

**MAT1830 - Discrete Mathematics for Computer Science**  
**Tutorial Sheet #7 and Additional Practice Questions**

**Tutorial Questions**

1. Calculate the following by hand.

(a)  $4!$   
 (b)  $\frac{10!}{8!}$   
 (c)  $\binom{10}{8}$   
 (d)  $\binom{7}{3}$

$$\begin{aligned} \text{(a)} \quad 4! &= 4 \times 3 \times 2 \times 1 = 24 & * \quad {}^nC_r &= \frac{n!}{(n-r)! r!} \\ \text{(b)} \quad \frac{10!}{8!} &= \frac{10 \times 9 \times 8 \times \dots}{8 \times 7 \times \dots} = 90 & * \quad {}^nPr &= {}^nCr \times r! \\ \text{(c)} \quad \binom{10}{8} &= {}^{10}C_8 = \frac{10!}{2!8!} = 45 & {}^nPr &= \frac{n!}{(n-r)!} \\ \text{(d)} \quad \binom{7}{3} &= {}^7C_3 = \frac{7!}{3!4!} = 35 \end{aligned}$$

2. (a) How many ways can a president, treasurer and secretary be chosen from a group of 10 people.  
 (They must be three separate people.)  $10P_3 \times 3!$   
 (b) How many ways can a team of three people be chosen from a group of 10 people.  ${}^{10}C_3$   
 (c) What's the essential difference between (a) and (b)? Which answer is larger? Could you have known this without doing any calculation? (b) answer is larger as (b) doesn't require arrangement  
 (d) How many ways can a bowl of three scoops of ice-cream be selected from 10 flavours?  
 (Multiple scoops of the same flavour are allowed.)  $[{}^{10-1} + 3] C_9 = {}^{12}C_9$   
 (e) How many ways can five different prizes be divided among Anastasia, Becky and Cadel?  
 (Not everyone has to get a prize.)  $5C_3$   $\hookrightarrow$  distinguishable  
 (f) In how many different orders can six horses finish a race?  
 (Assume there are no ties and they all do finish.)  $10P_3$

3. Yet another death star is firing missiles at a squadron of 16 rebel A-wings. Each missile will lock on to one of the A-wings but it cannot control which. What is the smallest number of missiles it must fire to ensure that at least one A-wing has at least 6 missiles locked on to it?

4. What is the coefficient of  $x^9$  in the expansions of the following? (Leave your answer as a mathematical expression rather than a number.)

(a)  $(x+2)^{20}$   
 (b)  $(3x+2)^{20}$   
 (c)  $(3x^3+2)^{20}$

	3				
	3	0	2	1	2
	3				

5. One side of a 6-sided die is marked "0", two sides are marked "2", and three sides are marked "3". Each side is equally likely to occur when the die is rolled. The die is rolled twice and the results are recorded.

Let  $A$  be the event that the first roll is 0.

Let  $B$  be the event that the second roll is 3.

Let  $C$  be the event that the sum of the two rolls is 5.

- (a) Find  $\Pr(A)$ ,  $\Pr(B)$ , and  $\Pr(C)$ .

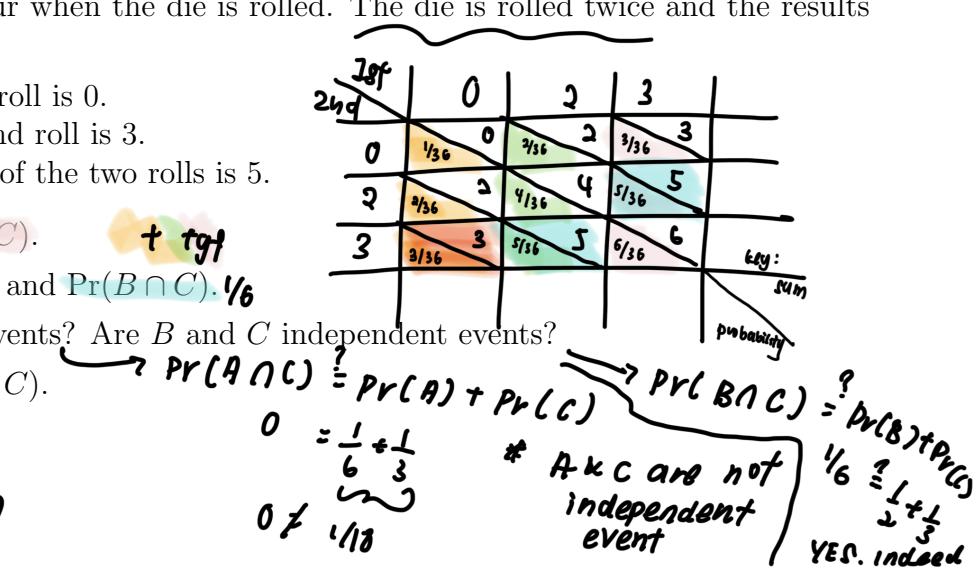
- (b) Find  $\Pr(A \cap B)$ ,  $\Pr(A \cap C)$ , and  $\Pr(B \cap C)$ .

- (c) Are  $A$  and  $C$  independent events? Are  $B$  and  $C$  independent events?

- (d) Find  $\Pr(A \cup C)$  and  $\Pr(B \cup C)$ .

(See over for practice questions.)

$$\Pr(A \cup C) = \Pr(A) + \Pr(C) - \Pr(A \cap C) \rightarrow \Pr(A \cap C) = \Pr(A) \cdot \Pr(C)$$



chef cares how the dishes are ordered  
↳ the quantity of each dish ordered

### Practice Questions

7 people order 7 dishes from menu of 12 dishes

1. A family of 7 are out to lunch and each of them orders one dish from a menu of 12 dishes.

- (a) From the waiter's perspective, how many orders are possible? (A)  $12^7$   
(b) From the chef's perspective, how many orders are possible? (B)  $12C7$   
(c) What is the effect of changing the perspective in the above? (C)

→ 12 ways each ppl can order

→ 0123456789

2. (a) How many PINs consisting of a string of five decimal digits are there? (A)  $10P_5$   
there is 26 alphabets.  
(b) How many passwords of length 5 using only decimal digits and upper and lower case letters (0–9, a–z and A–Z) are there?  $\text{Upper + Lower case} = 26 + 26$  total choices =  $52 + 10 = 62$  (B)  $62P_5$   
 $= 52$   
(c) What length of PIN is required to be more secure than the password in (b)?  
(C) a length longer than 5 like what we normally see is from 8 to 10
3. A tyrannical tutor demands that a tutorial of 16 oppressed students split themselves into four groups of 4 to work at whiteboards. In how many ways can this be accomplished?
4. A restaurant offers 4 different vegetarian dishes and 6 different meat dishes.
- (a) How many ways are there for the restaurant to serve a party of 20 different people one dish each so that 8 vegetarian and 12 meat dishes are served?  
(b) How many ways are there for the restaurant to serve a party of 20 different people one dish each so that 17 vegetarian and 3 meat dishes are served?  
(c) What do your answers above have to do with the expansion of  $(4v + 6m)^{20}$  and the binomial theorem?

2) (a) president, treasurer, secretary choose from 10 people

$$\cancel{^{10}C_1 \times ^9C_1 \times ^8C_1} \quad \text{OR } {}^{10}P_3 \text{ OR } 10 \times 9 \times 8 = 720$$

NOT  ${}^{10}C_3$  because 1 from the 10 is already chosen to be president



(b) team of 3 people choose from a group of 10 people

$${}^{10}C_3 \times 3!$$

(c) (a) have a specific requirement / arrangement but (b) doesn't.

Hence, (b) is larger. YES.

(d) Bowl of 3 scoops of ice-cream selected from 10 flavours

$${}^{10}C_3 \times 3!$$

(e) 5 different prize divided among Anastasia, Becky and Candi?  
(not everyone needs a prize)

$${}^5P_3$$

3. In a game a player rolls a standard die. If the number on the die is odd then that number is their final score for the turn. If the number on the die is even, they then flip a fair coin and their final score for the turn is half the number shown on the die if they flip tails and twice the number shown on the die if they flip heads.

Let  $A$  be the event that the player rolls a 2 or a 3 on the die.  
Let  $B$  be the event that the player's final score for the turn is 1.  
Let  $C$  be the event that the player's final score for the turn is 2.

(a) Find  $\Pr(A)$ ,  $\Pr(B)$  and  $\Pr(C)$ .  
(b) Find  $\Pr(A \cap B)$  and  $\Pr(A \cap C)$ .  
(c) Are  $A$  and  $B$  independent events? Are  $A$  and  $C$  independent events?

Give full reasoning and justifications in your submission.

① unordered repeated

↳ mom tell you to purchase drinks from Walmart

② unordered not repeated

↳ 10 people choose 3 people

③ ordered repeated

↳ PIN number, password

④ ordered not repeated

↳ President, treasurer, secretary choosed from 10 people