- A path is a walk with no repeated vertices
- A trail is a walk with no repeated edges

<u>Bipartite graph</u>

A graph is said to be bipartite if its vertex set can be partitioned into two sets such that edges only go between the two sets and not within them

- a complete bipartite graph is a bipartite graph in which each vertex in the first set is joined to each vertex in the second set by exactly one edge
- to determine what is the maximum number of edges that a bipartite graph with n vertices can have:
  (i) when n is even, maximum number of edges = n^2 / 4
  (ii) when n is odd, maximum number of edges = (n^2 - 1) / 4
- application of bipartite graph: web search engines, social network

<u>Adjacency matrix</u>

- Draw a graph according to the adjacency matrix given
- In the adjacency matrix, 1 means V_I is adjacent to V_j
- The (I, j) entry in the kth power of the adjacency matrix gives the number of walks of length k between V_i and V_j (the length of a walk is the number of edges between V_i and V_j)
- If V_i and V_j are adjacent (i.e. connected) to each other, odd length of walks is required to get from V_i to V_i whereas even length of walks is required to get from V_i to V_j

<u>Conditions to check whether a graph exists:</u>

(i) The sum of degrees is even
  - Sum of degrees = 2 x number of edges

- Properties of simple graph:
  - No loops
  - No multiple edges
  - Can be connected or disconnected
  - For a simple graph of n vertices, each vertex can only have at most n-1 degree

- Properties of tree:

(i) A tree with n vertices has n-1 edges
(ii) A simple graph which is connected (but removing any single edge will make it disconnected) and acyclic (i.e. no cycle but adding any single edge will form a single cycle)

- Properties of spanning tree
  - Is the subgraph of the graph that is a tree but hits every vertice
  - does not contain any cycles or loops
  - removing one edge makes it disconnected (the spanning tree is said to be minimally connected)
  - adding one edge makes it having a cycle (the spanning tree is said to be maximally acyclic)
  - has n-1 edges
  - all the vertices are connected together without any cycles and minimum possible total edge weight

(a) finding minimum spanning tree
   - ➔ start with the edge that has the lowest weight, connect the two vertices with that edge
   - ➔ look for the edge with the second lowest weight and check if there are alternative route that link the two edges, if yes then ignore this edge, if no then draw an edge between these two vertices
   - ➔ continue by finding the third lowest weight and so on and check if there is already an alternative route that joins the two vertices

<u>Conditions to check whether a Euler trail exists</u>:

(i) a graph that has not more than 2 odd degree vertices

- properties of Euler trail:
  - (a) a trail that uses every edge exactly once
  - (b) a graph with at most 2 odd degree vertices
  - (c) the start and end vertex are different

(i) properties of closed Euler trail:
   - ➔ a connected graph with no odd degree vertices has a CLOSED Euler trail
   - ➔ have the same start and end vertex

- algorithms to find a spanning tree of a graph:
  1) breadth-first algorithm (BFS)
     - is a vertex-based technique used to find the shortest path in the graph
     - start from head vertex (normally A), then add an edge to its child following alphabetical order, it all its chid present then the parent dies, the child will connect to their child following alphabetical order and so on
     - more suitable for searching vertices which are closer to the given source
     - slower than DFS
     - application: unweighted graphs, P2P networks, web crawlers, network broadcasting

  2) depth-first algorithm (DFS)
     - is a edge-based technique
     - it uses stack data structure
     - “if got road then walk, if no road then walk backwards(backtrack)”
     - more suitable when there are solutions away from source
     - application: weighted graph, detecting a cycle in a graph, path finding, topological sorting, searching strongly connected components of a graph, solving puzzles with only one solution