

Intersection

① point & line

1. sub point into line
2. get 3 equations (parametric form)
3. find t
↳ if t = same value \rightarrow intersects!

② line & line

$$l_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix} t$$

$$l_2 = \begin{pmatrix} m \\ n \\ o \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} s$$

1. $l_1 = l_2$ and get t, s
2. sub t, s to 2nd eq to check
3. sub t, s to eq get coordinate

③ plane & line

1. sub line to plane, find t
2. sub t back to line to get coordinate

④ plane & plane

$$\text{plane 1: } ax + by + cz = t$$

$$\text{plane 2: } dx + ey + fz = s$$

$$\begin{matrix} \rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} \\ \rightarrow \begin{pmatrix} d \\ e \\ f \end{pmatrix} \end{matrix} \left. \vphantom{\begin{matrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \\ \begin{pmatrix} d \\ e \\ f \end{pmatrix} \end{matrix}} \right\} \begin{matrix} n_1 \\ n_2 \end{matrix}$$

1. find $n_1 \times n_2 = \vec{n}$
2. plane 1 = plane 2 \rightarrow make p2 x/y/z as topic
3. sub $x=0 / y=0 / z=0$ to get coordinate
↳ random point on line
4. line of intersection = random point + $t(\vec{n})$
(equation)

Distance

① point & line

$$l = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix} t$$

$$\text{point} = (m, n, o)$$

1. Find $u = l - \text{point}$

$$2. \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot u = 0$$

Find t

3. Sub t to l to get coordinate

$$v = \text{coordinate} - \text{point}$$

$$4. \text{Find } v_d = \frac{v \cdot d}{|d|} \text{ where } d = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$5. \text{distance} : \sqrt{|v|^2 - (v_d)^2}$$

③ point & plane

1. Find random point (a)

$$\hookrightarrow y=0, z=0$$

$$2. v = \text{point} - a$$

* n from
plane
equation

$$3. v_n = \frac{v \cdot n}{|n|}$$

② line & line

$$l_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix} t$$

$$l_2 = \begin{pmatrix} m \\ n \\ o \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} s$$

$$n = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix}$$

$$v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} m \\ n \\ o \end{pmatrix}$$

$$v_n = \frac{v \cdot n}{|n|}$$

④ line & plane

① random point (a)

$$\hookrightarrow y=0, z=0$$

$$② v = \text{line's } n - a$$

$$③ v_n = \frac{v \cdot n}{|n|} \rightarrow \text{plane's } n$$

⑤ plane, plane

$$\text{plane 1} : ax + by + cz = t$$

$$\text{plane 2} : dx + ey + fz = s$$

* if parallel plane, n is the same,
use either 1

$$1. n = \begin{pmatrix} a \\ b \\ c \end{pmatrix} - \begin{pmatrix} d \\ e \\ f \end{pmatrix} \quad 2. v = p_1 - p_2$$

$$3. v_n = \frac{v \cdot n}{|n|}$$

random point $(p_1) \hookrightarrow y=z=0$

random point $(p_2) \hookrightarrow x=z=0$