Euler 2D-PCA for SAR Target Recognition

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***Abstract*—Euler-Principal Component Analysis (e-PCA) has been recently proposed and successfully applied to the classification frame works. By utilizing the robust dissimilarity measure e-PCA demonstrates better performance than standard PCA while dealing with nonlinear component analysis and suppressing outliers. In this letter, we define a two-Dimensional Euler-Principal Component Analysis (e-2DPCA) framework for SAR image processing. e-2DPCA is based on 2D image matrixes rather than 1D vector which could understand two dimensional (2D) images better and get rid of high dimensional data processing. Furthermore, we applied this algorithm to SAR target recognition. Finally, experiments on MSTAR database perform the usefulness of our method in robust classification towards different situation.**

***Index Terms*—Image Processing, Feature Extraction, Target Identification, Synthetic Aperture Radar.**

I. Introduction

Synthetic aperture radar (SAR) has been widely used in many applications due to the excellent working ability to work in all weather and all day. SAR automatic target recognition (ATR) play an important role in better exploring and understanding SAR images. A typical SAR ATR algorithm mainly consists of three stages: detection, discrimination, and classification[1].First two stages can find and select out the image chips, i.e., regions of interest (ROIS), containing candidate targets. The ROIS is classified by the last step which is studied in this paper.

Currently, the present methods used for classification in SAR ATR consists of template based methods[1], model based methods[2] and the others[3][4]. Template based methods include extraction features and comparing with the templates which are generated by the training images. The unknown is declared as the class of the matched template. For SAR image, the matching is difficult due to the clutter background. Moreover, the variations in pose and depression increase the match complexity.

As a classical data analysis tool, Principal component analysis (PCA) has been widely used in feature extraction, data representation and dimensionality reduction[5][6][7]. Generally speaking, PCA preserves the majority of total variance while transforms amount of training samples into an uncorrelated subset[8]. In the majority PCA-based image processing, the image matrixes should be transformed into very high dimensional vectors. Relatively, the number of corresponding training sample is quite small. Hence, dealing with small sample size image processing problems PCA faces inaccurate feature extraction and huge computational costs. In order to deal with small sample size problem two-dimensional principal component analysis (2DPCA) was firstly proposed which is based on 2D image matrixes[9]. 2DPCA evaluates the covariance matrix more accurately in the small size problem of image processing.

The original assumption of standard PCA is linear data dependencies. However, amounts of non-linear dependencies exist in nature. Considering these phenomenas, a kernel version extension of PCA (KPCA) was proposed[10]. KPCA exploits underlying non-linear nature of data in feature spaces via the use of kernel function. Furthermore, nonlinear Complex Reproducing Kernel Hilbert Spaces (CRKHS) was studied and a general Widely Linear Kernel Complex PCA (WLCK-PCA) was proposed which aimed at solve small sample size problem[11]. Complex kernel PCA was applied in multimodal biometric recognition and the performance demonstrated significant improvement[12].

As we all known, the KPCA is equally defined as l_2-norm function, which is optimal for adaptive Gaussian noise removal but not robust to outliers. In order to suppressing outliers, Euler Principal Component Analysis (e-PCA) was proposed[13][14]. In e-PCA euler representation of data is taken into account and a robust dissimilarity measure is utilized which could reduce the effect caused by outliers. However, there is limited research on component analysis and dimensionality reduction in Euler representation of complex numbers. Due to the inner product based kernel function, 2DPCA cant be used to explore the nonlinear dependencies. However, the kernel function defined by e-PCA which has a close form feature space representation retains this desirable properties. With this kernel function fast pre-image computation and explicit application of standard linear PCA algorithm could come true. As we know, no relevant research results have been published in this field and my research aims to close this gap. In this paper, we formulate a 2D image based two-Dimensional Euler-Principal Component Analysis (e-2DPCA) methodology which is more fast and robust. Particularly the main contributions of this paper are listed as the following

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- we describe the principle of e-2DPCA theory.
- utilizing the framework, we show how e-2DPCA works in SAR target recognition.
- we demonstrate the state-of-art results by applying the proposed e-2DPCA to SAR target recognition in different situations.

The rest of this paper is organized as follows. In Section 2, the theory of Euler Principal Component Analysis is briefly introduced. In Section 3, we formulate the framework of our proposed algorithm. In Section 4, experiment results on MSTAR database is discussed. At last, conclusions are drawn in Section 5.

II. Euler Principal Component Analysis

The theory of e-PCA was recently proposed in [13],[14]. In e-PCA euler representation of data is taken into account and a robust dissimilarity measure is utilized which could reduce the effects caused by outliers. Where the robust kernel is denoted by

$$k(\boldsymbol{x},\boldsymbol{y}) = \sum\nolimits_N exp(j\alpha\pi(x(i) - x(j))) \quad (1)$$

where $\boldsymbol{x},\boldsymbol{y} \in R^{N\times 1}$ are input vectors and $x(i)$ denotes the ith element of x. This kernel function lead to a closed form feature space representation

$$\phi(\boldsymbol{x}) = \frac{1}{\sqrt{2}} exp(j\alpha\pi\boldsymbol{x}) = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{(j\alpha\pi x(1))} \\ \vdots \\ e^{(j\alpha\pi x(N))} \end{bmatrix} \quad (2)$$

Particularly, the elements $x(i)$ is normalized in $[\ 0 \quad 1\ ]$ and α is restricted between $[\ 0 \quad 2\ ]$. The above representation is also Euler representation of complex numbers. $\phi(\boldsymbol{x})$ could be considered as the mapping vector of $\boldsymbol{x}$ in the feature space.

The distance between $\phi(\boldsymbol{x}_p)$ and $\phi(\boldsymbol{x}_q)$ could be calculated as follows

$$\begin{aligned} \|\phi(\boldsymbol{x}_p) - \phi(\boldsymbol{x}_q)\|_F^2 &= \frac{1}{2}\|(\cos(\alpha\pi\boldsymbol{x}_p) + i\sin(\alpha\pi\boldsymbol{x}_p)) \\ &\quad -(\cos(\alpha\pi\boldsymbol{x}_q) + i\sin(\alpha\pi\boldsymbol{x}_q))\|_F^2 \\ &= \sum_{i=1}^{N} 1 - \cos(\alpha\pi x_p(i) - \alpha\pi x_q(i)) \\ &= d(\boldsymbol{x}_p, \boldsymbol{x}_q) \end{aligned} \quad (3)$$

As we can see, e-PCA use the dissimilarity measure showed by 3 instead of l_2-norm which has a parameter α . Fig.1 shows that the change of dissimilarity function as a function of α . While α has a small value the proposed dissimilarity measure shows the same effect as the l_2-norm. The large value of dissimilarity function caused by outliers would be reduced while α increased. Visually, we can see the dissimilarity function changing with different α .

In the following we will briefly describe the framework of e-PCA works. Let us define a set of n training images as $\mathbf{I}_j, \quad j = 1, \cdots, n$, where n is the number of training

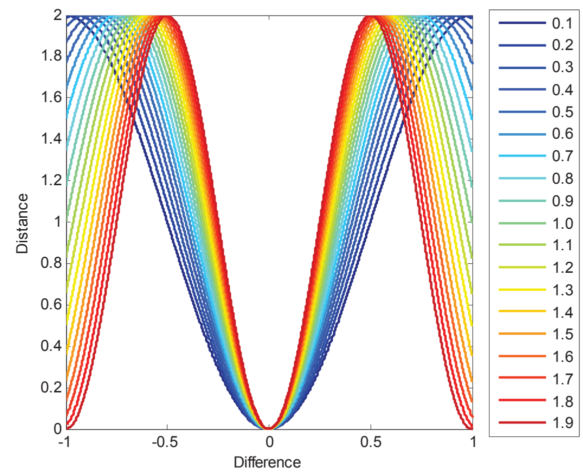

Fig. 1. The dissimilarity measure with changing α .

images. We assume $\mathbf{I}_j \in R^{M\times N}$. Stacking each training image matrixes $\mathbf{I}_j$ into a MN dimensional column vector $\boldsymbol{x}_j$. Further we arrange $\{\boldsymbol{x}_j\}_{j=1}^n$ into $MN \times n$ matrix $\mathbf{X}$ by columns. Let us calculate the explicit mapping $\mathbf{Z} = \phi(\mathbf{X})$ in feature space by (2) and each column $\boldsymbol{z}_j = \phi(\boldsymbol{x}_j)$. e-PCA aims at finding the orthogonal bases $\mathbf{U}_\phi$ that minimize the sum of recovery error of all training images. The optimization problem could be expressed as follows

$$\mathbf{U}_\phi = \arg\min_{\mathbf{U}_\phi} \sum_{i=1}^{n} \|\boldsymbol{z}_j - \mathbf{U}_\phi\mathbf{U}_\phi^H\boldsymbol{z}_j\|_2^2 \quad s.t. \mathbf{U}_\phi\mathbf{U}_\phi^H = \mathbf{I} \quad (4)$$

where $\mathbf{I}$ is the identity matrix and $\|\cdot\|_2^2$ denotes l_2-norm. (4)could be rewritten as

$$\mathbf{U}_\phi = \arg\min_{\mathbf{U}_\phi} \sum_{i=1}^{n} \|\mathbf{Z} - \mathbf{U}_\phi\mathbf{U}_\phi^H\mathbf{Z}\|_F^2 \quad s.t. \mathbf{U}_\phi\mathbf{U}_\phi^H = \mathbf{I} \quad (5)$$

where $\|\cdot\|_F^2$ denotes F-norm. Then (5) could be further simplified as follows

$$\begin{aligned} & \|\mathbf{Z} - \mathbf{U}_\phi\mathbf{U}_\phi^H\mathbf{Z}\|_F^2 \\ = \ & tr\left(\left(\mathbf{Z} - \mathbf{U}_\phi\mathbf{U}_\phi^H\mathbf{Z}\right)^H\left(\mathbf{Z} - \mathbf{U}_\phi\mathbf{U}_\phi^H\mathbf{Z}\right)\right) \\ = \ & tr(\mathbf{Z}^H\mathbf{Z}) - tr\left(\left(\mathbf{U}_\phi^H\mathbf{Z}\right)^H\left(\mathbf{U}_\phi^H\mathbf{Z}\right)\right) \\ = \ & \|\mathbf{Z}\|_F^2 - \|\mathbf{F}\|_F^2 \end{aligned} \quad (6)$$

where $\mathbf{F} = \mathbf{U}_\phi^H\mathbf{Z}$ is the feature matrixes which denote the result of $\mathbf{Z}$ projects to $\mathbf{U}_\phi$. The minimization problem of (5) equal to maximize the following function

$$\begin{aligned} \mathbf{U}_\phi &= \arg\max_{\mathbf{U}_\phi} \|\mathbf{F}\|_F^2 \\ &= \arg\max_{\mathbf{U}_\phi} tr\left(\mathbf{U}_\phi^H\mathbf{Z}\mathbf{Z}^H\mathbf{U}_\phi\right) \end{aligned} \quad (7)$$

Finally, performing **Theorem 1** could readily solve the high dimensional data problem. Calculating the solution of (7) by applying the eigenanalysis on the matrix $\mathbf{Z}^H\mathbf{Z} = \mathbf{B}_\phi \mathbf{\Lambda} \mathbf{B}_\phi^H$ and $\mathbf{U}_\phi = \mathbf{Z}\mathbf{B}_\phi \mathbf{\Lambda}^{-\frac{1}{2}}$.

Theorem 1 *Define matrixes* $\mathbf{A}$ *and* $\mathbf{B}$ *such that* $\mathbf{A} = \mathbf{\Phi}\mathbf{\Phi}^H$ *and* $\mathbf{B} = \mathbf{\Phi}^H\mathbf{\Phi}$ *, where computes the complex conjugate transposition of a matrix. Let* $\mathbf{U}_A$ *and* $\mathbf{U}_B$ *be the eigenvectors corresponding to the non-zero eigenvalues* $\mathbf{\Lambda}_A$ *and* $\mathbf{\Lambda}_B$ *of* $\mathbf{A}$ *and* $\mathbf{B}$ *, respectively. Then,* $\mathbf{\Lambda}_A = \mathbf{\Lambda}_B$ *and* $\mathbf{U}_A = \mathbf{\Phi}\mathbf{U}_B\mathbf{\Lambda}_A^{-\frac{1}{2}}$.

Due to the restriction of the value of α , this mapping showed in (2) is one-to-one. A closed form feature space representation lead to fast pre-image computation and explicit application of standard linear PCA algorithm. Reconstruction of data can be simply operated as follows

$$\tilde{\mathbf{Z}} = \mathbf{U}_\phi \mathbf{U}_\phi^H \mathbf{Z} \tag{8}$$

With the $\angle$-operator which outputs the angle of the complex number, the fast pre-image computation can be calculated as follows

$$\tilde{\mathbf{X}} = \frac{\angle \tilde{\mathbf{Z}}}{\alpha\pi} \tag{9}$$

III. Two-Dimensional Euler-Principal Component Analysis

In the above e-PCA theory image processing, the 2D images should be arranged into a 1D vectors. Obviously, the resulting vectors would have a large size. Relatively the number of training images would be small. It will dramatically increase the amount and complexity of calculating covariance matrix and eigenvectors. Furthermore the resulting eigenvectors cant be evaluated accurately which are statistically determined by the covariance matrix. Due to the inner product based kernel function, 2DPCA cant be used to explore the nonlinear dependencies. However, the kernel function defined by e-PCA which has a close form feature space representation retains this desirable properties. With this kernel function fast pre-image computation and explicit application of standard linear PCA algorithm could come true. As we know, no relevant research results have been published in this field and my research aims to close this gap. In the following, we proposed a directly image projection technique, namely two-dimensional principal component analysis (e-2DPCA), to deal with SAR image target recognition.

2D image matrix is taken into consideration instead of the transformed image vectors. Let us calculate the explicit mapping of each training image $\mathbf{Z}_i = \phi(\mathbf{I}_i)$ in feature space by (2). We aims to project image $\mathbf{Z}_i$ onto the orthogonal bases $\mathbf{U}_\phi$ by the following linear transformation

$$\mathbf{Y}_i = \mathbf{Z}_i \mathbf{U}_\phi \tag{10}$$

where $\mathbf{Y}_i$ is the projected vector, namely projected feature vector of image $\mathbf{Z}_i$. e-2DPCA aims at finding the orthogonal bases $\mathbf{U}_\phi$ that minimize the sum of recovery error of all training images. The optimization problem could be expressed as follows

$$\mathbf{U}_\phi = \arg\min_{\mathbf{U}_\phi} \sum_{i=1}^{n} \|\mathbf{Z}_i - \mathbf{Z}_i\mathbf{U}_\phi\mathbf{U}_\phi^H\|_F^2 \quad s.t. \mathbf{U}_\phi\mathbf{U}_\phi^H = \mathbf{I} \tag{11}$$

Because of

$$\sum_{i=1}^{n} \|\mathbf{Z}_i - \mathbf{Z}_i\mathbf{U}_\phi\mathbf{U}_\phi^H\|_F^2 = \sum_{i=1}^{n} \|\mathbf{Z}_i\|_F^2 - \sum_{i=1}^{n} \|\mathbf{Z}_i\mathbf{U}_\phi\|_F^2 \tag{12}$$

Hence the minimization problem of (11) equal to maximize the following function

$$\begin{aligned} \mathbf{U}_\phi &= \arg\max_{\mathbf{U}_\phi} \sum_{i=1}^{n} \|\mathbf{Z}_i\mathbf{U}_\phi\|_F^2 \\ &= \arg\max_{\mathbf{U}_\phi} \sum_{i=1}^{n} tr(\mathbf{U}_\phi^H\mathbf{Z}_i^H\mathbf{Z}_i\mathbf{U}_\phi) \\ &= \arg\max_{\mathbf{U}_\phi} tr(\mathbf{U}_\phi^H\mathbf{G}_t\mathbf{U}_\phi) \end{aligned} \tag{13}$$

where $\mathbf{G}_t = \sum_{i=1}^{n}\mathbf{Z}_i^H\mathbf{Z}_i$ is the image covariance matrix. It is verified that $\mathbf{G}_t$ is nonnegative definite matrix. Finally, performing eigenanalysis on the matrix $\mathbf{G}_t = \mathbf{U}_\phi\mathbf{\Lambda}\mathbf{U}_\phi^H$ to solve the problem in (13). The optimal orthogonal bases $\mathbf{U}_\phi$ are used for feature extraction. Reconstruction of data can be simply operated as follows

$$\tilde{\mathbf{Z}}_i = \mathbf{Z}_i\mathbf{U}_\phi\mathbf{U}_\phi^H, \quad i = 1, \cdots, n \tag{14}$$

According to (9) the fast pre-image computation can be calculated as follows

$$\tilde{\mathbf{I}}_i = \frac{\angle \tilde{\mathbf{Z}}_i}{\alpha\pi} \tag{15}$$

Next we discuss image classification method. At first let us assume that there are m kinds of SAR targets. Let us define a set of n^i training images in class $i, \quad i = 1, \cdots, m$ as $\mathbf{I}_i^i, \quad j = 1, \cdots, n$, where n is the number of training images. In feature space it is assumed that a test image can be linearly represented by components in the same class so given a test image $\mathbf{I}_0$. We assume $\mathbf{I}_j^i \in R^{M\times N}$. The optimal projection vectors $\mathbf{U}_\phi^i$ of training images in class i is obtained by performing e-2DPCA algorithm. We define the overall dictionary which is formed by concatenating m classes dictionary, denoting

$$\mathbf{D} = \begin{bmatrix} \mathbf{U}_\phi^1, & \mathbf{U}_\phi^2, & \cdots, & \mathbf{U}_\phi^m \end{bmatrix} \tag{16}$$

According to (10) the projected vector $\mathbf{Y}_{train}^i, \quad i = 1, \cdots, n$ of training images in feature space is obtained by projecting the training images to dictionary $\mathbf{D}$. The projected vector of the test image is denoted as $\mathbf{Y}_{test}$. Then, the nearest neighbor classifier is applied to target classification.

IV. SAR Target Recognition with e-2DPCA

This section verifies the proposed method on MSTAR SAR database, a collection captured using a 10GHz SAR sensor in one-foot resolution spotlight mode. The involved data is a subset of the MSTAR containing five different targets BRDM2(tank), BTR70(armored car), D7(bulldozer), T72(tank) and ZSU(truck). We utilize the prototype configurations at 15° depression as training images, while at 17° depression angle are utilized as testing images. The number of training images and testing images of different targets are shown in Tab.IV. In order to exclude the clutter background the center 64×64-pixel patch of the raw image is cropped as shown in Fig.2.

TABLE I

	BRDM2	BTR70	D7	T72	ZSU
15°	274	196	274	196	274
17°	298	233	299	232	299

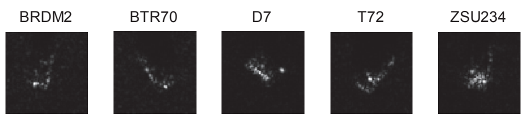

Fig. 2. Cropped example images of different targets from MSTAR database

To evaluate the robustness and accuracy of e-2DPCA, we perform two sets of experiments which contain noise corruption and the number of components variation. Under the same situations a number of classical and state-of-the-art approaches such as PCA, 2DPCA and e-PCA are selected as comparisons.

The performance of our proposed method is presented in Fig.3. The accurate recognition rate is the average accurate recognition rate of five targets. Fig.3 plots accurate recognition rate as a function of the number of principal component vectors. It is clearly that e-2DPCA and 2DPCA significantly outperforms eCPCA and PCA. The value of e-2DPCA remains above 0.9 for only 2 principal component vectors while eCPCA needs 6 and PCA needs 8. With only one components the accurate recognition rate of e-2DPCA and 2DPCA reach 0.85 while eCPCA is 0.43and PCA is 0.23. Even though the accurate recognition rate of eCPCA and PCA increase rapidly as the number of principal component vectors raises, it still less than -2DPCA and 2DPCA when 10 principal component vectors are taken into account. The experimental results shown in Fig.3 proved that e-2DPCA is more suitable for solving small sample size problems than eCPCA.

As we can see, with clean training images the performance of e-2DPCA is the same as 2DPCA. We further evaluate the performance of our method and the comparison methods when the training images are corrupted by noise. The corrupted training images are obtained by randomly choose a percentage of pixels from each training image and replace its value with independent and identically distributed complex Gaussian noise. The example of corrupted training images are as shown in Fig.4.

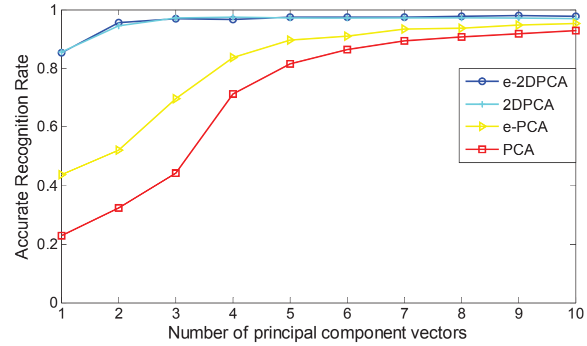

Fig. 3. Accurate recognition rate of e-2DPCA, 2DPCA, e-PCA and PCA.

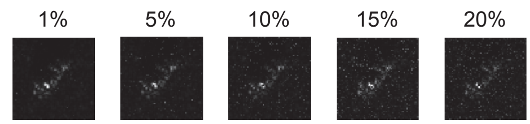

Fig. 4. Illustration of random noise corruption

Fig.5 shows the accurate recognition rate under different configuration. The worst is PCA and e-PCA performs a little better, especially when the training images are 20% corrupted by noise. For 1% corrupted and 5% corrupted training images the performance of e-2DPCA is equal to 2DPCA. However, as we can see in subfigure (c), (d) and (e) of Fig.5, e-2DPCA perform slightly better than 2DPCA especially when the training images are 20% corrupted by noise. In subfigure (e) of Fig.5, for the first 4 component vectors, e-2DPCA significantly outperforms 2DPCA. The experimental results shown in Fig.5 proved that e-2DPCA has better denoising performance than 2DPCA. Finally, the subfigure (e) of Fig.5 demonstrates the accurate recognition rate of e-2DPCA and e-PCA with different configuration. It is clearly that e-2DPCA significantly outperforms e-PCA.

V. Conclusion

In this paper, we performed 2DPCA with euler kernel and applied it to SAR target recognition. Extensive experimental comparisons and e-2DPCA were verified on MSTAR database. Empirical results proved that e-2DPCA is more suitable for solving small sample size problems than eCPCA. With noise corruption training images, e-2DPCA obtained better denoising performance than 2DPCA. Future research work on the topic will contain the extension of feature extraction theory based on euler kernel.

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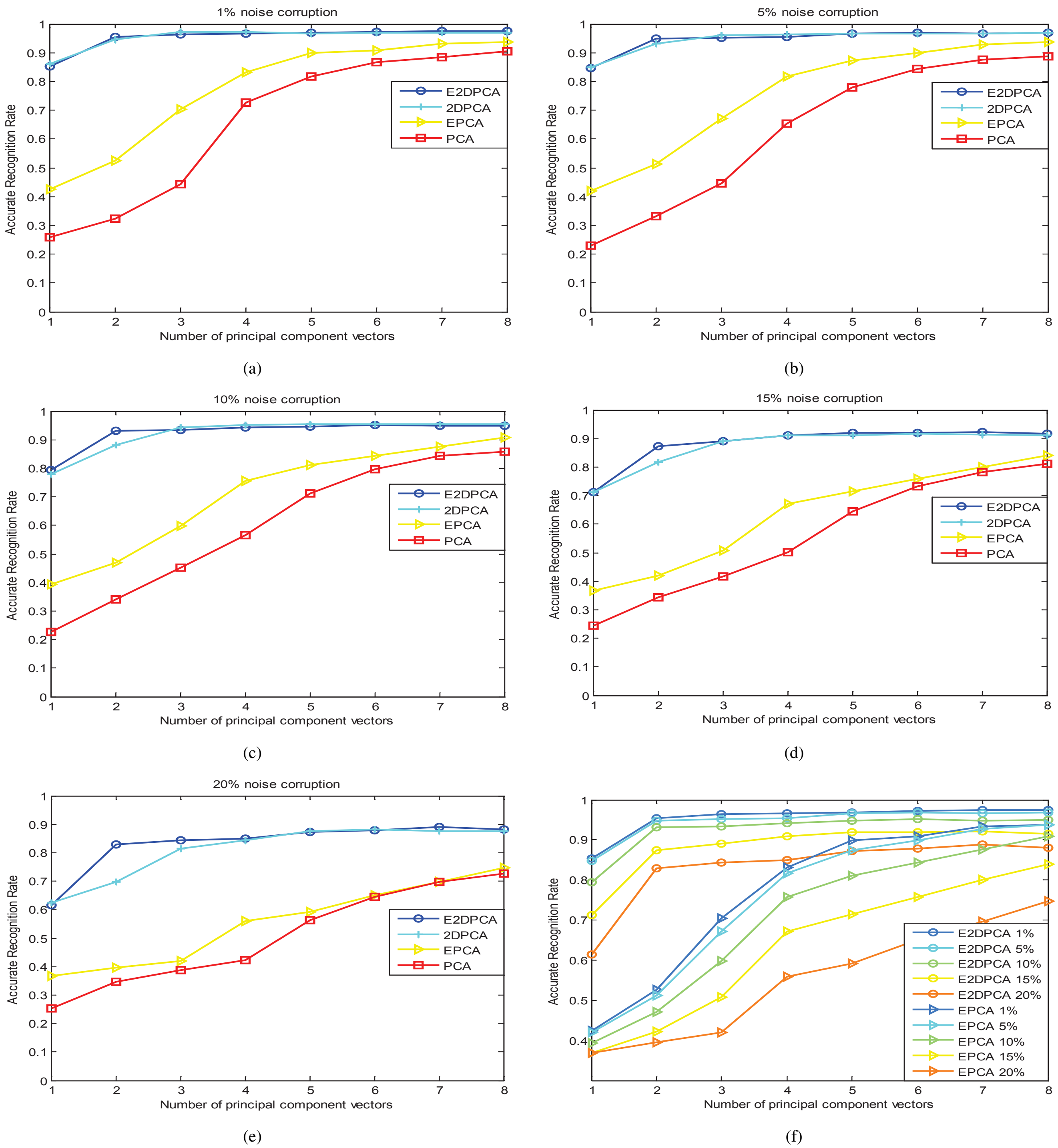

Fig. 5. Accurate recognition rate with different rate of noise corrupted images

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