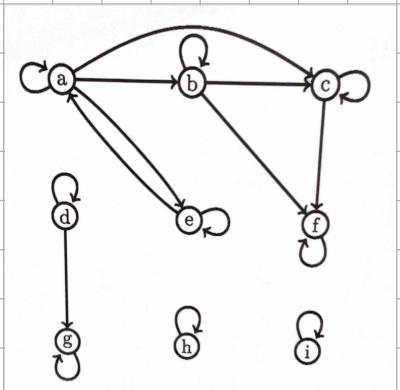
Continuation :

$A = \{a, b, c, d, e, f, g, h, i\}$

$S = \mathbb{Z} \times \mathbb{Z}$ by $(u, x) S (y, z)$ if and only if $u + x - y - z$ is even



19) reflexive = ✓
symmetric = X

20) antisymmetric : X
transitive : X ⇝ aRb, bRf but a $\not R$ f

21) reflexive = ✓ ⇝ $(u, x) S (w, x)$ if $u = 1, x = 2$
$1 + 2 - 1 - 2 = 0$
$2 + 1 - 2 - 1 = 0$
} so ✓ reflexive

symmetric = ✓
$(u, x) S (y, z)$ if $u = 1, x = 2, y = 3, z = 4$
$1 + 2 - 3 - 4 = 3 - 3 - 4 = -4 \Rightarrow$ even
$3 + 4 - 1 - 2 = 7 - 3 = 4 \Rightarrow$ even
} so ✓ symmetry

22) antisymmetric = X
transitive = ✓ ⇝ for all $(u, v), (u, x), (y, z) \in \mathbb{Z} \times \mathbb{Z}$

23) $S = RST$
4 equivalence relation
2 equivalence class : $\{(w, x) : (u, x) \in \mathbb{Z} \times \mathbb{Z}$ and $u + x$ is even$\}$
$\{(w, x) : (u, x) \in \mathbb{Z} \times \mathbb{Z}$ and $u + x$ is odd$\}$

24) written as sum of 1, 3, 4 only
$S_3 = 1 + 1 + 1$ or 3

25) $S_n = S_{n-1} + S_{n-3} + S_{n-4}$
because only can written as sum of 1, 3, 4 only

26) $Pr[X_1 = 2 \wedge Y = 2) \longrightarrow \{2,2\}; \{1,2\}$
$Pr(X_1 = 1 \wedge X_2 = 2) + Pr(X_1 = 2 \wedge X_2 = 2) = (1/3 \times 1/3) + (1/3 \times 1/3) = 2/9$

27) $E[Y] = Pr(1) + Pr(2) + Pr(3)$
$Pr(Y = 1) = \{1,1\} = Pr(X_1 = 1 \wedge X_2 = 1) = 1/9$
$Pr(Y = 2) = \{2,1\}, \{2,2\}, \{1,2\} = 3/9$
$Pr(Y = 3) = \{1,3\}, \{3,3\}, \{3,1\}, \{3,2\}, \{2,3\} = 5/9$
$Pr(1) + Pr(2) + Pr(3) = 1/9 + (2 \times 3/9) + (3 \times 5/9)$
$= 22/9$

AND = x
OR = +

28) probability : $\{1\dots100\} \times \{1\dots100\} = 1/1000$

29) Poisson distribution , 3 call per minute
- $E(X) = \lambda$
- $\text{var}(X) = \lambda$
- $Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$

no calls in 1 min probability
$Pr(X=0) = 3^0 e^{-3} / 0!$
$= e^{-3}$

30) 2 calls in 2 min
$Pr(X=2) = e^{-3} 3^2 / 2!$
$= e^{-3}(9)/2$

$Pr(Y \geq 2) = 1 - Pr(Y=1) - Pr(Y=0)$
$= 1 - (e^{-6} 6^1/1!) - (e^{-6} 6^0/0!)$
$= 1 - 6e^{-6} - e^{-6}$
$= 1 - 7e^{-6}$

31) 8

32) Closed Euler trail $\overset{?}{\Rightarrow}$ SPRQTS
BUT ODD DEGREES SO NO!

33) QS, RS must include ST OR QT AND one from PQ, PR, PS so $2 \times 3 = 6$

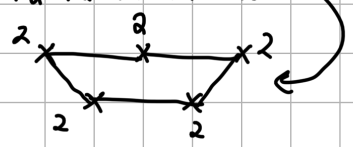
34)

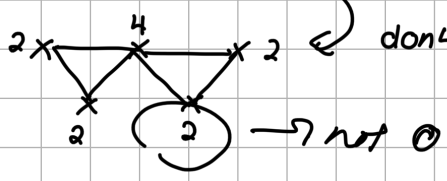
	v_1	v_2	v_3	v_4	
v_1	0	1	1	1	v_1v_4
v_2	1	0	1	1	v_2v_4 } 2
v_3	1	1	0	0	← start = row
v_4	1	1	0	0	

35) need m with the power of 6

36) 100 vertices = 99 edges
↳ disconnected = exist but no tree

38) (A) $5+3+2+2+2 = 14$ → 5 vertice, 1 vertex max 4 edge
(B) have 0 = disconnected ; $4+2+2+2+0 = 10$
(C) $2+2+2+2+2 = 10$

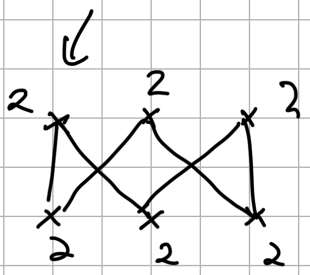


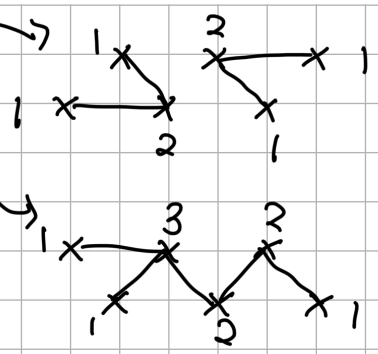


39) (A) $2+2+1+1+1=8$ → disconnected

(B) $3+2+2+1+1+1=10$

(C) $2+2+2+2+2+2=12$





6 vertice + 5 edge HENCE sum of degree $= 2 \times 5 = 10$ (B)

40) $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n}{n-r} \times \frac{(n-1)!}{r!(n-1-r)!} = \frac{n}{n-r}\binom{n-1}{r}$

41) $3 \times 5 \times \binom{4}{2} = 15 \times \frac{4!}{2!(4-2)!}$

$= 15 \times \frac{4 \times 3 \times 2!}{2!(2!)}$

$= 15 \times \frac{12}{2}\ 6$

$= 90$ #

42) simple induction: $19^n - 12^n \mid 7$ for all integers $n \geq 1$