Target Recognition in Radar Images Using Weighted Statistical Dictionary-Based Sparse Representation

Ayoub Karine, Abdelmalek Toumi, Ali Khenchaf, and Mohammed El Hassouni

Abstract— **In this letter, we present a novel generic approach for radar automatic target recognition in either inverse synthetic aperture radar (ISAR) or synthetic aperture radar (SAR) images. For this purpose, the radar image is described by a statistical modeling in the complex wavelet domain. Thus, the radar image is transformed into a complex wavelet domain using the dual-tree complex wavelet transform. Afterward, the magnitudes of the complex sub-bands are modeled by Weibull or Gamma distributions. The estimated parameters of these models are stacked together to create a statistical dictionary in training step. For the recognition task, we use the weighted sparse representation-based classification method that captures the linearity and locality information of image features. In this context, we propose to use the Kullback–Leibler divergence between the parametric statistical models of training and test sets in order to assign a weight for each training sample. Experiments conducted on both ISAR and SAR images' databases demonstrate that the proposed approach leads to an improvement in the recognition rate.**

Index Terms— **Automatic target recognition (ATR), dual-tree complex wavelet transform (DT-CWT), inverse synthetic aperture radar (ISAR)/synthetic aperture radar (SAR), Kullback–Leibler divergence (KLD), non-Gaussian statistical modeling, weighted sparse representation.**

I. Introduction

RADAR target recognition is one of the most flourishing research topics in remote sensing. Due to the ability of radar sensor to work against inclement weather, the automatic target recognition (ATR) based on radar images has become an active research field for several military and civilian applications. The ultimate goal of radar ATR system is to detect and recognize unknown radar targets. In this letter, we focus on the automatic radar target recognition using inverse synthetic aperture radar (ISAR) and synthetic aperture radar (SAR) images. The main difference between these two radar image types is that ISAR images exploit the motion of the target to create the synthetic aperture whereas the SAR images utilize the motion of the radar (emitter). We underline here that the ISAR images are exploited in our work to recognize the aerial targets whereas the SAR images are used to recognize the ground targets. Generally, the extraction of good features and appropriate classifier are the fundamental parts of the ATR system success.

Recently, several works [1], [2] give a special attention to characterize a radar image using the wavelet transform that captures details of an image at different levels and orientations. Moreover, in other image processing applications, several researchers [3], [4] adopt a scheme that consists in representing each wavelet sub-band by a parametric probabilistic model that describes the marginal statistics of sub-bands. In our research, we follow the same philosophy and we aim to extract a new information from radar images that is the magnitude of dual-tree complex wavelet transform (DT-CWT) coefficients and model it statistically. For this end, different adequate distributions have been investigated, which are Gamma and Weibull models. More precisely, we perform the DT-CWT on ISAR/SAR images. Then, the produced complex sub-bands magnitudes are modeled by a non-Gaussian statistical model. The resulting statistical parameters are combined to build the feature vector. This representation of radar images is characterized by a few and pertinent sets of parameters that speed up the classification process.

On the other hand, for the classification task, the sparse representation-based classification (SRC) [5] has aroused a considerable resurgence of interest due to its state-of-the-art performance in various fields, including face recognition [5] and target recognition in SAR images [2], [6]. The idea behind using SRC is to represent a test sample as a linear combination of training samples. Based on sparse reconstruction error, the SRC determines the class of the test sample. However, this classification method employs only the linearity structure of data and suffers from the locality one. To overcome this limit, Lu *et al.* [7] and Fan *et al.* [8] proposed a weighted SRC (WSRC) method. According to the distance between a test sample and training samples, the WSRC assigns a weight for the training samples. In this way, the test sample will be represented by the significant training samples and both linearity and locality are taken into account. In this letter, we modify the WSRC method by integrating the Kullback–Leibler divergence (KLD) between statistical models. Specifically, to recognize unknown features vectors of targets, we compute its distances with the training samples using the KLD. Then, the weights of the training samples are determined by considering the distance information. After that, we represent the

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A. Karine is with the Lab-STICC UMR CNRS 6285, ENSTA, 29806 Bretagne, France, and also with the LRIT-CNRST, URAC 29, Rabat IT Center, Faculty of Sciences, Mohammed V University in Rabat, Rabat, Morocco (e-mail: ayoub.karine@ensta-bretagne.org).

A. Toumi and A. Khenchaf are with the Lab-STICC UMR CNRS 6285, ENSTA, 29806 Bretagne, France (e-mail: abdelmalek.toumi@ensta-bretagne.fr; ali.khenchaf@ensta-bretagne.fr).

M. El Hassouni is with the LRIT-CNRST, URAC 29, Rabat IT Center, FLSH, Mohammed V University in Rabat, Rabat, Morocco (e-mail: mohammed.elhassouni@um5.ac.ma).

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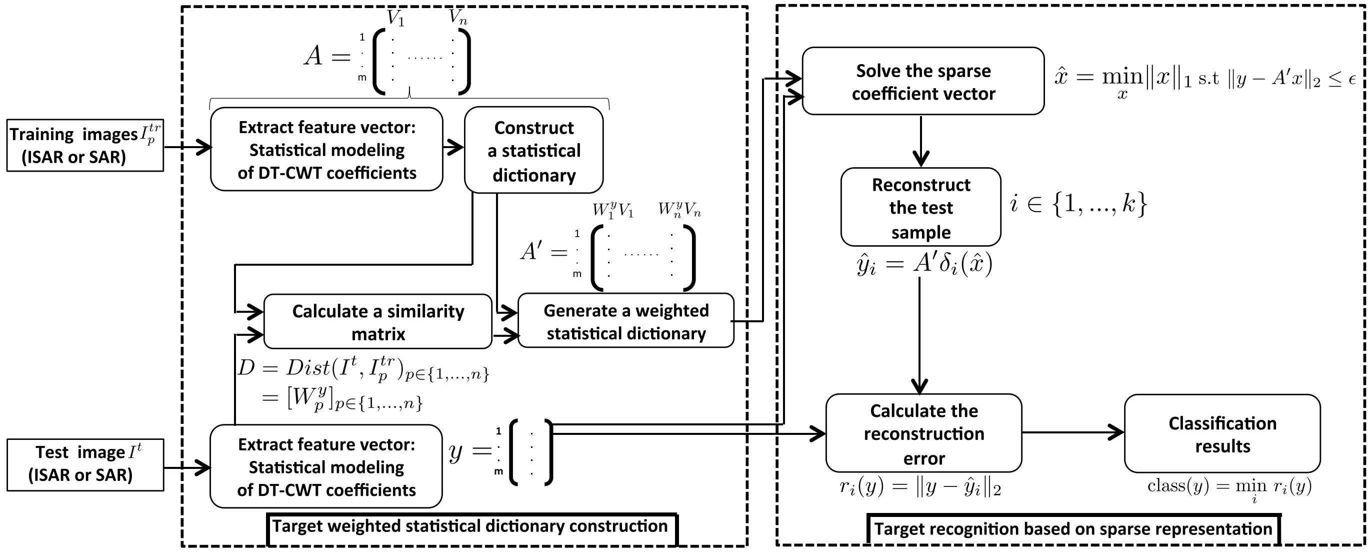

Fig. 1. Flowchart of the proposed approach.

test sample using the weighted statistical dictionary. Finally, the class label of the test sample is the one that has the minimum residual in the sparse reconstruction errors.

The rest of this letter is organized as follows. Section II gives an overview of the proposed radar target recognition approach. Section III reports the experiment results on two radar images database. Finally, we conclude this letter in Section IV.

II. Proposed Target Recognition Method

In this letter, we adopt a sparse classifier to recognize radar target. Despite that this classification method proves its performance in pattern recognition, it suffers by its time consuming specially in the case of feature with a huge dimension like the use of pixels information or the whole wavelet coefficients as feature vectors. To overcome this limit, one of the aims of this letter is to propose a feature vector with a reduced dimension, which will be suitable with the sparse framework and consequently speed up the classification process. For this end, we use an approach that models the magnitude of DT-CWT coefficients, which provides a feature vector with reduced dimension. The flowchart of the proposed approach is depicted in Fig. 1. It is composed by two major sequential steps: the construction of weighted statistical dictionary and the recognition based on SRC. These two steps compose the WSRC algorithm. In the first step, we compute the feature vector of training samples using the statistical modeling of the magnitude of DT-CWT coefficients. The obtained feature vectors are after stacked together to create a statistical dictionary. For a given test sample, we compute its feature vector according to the same approach used for training samples. After that, we calculate its similarity with all training samples. This similarity information represents the weight of each training sample according to the test sample. Each training sample is then multiplied by its weight to build a weighted statistical dictionary. By solving an l^1 minimization problem, a sparse vector is obtained to code the test sample. Finally, this sparse vector leads to recognize the test sample as the class with the lowest reconstruction error. The different steps of the proposed approach are detailed in the next section.

A. Target Weighted Statistical Dictionary Construction

1) Feature Extraction and Statistical Dictionary Construction: To characterize each radar target, we transform the ISAR/SAR image into the complex wavelet domain using the DT-CWT method. This transformation captures the image details at different levels and orientations and is extensively used in image retrieval and pattern recognition. The DT-CWT produces six complex wavelet sub-bands (orientations) per decomposition level. The flowchart and more details about DT-CWT can be found in [9]. The magnitude of the resulting complex wavelet coefficients are considered as a set of random variables RV

$$\mathrm{RV} = (\mathrm{RV}^j)_{1\le j\le 6J} \tag{1}$$

where j is the index of a complex sub-band and J is the number of decomposition levels. After that, we estimate the statistical parameters (shape and scale) denoted $\theta^j_{1\le j\le 6J}$ of the magnitude of each produced complex sub-band (each RV^j) using the maximum-likelihood method. Motivated by the work in [4], which concludes that Gamma and Weibull distributions are suitable to model the statistical behavior of the magnitude of DT-CWT coefficients, we investigate these two models in order to select the most adequate for our data. The probability density functions (PDFs) of the Gamma and Weibull distributions can be found in Table I. Finally, the concatenation of the estimated statistical parameters of all produced sub-bands for all levels of decomposition leads to a feature vector of the radar image. In this letter, we denote $V_p \in \mathbb{R}^{m\times 1}$ and $y \in \mathbb{R}^{m\times 1}$ to refer to the feature vectors of train and test radar images, respectively. Each training feature vector V_p is stacked as a column of the statistical dictionary

$$A = [V_1, \ldots, V_n] \in \mathbb{R}^{m\times n} \tag{2}$$

where n is the number of training images, m is the feature vector dimension that is equal to $s * 6J$, where s is the number

TABLE I

PDF AND KLD OF GAMMA AND WEIBULL DISTRIBUTIONS

Model	Probability density function	Kullback-Leibler divergence
Gamma	$f(RV;\theta)=\frac{a^{-b}x^{b-1}}{\Gamma(b)}\exp\left(-\frac{x}{a}\right) \quad x\in\mathbb{R}^+$ with $\theta=(b,a)$ $b>0$: the shape parameter $a>0$: the scale parameter $\Gamma(z)=\int_0^\infty t^{z-1}e^{-t}dt$: the Gamma function	$\underset{Gamma}{\mathrm{KLD}}(f_1(RV_1;\theta_1)\Vert f_2(RV_2;\theta_2))=\psi(b_1)(b_1-b2)-b_1+\log(\frac{\Gamma(b_2)}{\Gamma(b_1)})+b_2\log(\frac{a_1}{a_2})+\frac{b_1a_1}{\theta_2}$
Weibull	$f(RV;\theta)=\frac{\tau}{\mu}\left(\frac{x}{\mu}\right)^{\tau-1}\exp\left(-\left(\frac{x}{\mu}\right)^{\tau}\right) \quad x\in\mathbb{R}^+$ with $\theta=(\tau,\mu)$ $\tau>0$: the shape parameter $\mu>0$: the scale parameter	$\underset{Weibull}{\mathrm{KLD}}(f_1(RV_1;\theta_1)\Vert f_2(RV_2;\theta_2))=\Gamma(\frac{\tau_2}{\tau_1}+1)(\frac{\mu_1}{\mu_2})^{\tau_2}+\log(\mu_1^{-\tau_1}\tau_1)-\log(\mu_2^{-\tau_2}\tau_2)+\log(\mu_1)\tau_1\log(\mu_1)\tau_2+\frac{(\gamma\tau_2)}{\tau_1}-\gamma-1$ where γ: the negative of the digamma function$\psi(x)=\Gamma'(x)/\Gamma(x)$

of the estimated statistical parameters per sub-band, and p is the index of a training image ($1\leq p\leq n$).

2) Similarity Measurement and Weighted Statistical Dictionary Construction: To measure the significance of each training samples according to the test sample, we propose to transform the statistical dictionary A to a weighted one A'. The training targets that are similar (closer) to the target test should have a larger weight (most significant) than the nonsimilar ones. For this end, we assign a weight to each atom in the dictionary A through the similarity information. Since a statistical dictionary is adopted in our method, this enables us to employ an appropriate distance measure between PDFs. Then we propose to use a distance formula that integrates the well-known KLD. The mathematical formulas of the KLD between Gamma and Weibull PDFs are recorded in Table I. Thus, the distance $Dist$ between a test radar image I^t represented by y and a train radar image I_p^{tr} represented by V_p is given by

$$\mathrm{Dist}\big(I^t, I_p^{\mathrm{tr}}\big)=\exp(-d_{\mathrm{KLD}}(I^t, I^{\mathrm{tr}}))=W_p^y \tag{3}$$

where W_p^y is the weight of a training target V_p according to the test target y. d_{KLD} is the KLD between the PDFs of test sub-bands ($\mathrm{RV}^{\mathrm{test},j}$) of I^t and the PDFs of training sub-bands ($\mathrm{RV}^{\mathrm{train},j}$) of I^{tr} and is expressed as follows:

$$d_{\mathrm{KLD}}(I^t, I^{\mathrm{tr}}) = \sum_{j=1}^{6J}\mathrm{KLD}(f_1(\mathrm{RV}^{\mathrm{test},j};\theta^{\mathrm{test},j})\Vert f_2(\mathrm{RV}^{\mathrm{train},j};\theta^{\mathrm{train},j})) \tag{4}$$

with f_1 and f_2 the two PDFs of Gamma or Weibull (see Table I) of $\mathrm{RV}^{\mathrm{test},j}$ and $\mathrm{RV}^{\mathrm{train},j}$ respectively. The produced distances can be directly used as a weight because their values are comprised between zero and one. For instance, the distance between an image and its self is zero, so its corresponding weight is equal to $\exp(0)=1$. After computing the weight of each training sample, the new weighted statistical dictionary $A'\in\mathbb{R}^{m\times n}$ is expressed as

$$A'=\big[W_p^yV_p\big]_{p\in\{1,\ldots,n\}}. \tag{5}$$

Following this strategy, instead of using the overall dictionary A to identify all test samples, a specific dictionary A' of each test sample is generated. This weighted dictionary records the linearity and locality (similarity) information between test and training samples. The obtained weighted statistical dictionary A' is used in the next step to identify the class of the given test sample y based on sparse representation.

B. Target Recognition Based on Sparse Representation

Once we obtain the weighted statistical dictionary A' for the test sample to identify, the second step relies on its classification according to the SRC method and A'. This algorithm aims to code the test sample as follows: $y=A'x$. The vector $x\in\mathbb{R}^n$ should be as sparse as possible. To this end, the following optimization problem should be solved:

$$\hat{x}=\min_x\ \Vert x\Vert_1\quad \text{s.t. } \Vert y-A'x\Vert_2\leq\epsilon \tag{6}$$

where $\Vert x\Vert_1=(\sum_{i=1}^n\mid x_i\mid)^{1/2}$ denotes the l^1−norm and ϵ is the error tolerance. Equation (6) can be efficiently solved via second-order cone programming. For each class $i\in\{1,\ldots,k\}$ where k is the number of classes, let $\delta_i:\mathbb{R}^n\rightarrow\mathbb{R}^n$ be the characteristic function that selects the coefficients associated with the ith class. In other words, the nonzero entries of $\delta_i(\hat{x})\in\mathbb{R}^n$ are the entries in $\hat{x}$ that are associated with class i. After the optimal solution $\hat{x}$ for (6) is found, the reconstruction of a test sample becomes $\hat{y}_i=A'\delta_i(\hat{x})$. Finally, the identity of the test sample y is the class that has the minimum residual r between y and $\hat{y}_i$

$$\mathrm{class}(y)=\min_i\ r_i(y)=\min_i\ \Vert y-\hat{y}_i\Vert_2. \tag{7}$$

Whereas the SRC is based only on the linearity information, the WSRC one preserves the sparse linear representation and also takes advantage from the locality aspect using the similarity between the test sample and its neighboring training samples.

III. EXPERIMENTAL RESULTS AND DISCUSSION

A. Radar Images Databases Description

In this section, we investigate the performance of our proposed approach for radar ATR using two radar images databases that are ISAR and SAR images. The first one is acquired from an anechoic chamber of ENSTA Bretagne (Brest, France) using 12 reduced-scale (1/48) aircraft models: Harrier, Rafale, Tornado, F104, F117, A10, F14, F15, F16, MIG29, F18, and F4. Each target is illuminated in the acquisition level by a frequency stepped signal with a bandwidth B between 11.65 and 18 GHz. Then, a sequence of $N+1$ pulses is emitted at linearly increasing frequencies $f_n=f_0+n\Delta f$ at time moments t_n, where n runs from 0 to N. The frequency increment in our case is $\Delta f=50$ MHz. To construct the ISAR images, the inverse fast Fourier transform is used [10]. As a result, 162 ISAR images of 256×256 grayscale pixels

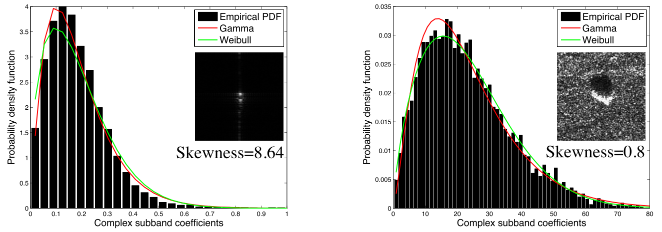

Fig. 2. Fitting of the complex wavelet coefficients at level 3 corresponding to an example of (Left) ISAR image and (Right) SAR image.

TABLE II
KS GOF TEST ON DT-CWT OF RADAR IMAGES USING TWO NON-GAUSSIAN STATISTICAL MODELS

	Statistical models			
	Weibull		Gamma	
Databases	ISAR	SAR	ISAR	SAR
KS	0.1384	0.1643	0.1325	0.1592

have been produced for each target. Consequently, 1944 ISAR images are generated. For more details about the used anechoic chamber, the reader is referred to [10]. For the second database, we use the moving and stationary target acquisition and recognition (MSTAR) public data set.[1] This data set includes 10 ground targets: four armored personnel carriers (BMP2, BRDM2, BTR70, and BTR60), two tanks (T72 and T62), a rocket launcher (2S1), a truck (ZIL131), a bulldozer (D7), and an air defense unit (ZSU234). These radar images are collected using an X-band SAR sensor in spotlight mode. This database is divided into test (captured at the 15° depressing angle) and training sets (captured at the 17° depressing angle). The total number of SAR images in training set is 2747 whereas it is 2425 in test set.

B. Statistical Fitting of ISAR/SAR Images

We study here the fitting of the magnitude of DT-CWT coefficients. We first show in Fig. 2 the empirical PDF (histogram) as well as their Gamma and Weibull distributions' fit. It is obvious that the considered empirical PDF has a leptokurtic shape (heavy tails), which motivates the use of non-Gaussian distributions. In addition, the empirical PDF is positive and asymmetric. To highlight this asymmetry of the whole two databases, we compute the skewness values. The skewness is between 1.72 and 38.02 for ISAR images and it is between 0.23 and 18.35 for SAR images. Then, for the both databases, the skewness values are positive, which demonstrate that the PDFs of all sub-bands are asymmetric and spread out more to the right. All these issues enable us to use the two families: Gamma and Weibull. In order to check the goodness-of-fit (GOF) of the selected statistical models, we use the Kolmogorov–Smirnov (KS) test as a quantitative analysis method as recorder in Table II. For the both radar images databases, the low values of KS for the Gamma and Weibull distributions demonstrate that they well describe the statistical behavior of the magnitude of the DT-CWT coefficients with a slight superiority of the Gamma distribution. Nonetheless, the ISAR images are well fitted than the SAR images.

[1] https://www.sdms.afrl.af.mil/index.php?collection=mstar

C. Target Recognition Results

We mention that all experiments are performed in the MATLAB 2016 environment with 3.10-GHz Intel processor and 8 GB of memory. For each radar image, we conduct a DT-CWT transform with the Kinkgsburys Q-Shift (14, 14)-tap filters in combination with (13, 19)-tap near-orthogonal filters with different levels of decomposition. Thereafter, we estimate the two Gamma or Weibull parameters for the magnitude of each complex sub-band. As a result, each radar target is presented by $6 \times J \times 2$ statistical parameters, where J is the level of decomposition. In the case of ISAR images' database, we conduct the hold-out cross validation to generate training and testing samples. We compared several quotients of training and test sets. For instance, with only 10% of training samples, our approach achieves a recognition rate of 83.73%. We also remark that when the percentage of training samples increases, the recognition rates rise as well but the runtime also increases due to the size of dictionary, which affects the speed of the solution of the optimization problem given in (6). For this end, 40% of the database is used for train and remaining for test. For SAR images, the MSTAR database is already partitioned (53% of the database for training and 47% for the test). Then, we used it in this form in order to be in compliance with the target recognition state-of-the-art results [2], [11]. To make a fair comparison, the classification results of our approach are compared with five classification methods: k-nearest neighbors (KNNs), support vector machine (SVM), SRC, WSRC1, and WSRC2 using the same descriptor. The tuning parameters of these classifiers are selected empirically. For KNN, we varied the K between 3 and 13 and select the K that leads to high recognition rate. We maintain $K = 3$ for ISAR images and $K = 9$ for SAR images. For SVM classifier, the linear kernel function is used for both ISAR and SAR images. The WSRC1 denotes the method proposed in [7] and WSRC2 stands for the contribution in [8]. The main difference between the WSRC1 and WSRC2 is the distance method. In the first one, Lu *et al.* [7] use $\|y - V_p\|$ as the weight. In the second one, Fan *et al.* [8] propose to measure the weight according to the following distance: $\exp((-d_{\mathrm{ED}}(y, V_p)^2)/(2\sigma^2))$, where ED is the Euclidean distance and σ is the width that is set to be the average Euclidean distance of the training samples. In our experiment, σ equals 1.65 for ISAR images and it equals 0.21 for SAR images. The proposed approach has different degrees of freedom such as the choice of level of decomposition, the selection of the statistical models, and the adopted classification methods. We study all of them in Table III for the both radar images databases. From these results, many common observations can be raised between the two tested databases. First, the increasing of levels of decomposition generally leads to an improvement in the recognition rates. We note here that three levels mean the concatenation of signatures of first, second, and three levels of decomposition. For the fourth level, although that provides a high recognition rate, the classification works in a low runtime

TABLE III

RECOGNITION RATES (%) OF DIFFERENT METHODS COMPARED WITH THE PROPOSED METHOD

	Levels of decomposition											
	1 Level				2 Levels				3 Levels			
Statistical models	Weibull		Gamma		Weibull		Gamma		Weibull		Gamma	
Databases	ISAR	SAR	ISAR	SAR	ISAR	SAR	ISAR	SAR	ISAR	SAR	ISAR	SAR
KNN	79.25	66.23	73.93	64.33	92.11	67.58	86.36	68.73	93.65	68.66	90.48	69.20
SVM	45.45	62.07	48.71	57.56	66.29	69.32	72.81	66.96	82.42	71.75	87.22	73.81
SRC [4]	30.96	23.14	42.19	34.03	73.67	62.89	80.79	62.58	93.56	78.43	95.19	79.68
WSRC1 [6]	32.42	24.86	41.20	34.20	73.67	63.04	81.39	63.75	94.17	80.03	95.45	80.17
WSRC2 [7]	30.27	27.13	42.28	35.24	72.73	64.29	79.41	66.32	93.91	80.11	95.79	80.26
Proposed method	30.96	28.01	42.02	36.57	73.76	65.73	81.39	69.07	93.74	80.95	**96.91**	**81.04**

due to the augmented size of the feature vector. Second, the Gamma distribution performs better than the Weibull distribution. This is in compliance with the low KS value of Gamma presented in Table II. Third, using Gamma distribution and three levels of decomposition, the low recognition rates are 87.22% and 69.20% for ISAR and SAR images, respectively. These recognition rates prove that the used descriptor is suitable to characterize radar images. Fourth, the sparse classifiers (SRC, WSRC1, and WSRC2) perform worst in the case of level 1 of decomposition. That is due to the small dimension of feature descriptor in this case that is 12 values. For this reason, we adopt the three levels of decomposition with Gamma distribution. In this case, the sparse classifiers work better than the KNN and SVM ones. Fifth, the different versions of WSRC outperform the typical SRC algorithm. This may be explained by the fact that the WSRC captures the locality information of the data. Finally, we underline that the proposed method provides a high recognition rate compared with the WSRC1 and WSRC2 methods. The two distances used in WSRC1 and WSRC2 are generic and not specific of a given built feature vector. Then an appropriate distance between statistical models is adopted, which assigns a meaningful weight to each atom in the statistical dictionary and leads to an excellent performance. Using the proposed method, the Gamma model with three levels of decomposition leads to 96.91% for ISAR images and to 81.04% for SAR images. Hence, the obtained results stressed out the superiority of the proposed method, which enhances the recognition rates. Regarding the computational complexity, the adopted feature extraction method is $\mathcal{O}(N)$, where N is the number of pixels of the radar image. The time of classification task has a complexity of $\mathcal{O}(m^2 \times n \times nt)$, where m is the dimension of feature vector and n and nt are the number of training and test samples, respectively. For the time consuming, to extract a feature vector, our approach needs a mean runtime of 0.08 and 0.06 s for each ISAR and SAR image, respectively. The classification of each ISAR and SAR image is done in a mean runtime of 0.27 and 6.86 s, respectively. We can observe that the runtime for classifying the SAR images is relatively higher than the ISAR images. That is due to the high number of SAR images in train and test sets that enhance the computational load of calculating similarity matrix [see (4)].

IV. CONCLUSION

In this letter, a generic novel approach for ISAR/SAR target recognition is proposed based on statistical model and WSRC. First, a statistical dictionary is constructed using the training set. It is done by estimating the statistical parameters of the magnitude of DT-CWT coefficients using a non-Gaussian model. In this context, we compared the Gamma and Weibull distributions. Given a test sample to classify, we computed the weight for a training sample according to the KLD between the statistical parameters of the test sample and the training sample. As a result, a weighted statistical dictionary is generated and fed into an SRC classifier. In this way, the WSRC integrates the data linearity and locality. Empirical results on two databases confirm that the used descriptor and the integration of the KLD information in WSRC lead to an improvement in the recognition rate. However, our approach does not work well in the case of SAR images. That is due to the clutter highly existed in these images. To omit this limitation, our future venues of exploration will concern the use of a segmentation method to locate the target and delete the clutter.

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