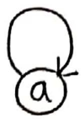
aRb means a is R-related to b when (a, b) is in the set

* order matters because aRb but $b \not R a$

every function is a relation but
every relation may not be a function

Properties of equivalence relation (RST)



① Reflexitivity if $(a,a) \in R$ for every element $a \in A$

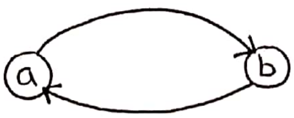
→ property where every element of the set is related to itself

→ to prove R is reflexive, show that for all $x \in A$, xRx

→ to prove R is NOT reflexive, show that there are some $y \in A$, $y \not R y$

② Symmetry if $(b,a) \in R$ whenever $(a,b) \in R$ for all $a,b \in A$

→ property where any element is related to another element and the element is also related to itself for every pair of element in the set



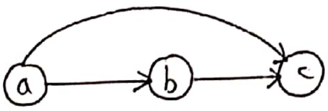
→ to prove R is symmetric, show that for all $x,y \in A$, if xRy then yRx

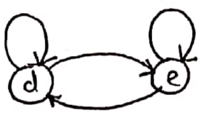
→ to prove R is NOT symmetric, show that there are some $x,y \in A$ such that xRy but yRx

③ Transitivity if whenever $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$ for all $a,b,c \in A$

→ if a is related to b and b is related to c, then a must be related to c

→ to prove R is transitive, show that for all, $x,y,z \in A$, if xRy and yRz then xRz





→ to prove R is NOT transitive, show that there are some $x,y,z \in A$ such that xRy and yRz but $x \not R z$

How many relations are there on a set of n elements?

2^{n^2} relations

Properties of partial order relation (RAT)

① Reflexitivity

② Antisymmetric

③ Transitive

$a \longrightarrow b$

→ to prove R is antisymmetric, show that for all $x, y \in A$, if xRy and yRx then $x = y$ for all $a, b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a = b$

→ to prove R is NOT antisymmetric, show that there are some $x, y \in A$ such that $x \neq y$, xRy and yRx

* if a relation is symmetric then it is not antisymmetric
if a relation is NOT symmetric then it may/may not be antisymmetric

Total order relation

→ is a partial order relation that has the property aRb or bRa for all $a, b \in A$

Well-order relation

→ is a total order relation with the property that every non-empty subset of A has a least element

Summary

Reflexive : aRa for all a

Symmetric : $aRb \rightarrow bRa$ for all a, b

Antisymmetric : $aRb \wedge bRa \rightarrow a = b$

Transitive : $aRb \wedge bRc \rightarrow aRc$