Math Journal

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Combinatorial Proof

Proposition 1. *For positive integers n and k with $n = 2k$, $\frac{n!}{2!^k}$ is an integer.*

Proof. Consider the n symbols: $x_1, x_1, x_2, x_2, \cdots, x_k, x_k$. The number of arrangements of all these $n = 2k$ symbols is an integer that equals

$$\frac{n!}{\underbrace{2!2!\cdots 2!}_{\text{k factors of 2!}}} = \frac{n!}{2!^k}$$

I learned this from the first day of class in MACM 101. This is an example of proving that a value is an integer by obtaining that value from counting something. You can also use combinatorics in proof by double counting:

Identity 1.

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

This can be proved by counting the number of subsets of a set with n elements.