Carnegie Mellon University

CMU 2

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Contents

Contest (1)

template.cpp
31 lines

```
#include <bits/stdc++.h>

using namespace std;

#define f first
#define s second
#define pb push_back
#define mp make_pair
#define all(v) v.begin(), v.end()
#define sz(v) (int)v.size()

#define MOO(i, a, b) for(int i=a; i<b; i++)
#define M00(i, a) for(int i=0; i<a; i++)
#define MOOd(i,a,b) for(int i = (b)-1; i >= a; i--)
#define M00d(i,a) for(int i = (a)-1; i>=0; i--)

#define FAST ios::sync_with_stdio(0); cin.tie(0);
#define finish(x) return cout << x << '\n', 0;
#define dbg(x) cerr << ">>> " << #x << " = " << x << "\n";
#define _ << " _ " <<

typedef long long ll;
typedef long double ld;
typedef vector<int> vi;
typedef pair<int,int> pi;
typedef pair<ld,ld> pd;
typedef complex<ld> cd;

int main() { FAST

}
```

.bashrc
16 lines

```
co() {
  g++ -std=c++11 $1.cpp -o $1
}

run() {
  if [ $# -eq 2 ]
   then
     ./$1 < $1$2.in
   else
     ./$1
   fi
}

crun() {
  co $1 && echo "Compiled!" && run $1 $2
}
```

.vimrc
4 lines

```
set nocp backspace=indent,eol,start nu ru si ts=4 sw=4 is
   ↪hls sm mouse=a
syntax on
filetype plugin indent on
colorscheme slate
```

cppreference.txt
7 lines

```
atan(m) -> angle from -pi/2 to pi/2
atan2(y,x) -> angle from -pi to pi
acos(x) -> angle from 0 to pi
asin(y) -> angle from -pi/2 to pi/2

lower_bound -> first element >= val
upper_bound -> first element > val
```

Data Structures (2)

2.1 STL

MapComparator.h

Description: custom comparator for map / set

8 lines

```
struct cmp {
  bool operator()(const int& l, const int& r) const {
    return l > r;
  }
};

set<int,cmp> s; // FOR(i,10) s.insert(rand()); trav(i,s)
   ↪ps(i);
map<int,int,cmp> m;
```

CustomHash.h

Description: faster than standard unordered map

23 lines

```
struct chash {
  static uint64_t splitmix64(uint64_t x) {
    // http://xorshift.di.unimi.it/splitmix64.c
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }

  size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
      chrono::steady_clock::now()
      .time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
  }
};

template<class K, class V> using um = unordered_map<K, V,
   ↪chash>;
template<class K, class V> using ht = gp_hash_table<K, V,
   ↪chash>;

template<class K, class V> V get(ht<K,V>& u, K x) {
  return u.find(x) == end(u) ? 0 : u[x];
}
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element.

Time: $\mathcal{O}(\log N)$

<ext/pb_ds/tree_policy.hpp>, <ext/pb_ds/assoc_container.hpp> 18 lines

```
using namespace __gnu_pbds;

template <class T> using Tree = tree<T, null_type, less<T
   ↪>,
  rb_tree_tag, tree_order_statistics_node_update>;
// to get a map, change null_type

#define ook order_of_key
#define fbo find_by_order

void treeExample() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).f;
  assert(it == t.lb(9));
  assert(t.ook(10) == 1);
  assert(t.ook(11) == 2);
  assert(*t.fbo(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into
     ↪t
}
```

Rope.h

Description: insert element at n-th position, cut a substring and re-insert somewhere else

Time: $\mathcal{O}(\log N)$ per operation? not well tested

<ext/rope> 13 lines

```
using namespace __gnu_cxx;

void ropeExample() {
```

```
  rope<int> v(5, 0);
  FOR(i,sz(v)) v.mutable_reference_at(i) = i+1; // or push_back
  rope<int> cur = v.substr(1,2); v.erase(1,2);
  FOR(i,sz(v)) cout << v[i] << " ";  // 1 4 5
  cout << "\n";
  v.insert(v.mutable_begin()+2,cur);
  for (rope<int>::iterator it = v.mutable_begin(); it != v.mutable_end(); ++it)
    cout << *it << " "; // 1 4 2 3 5
  cout << "\n";
}
```

LineContainer.h

Description: Given set of lines, computes greatest y-coordinate for any x

Time: $\mathcal{O}(\log N)$ 31 lines

```
struct Line {
  mutable ll k, m, p; // slope, y-intercept, last optimal x
  ll eval (ll x) { return k*x+m; }
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LC : multiset<Line,less<>> {
  // for doubles, use inf = 1/.0, div(a,b) = a/b
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { return a/b-((a^b) < 0 && a%b); } // floored division
  ll bet(const Line& x, const Line& y) { // last x such that first line is better
    if (x.k == y.k) return x.m >= y.m ? inf : -inf;
    return div(y.m-x.m,x.k-y.k);
  }
  bool isect(iterator x, iterator y) { // updates x->p, determines if y is unneeded
    if (y == end()) { x->p = inf; return 0; }
    x->p = bet(*x,*y); return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k,m,0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p) isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lb(x);
    return l.k*x+l.m;
  }
};
```

2.2 1D Range Queries

RMQ.h

Description: 1D range minimum query

Time: $\mathcal{O}(N\log N)$ build, $\mathcal{O}(1)$ query 25 lines

```
template<class T> struct RMQ {
  constexpr static int level(int x) {
    return 31-__builtin_clz(x);
  } // floor(log_2(x))
  vector<vi> jmp;
  vector<T> v;
  int comb(int a, int b) {
    return v[a] == v[b] ? min(a,b) : (v[a] < v[b] ? a : b);
  } // index of minimum

  void init(const vector<T>& _v) {
    v = _v; jmp = {vi(sz(v))}; iota(all(jmp[0]),0);
    for (int j = 1; 1<<j <= sz(v); ++j) {
      jmp.pb(vi(sz(v)-(1<<j)+1));
      FOR(i,sz(jmp[j])) jmp[j][i] = comb(jmp[j-1][i],
                jmp[j-1][i+(1<<(j-1))]);
    }
  }

  int index(int l, int r) { // get index of min element
    int d = level(r-l+1);
    return comb(jmp[d][l],jmp[d][r-(1<<d)+1]);
  }
  T query(int l, int r) { return v[index(l,r)]; }
};
```

BIT.h

Description: N-D range sum query with point update

Time: $\mathcal{O}\left((\log N)^D\right)$ 19 lines

```
template <class T, int ...Ns> struct BIT {
  T val = 0;
  void upd(T v) { val += v; }
  T query() { return val; }
};

template <class T, int N, int... Ns> struct BIT<T, N, Ns...> {
  BIT<T,Ns...> bit[N+1];
  template<typename... Args> void upd(int pos, Args... args) {
    for (; pos <= N; pos += (pos&-pos)) bit[pos].upd(args...);
  }
  template<typename... Args> T sum(int r, Args... args) {
    T res = 0; for (; r; r -= (r&-r)) res += bit[r].query(args...);
    return res;
  }
  template<typename... Args> T query(int l, int r, Args... args) {
    return sum(r,args...)-sum(l-1,args...);
  }
}; // BIT<int,10,10> gives a 2D BIT
```

BITrange.h

Description: 1D range increment and sum query

Time: $\mathcal{O}(\log N)$

"BIT.h" 11 lines

```
template<class T, int SZ> struct BITrange {
  BIT<T,SZ> bit[2]; // piecewise linear functions
  // let cum[x] = sum_{i=1}^{x}a[i]
  void upd(int hi, T val) { // add val to a[1..hi]
    bit[1].upd(1,val), bit[1].upd(hi+1,-val); // if x <= hi, cum[x] += val*x
    bit[0].upd(hi+1,hi*val); // if x > hi, cum[x] += val*hi
  }
  void upd(int lo, int hi, T val) { upd(lo-1,-val), upd(hi,val); }
  T sum(int x) { return bit[1].sum(x)*x+bit[0].sum(x); } // get cum[x]
  T query(int x, int y) { return sum(y)-sum(x-1); }
};
```

SegTree.h

Description: 1D point update, range query

Time: $\mathcal{O}(\log N)$ 33 lines

```
template<class T, int SZ> struct segtree {
    // modify these
    T identity = 0;
    T comb(T l, T r) {
        return l + r;
    }
    void updLeaf(T& l, T val) {
        l = val;
    }

    T tree[2*SZ+1];
    segtree() {
        M00(i, 2*SZ+1) tree[i] = identity;
    }
    void upd(int pos, T val) {
        pos += SZ+1;
        updLeaf(tree[pos], val);
        for(pos >>= 1; pos >= 1; pos >>= 1) {
            tree[pos] = comb(tree[2*pos], tree[2*pos+1]);
        }
    }
    T query(int l, int r) {
        l += SZ+1;
        r += SZ+1;
        T res = identity;
        while(l <= r) {
            if(l&1) res = comb(res, tree[l++]);
            if(!(r&1)) res = comb(res, tree[r--]);
            l >>= 1; r >>= 1;
        }
        return res;
    }
};
```

SegTreeBeats.h

Description: supports modifications in the form `ckmin(a_i,t)` for all $l \leq i \leq r$, range max and sum queries

Time: $\mathcal{O}(\log N)$

65 lines

```
template<int SZ> struct SegTreeBeats {
  int N;
  ll sum[2*SZ];
  int mx[2*SZ][2], maxCnt[2*SZ];

  void pull(int ind) {
    FOR(i,2) mx[ind][i] = max(mx[2*ind][i],mx[2*ind+1][i])
      ↪;
    maxCnt[ind] = 0;
    FOR(i,2) {
      if (mx[2*ind+i][0] == mx[ind][0])
        maxCnt[ind] += maxCnt[2*ind+i];
      else ckmax(mx[ind][1],mx[2*ind+i][0]);
    }
    sum[ind] = sum[2*ind]+sum[2*ind+1];
  }
  void build(vi& a, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) { R = (N = sz(a))-1; }
    if (L == R) {
      mx[ind][0] = sum[ind] = a[L];
      maxCnt[ind] = 1; mx[ind][1] = -1;
      return;
    }
    int M = (L+R)/2;
    build(a,2*ind,L,M); build(a,2*ind+1,M+1,R); pull(ind);
  }

  void push(int ind, int L, int R) {
    if (L == R) return;
    FOR(i,2)
      if (mx[2*ind^i][0] > mx[ind][0]) {
        sum[2*ind^i] -= (ll)maxCnt[2*ind^i]*
                (mx[2*ind^i][0]-mx[ind][0]);
        mx[2*ind^i][0] = mx[ind][0];
      }
  }
  void upd(int x, int y, int t, int ind = 1, int L = 0,
    ↪int R = -1) {
    if (R == -1) R += N;
    if (R < x || y < L || mx[ind][0] <= t) return;
    push(ind,L,R);
    if (x <= L && R <= y && mx[ind][1] < t) {
      sum[ind] -= (ll)maxCnt[ind]*(mx[ind][0]-t);
      mx[ind][0] = t;
      return;
    }
    if (L == R) return;
    int M = (L+R)/2;
    upd(x,y,t,2*ind,L,M); upd(x,y,t,2*ind+1,M+1,R); pull(
      ↪ind);
  }
  ll qsum(int x, int y, int ind = 1, int L = 0, int R =
    ↪-1) {
    if (R == -1) R += N;
    if (R < x || y < L) return 0;
    push(ind,L,R);
    if (x <= L && R <= y) return sum[ind];
    int M = (L+R)/2;
    return qsum(x,y,2*ind,L,M)+qsum(x,y,2*ind+1,M+1,R);
  }
  int qmax(int x, int y, int ind = 1, int L = 0, int R =
    ↪-1) {
    if (R == -1) R += N;
    if (R < x || y < L) return -1;
    push(ind,L,R);
    if (x <= L && R <= y) return mx[ind][0];
    int M = (L+R)/2;
    return max(qmax(x,y,2*ind,L,M), qmax(x,y,2*ind+1,M+1,R
      ↪));
  }
};
```

Lazy SegTree.h

Description: 1D range update, range query

85 lines

```
template<class T> struct node {
    T val;
    T lazy;
    int l, r;
    node* left;
    node* right;
    node(int l, int r) {
        this -> l = l;
        this -> r = r;
        this -> left = nullptr;
        this -> right = nullptr;
    }
};

template<class T, int SZ> struct segtree {
    // modify these
    T combIdentity = 1e9;
    T comb(T l, T r) {
        return min(l,r);
    }
    T pushIdentity = 0;
    void push(node<T>* n) {
        n->val += n->lazy;
        if(n->l != n->r) {
            n->left->lazy += n->lazy;
            n->right->lazy += n->lazy;
        }
        n->lazy = pushIdentity;
    }

    node<T>* root;
    segtree() {
        int ub = 1;
        while(ub < SZ) ub *= 2;
        root = new node<T>(0, ub-1);
        root->val = pushIdentity;
        root->lazy = pushIdentity;
    }
    void propagate(node<T>* n) {
        if(n->l != n->r) {
            int mid = ((n->l) + (n->r))/2;
            if(n->left == nullptr) {
                n->left = new node<T>(n->l, mid);
                n->left->val = pushIdentity;
                n->left->lazy = pushIdentity;
            }
            if(n->right == nullptr) {
                n->right = new node<T>(mid+1, n->r);
                n->right->val = pushIdentity;
                n->right->lazy = pushIdentity;
            }
        }
        push(n);
    }
    void updN(node<T>* n, int i1, int i2, T val) {
        propagate(n);
        if(i2 < n->l || i1 > n->r) return;
        if(i1 <= n->l && i2 >= n->r) {
            n->lazy = val;
            push(n);
            return;
        }

        updN(n->left, i1, i2, val);
        updN(n->right, i1, i2, val);
        n->val = comb(n->left->val, n->right->val);
    }
    void upd(int i1, int i2, T val) {
        updN(root, i1, i2, val);
    }
    T queryN(node<T>* n, int i1, int i2) {
        propagate(n);
        if(i2 < n->l || i1 > n->r) return combIdentity;
        if(n->l >= i1 && n->r <= i2) return n->val;

        T a = combIdentity;
        if(n->left != nullptr) a = comb(a, queryN(n->left,
          ↪ i1, i2));
        if(n->right != nullptr) a = comb(a, queryN(n->
          ↪right, i1, i2));

        return a;
    }
    T query(int i1, int i2) {
        return queryN(root, i1, i2);
    }
};
```

Sparse SegTree.h

Description: Does not allocate storage for nodes with no data

75 lines

```
template<class T> struct node {
    T val;
    int l, r;
    node* left;
    node* right;
    node(int l, int r) {
        this -> l = l;
        this -> r = r;
        this -> left = nullptr;
```

```
        this -> right = nullptr;
    }
};

template<class T, int SZ> struct segtree {
    // modify these
    T identity = 0;
    T comb(T l, T r) {
        return l + r;
    }
    void updLeaf(T& l, T val) {
        l = val;
    }

    node<T>* root;
    segtree() {
        int ub = 1;
        while(ub < SZ) ub *= 2;
        root = new node<T>(0, ub-1);
        root->val = identity;
    }
    void updN(node<T>* n, int pos, T val) {
        if(pos < n->l || pos > n->r) return;
        if(n->l == n->r) {
            updLeaf(n->val, val);
            return;
        }

        int mid = (n->l + n->r)/2;
        if(pos > mid) {
            if(n->right == nullptr) {
                n->right = new node<T>(mid+1, n->r);
                n->right->val = identity;
            }
            updN(n->right, pos, val);
        }
        else {
            if(n->left == nullptr) {
                n->left = new node<T>(n->l, mid);
                n->left->val = identity;
            }
            updN(n->left, pos, val);
        }

        T lv = (n->left == nullptr) ? identity : n->left->
            ↪val;
        T rv = (n->right == nullptr) ? identity : n->right
            ↪->val;
        n->val = comb(lv, rv);
    }
    void upd(int pos, T val) {
        updN(root, pos, val);
    }
    T queryN(node<T>* n, int i1, int i2) {
        if(i2 < n->l || i1 > n->r) return identity;
        if(n->l == n->r) return n->val;
        if(n->l >= i1 && n->r <= i2) return n->val;

        T a = identity;
        if(n->left != nullptr) a = comb(a, queryN(n->left,
            ↪ i1, i2));
        if(n->right != nullptr) a = comb(a, queryN(n->
            ↪right, i1, i2));

        return a;
    }
    T query(int i1, int i2) {
        return queryN(root, i1, i2);
    }
};
```

PersSegTree.h

Description: persistent segtree with lazy updates, assumes that `lazy[cur]` is included in `val[cur]` before propagating `cur`
Time: $\mathcal{O}(\log N)$ 60 lines

```
template<class T, int SZ> struct pseg {
  static const int LIMIT = 10000000; // adjust
  int l[LIMIT], r[LIMIT], nex = 0;
  T val[LIMIT], lazy[LIMIT];

  int copy(int cur) {
    int x = nex++;
    val[x] = val[cur], l[x] = l[cur], r[x] = r[cur], lazy[
      ↪x] = lazy[cur];
    return x;
  }
  T comb(T a, T b) { return min(a,b); }
  void pull(int x) { val[x] = comb(val[l[x]],val[r[x]]); }
  void push(int cur, int L, int R) {
    if (!lazy[cur]) return;
    if (L != R) {
      l[cur] = copy(l[cur]);
      val[l[cur]] += lazy[cur];
      lazy[l[cur]] += lazy[cur];

      r[cur] = copy(r[cur]);
      val[r[cur]] += lazy[cur];
      lazy[r[cur]] += lazy[cur];
    }
    lazy[cur] = 0;
  }

  T query(int cur, int lo, int hi, int L, int R) {
    if (lo <= L && R <= hi) return val[cur];
    if (R < lo || hi < L) return INF;
    int M = (L+R)/2;
    return lazy[cur]+comb(query(l[cur],lo,hi,L,M), query(r
      ↪[cur],lo,hi,M+1,R));
  }
  int upd(int cur, int lo, int hi, T v, int L, int R) {
    if (R < lo || hi < L) return cur;

    int x = copy(cur);
    if (lo <= L && R <= hi) { val[x] += v, lazy[x] += v;
      ↪return x; }
    push(x,L,R);

    int M = (L+R)/2;
    l[x] = upd(l[x],lo,hi,v,L,M), r[x] = upd(r[x],lo,hi,v,
      ↪M+1,R);
    pull(x); return x;
  }
  int build(vector<T>& arr, int L, int R) {
    int cur = nex++;
    if (L == R) {
      if (L < sz(arr)) val[cur] = arr[L];
      return cur;
    }

    int M = (L+R)/2;
    l[cur] = build(arr,L,M), r[cur] = build(arr,M+1,R);
    pull(cur); return cur;
  }

  vi loc;
  void upd(int lo, int hi, T v) { loc.pb(upd(loc.back(),lo
    ↪,hi,v,0,SZ-1)); }
  T query(int ti, int lo, int hi) { return query(loc[ti],
    ↪lo,hi,0,SZ-1); }
  void build(vector<T>& arr) { loc.pb(build(arr,0,SZ-1));
    ↪}
};
```

Treap.h

Description: easy BBST, use split and merge to implement insert and delete
Time: $\mathcal{O}(\log N)$ 77 lines

```
typedef struct tnode* pt;

struct tnode {
  int pri, val; pt c[2]; // essential
  int sz; ll sum; // for range queries
  bool flip; // lazy update

  tnode (int _val) {
    pri = rand()+(rand()<<15); val = _val; c[0] = c[1] =
      ↪NULL;
    sz = 1; sum = val;
    flip = 0;
  }
};

int getsz(pt x) { return x?x->sz:0; }
ll getsum(pt x) { return x?x->sum:0; }

pt prop(pt x) {
  if (!x || !x->flip) return x;
  swap(x->c[0],x->c[1]);
  x->flip = 0;
  FOR(i,2) if (x->c[i]) x->c[i]->flip ^= 1;
  return x;
}

pt calc(pt x) {
  assert(!x->flip);
  prop(x->c[0]), prop(x->c[1]);
  x->sz = 1+getsz(x->c[0])+getsz(x->c[1]);
```

```
  x->sum = x->val+getsum(x->c[0])+getsum(x->c[1]);
  return x;
}
void tour(pt x, vi& v) {
  if (!x) return;
  prop(x);
  tour(x->c[0],v); v.pb(x->val); tour(x->c[1],v);
}

pair<pt,pt> split(pt t, int v) { // >= v goes to the right
  if (!t) return {t,t};
  prop(t);
  if (t->val >= v) {
    auto p = split(t->c[0], v); t->c[0] = p.s;
    return {p.f, calc(t)};
  } else {
    auto p = split(t->c[1], v); t->c[1] = p.f;
    return {calc(t), p.s};
  }
}
pair<pt,pt> splitsz(pt t, int sz) { // leftmost sz nodes
   ↪go to left
  if (!t) return {t,t};
  prop(t);
  if (getsz(t->c[0]) >= sz) {
    auto p = splitsz(t->c[0], sz); t->c[0] = p.s;
    return {p.f, calc(t)};
  } else {
    auto p = splitsz(t->c[1], sz-getsz(t->c[0])-1); t->c
       ↪[1] = p.f;
    return {calc(t), p.s};
  }
}

pt merge(pt l, pt r) {
  if (!l || !r) return l ? l : r;
  prop(l), prop(r);
  pt t;
  if (l->pri > r->pri) l->c[1] = merge(l->c[1],r), t = l;
  else r->c[0] = merge(l,r->c[0]), t = r;
  return calc(t);
}
pt ins(pt x, int v) { // insert v
  auto a = split(x,v), b = split(a.s,v+1);
  return merge(a.f,merge(new tnode(v),b.s));
}
pt del(pt x, int v) { // delete v
  auto a = split(x,v), b = split(a.s,v+1);
  return merge(a.f,b.s);
}
```

SqrtDecomp.h

Description: 1D point update, range query

Time: $\mathcal{O}\left(\sqrt{N}\right)$ 44 lines

```
struct sqrtDecomp {
    const static int blockSZ = 10; //change this
    int val[blockSZ*blockSZ];
    int lazy[blockSZ];

    sqrtDecomp() {
        M00(i, blockSZ*blockSZ) val[i] = 0;
        M00(i, blockSZ) lazy[i] = 0;
    }
    void upd(int l, int r, int v) {
        int ind = l;
        while(ind%blockSZ && ind <= r) {
            val[ind] += v;
            lazy[ind/blockSZ] += v;
            ind++;
        }
        while(ind + blockSZ <= r) {
            lazy[ind/blockSZ] += v*blockSZ;
            ind += blockSZ;
        }
        while(ind <= r) {
            val[ind] += v;
            lazy[ind/blockSZ] += v;
            ind++;
        }
    }
    int query(int l, int r) {
        int res = 0;
        int ind = l;
        while(ind%blockSZ && ind <= r) {
            res += val[ind];
            ind++;
        }
        while(ind + blockSZ <= r) {
            res += lazy[ind/blockSZ];
            ind += blockSZ;
        }
        while(ind <= r) {
            res += val[ind];
            ind++;
        }
        return res;
    }
};
```

Mo.h

Description: Answers queries offline in (N+Q)sqrt(N) Also see Mo's on trees 33 lines

```
int N, A[MX];
int ans[MX], oc[MX], BLOCK;
vector<array<int,3>> todo; // store left, right, index of
   ↪ans

bool cmp(array<int,3> a, array<int,3> b) { // sort queries
  if (a[0]/BLOCK != b[0]/BLOCK) return a[0] < b[0];
  return a[1] < b[1];
}

int l = 0, r = -1, cans = 0;

void modify(int x, int y = 1) {
  x = A[x];
  // if condition: cans --;
  oc[x] += y;
  // if condition: cans ++;
}

int answer(int L, int R) { // modifyjust interval
  while (l > L) modify(--l);
  while (r < R) modify(++r);
  while (l < L) modify(l++,-1);
  while (r > R) modify(r--,-1);
  return cans;
}

void solve() {
  BLOCK = sqrt(N); sort(all(todo),cmp);
  trav(x,todo) {
    answer(x[0],x[1]);
    ans[x[2]] = cans;
  }
}
```

MaxQueue.h

Description: queue, but get() returns max element

Time: $\mathcal{O}(1)$ 16 lines

```
struct maxQueue {
    queue<int> q;
    deque<int> dq;
    void push(int v) {
        q.push(v);
        if(q.empty()) {dq.push_back(v); return;}
        while(!dq.empty() && dq.back() < v) dq.pop_back();
        dq.push_back(v);
    }
    void pop() {
        if(q.front() == dq.front()) dq.pop_front();
        q.pop();
    }
    int get() {return dq.front();}
    int size() {return (int)q.size();}
};
```

2.3 2D Range Queries

2D Sumtree.h

Description: Lawrence's 2d sum segment tree 104 lines

```
struct sumtreenode{
    node* root;
    sumtreenode* left;
    sumtreenode* right;
    int l, r;
    sumtreenode(int l, int r, int SZ) {
        int ub = 1;
        while(ub < SZ) ub *= 2;
        root = new node(0, ub-1);
        this -> l = l;
        this -> r = r;
        this->left = nullptr;
        this->right = nullptr;
    }
```

```
    void updN(node* n, int pos, int val) {
        if(pos < n->l || pos > n->r) return;
        if(n->l == n->r) {
            n->val = val;
            return;
        }

        int mid = (n->l + n->r)/2;
        if(pos > mid) {
            if(n->right == nullptr) n->right = new node(mid+1, n->r);
            updN(n->right, pos, val);
        }
        else {
            if(n->left == nullptr) n->left = new node(n->l, mid);
            updN(n->left, pos, val);
        }

        int s = 0;
        if(n->right != nullptr) s += n->right->val;
        if(n->left != nullptr) s += n->left->val;
        n->val = s;
    }
    void upd(int pos, int val) {
        updN(root, pos, val);
    }
    int queryN(node* n, int i1, int i2) {
        if(i2 < n->l || i1 > n->r) return 0;
        if(n->l == n->r) return n->val;
        if(n->l >= i1 && n->r <= i2) return n->val;

        int s = 0;
        if(n->left != nullptr) s += queryN(n->left, i1, i2);
        if(n->right != nullptr) s += queryN(n->right, i1, i2);

        return s;
    }
    int query(int i1, int i2) {
        return queryN(root, i1, i2);
    }
};

template<int w, int h> struct sumtree2d{
    sumtreenode* root;
    sumtree2d() {
        int ub = 1;
        while(ub < w) ub *= 2;
        this->root = new sumtreenode(0, ub-1, h);
        root->left = nullptr;
        root->right = nullptr;
    }
    void updN(sumtreenode* n, int x, int y, int val) {
        if(x < n->l || x > n->r) return;
        if(n->l == n->r) {
            n->upd(y, val);
            return;
        }

        int mid = (n->l + n->r)/2;
        if(x > mid) {
            if(n->right == nullptr) n->right = new sumtreenode(mid+1, n->r, h);
            updN(n->right, x, y, val);
        }
        else {
            if(n->left == nullptr) n->left = new sumtreenode(n->l, mid, h);
            updN(n->left, x, y, val);
        }

        int s = 0;
        if(n->left != nullptr) s += n->left->query(y, y);
        if(n->right != nullptr) s += n->right->query(y, y);
        n->upd(y, s);
    }
    void upd(int x, int y, int val) {
        updN(root, x, y, val);
    }
    int queryN(sumtreenode* n, int x1, int y1, int x2, int y2) {
        if(x2 < n->l || x1 > n->r) return 0;
        if(n->l == n->r) return n->query(y1, y2);
        if(n->l >= x1 && n->r <= x2) return n->query(y1, y2);

        int s = 0;
        if(n->left != nullptr) s += queryN(n->left, x1, y1, x2, y2);
        if(n->right != nullptr) s += queryN(n->right, x1, y1, x2, y2);

        return s;
    }
    int query(int x1, int y1, int x2, int y2) {
        return queryN(root, x1, y1, x2, y2);
    }
};
```

Number Theory (3)

3.1 Modular Arithmetic

Modular.h

Description: modular arithmetic operations 41 lines

```
template<class T> struct modular {
  T val;
  explicit operator T() const { return val; }
  modular() { val = 0; }
  modular(const ll& v) {
    val = (-MOD <= v && v <= MOD) ? v : v % MOD;
    if (val < 0) val += MOD;
  }
  // friend ostream& operator<<(ostream& os, const modular& a) { return os << a.val; }
  friend void pr(const modular& a) { pr(a.val); }
  friend void re(modular& a) { ll x; re(x); a = modular(x); }

  friend bool operator==(const modular& a, const modular& b) { return a.val == b.val; }
  friend bool operator!=(const modular& a, const modular& b) { return !(a == b); }
  friend bool operator<(const modular& a, const modular& b) { return a.val < b.val; }

  modular operator-() const { return modular(-val); }
  modular& operator+=(const modular& m) { if ((val += m.val) >= MOD) val -= MOD; return *this; }
  modular& operator-=(const modular& m) { if ((val -= m.val) < 0) val += MOD; return *this; }
  modular& operator*=(const modular& m) { val = (ll)val*m.val%MOD; return *this; }
  friend modular pow(modular a, ll p) {
    modular ans = 1; for (; p; p /= 2, a *= a) if (p&1) ans *= a;
    return ans;
  }
  friend modular inv(const modular& a) {
    assert(a != 0); return exp(a,MOD-2);
  }
  modular& operator/=(const modular& m) { return (*this) *= inv(m); }

  friend modular operator+(modular a, const modular& b) { return a += b; }
  friend modular operator-(modular a, const modular& b) { return a -= b; }
  friend modular operator*(modular a, const modular& b) { return a *= b; }

  friend modular operator/(modular a, const modular& b) { return a /= b; }
};

typedef modular<int> mi;
typedef pair<mi,mi> pmi;
typedef vector<mi> vmi;
typedef vector<pmi> vpmi;
```

ModFact.h

Description: pre-compute factorial mod inverses for MOD, assumes MOD is prime and $SZ < MOD$

Time: $\mathcal{O}(SZ)$ 10 lines

```
vl inv, fac, ifac;
void genInv(int SZ) {
  inv.rsz(SZ), fac.rsz(SZ), ifac.rsz(SZ);
  inv[1] = 1; FOR(i,2,SZ) inv[i] = MOD-MOD/i*inv[MOD%i]%MOD;
  fac[0] = ifac[0] = 1;
  FOR(i,1,SZ) {
    fac[i] = fac[i-1]*i%MOD;
```

```
    ifac[i] = ifac[i-1]*inv[i]%MOD;
  }
}
```

ModMulLL.h

Description: multiply two 64-bit integers mod another if 128-bit is not available works for $0 \le a, b < mod < 2^{63}$

14 lines

```
typedef unsigned long long ul;

// equivalent to (ul)(__int128(a)*b%mod)
ul modMul(ul a, ul b, const ul mod) {
  ll ret = a*b-mod*(ul)((ld)a*b/mod);
  return ret+((ret<0)-(ret>=(ll)mod))*mod;
}
ul modPow(ul a, ul b, const ul mod) {
  if (b == 0) return 1;
  ul res = modPow(a,b/2,mod);
  res = modMul(res,res,mod);
  if (b&1) return modMul(res,a,mod);
  return res;
}
```

ModSqrt.h

Description: find sqrt of integer mod a prime

Time: ?

"Modular.h" 26 lines

```
template<class T> T sqrt(modular<T> a) {
  auto p = pow(a,(MOD-1)/2); if (p != 1) return p == 0 ? 0
    : -1; // check if zero or does not have sqrt
  T s = MOD-1, e = 0; while (s % 2 == 0) s /= 2, e ++;
  modular<T> n = 1; while (pow(n,(MOD-1)/2) == 1) n = (T)(
    n)+1; // find non-square residue

  auto x = pow(a,(s+1)/2), b = pow(a,s), g = pow(n,s);
  int r = e;
  while (1) {
    auto B = b; int m = 0; while (B != 1) B *= B, m ++;
    if (m == 0) return min((T)x,MOD-(T)x);
    FOR(i,r-m-1) g *= g;
    x *= g; g *= g; b *= g; r = m;
  }
}

/* Explanation:
 * Initially, x^2=ab, ord(b) = 2^m, ord(g) = 2^r where m<r
 * g = g^{2^{r-m-1}} -> ord(g) = 2^{m+1}
 * if x'=x*g, then b' = b*g^2
     (b')^{2^{m-1}} = (b*g^2)^{2^{m-1}}
                    = b^{2^{m-1}}*g^{2^m}
                    = -1*-1
                    = 1
 -> ord(b')|ord(b)/2
 * m decreases by at least one each iteration
 */
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions

15 lines

```
typedef unsigned long long ul;

ul sumsq(ul to) { return (to-1)*to/2; } // sum of 0..to-1

ul divsum(ul to, ul c, ul k, ul m) { // sum_{i=0}^{to-1}
    floor((ki+c)/m)
  ul res = k/m*sumsq(to)+c/m*to;
  k %= m; c %= m; if (!k) return res;
  ul to2 = (to*k+c)/m;
  return res+(to-1)*to2-divsum(to2,m-1-c,m,k);
}

ll modsum(ul to, ll c, ll k, ll m) {
  c = (c%m+m)%m, k = (k%m+m)%m;
  return to*c+k*sumsq(to)-m*divsum(to,c,k,m);
}
```

3.2 Primality

PrimeSieve.h

Description: tests primality up to SZ

Time: $\mathcal{O}(SZ \log\log SZ)$

11 lines

```
template<int SZ> struct Sieve {
  bitset<SZ> isprime;
  vi pr;
  Sieve() {
    isprime.set(); isprime[0] = isprime[1] = 0;
    for (int i = 4; i < SZ; i += 2) isprime[i] = 0;
    for (int i = 3; i*i < SZ; i += 2) if (isprime[i])
      for (int j = i*i; j < SZ; j += i*2) isprime[j] = 0;
    FOR(i,2,SZ) if (isprime[i]) pr.pb(i);
  }
};
```

FactorFast.h

Description: Factors integers up to 2^{60}

Time: ?

"PrimeSieve.h" 46 lines

```
Sieve<1<<20> S = Sieve<1<<20>(); // should take care of
    all primes up to n^(1/3)

bool millerRabin(ll p) { // test primality
  if (p == 2) return true;
  if (p == 1 || p % 2 == 0) return false;
  ll s = p - 1; while (s % 2 == 0) s /= 2;
  FOR(i,30) { // strong liar with probability <= 1/4
    ll a = rand() % (p - 1) + 1, tmp = s;
    ll mod = mod_pow(a, tmp, p);
    while (tmp != p - 1 && mod != 1 && mod != p - 1) {
      mod = mod_mul(mod, mod, p);
      tmp *= 2;
    }
    if (mod != p - 1 && tmp % 2 == 0) return false;
  }
  return true;
}

ll f(ll a, ll n, ll &has) { return (mod_mul(a, a, n) + has
    ) % n; }

vpl pollardsRho(ll d) {
  vpl res;
  auto& pr = S.pr;
  for (int i = 0; i < sz(pr) && pr[i]*pr[i] <= d; i++) if
      (d % pr[i] == 0) {
    int co = 0; while (d % pr[i] == 0) d /= pr[i], co ++;
    res.pb({pr[i],co});
  }
  if (d > 1) { // d is now a product of at most 2 primes.
    if (millerRabin(d)) res.pb({d,1});
    else while (1) {
      ll has = rand() % 2321 + 47;
      ll x = 2, y = 2, c = 1;
      for (; c == 1; c = __gcd(abs(x-y), d)) {
        x = f(x, d, has);
        y = f(f(y, d, has), d, has);
      } // should cycle in ~sqrt(smallest nontrivial
          divisor) turns
      if (c != d) {
        d /= c; if (d > c) swap(d,c);
        if (c == d) res.pb({c,2});
        else res.pb({c,1}), res.pb({d,1});
        break;
      }
    }
  }
  return res;
}
```

3.3 Divisibility

Euclid.h

Description: Euclidean Algorithm

9 lines

```
pl euclid(ll a, ll b) { // returns {x,y} such that a*x+b*y
    =gcd(a,b)
  if (!b) return {1,0};
  pl p = euclid(b,a%b);
  return {p.s,p.f-a/b*p.s};
}
ll invGeneral(ll a, ll b) {
  pl p = euclid(a,b); assert(p.f*a+p.s*b == 1);
  return p.f+(p.f<0)*b;
}
```

CRT.h

Description: Chinese Remainder Theorem

"Euclid.h" 7 lines

```
pl solve(pl a, pl b) {
  auto g = __gcd(a.s,b.s), l = a.s/g*b.s;
  if ((b.f-a.f) % g != 0) return {-1,-1};
  auto A = a.s/g, B = b.s/g;
  auto mul = (b.f-a.f)/g*invGeneral(A,B) % B;
  return {((mul*a.s+a.f)%l+l)%l,l};
}
```

Combinatorial (4)

IntPerm.h

Description: convert permutation of $\{0, 1, ..., N-1\}$ to integer in $[0, N!)$
Usage: assert(encode(decode(5,37)) == 37);
Time: $\mathcal{O}(N)$

20 lines

```
vi decode(int n, int a) {
  vi el(n), b; iota(all(el),0);
  FOR(i,n) {
    int z = a%sz(el);
    b.pb(el[z]); a /= sz(el);
    swap(el[z],el.back()); el.pop_back();
  }
  return b;
}

int encode(vi b) {
  int n = sz(b), a = 0, mul = 1;
  vi pos(n); iota(all(pos),0); vi el = pos;
  FOR(i,n) {
    int z = pos[b[i]]; a += mul*z; mul *= sz(el);
    swap(pos[el[z]],pos[el.back()]);
    swap(el[z],el.back()); el.pop_back();
  }
  return a;
}
```

MatroidIntersect.h

Description: computes a set of maximum size which is independent in both graphic and colorful matroids, aka a spanning forest where no two edges are of the same color
Time: $\mathcal{O}\left(GI^{1.5}\right)$ calls to oracles, where G is the size of the ground set and I is the size of the independent set

"DSU.h" 108 lines

```
int R;
map<int,int> m;

struct Element {
  pi ed;
  int col;
  bool in_independent_set = 0;
  int independent_set_position;
  Element(int u, int v, int c) { ed = {u,v}; col = c; }
};

vi independent_set;
vector<Element> ground_set;
bool col_used[300];

struct GBasis {
  DSU D;
  void reset() { D.init(sz(m)); }
  void add(pi v) { assert(D.unite(v.f,v.s)); }
  bool independent_with(pi v) { return !D.sameSet(v.f,v.s)
    ; }
};
GBasis basis, basis_wo[300];

bool graph_oracle(int inserted) {
  return basis.independent_with(ground_set[inserted].ed);
}
bool graph_oracle(int inserted, int removed) {
  int wi = ground_set[removed].independent_set_position;
  return basis_wo[wi].independent_with(ground_set[inserted
    ].ed);
}
void prepare_graph_oracle() {
  basis.reset();
  FOR(i,sz(independent_set)) basis_wo[i].reset();
  FOR(i,sz(independent_set)) {
    pi v = ground_set[independent_set[i]].ed; basis.add(v)
      ;
    FOR(j,sz(independent_set)) if (i != j) basis_wo[j].add
      (v);
  }
}

bool colorful_oracle(int ins) {
  ins = ground_set[ins].col;
  return !col_used[ins];
}
bool colorful_oracle(int ins, int rem) {
  ins = ground_set[ins].col;
  rem = ground_set[rem].col;
  return !col_used[ins] || ins == rem;
}
void prepare_colorful_oracle() {
  FOR(i,R) col_used[i] = 0;
  trav(t,independent_set) col_used[ground_set[t].col] = 1;
}

bool augment() {
  prepare_graph_oracle();
  prepare_colorful_oracle();

  vi par(sz(ground_set),MOD);
  queue<int> q;
  FOR(i,sz(ground_set)) if (colorful_oracle(i)) {
    assert(!ground_set[i].in_independent_set);
    par[i] = -1; q.push(i);
  }
  int lst = -1;
  while (sz(q)) {
    int cur = q.front(); q.pop();
    if (ground_set[cur].in_independent_set) {
      FOR(to,sz(ground_set)) if (par[to] == MOD) {
        if (!colorful_oracle(to,cur)) continue;
        par[to] = cur; q.push(to);
      }
    } else {
      if (graph_oracle(cur)) { lst = cur; break;  }
      trav(to,independent_set) if (par[to] == MOD) {
        if (!graph_oracle(cur,to)) continue;
        par[to] = cur; q.push(to);
      }
    }
  }
  if (lst == -1) return 0;
  do {
    ground_set[lst].in_independent_set ^= 1;
    lst = par[lst];
  } while (lst != -1);
  independent_set.clear();
  FOR(i,sz(ground_set)) if (ground_set[i].
    in_independent_set) {
    ground_set[i].independent_set_position = sz(
      independent_set);
    independent_set.pb(i);
  }
  return 1;
}

void solve() {
  re(R); if (R == 0) exit(0);
  m.clear(); ground_set.clear(); independent_set.clear();
  FOR(i,R) {
    int a,b,c,d; re(a,b,c,d);
    ground_set.pb(Element(a,b,i));
    ground_set.pb(Element(c,d,i));
    m[a] = m[b] = m[c] = m[d] = 0;
  }
  int co = 0;
  trav(t,m) t.s = co++;
  trav(t,ground_set) t.ed.f = m[t.ed.f], t.ed.s = m[t.ed.s
    ];
  while (augment());
  ps(2*sz(independent_set));
}
```

PermGroup.h

Description: Schreier-Sims, count number of permutations in group and test whether permutation is a member of group
Time: ?

51 lines

```
const int N = 15;
int n;

vi inv(vi v) { vi V(sz(v)); FOR(i,sz(v)) V[v[i]] = i;
  return V; }
vi id() { vi v(n); iota(all(v),0); return v; }
vi operator*(const vi& a, const vi& b) {
  vi c(sz(a)); FOR(i,sz(a)) c[i] = a[b[i]];
  return c;
}

struct Group {
  bool flag[N];
  vi sigma[N]; // sigma[t][k] = t, sigma[t][x] = x if x >
    k
  vector<vi> gen;
  void clear(int p) {
    memset(flag,0, sizeof flag);
    flag[p] = 1; sigma[p] = id();
    gen.clear();
  }
} g[N];
```

```
bool check(const vi& cur, int k) {
  if (!k) return 1;
  int t = cur[k];
  return g[k].flag[t] ? check(inv(g[k].sigma[t])*cur,k-1) : 0;
}
void updateX(const vi& cur, int k);
void ins(const vi& cur, int k) {
  if (check(cur,k)) return;
  g[k].gen.pb(cur);
  F0R(i,n) if (g[k].flag[i]) updateX(cur*g[k].sigma[i],k);
}
void updateX(const vi& cur, int k) {
  int t = cur[k];
  if (g[k].flag[t]) ins(inv(g[k].sigma[t])*cur,k-1); // fixes k -> k
  else {
    g[k].flag[t] = 1, g[k].sigma[t] = cur;
    trav(x,g[k].gen) updateX(x*cur,k);
  }
}

ll order(vector<vi> gen) {
  assert(sz(gen)); n = sz(gen[0]); F0R(i,n) g[i].clear(i);
  trav(a,gen) ins(a,n-1); // insert perms into group one by one
  ll tot = 1;
  F0R(i,n) {
    int cnt = 0; F0R(j,i+1) cnt += g[i].flag[j];
    tot *= cnt;
  }
  return tot;
}
```

Numerical (5)

5.1 Matrix

Matrix.h

Description: 2D matrix operations

36 lines

```
template<class T> struct Mat {
  int r,c;
  vector<vector<T>> d;
  Mat(int _r, int _c) : r(_r), c(_c) { d.assign(r,vector<T>(c)); }
  Mat() : Mat(0,0) {}
  Mat(const vector<vector<T>>& _d) : r(sz(_d)), c(sz(_d[0])) { d = _d; }
  friend void pr(const Mat& m) { pr(m.d); }

  Mat& operator+=(const Mat& m) {
    assert(r == m.r && c == m.c);
    F0R(i,r) F0R(j,c) d[i][j] += m.d[i][j];
    return *this;
  }
  Mat& operator-=(const Mat& m) {
    assert(r == m.r && c == m.c);
    F0R(i,r) F0R(j,c) d[i][j] -= m.d[i][j];
    return *this;
  }
  Mat operator*(const Mat& m) {
    assert(c == m.r); Mat x(r,m.c);
    F0R(i,r) F0R(j,c) F0R(k,m.c) x.d[i][k] += d[i][j]*m.d[j][k];
    return x;
  }

  Mat operator+(const Mat& m) { return Mat(*this)+=m; }
  Mat operator-(const Mat& m) { return Mat(*this)-=m; }
  Mat& operator*=(const Mat& m) { return *this = (*this)*m; }

  friend Mat pow(Mat m, ll p) {
    assert(m.r == m.c);
    Mat r(m.r,m.c);
    F0R(i,m.r) r.d[i][i] = 1;
    for (; p; p /= 2, m *= m) if (p&1) r *= m;
    return r;
  }
};
```

MatrixInv.h

Description: calculates determinant via gaussian elimination

Time: $\mathcal{O}\left(N^3\right)$

"Matrix.h" 31 lines

```
template<class T> T gauss(Mat<T>& m) { // determinant of 1000x1000 Matrix in ~1s
  int n = m.r;
  T prod = 1; int nex = 0;
  F0R(i,n) {
    int row = -1; // for ld use EPS rather than 0
    FOR(j,nex,n) if (m.d[j][i] != 0) { row = j; break; }
    if (row == -1) { prod = 0; continue; }
    if (row != nex) prod *= -1, swap(m.d[row],m.d[nex]);
    prod *= m.d[nex][i];
    auto x = 1/m.d[nex][i]; FOR(k,i,m.c) m.d[nex][k] *= x;
    F0R(j,n) if (j != nex) {
      auto v = m.d[j][i];
      if (v != 0) FOR(k,i,m.c) m.d[j][k] -= v*m.d[nex][k];
    }
    nex ++;
  }
  return prod;
}

template<class T> Mat<T> inv(Mat<T> m) {
  int n = m.r;
  Mat<T> x(n,2*n);
  F0R(i,n) {
    x.d[i][i+n] = 1;
    F0R(j,n) x.d[i][j] = m.d[i][j];
  }
  if (gauss(x) == 0) return Mat<T>(0,0);
  Mat<T> r(n,n);
  F0R(i,n) F0R(j,n) r.d[i][j] = x.d[i][j+n];
  return r;
}
```

MatrixTree.h

Description: Kirchhoff's Matrix Tree Theorem: given adjacency matrix, calculates # of spanning trees

"MatrixInv.h" 13 lines

```
mi numSpan(Mat<mi> m) {
  int n = m.r;
  Mat<mi> res(n-1,n-1);
  F0R(i,n) FOR(j,i+1,n) {
    mi ed = m.d[i][j];
    res.d[i][i] += ed;
    if (j != n-1) {
      res.d[j][j] += ed;
      res.d[i][j] -= ed, res.d[j][i] -= ed;
    }
  }
  return gauss(res);
}
```

5.2 Polynomials

VecOp.h

Description: arithmetic + misc polynomial operations with vectors

73 lines

```
namespace VecOp {
  template<class T> vector<T> rev(vector<T> v) { reverse(all(v)); return v; }
  template<class T> vector<T> shift(vector<T> v, int x) { v.insert(v.begin(),x,0); return v; }
  template<class T> vector<T> integ(const vector<T>& v) {
    vector<T> res(sz(v)+1);
    F0R(i,sz(v)) res[i+1] = v[i]/(i+1);
    return res;
  }
  template<class T> vector<T> dif(const vector<T>& v) {
    if (!sz(v)) return v;
    vector<T> res(sz(v)-1); FOR(i,1,sz(v)) res[i-1] = i*v[i];
    return res;
  }
  template<class T> vector<T>& remLead(vector<T>& v) {
    while (sz(v) && v.back() == 0) v.pop_back();
    return v;
  }
  template<class T> T eval(const vector<T>& v, const T& x) {
    T res = 0; R0F(i,sz(v)) res = x*res+v[i];
    return res;
  }

  template<class T> vector<T>& operator+=(vector<T>& l, const vector<T>& r) {
    l.rsz(max(sz(l),sz(r))); F0R(i,sz(r)) l[i] += r[i];
    return l;
  }
  template<class T> vector<T>& operator-=(vector<T>& l, const vector<T>& r) {
```

```
  l.rsz(max(sz(l),sz(r))); FOR(i,sz(r)) l[i] -= r[i]; return l;
}
template<class T> vector<T>& operator*=(vector<T>& l, const T& r) { trav(t,l) t *= r; return l;  }
template<class T> vector<T>& operator/=(vector<T>& l, const T& r) { trav(t,l) t /= r; return l; }

template<class T> vector<T> operator+(vector<T> l, const vector<T>& r) { return l += r; }
template<class T> vector<T> operator-(vector<T> l, const vector<T>& r) { return l -= r; }
template<class T> vector<T> operator*(vector<T> l, const T& r) { return l *= r; }
template<class T> vector<T> operator*(const T& r, const vector<T>& l) { return l*r; }
template<class T> vector<T> operator/(vector<T> l, const T& r) { return l /= r;  }

template<class T> vector<T> operator*(const vector<T>& l, const vector<T>& r) {
  if (min(sz(l),sz(r)) == 0) return {};
  vector<T> x(sz(l)+sz(r)-1); FOR(i,sz(l)) FOR(j,sz(r)) x[i+j] += l[i]*r[j];
  return x;
}
template<class T> vector<T>& operator*=(vector<T>& l, const vector<T>& r) { return l = l*r; }

template<class T> pair<vector<T>,vector<T>> qr(vector<T> a, vector<T> b) { // quotient and remainder
  assert(sz(b)); auto B = b.back(); assert(B != 0);
  B = 1/B; trav(t,b) t *= B;

  remLead(a); vector<T> q(max(sz(a)-sz(b)+1,0));
  while (sz(a) >= sz(b)) {
    q[sz(a)-sz(b)] = a.back();
    a -= a.back()*shift(b,sz(a)-sz(b));
    remLead(a);
  }

  trav(t,q) t *= B;
  return {q,a};
}
template<class T> vector<T> quo(const vector<T>& a, const vector<T>& b) { return qr(a,b).f; }
template<class T> vector<T> rem(const vector<T>& a, const vector<T>& b) { return qr(a,b).s; }

template<class T> vector<T> interpolate(vector<pair<T,T>> v) {
  vector<T> ret, prod = {1};
  FOR(i,sz(v)) prod *= vector<T>({-v[i].f,1});
  FOR(i,sz(v)) {
    T todiv = 1; FOR(j,sz(v)) if (i != j) todiv *= v[i].f-v[j].f;
    ret += qr(prod,{-v[i].f,1}).f*(v[i].s/todiv);
  }
  return ret;
}
}

using namespace VecOp;
```

PolyRoots.h

Description: Finds the real roots of a polynomial.

Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`

Time: $\mathcal{O}\left(N^2 \log(1/\epsilon)\right)$

"VecOp.h" 19 lines

```
vd polyRoots(vd p, ld xmin, ld xmax) {
  if (sz(p) == 2) { return {-p[0]/p[1]}; }
  auto dr = polyRoots(dif(p),xmin,xmax);
  dr.pb(xmin-1); dr.pb(xmax+1); sort(all(dr));
  vd ret;
  FOR(i,sz(dr)-1) {
    auto l = dr[i], h = dr[i+1];
    bool sign = eval(p,l) > 0;
    if (sign ^ (eval(p,h) > 0)) {
      FOR(it,60) { // while (h - l > 1e-8)
        auto m = (l+h)/2, f = eval(p,m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.pb((l+h)/2);
    }
  }
  return ret;
}
```

Karatsuba.h

Description: multiply two polynomials

Time: $\mathcal{O}\left(N^{\log_2 3}\right)$

26 lines

```
int size(int s) { return s > 1 ? 32-__builtin_clz(s-1) : 0; }

void karatsuba(ll *a, ll *b, ll *c, ll *t, int n) {
  int ca = 0, cb = 0; FOR(i,n) ca += !!a[i], cb += !!b[i];
  if (min(ca, cb) <= 1500/n) { // few numbers to multiply
    if (ca > cb) swap(a, b);
    FOR(i,n) if (a[i]) FOR(j,n) c[i+j] += a[i]*b[j];
  } else {
    int h = n >> 1;
    karatsuba(a, b, c, t, h); // a0*b0
    karatsuba(a+h, b+h, c+n, t, h); // a1*b1
    FOR(i,h) a[i] += a[i+h], b[i] += b[i+h];
    karatsuba(a, b, t, t+n, h); // (a0+a1)*(b0+b1)
    FOR(i,h) a[i] -= a[i+h], b[i] -= b[i+h];
    FOR(i,n) t[i] -= c[i]+c[i+n];
    FOR(i,n) c[i+h] += t[i], t[i] = 0;
  }
}

vl conv(vl a, vl b) {
  int sa = sz(a), sb = sz(b); if (!sa || !sb) return {};
  int n = 1<<size(max(sa,sb)); a.rsz(n), b.rsz(n);
  vl c(2*n), t(2*n); FOR(i,2*n) t[i] = 0;
  karatsuba(&a[0], &b[0], &c[0], &t[0], n);
  c.rsz(sa+sb-1); return c;
}
```

FFT.h

Description: multiply two polynomials

Time: $\mathcal{O}(N \log N)$

"Modular.h" 40 lines

```
typedef complex<db> cd;
const int MOD = (119 << 23) + 1, root = 3; // = 998244353
// NTT: For p < 2^30 there is also e.g. (5 << 25, 3), (7 << 26, 3),
// (479 << 21, 3) and (483 << 21, 5). The last two are > 10^9.

constexpr int size(int s) { return s > 1 ? 32-__builtin_clz(s-1) : 0; }
void genRoots(vmi& roots) { // primitive n-th roots of unity
  int n = sz(roots); mi r = pow(mi(root),(MOD-1)/n);
  roots[0] = 1; FOR(i,1,n) roots[i] = roots[i-1]*r;
}
void genRoots(vcd& roots) { // change cd to complex<double> instead?
  int n = sz(roots); double ang = 2*PI/n;
  FOR(i,n) roots[i] = cd(cos(ang*i),sin(ang*i)); // is there a way to do this more quickly?
}

template<class T> void fft(vector<T>& a, const vector<T>& roots, bool inv = 0) {
  int n = sz(a);
  for (int i = 1, j = 0; i < n; i++) { // sort by reverse bit representation
    int bit = n >> 1;
    for (; j&bit; bit >>= 1) j ^= bit;
    j ^= bit; if (i < j) swap(a[i], a[j]);
  }
  for (int len = 2; len <= n; len <<= 1)
    for (int i = 0; i < n; i += len)
      FOR(j,len/2) {
        int ind = n/len*j; if (inv && ind) ind = n-ind;
        auto u = a[i+j], v = a[i+j+len/2]*roots[ind];
        a[i+j] = u+v, a[i+j+len/2] = u-v;
      }
  if (inv) { T i = T(1)/T(n); trav(x,a) x *= i; }
}

template<class T> vector<T> mult(vector<T> a, vector<T> b) {
  int s = sz(a)+sz(b)-1, n = 1<<size(s);
  vector<T> roots(n); genRoots(roots);
  a.rsz(n), fft(a,roots);
  b.rsz(n), fft(b,roots);
  FOR(i,n) a[i] *= b[i];
  fft(a,roots,1); return a;
}
```

FFTmod.h

Description: multiply two polynomials with arbitrary MOD ensures precision by splitting in half

"FFT.h" 27 lines

```
vl multMod(const vl& a, const vl& b) {
  if (!min(sz(a),sz(b))) return {};
  int s = sz(a)+sz(b)-1, n = 1<<size(s), cut = sqrt(MOD);
  vcd roots(n); genRoots(roots);

  vcd ax(n), bx(n);
  FOR(i,sz(a)) ax[i] = cd((int)a[i]/cut, (int)a[i]%cut);
    ↪// ax(x)=a1(x)+i*a0(x)
  FOR(i,sz(b)) bx[i] = cd((int)b[i]/cut, (int)b[i]%cut);
    ↪// bx(x)=b1(x)+i*b0(x)
  fft(ax,roots), fft(bx,roots);

  vcd v1(n), v0(n);
  FOR(i,n) {
    int j = (i ? (n-i) : i);
    v1[i] = (ax[i]+conj(ax[j]))*cd(0.5,0)*bx[i]; // v1 =
      ↪a1*(b1+b0*cd(0,1));
    v0[i] = (ax[i]-conj(ax[j]))*cd(0,-0.5)*bx[i]; // v0 =
      ↪a0*(b1+b0*cd(0,1));
  }
  fft(v1,roots,1), fft(v0,roots,1);

  vl ret(n);
  FOR(i,n) {
    ll V2 = (ll)round(v1[i].real()); // a1*b1
    ll V1 = (ll)round(v1[i].imag())+(ll)round(v0[i].real()
      ↪); // a0*b1+a1*b0
    ll V0 = (ll)round(v0[i].imag()); // a0*b0
    ret[i] = ((V2%MOD*cut+V1)%MOD*cut+V0)%MOD;
  }
  ret.rsz(s); return ret;
} // ~0.8s when sz(a)=sz(b)=1<<19
```

PolyInv.h

Description: ?
Time: ?

"FFT.h" 11 lines

```
template<class T> vector<T> inv(vector<T> v, int p) { //
  ↪compute inverse of v mod x^p, where v[0] = 1
  v.rsz(p);  vector<T> a = {T(1)/v[0]};
  for (int i = 1; i < p; i *= 2) {
    if (2*i > p) v.rsz(2*i);
    auto l = vector<T>(begin(v),begin(v)+i), r = vector<T
      ↪>(begin(v)+i,begin(v)+2*i);
    auto c = mult(a,l); c = vector<T>(begin(c)+i,end(c));
    auto b = mult(a*T(-1),mult(a,r)+c); b.rsz(i);
    a.insert(end(a),all(b));
  }
  a.rsz(p); return a;
}
```

PolyDiv.h

Description: divide two polynomials
Time: $\mathcal{O}(N\log N)$?

"PolyInv.h" 7 lines

```
template<class T> pair<vector<T>,vector<T>> divi(const
  ↪vector<T>& f, const vector<T>& g) { // f = q*g+r
  if (sz(f) < sz(g)) return {{},f};
  auto q = mult(inv(rev(g),sz(f)-sz(g)+1),rev(f));
  q.rsz(sz(f)-sz(g)+1); q = rev(q);
  auto r = f-mult(q,g); r.rsz(sz(g)-1);
  return {q,r};
}
```

PolySqrt.h

Description: find sqrt of polynomial
Time: $\mathcal{O}(N\log N)$?

"PolyInv.h" 8 lines

```
template<class T> vector<T> sqrt(vector<T> v, int p) { //
  ↪S*S = v mod x^p, p is power of 2
  assert(v[0] == 1); if (p == 1) return {1};
  v.rsz(p);
  auto S = sqrt(v,p/2);
  auto ans = S+mult(v,inv(S,p));
  ans.rsz(p); ans *= T(1)/T(2);
  return ans;
}
```

5.3 Misc

LinRec.h

Description: Berlekamp-Massey: computes linear recurrence of order n for sequence of $2n$ terms
Time: ?

35 lines

```
using namespace vecOp;

struct LinRec {
  vmi x; // original sequence
  vmi C, rC;
  void init(const vmi& _x) {
    x = _x; int n = sz(x), m = 0;
    vmi B; B = C = {1}; // B is fail vector

    mi b = 1; // B gives 0,0,0,...,b
    FOR(i,n) {
      m ++;
      mi d = x[i]; FOR(j,1,sz(C)) d += C[j]*x[i-j];
      if (d == 0) continue; // recurrence still works
      auto _B = C; C.rsz(max(sz(C),m+sz(B)));
      mi coef = d/b; FOR(j,m,m+sz(B)) C[j] -= coef*B[j-m];
        ↪ // recurrence that gives 0,0,0,...,d
      if (sz(_B) < m+sz(B)) { B = _B; b = d; m = 0; }
    }

    rC = C; reverse(all(rC)); // polynomial for getPo
    C.erase(begin(C)); trav(t,C) t *= -1; // x[i]=sum_{j
      ↪=0}^{sz(C)-1}C[j]*x[i-j-1]
  }

  vmi getPo(int n) {
    if (n == 0) return {1};
    vmi x = getPo(n/2); x = rem(x*x,rC);
    if (n&1) { vmi v = {0,1}; x = rem(x*v,rC); }
    return x;
  }
  mi eval(int n) {
    vmi t = getPo(n);
    mi ans = 0; FOR(i,sz(t)) ans += t[i]*x[i];
    return ans;
  }
};
```

Integrate.h

Description: ?

8 lines

```
// db f(db x) { return x*x+3*x+1; }

db quad(db (*f)(db), db a, db b) {
  const int n = 1000;
  db dif = (b-a)/2/n, tot = f(a)+f(b);
  FOR(i,1,2*n) tot += f(a+i*dif)*(i&1?4:2);
  return tot*dif/3;
}
```

IntegrateAdaptive.h

Description: ?

19 lines

```
// db f(db x) { return x*x+3*x+1; }

db simpson(db (*f)(db), db a, db b) {
  db c = (a+b) / 2;
  return (f(a) + 4*f(c) + f(b)) * (b-a) / 6;
}

db rec(db (*f)(db), db a, db b, db eps, db S) {
  db c = (a+b) / 2;
  db S1 = simpson(f, a, c);
  db S2 = simpson(f, c, b), T = S1 + S2;
  if (abs(T - S) <= 15*eps || b-a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps/2, S1) + rec(f, c, b, eps/2, S2)
    ↪;
}

db quad(db (*f)(db), db a, db b, db eps = 1e-8) {
  return rec(f, a, b, eps, simpson(f, a, b));
}
```

Simplex.h

Description: Simplex algorithm for linear programming, maximize $c^T x$ subject to $Ax \le b, x \ge 0$
Time: ?

73 lines

```
typedef double T;
typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1/.0;

#define ltj(X) if (s == -1 || mp(X[j],N[j]) < mp(X[s],N[s
  ↪])) s=j
```

```
struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      FOR(i,m) FOR(j,n) D[i][j] = A[i][j];
      FOR(i,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i
        ]; } // B[i] -> basic variables, col n+1 is for
        constants, why D[i][n]=-1?
      FOR(j,n) { N[j] = j; D[m][j] = -c[j]; } // N[j] ->
        non-basic variables, all zero
      N[n] = -1; D[m+1][n] = 1;
    }

  void print() {
    ps("D");
    trav(t,D) ps(t);
    ps();
    ps("B",B);
    ps("N",N);
    ps();
  }

  void pivot(int r, int s) { // row, column
    T *a = D[r].data(), inv = 1/a[s]; // eliminate col s
      from consideration
    FOR(i,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s]*inv;
      FOR(j,n+2) b[j] -= a[j]*inv2;
      b[s] = a[s]*inv2;
    }
    FOR(j,n+2) if (j != s) D[r][j] *= inv;
    FOR(i,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv; swap(B[r], N[s]); // swap a basic and
      non-basic variable
  }

  bool simplex(int phase) {
    int x = m+phase-1;
    for (;;) {
      int s = -1; FOR(j,n+1) if (N[j] != -phase) ltj(D[x])
        ; // find most negative col
      if (D[x][s] >= -eps) return true; // have best
        solution
      int r = -1;
      FOR(i,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || mp(D[i][n+1] / D[i][s], B[i])
              < mp(D[r][n+1] / D[r][s], B[r])) r = i; //
                find smallest positive ratio
      }
      if (r == -1) return false; // unbounded
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0; FOR(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i
      ;
    if (D[r][n+1] < -eps) { // x=0 is not a solution
      pivot(r, n); // -1 is artificial variable, initially
        set to smth large but want to get to 0
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
        // no solution
      // D[m+1][n+1] is max possible value of the negation
        of artificial variable, starts negative but
        should get to zero
      FOR(i,m) if (B[i] == -1) {
        int s = 0; FOR(j,1,n+1) ltj(D[i]);
        pivot(i,s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    FOR(i,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

Graphs (6)

6.1 Fundamentals

DSU.h

Description: ?
Time: $\mathcal{O}(N\alpha(N))$

29 lines

```
template<int SZ> struct DSU {
    int par[SZ];
    int size[SZ];
    DSU() {
        M00(i, SZ) par[i] = i, size[i] = 1;
    }
    int get(int node) {
        if(par[node] != node) par[node] = get(par[node]);
        return par[node];
    }
    bool connected(int n1, int n2) {
        return (get(n1) == get(n2));
    }
    int sz(int node) {
        return size[get(node)];
    }
    void unite(int n1, int n2) {
        n1 = get(n1);
        n2 = get(n2);
        if(n1 == n2) return;
        if(rand()%2) {
            par[n1] = n2;
            size[n2] += size[n1];
        } else {
            par[n2] = n1;
            size[n1] += size[n2];
        }
    }
};
```

ManhattanMST.h

Description: Compute minimum spanning tree of points where edges are manhattan distances
Time: $\mathcal{O}(N \log N)$

"MST.h" 60 lines

```
int N;
vector<array<int,3>> cur;
vector<pair<ll,pi>> ed;
vi ind;

struct {
  map<int,pi> m;
  void upd(int a, pi b) {
    auto it = m.lb(a);
    if (it != m.end() && it->s <= b) return;
    m[a] = b; it = m.find(a);
    while (it != m.begin() && prev(it)->s >= b) m.erase(
      prev(it));
  }
  pi query(int y) { // for all a > y find min possible
    value of b
    auto it = m.ub(y);
    if (it == m.end()) return {2*MOD,2*MOD};
    return it->s;
  }
} S;

void solve() {
  sort(all(ind),[](int a, int b) { return cur[a][0] > cur[
    b][0]; });
  S.m.clear();
  int nex = 0;
  trav(x,ind) { // cur[x][0] <= ?, cur[x][1] < ?
    while (nex < N && cur[ind[nex]][0] >= cur[x][0]) {
      int b = ind[nex++];
      S.upd(cur[b][1],{cur[b][2],b});
    }
    pi t = S.query(cur[x][1]);
    if (t.s != 2*MOD) ed.pb({(ll)t.f-cur[x][2],{x,t.s}});
  }
}

ll mst(vpi v) {
  N = sz(v); cur.resz(N); ed.clear();
  ind.clear(); FOR(i,N) ind.pb(i);
  sort(all(ind),[&v](int a, int b) { return v[a] < v[b];
    });
  FOR(i,N-1) if (v[ind[i]] == v[ind[i+1]]) ed.pb({0,{ind[i
    ],ind[i+1]}});
  FOR(i,2) { // it's probably ok to consider just two
    quadrants?
    FOR(i,N) {
      auto a = v[i];
      cur[i][2] = a.f+a.s;
    }
    FOR(i,N) { // first octant
      auto a = v[i];
      cur[i][0] = a.f-a.s;
      cur[i][1] = a.s;
```

```
    }
    solve();
    FOR(i,N) { // second octant
      auto a = v[i];
      cur[i][0] = a.f;
      cur[i][1] = a.s-a.f;
    }
    solve();
    trav(a,v) a = {a.s,-a.f}; // rotate 90 degrees, repeat
  }
  return kruskal(ed);
}
```

Dijkstra.h

Description: Dijkstra's algorithm for shortest path
Time: $\mathcal{O}(E \log V)$ 31 lines

```
template<int SZ> struct dijkstra {
    vector<pair<int, ll>> adj[SZ];
    bool vis[SZ];
    ll d[SZ];

    void addEdge(int u, int v, ll l) {
        adj[u].PB(MP(v, l));
    }
    ll dist(int v) {
        return d[v];
    }
    void build(int u) {
        M00(i, SZ) vis[i] = 0;
        priority_queue<pair<ll, int>, vector<pair<ll, int
            ↪>>, greater<pair<ll, int>>> pq;
        M00(i, SZ) d[i] = 1e17;
        d[u] = 0;
        pq.push(MP(0, u));
        while(!pq.empty()) {
            pair<ll, int> t = pq.top(); pq.pop();
            while(!pq.empty() && vis[t.S]) t = pq.top(),
                ↪pq.pop();
            vis[t.S] = 1;

            for(auto& v: adj[t.S]) if(!vis[v.F]) {
                if(d[v.F] > d[t.S] + v.S) {
                    d[v.F] = d[t.S] + v.S;
                    pq.push(MP(d[v.F], v.F));
                }
            }
        }
    }
};
```

FloydWarshall.h

Description: Floyd Warshall's algorithm for all pairs shortest path
Time: $\mathcal{O}(V^3)$ 13 lines

```
/*
 let dist be a |V| * |V| array of minimum distances
    ↪initialized to inf
 for each edge (u, v) do
    dist[u][v] <- w(u, v)  // The weight of the edge (u, v
        ↪)
 for each vertex v do
    dist[v][v] <- 0
 for k from 1 to |V|
    for i from 1 to |V|
        for j from 1 to |V|
            if dist[i][j] > dist[i][k] + dist[k][j]
                dist[i][j] <- dist[i][k] + dist[k][j]
            end if
*/
```

6.2 Trees

LCAjumps.h

Description: calculates least common ancestor in tree with binary jumping
Time: $\mathcal{O}(N \log N)$ 44 lines

```
template<int SZ> struct tree {
    vector<pair<int, ll>> adj[SZ];
    const static int LGSZ = 32-__builtin_clz(SZ-1);
    pair<int, ll> ppar[SZ][LGSZ];
    int depth[SZ];
    ll distfromroot[SZ];

    void addEdge(int u, int v, int d) {
        adj[u].PB(MP(v, d));
        adj[v].PB(MP(u, d));
    }
    void dfs(int u, int dep, ll dis) {
        depth[u] = dep;
        distfromroot[u] = dis;
        for(auto& v: adj[u]) if(ppar[u][0].F != v.F) {
            ppar[v.F][0] = MP(u, v.S);
            dfs(v.F, dep + 1, dis + v.S);
        }
    }
    void build() {
        ppar[0][0] = MP(0, 0);
        M00(i, SZ) depth[i] = 0;
        dfs(0, 0, 0);
        MOO(i, 1, LGSZ) M00(j, SZ) {
            ppar[j][i].F = ppar[ppar[j][i-1].F][i-1].F;
            ppar[j][i].S = ppar[j][i-1].S + ppar[ppar[j][i
                ↪-1].F][i-1].S;
        }
    }
    int lca(int u, int v) {
        if(depth[u] < depth[v]) swap(u, v);
        M00d(i, LGSZ) if(depth[ppar[u][i].F] >= depth[v])
            ↪u = ppar[u][i].F;
        if(u == v) return u;
        M00d(i, LGSZ) {
            if(ppar[u][i].F != ppar[v][i].F) {
                u = ppar[u][i].F;
                v = ppar[v][i].F;
            }
        }
        return ppar[u][0].F;
    }
    ll dist(int u, int v) {
        return distfromroot[u] + distfromroot[v] - 2*
            ↪distfromroot[lca(u, v)];
    }
};
```

LCArmq.h

Description: Euler Tour LCA w/ O(1) query 58 lines

```
template<int SZ> struct tree {
    vector<pair<int, ll>> adj[SZ];
    pair<int, ll> par[SZ];
    const static int LGSZ = 33-__builtin_clz(SZ-1);
    ll distfromroot[SZ];
    int depth[SZ], t, tin[SZ], RMQ[2*SZ-1][LGSZ], oldToNew
        ↪[SZ], newToOld[SZ], numNodes;

    void addEdge(int u, int v, int d) {
        adj[u].PB(MP(v, d));
        adj[v].PB(MP(u, d));
    }
    void dfs(int u, int dep, ll dis) {
        depth[u] = dep;
        distfromroot[u] = dis;
        for(auto& v: adj[u]) if(par[u].F != v.F) {
            par[v.F] = MP(u, v.S);
            dfs(v.F, dep + 1, dis + v.S);
        }
    }
    void buildtarr(int u) {
        RMQ[t][0] = oldToNew[u], tin[oldToNew[u]] = t++;
        for(auto& v: adj[u]) if(par[u].F != v.F) {
            buildtarr(v.F);
            RMQ[t++][0] = oldToNew[u];
        }
    }
    void build(int n) {
        this->numNodes = n;
        par[0] = MP(0, 0);
        M00(i, numNodes) depth[i] = 0;
        dfs(0, 0, 0);
        t = 0;
        queue<int> q;
        q.push(0);
        while(!q.empty()) {
            int u = q.front(); q.pop();
            oldToNew[u] = t++;
            for(auto& v: adj[u]) if(par[u].F != v.F) q.
                ↪push(v.F);
        }
        M00(i, numNodes) newToOld[oldToNew[i]] = i;
        t = 0;
        buildtarr(0);
        MOO(j, 1, LGSZ) M00(i, 2*numNodes-1) if(i+(1<<(j
            ↪-1)) < 2*numNodes-1)
            RMQ[i][j] = min(RMQ[i][j-1], RMQ[i+(1<<(j-1))
                ↪][j-1]);
    }
    int lca(int u, int v) {
```

```
        u = oldToNew[u], v = oldToNew[v];
        if(tin[u] > tin[v]) swap(u, v);
        int l = tin[u];
        int r = tin[v];
        int len = r-l+1;
        int hl = 31-__builtin_clz(len-1);
        return newToOld[min(RMQ[l][hl], RMQ[r-(1<<hl)+1][
            ↪hl])];
    }
    ll dist(int u, int v) {
        return distfromroot[u]+distfromroot[v]-2*
            ↪distfromroot[lca(u, v)];
    }
};
```

CentroidDecomp.h

Description: can support tree path queries and updates
Time: $\mathcal{O}(N \log N)$ 47 lines

```
template<int SZ> struct centroidDecomp {
    vi neighbor[SZ];
    int subsize[SZ];
    bool vis[SZ];
    int p[SZ];
    int par[SZ];
    vi child[SZ];
    int numNodes;

    centroidDecomp(int num) {
        this->numNodes = num;
    }
    void addEdge(int u, int v) {
        neighbor[u].PB(v);
        neighbor[v].PB(u);
    }
    void build() {
        M00(i, numNodes) vis[i] = 0, par[i] = -1;
        solve(0);
        M00(i, numNodes) if(par[i] != -1) child[par[i]].PB
            ↪(i);
    }
    void getSizes(int node) {
        subsize[node] = 1;
        for(int ch: neighbor[node]) if(!vis[ch] && ch != p
            ↪[node]) {
            p[ch] = node;
            getSizes(ch);
            subsize[node] += subsize[ch];
        }
    }
    int getCentroid(int root) {
        p[root] = -1;
        getSizes(root);
        int cur = root;
        while(1) {
            pi hi = MP(subsize[root]-subsize[cur], cur);
            for(int v: neighbor[cur]) if(!vis[v] && v != p
                ↪[cur]) hi = max(hi, MP(subsize[v], v));
            if(hi.F <= subsize[root]/2) return cur;
            cur = hi.S;
        }
    }
    int solve(int node) {
        node = getCentroid(node);
        vis[node] = 1;
        for(int ch: neighbor[node]) if(!vis[ch]) par[solve
            ↪(ch)] = node;
        return node;
    }
};
```

HLD.h

Description: Heavy Light Decomposition
Time: $\mathcal{O}(\log^2 N)$ per path operations 50 lines

```
template<int SZ, bool VALUES_IN_EDGES> struct HLD {
  int N; vi adj[SZ];
  int par[SZ], sz[SZ], depth[SZ];
  int root[SZ], pos[SZ];
  LazySegTree<ll,SZ> tree;
  void addEdge(int a, int b) { adj[a].pb(b), adj[b].pb(a);
    ↪ }

  void dfs_sz(int v = 1) {
    if (par[v]) adj[v].erase(find(all(adj[v]),par[v]));
    sz[v] = 1;
    trav(u,adj[v]) {
      par[u] = v; depth[u] = depth[v]+1;
      dfs_sz(u); sz[v] += sz[u];
      if (sz[u] > sz[adj[v][0]]) swap(u, adj[v][0]);
    }
  }
  void dfs_hld(int v = 1) {
    static int t = 0;
    pos[v] = t++;
    trav(u,adj[v]) {
      root[u] = (u == adj[v][0] ? root[v] : u);
      dfs_hld(u);
    }
  }
  void init(int _N) {
    N = _N; par[1] = depth[1] = 0; root[1] = 1;
    dfs_sz(); dfs_hld();
  }

  template <class BinaryOperation>
  void processPath(int u, int v, BinaryOperation op) {
    for (; root[u] != root[v]; v = par[root[v]]) {
      if (depth[root[u]] > depth[root[v]]) swap(u, v);
      op(pos[root[v]], pos[v]);
    }
    if (depth[u] > depth[v]) swap(u, v);
    op(pos[u]+VALUES_IN_EDGES, pos[v]);
  }

  void modifyPath(int u, int v, int val) { // add val to
    ↪vertices/edges along path
    processPath(u, v, [this, &val](int l, int r) { tree.
      ↪upd(l, r, val); });
  }

  void modifySubtree(int v, int val) { // add val to
    ↪vertices/edges in subtree
    tree.upd(pos[v]+VALUES_IN_EDGES,pos[v]+sz[v]-1,val);
  }
  ll queryPath(int u, int v) { // query sum of path
    ll res = 0; processPath(u, v, [this, &res](int l, int
      ↪r) { res += tree.qsum(l, r); });
    return res;
  }
};
```

6.3 DFS Algorithms

SCC.h

Description: Kosaraju's Algorithm: DFS two times to generate SCCs in topological order
Time: $\mathcal{O}(N + M)$ 24 lines

```
template<int SZ> struct SCC {
  int N, comp[SZ];
  vi adj[SZ], radj[SZ], todo, allComp;
  bitset<SZ> visit;
  void addEdge(int a, int b) { adj[a].pb(b), radj[b].pb(a)
    ↪; }

  void dfs(int v) {
    visit[v] = 1;
    trav(w,adj[v]) if (!visit[w]) dfs(w);
    todo.pb(v);
  }
  void dfs2(int v, int val) {
    comp[v] = val;
    trav(w,radj[v]) if (comp[w] == -1) dfs2(w,val);
  }

  void init(int _N) { // fills allComp
    N = _N;
    F0R(i,N) comp[i] = -1, visit[i] = 0;
    F0R(i,N) if (!visit[i]) dfs(i);
    reverse(all(todo)); // now todo stores vertices in
      ↪order of topological sort
    trav(i,todo) if (comp[i] == -1) dfs2(i,i), allComp.pb(
      ↪i);
  }
};
```

TopoSort.h

Description: sorts vertices such that if there exists an edge x->y, then x goes before y 14 lines

```
template<int SZ> struct TopoSort {
    int N, in[SZ];
    vi res, adj[SZ];
    void ae(int x, int y) { adj[x].pb(y), in[y] ++; }
    bool sort(int _N) {
        N = _N; queue<int> todo;
        FOR(i,1,N+1) if (!in[i]) todo.push(i);
        while (sz(todo)) {
            int x = todo.front(); todo.pop(); res.pb(x);
            trav(i,adj[x]) if (!(--in[i])) todo.push(i);
```

```
        }
        return sz(res) == N;
    }
};
```

2SAT.h

Description: ?

"SCC.h" 38 lines

```
template<int SZ> struct TwoSat {
  SCC<2*SZ> S;
  bitset<SZ> ans;
  int N = 0;
  int addVar() { return N++; }

  void either(int x, int y) {
    x = max(2*x,-1-2*x), y = max(2*y,-1-2*y);
    S.addEdge(x^1,y); S.addEdge(y^1,x);
  }
  void implies(int x, int y) { either(~x,y); }
  void setVal(int x) { either(x,x); }
  void atMostOne(const vi& li) {
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    FOR(i,2,sz(li)) {
      int next = addVar();
      either(cur,~li[i]);
      either(cur,next);
      either(~li[i],next);
      cur = ~next;
    }
    either(cur,~li[1]);
  }

  bool solve(int _N) {
    if (_N != -1) N = _N;
    S.init(2*N);
    for (int i = 0; i < 2*N; i += 2)
      if (S.comp[i] == S.comp[i^1]) return 0;
    reverse(all(S.allComp));
    vi tmp(2*N);
    trav(i,S.allComp) if (tmp[i] == 0)
      tmp[i] = 1, tmp[S.comp[i^1]] = -1;
    FOR(i,N) if (tmp[S.comp[2*i]] == 1) ans[i] = 1;
    return 1;
  }
};
```

EulerPath.h

Description: Eulerian Path for both directed and undirected graphs
Time: $\mathcal{O}(N+M)$

30 lines

```
template<int SZ, bool directed> struct Euler {
  int N, M = 0;
  vpi adj[SZ];
  vpi::iterator its[SZ];
  vector<bool> used;

  void addEdge(int a, int b) {
    if (directed) adj[a].pb({b,M});
    else adj[a].pb({b,M}), adj[b].pb({a,M});
    used.pb(0); M ++;
  }

  vpi solve(int _N, int src = 1) {
    N = _N;
    FOR(i,1,N+1) its[i] = begin(adj[i]);
    vector<pair<pi,int>> ret, s = {{{src,-1},-1}};
    while (sz(s)) {
      int x = s.back().f.f;
      auto& it = its[x], end = adj[x].end();
      while (it != end && used[it->s]) it ++;
      if (it == end) {
        if (sz(ret) && ret.back().f.s != s.back().f.f)
          ↪return {}; // path isn't valid
        ret.pb(s.back()), s.pop_back();
      } else { s.pb({{it->f,x},it->s}); used[it->s] = 1; }
    }
    if (sz(ret) != M+1) return {};
    vpi ans; trav(t,ret) ans.pb({t.f.f,t.s});
    reverse(all(ans)); return ans;
  }
};
```

BCC.h

Description: computes biconnected components
Time: $\mathcal{O}(N+M)$

37 lines

```
template<int SZ> struct BCC {
  int N;
  vpi adj[SZ], ed;
  void addEdge(int u, int v) {
    adj[u].pb({v,sz(ed)}), adj[v].pb({u,sz(ed)});
    ed.pb({u,v});
  }

  int disc[SZ];
  vi st; vector<vi> fin;
  int bcc(int u, int p = -1) { // return lowest disc
    static int ti = 0;
    disc[u] = ++ti; int low = disc[u];
    int child = 0;
    trav(i,adj[u]) if (i.s != p)
      if (!disc[i.f]) {
        child ++; st.pb(i.s);
        int LOW = bcc(i.f,i.s); ckmin(low,LOW);
        // disc[u] < LOW -> bridge
        if (disc[u] <= LOW) {
          // if (p != -1 || child > 1) -> u is
            ↪articulation point
          vi tmp; while (st.back() != i.s) tmp.pb(st.back
            ↪()), st.pop_back();
          tmp.pb(st.back()), st.pop_back();
          fin.pb(tmp);
        }
      } else if (disc[i.f] < disc[u]) {
        ckmin(low,disc[i.f]);
        st.pb(i.s);
      }
    return low;
  }

  void init(int _N) {
    N = _N; FOR(i,N) disc[i] = 0;
    FOR(i,N) if (!disc[i]) bcc(i); // st should be empty
      ↪after each iteration
  }
};
```

6.4 Flows

Dinic.h

Description: faster flow
Time: $\mathcal{O}(N^2M)$ flow, $\mathcal{O}\left(M\sqrt{N}\right)$ bipartite matching

45 lines

```
template<int SZ> struct Dinic {
  typedef ll F; // flow type
  struct Edge { int to, rev; F flow, cap; };

  int N,s,t;
  vector<Edge> adj[SZ];
  typename vector<Edge>::iterator cur[SZ];
  void addEdge(int u, int v, F cap) {
    assert(cap >= 0); // don't try smth dumb
    Edge a{v, sz(adj[v]), 0, cap}, b{u, sz(adj[u]), 0, 0};
    adj[u].pb(a), adj[v].pb(b);
  }

  int level[SZ];
  bool bfs() { // level = shortest distance from source
    // after computing flow, edges {u,v} such that level[u
      ↪] \neq -1, level[v] = -1 are part of min cut
    M00(i,N) level[i] = -1, cur[i] = begin(adj[i]);
    queue<int> q({s}); level[s] = 0;
    while (sz(q)) {
      int u = q.front(); q.pop();
            for(Edge e: adj[u]) if (level[e.to] < 0 && e.
              ↪flow < e.cap)
        q.push(e.to), level[e.to] = level[u]+1;
    }
    return level[t] >= 0;
  }
  F sendFlow(int v, F flow) {
    if (v == t) return flow;
    for (; cur[v] != end(adj[v]); cur[v]++) {
      Edge& e = *cur[v];
      if (level[e.to] != level[v]+1 || e.flow == e.cap)
        ↪continue;
      auto df = sendFlow(e.to,min(flow,e.cap-e.flow));
      if (df) { // saturated at least one edge
        e.flow += df; adj[e.to][e.rev].flow -= df;
        return df;
      }
    }
    return 0;
  }
  F maxFlow(int _N, int _s, int _t) {
    N = _N, s = _s, t = _t; if (s == t) return -1;
    F tot = 0;
```

```
    while (bfs()) while (auto df = sendFlow(s,numeric_limits<F>::max())) tot += df;
    return tot;
  }
};
```

MCMF.h

Description: Min-Cost Max Flow, no negative cycles allowed
Time: $\mathcal{O}\left(NM^2 \log M\right)$ 53 lines

```
template<class T> using pqg = priority_queue<T,vector<T>,greater<T>>;
template<class T> T poll(pqg<T>& x) {
  T y = x.top(); x.pop();
  return y;
}

template<int SZ> struct mcmf {
  typedef ll F; typedef ll C;
  struct Edge { int to, rev; F flow, cap; C cost; int id; };
  vector<Edge> adj[SZ];
  void addEdge(int u, int v, F cap, C cost) {
    assert(cap >= 0);
    Edge a{v, sz(adj[v]), 0, cap, cost}, b{u, sz(adj[u]), 0, 0, -cost};
    adj[u].pb(a), adj[v].pb(b);
  }

  int N, s, t;
  pi pre[SZ]; // previous vertex, edge label on path
  pair<C,F> cost[SZ]; // tot cost of path, amount of flow
  C totCost, curCost; F totFlow;
  void reweight() { // makes all edge costs non-negative
    // all edges on shortest path become 0
    FOR(i,N) trav(p,adj[i]) p.cost += cost[i].f-cost[p.to].f;
  }
  bool spfa() { // reweight ensures that there will be negative weights
    // only during the first time you run this
    FOR(i,N) cost[i] = {INF,0}; cost[s] = {0,INF};
    pqg<pair<C,int>> todo; todo.push({0,s});
    while (sz(todo)) {
      auto x = poll(todo); if (x.f > cost[x.s].f) continue;
      trav(a,adj[x.s]) if (x.f+a.cost < cost[a.to].f && a.flow < a.cap) {
        // if costs are doubles, add some EPS to ensure that
        // you do not traverse some 0-weight cycle repeatedly
        pre[a.to] = {x.s,a.rev};
        cost[a.to] = {x.f+a.cost,min(a.cap-a.flow,cost[x.s].s)};
        todo.push({cost[a.to].f,a.to});
      }
    }
    curCost += cost[t].f; return cost[t].s;
  }
  void backtrack() {
    F df = cost[t].s; totFlow += df, totCost += curCost*df;
    for (int x = t; x != s; x = pre[x].f) {
      adj[x][pre[x].s].flow -= df;
      adj[pre[x].f][adj[x][pre[x].s].rev].flow += df;
    }
  }
  pair<F,C> calc(int _N, int _s, int _t) {
    N = _N; s = _s, t = _t; totFlow = totCost = curCost = 0;
    while (spfa()) reweight(), backtrack();
    return {totFlow, totCost};
  }
};
```

GomoryHu.h

Description: Compute max flow between every pair of vertices of undirected graph
"Dinic.h" 56 lines

```
template<int SZ> struct GomoryHu {
  int N;
  vector<pair<pi,int>> ed;
  void addEdge(int a, int b, int c) { ed.pb({{a,b},c}); }

  vector<vi> cor = {{}}; // groups of vertices
  map<int,int> adj[2*SZ]; // current edges of tree
  int side[SZ];

  int gen(vector<vi> cc) {
    Dinic<SZ> D = Dinic<SZ>();
    vi comp(N+1); FOR(i,sz(cc)) trav(t,cc[i]) comp[t] = i;
    trav(t,ed) if (comp[t.f.f] != comp[t.f.s]) {
      D.addEdge(comp[t.f.f],comp[t.f.s],t.s);
      D.addEdge(comp[t.f.s],comp[t.f.f],t.s);
    }
    int f = D.maxFlow(0,1);
    FOR(i,sz(cc)) trav(j,cc[i]) side[j] = D.level[i] >= 0; // min cut
    return f;
  }

  void fill(vi& v, int a, int b) {
    trav(t,cor[a]) v.pb(t);
    trav(t,adj[a]) if (t.f != b) fill(v,t.f,a);
  }

  void addTree(int a, int b, int c) { adj[a][b] = c, adj[b][a] = c; }
  void delTree(int a, int b) { adj[a].erase(b), adj[b].erase(a); }

  vector<pair<pi,int>> init(int _N) { // returns edges of Gomory-Hu Tree
    N = _N;
    FOR(i,1,N+1) cor[0].pb(i);
    queue<int> todo; todo.push(0);
    while (sz(todo)) {
      int x = todo.front(); todo.pop();
      vector<vi> cc; trav(t,cor[x]) cc.pb({t});
      trav(t,adj[x]) {
        cc.pb({});
        fill(cc.back(),t.f,x);
      }
      int f = gen(cc); // run max flow
      cor.pb({}), cor.pb({});
      trav(t,cor[x]) cor[sz(cor)-2+side[t]].pb(t);
      FOR(i,2) if (sz(cor[sz(cor)-2+i]) > 1) todo.push(sz(cor)-2+i);
      FOR(i,sz(cor)-2) if (i != x && adj[i].count(x)) {
        addTree(i,sz(cor)-2+side[cor[i][0]],adj[i][x]);
        delTree(i,x);
      } // modify tree edges
      addTree(sz(cor)-2,sz(cor)-1,f);
    }
    vector<pair<pi,int>> ans;
    FOR(i,sz(cor)) trav(j,adj[i]) if (i < j.f)
      ans.pb({{cor[i][0],cor[j.f][0]},j.s});
    return ans;
  }
};
```

6.5 Matching

DFSmatch.h

Description: naive bipartite matching
Time: $\mathcal{O}(NM)$ 26 lines

```
template<int SZ> struct MaxMatch {
  int N, flow = 0, match[SZ], rmatch[SZ];
  bitset<SZ> vis;
  vi adj[SZ];
  MaxMatch() {
    memset(match,0,sizeof match);
    memset(rmatch,0,sizeof rmatch);
  }

  void connect(int a, int b, bool c = 1) {
    if (c) match[a] = b, rmatch[b] = a;
    else match[a] = rmatch[b] = 0;
  }
  bool dfs(int x) {
    if (!x) return 1;
    if (vis[x]) return 0;
    vis[x] = 1;
    trav(t,adj[x]) if (t != match[x] && dfs(rmatch[t]))
      return connect(x,t),1;
    return 0;
  }
  void tri(int x) { vis.reset(); flow += dfs(x); }
  void init(int _N) {
    N = _N; FOR(i,1,N+1) if (!match[i]) tri(i);
  }
};
```

Hungarian.h

Description: finds min cost to complete n jobs w/ m workers each worker is assigned to at most one job (n <= m)

Time: ?

28 lines

```
int HungarianMatch(const vector<vi>& a) { // cost array,
    ↪negative values are ok
  int n = sz(a)-1, m = sz(a[0])-1; // jobs 1..n, workers
      ↪1..m
  vi u(n+1), v(m+1), p(m+1); // p[j] -> job picked by
      ↪worker j
  FOR(i,1,n+1) { // find alternating path with job i
    p[0] = i; int j0 = 0;
    vi dist(m+1, MOD), pre(m+1,-1); // dist, previous
        ↪vertex on shortest path
    vector<bool> done(m+1, false);
    do {
      done[j0] = true;
      int i0 = p[j0], j1; int delta = MOD;
      FOR(j,1,m+1) if (!done[j]) {
        auto cur = a[i0][j]-u[i0]-v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      FOR(j,m+1) // just dijkstra with potentials
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      j0 = j1;
    } while (p[j0]);
    do { // update values on alternating path
      int j1 = pre[j0];
      p[j0] = p[j1];
      j0 = j1;
    } while (j0);
  }
  return -v[0]; // min cost
}
```

UnweightedMatch.h

Description: general unweighted matching
Time: ?

79 lines

```
template<int SZ> struct UnweightedMatch {
  int vis[SZ], par[SZ], orig[SZ], match[SZ], aux[SZ], t, N
      ↪; // 1-based index
  vi adj[SZ];
  queue<int> Q;
  void addEdge(int u, int v) {
    adj[u].pb(v); adj[v].pb(u);
  }

  void init(int n) {
    N = n; t = 0;
    FOR(i,N+1) {
      adj[i].clear();
      match[i] = aux[i] = par[i] = 0;
    }
  }

  void augment(int u, int v) {
    int pv = v, nv;
    do {
      pv = par[v]; nv = match[pv];
      match[v] = pv; match[pv] = v;
      v = nv;
    } while(u != pv);
  }

  int lca(int v, int w) {
    ++t;
    while (1) {
      if (v) {
        if (aux[v] == t) return v; aux[v] = t;
        v = orig[par[match[v]]];
      }
      swap(v, w);
    }
  }

  void blossom(int v, int w, int a) {
    while (orig[v] != a) {
      par[v] = w; w = match[v];
      if (vis[w] == 1) Q.push(w), vis[w] = 0;
      orig[v] = orig[w] = a;
      v = par[w];
    }
  }

  bool bfs(int u) {
    fill(vis+1, vis+1+N, -1); iota(orig + 1, orig + N + 1,
        ↪ 1);
    Q = queue<int> (); Q.push(u); vis[u] = 0;
    while (sz(Q)) {
      int v = Q.front(); Q.pop();
      trav(x,adj[v]) {
        if (vis[x] == -1) {
          par[x] = v; vis[x] = 1;
          if (!match[x]) return augment(u, x), true;
          Q.push(match[x]); vis[match[x]] = 0;
        } else if (vis[x] == 0 && orig[v] != orig[x]) {
          int a = lca(orig[v], orig[x]);
          blossom(x, v, a); blossom(v, x, a);
        }
      }
    }
    return false;
  }

  int match() {
    int ans = 0;
    // find random matching (not necessary, constant
        ↪improvement)
    vi V(N-1); iota(all(V), 1);
    shuffle(all(V), mt19937(0x94949));
    trav(x,V) if(!match[x])
      trav(y,adj[x]) if (!match[y]) {
        match[x] = y, match[y] = x;
        ++ans; break;
      }

    FOR(i,1,N+1) if (!match[i] && bfs(i)) ++ans;
    return ans;
  }
};
```

6.6 Misc

MaximalCliques.h

Description: Used only once. Finds all maximal cliques.
Time: $\mathcal{O}\left(3^{N/3}\right)$

21 lines

```
typedef bitset<128> B;
int N;
B adj[128];

// possibly in clique, not in clique, in clique
void cliques(B P = ~B(), B X={}, B R={}) {
    if (!P.any()) {
        if (!X.any()) {
            // do smth with R
        }
        return;
    }
    int q = (P|X)._Find_first();
    // clique must contain q or non-neighbor of q
    B cands = P&~adj[q];
    FOR(i,N) if (cands[i]) {
        R[i] = 1;
        cliques(P&adj[i],X&adj[i],R);
        R[i] = P[i] = 0; X[i] = 1;
    }
}
```

LCT.h

Description: Link-Cut Tree, use vir for subtree size queries
Time: $\mathcal{O}(\log N)$

96 lines

```
typedef struct snode* sn;

struct snode {
  sn p, c[2]; // parent, children
  int val; // value in node
  int sum, mn, mx; // sum of values in subtree, min and
      ↪max prefix sum
  bool flip = 0;
  // int vir = 0; stores sum of virtual children

  snode(int v) {
    p = c[0] = c[1] = NULL;
    val = v; calc();
  }

  friend int getSum(sn x) { return x?x->sum:0; }
  friend int getMn(sn x) { return x?x->mn:0; }
  friend int getMx(sn x) { return x?x->mx:0; }

  void prop() {
    if (!flip) return;
    swap(c[0],c[1]); tie(mn,mx) = mp(sum-mx,sum-mn);
    FOR(i,2) if (c[i]) c[i]->flip ^= 1;
    flip = 0;
  }
  void calc() {
```

```
  FOR(i,2) if (c[i]) c[i]->prop();
  int s0 = getSum(c[0]), s1 = getSum(c[1]); sum = s0+val
    ↪+s1; // +vir
  mn = min(getMn(c[0]),s0+val+getMn(c[1]));
  mx = max(getMx(c[0]),s0+val+getMx(c[1]));
}

int dir() {
  if (!p) return -2;
  FOR(i,2) if (p->c[i] == this) return i;
  return -1; // p is path-parent pointer, not in current
    ↪ splay tree
}
bool isRoot() { return dir() < 0; }

friend void setLink(sn x, sn y, int d) {
  if (y) y->p = x;
  if (d >= 0) x->c[d] = y;
}
void rot() { // assume p and p->p propagated
  assert(!isRoot()); int x = dir(); sn pa = p;
  setLink(pa->p, this, pa->dir());
  setLink(pa, c[x^1], x);
  setLink(this, pa, x^1);
  pa->calc(); calc();
}
void splay() {
  while (!isRoot() && !p->isRoot()) {
    p->p->prop(), p->prop(), prop();
    dir() == p->dir() ? p->rot() : rot();
    rot();
  }
  if (!isRoot()) p->prop(), prop(), rot();
  prop();
}

void access() { // bring this to top of tree
  for (sn v = this, pre = NULL; v; v = v->p) {
    v->splay();
    // if (pre) v->vir -= pre->sz;
    // if (v->c[1]) v->vir += v->c[1]->sz;
    v->c[1] = pre; v->calc();
    pre = v;
    // v->sz should remain the same if using vir
  }
  splay(); assert(!c[1]); // left subtree of this is now
    ↪ path to root, right subtree is empty
}
void makeRoot() { access(); flip ^= 1; }
void set(int v) { splay(); val = v; calc(); } // change
  ↪value in node, splay suffices instead of access
  ↪because it doesn't affect values in nodes above it

friend sn lca(sn x, sn y) {
  if (x == y) return x;
  x->access(), y->access(); if (!x->p) return NULL; //
    ↪access at y did not affect x, so they must not be
    ↪ connected
  x->splay(); return x->p ? x->p : x;
}
friend bool connected(sn x, sn y) { return lca(x,y); }
friend int balanced(sn x, sn y) {
  x->makeRoot(); y->access();
  return y->sum-2*y->mn;
}

friend bool link(sn x, sn y) { // make x parent of y
  if (connected(x,y)) return 0; // don't induce cycle
  y->makeRoot(); y->p = x;
  // x->access(); x->sz += y->sz; x->vir += y->sz;
  return 1; // success!
}
friend bool cut(sn x, sn y) { // x is originally parent
  ↪of y
  x->makeRoot(); y->access();
  if (y->c[0] != x || x->c[0] || x->c[1]) return 0; //
    ↪splay tree with y should not contain anything
    ↪else besides x
  x->p = y->c[0] = NULL; y->calc(); return 1; // calc is
    ↪ redundant as it will be called elsewhere anyways
    ↪?
}
};
```

DirectedMST.h

Description: computes minimum weight directed spanning tree, edge from $inv[i] \to i$ for all $i \neq r$

Time: $\mathcal{O}(M \log M)$

"DSUrb.h" 64 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll,vi> dmst(int n, int r, const vector<Edge>& g) {
  DSUrb dsu; dsu.init(n); // DSU with rollback if need to
    ↪return edges
  vector<Node*> heap(n); // store edges entering each
    ↪vertex in increasing order of weight
  trav(e,g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0; vi seen(n,-1); seen[r] = r;
  vpi in(n,{-1,-1});
  vector<pair<int,vector<Edge>>> cycs;
  FOR(s,n) {
    int u = s, w;
    vector<pair<int,Edge>> path;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      seen[u] = s;
      Edge e = heap[u]->top(); path.pb({u,e});
      heap[u]->delta -= e.w, pop(heap[u]);
      res += e.w, u = dsu.get(e.a);
      if (seen[u] == s) { // compress verts in cycle
        Node* cyc = 0; cycs.pb({u,{}});
        do {
          cyc = merge(cyc, heap[w = path.back().f]);
          cycs.back().s.pb(path.back().s);
          path.pop_back();
        } while (dsu.unite(u, w));
        u = dsu.get(u); heap[u] = cyc, seen[u] = -1;
      }
    }
    trav(t,path) in[dsu.get(t.s.b)] = {t.s.a,t.s.b}; //
      ↪found path from root
  }
  while (sz(cycs)) { // expand cycs to restore sol
    auto c = cycs.back(); cycs.pop_back();
    pi inEdge = in[c.f];
    trav(t,c.s) dsu.rollback();
    trav(t,c.s) in[dsu.get(t.b)] = {t.a,t.b};
    in[dsu.get(inEdge.s)] = inEdge;
  }
  vi inv;
  FOR(i,n) {
    assert(i == r ? in[i].s == -1 : in[i].s == i);
    inv.pb(in[i].f);
  }
  return {res,inv};
}
```

DominatorTree.h

Description: a dominates b iff every path from 1 to b passes through a

Time: $\mathcal{O}(M \log N)$

46 lines

```
template<int SZ> struct Dominator {
  vi adj[SZ], ans[SZ]; // input edges, edges of dominator
    ↪tree
  vi radj[SZ], child[SZ], sdomChild[SZ];
  int label[SZ], rlabel[SZ], sdom[SZ], dom[SZ], co;
  int root = 1;

  int par[SZ], bes[SZ];
  int get(int x) {
    // DSU with path compression
    // get vertex with smallest sdom on path to root
    if (par[x] != x) {
      int t = get(par[x]); par[x] = par[par[x]];
      if (sdom[t] < sdom[bes[x]]) bes[x] = t;
    }
    return bes[x];
  }
```

```
  void dfs(int x) { // create DFS tree
    label[x] = ++co; rlabel[co] = x;
    sdom[co] = par[co] = bes[co] = co;
    trav(y,adj[x]) {
      if (!label[y]) {
        dfs(y);
        child[label[x]].pb(label[y]);
      }
      radj[label[y]].pb(label[x]);
    }
  }
  void init() {
    dfs(root);
    ROF(i,1,co+1) {
      trav(j,radj[i]) ckmin(sdom[i],sdom[get(j)]);
      if (i > 1) sdomChild[sdom[i]].pb(i);
      trav(j,sdomChild[i]) {
        int k = get(j);
        if (sdom[j] == sdom[k]) dom[j] = sdom[j];
        else dom[j] = k;
      }
      trav(j,child[i]) par[j] = i;
    }
    FOR(i,2,co+1) {
      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
      ans[rlabel[dom[i]]].pb(rlabel[i]);
    }
  }
};
```

EdgeColor.h

Description: naive implementation of Misra & Gries edge coloring, by Vizing's Theorem a simple graph with max degree d can be edge colored with at most $d+1$ colors

Time: $\mathcal{O}\left(MN^2\right)$ 54 lines

```
template<int SZ> struct EdgeColor {
  int N = 0, maxDeg = 0, adj[SZ][SZ], deg[SZ];
  EdgeColor() {
    memset(adj,0,sizeof adj);
    memset(deg,0,sizeof deg);
  }
  void addEdge(int a, int b, int c) {
    adj[a][b] = adj[b][a] = c;
  }
  int delEdge(int a, int b) {
    int c = adj[a][b];
    adj[a][b] = adj[b][a] = 0;
    return c;
  }
  vector<bool> genCol(int x) {
    vector<bool> col(N+1); FOR(i,N) col[adj[x][i]] = 1;
    return col;
  }
  int freeCol(int u) {
    auto col = genCol(u);
    int x = 1; while (col[x]) x ++; return x;
  }
  void invert(int x, int d, int c) {
    FOR(i,N) if (adj[x][i] == d)
      delEdge(x,i), invert(i,c,d), addEdge(x,i,c);
  }
  void addEdge(int u, int v) { // follows wikipedia steps
    // check if you can add edge w/o doing any work
    assert(N); ckmax(maxDeg,max(++deg[u],++deg[v]));
    auto a = genCol(u), b = genCol(v);
    FOR(i,1,maxDeg+2) if (!a[i] && !b[i]) return addEdge(u
      ↪,v,i);

    // 2. find maximal fan of u starting at v
    vector<bool> use(N); vi fan = {v}; use[v] = 1;
    while (1) {
      auto col = genCol(fan.back());
      if (sz(fan) > 1) col[adj[fan.back()][u]] = 0;
      int i = 0; while (i < N && (use[i] || col[adj[u][i
        ↪]])) i ++;
      if (i < N) fan.pb(i), use[i] = 1;
      else break;
    }

    // 3/4. choose free cols for endpoints of fan, invert
      ↪cd_u path
    int c = freeCol(u), d = freeCol(fan.back()); invert(u,
      ↪d,c);
    // 5. find i such that d is free on fan[i]
    int i = 0; while (i < sz(fan) && genCol(fan[i])[d]
      && adj[u][fan[i]] != d) i ++;
    assert (i != sz(fan));
    // 6. rotate fan from 0 to i
    FOR(j,i) addEdge(u,fan[j],delEdge(u,fan[j+1]));
    // 7. add new edge
    addEdge(u,fan[i],d);
  }
};
```

Geometry (7)

7.1 Primitives

Point.h

Description: Easy Geo 44 lines

```
typedef ld T;
template <class T> int sgn(T x) { return (x > 0) - (x < 0)
  ↪; }

namespace Point {
  typedef pair<T,T> P;
  typedef vector<P> vP;

  P dir(T ang) {
    auto c = exp(ang*complex<T>(0,1));
    return P(c.real(),c.imag());
  }

  T norm(P x) { return x.f*x.f+x.s*x.s; }
  T abs(P x) { return sqrt(norm(x)); }
  T angle(P x) { return atan2(x.s,x.f); }
  P conj(P x) { return P(x.f,-x.s); }

  P operator+(const P& l, const P& r) { return P(l.f+r.f,l
    ↪.s+r.s); }
  P operator-(const P& l, const P& r) { return P(l.f-r.f,l
    ↪.s-r.s); }
  P operator*(const P& l, const T& r) { return P(l.f*r,l.s
    ↪*r); }
  P operator*(const T& l, const P& r) { return r*l; }
  P operator/(const P& l, const T& r) { return P(l.f/r,l.s
    ↪/r); }
  P operator*(const P& l, const P& r) { return P(l.f*r.f-l
    ↪.s*r.s,l.s*r.f+l.f*r.s); }
  P operator/(const P& l, const P& r) { return l*conj(r)/
    ↪norm(r); }

  P& operator+=(P& l, const P& r) { return l = l+r; }
  P& operator-=(P& l, const P& r) { return l = l-r; }
  P& operator*=(P& l, const T& r) { return l = l*r; }
  P& operator/=(P& l, const T& r) { return l = l/r; }
  P& operator*=(P& l, const P& r) { return l = l*r; }
  P& operator/=(P& l, const P& r) { return l = l/r; }

  P unit(P x) { return x/abs(x); }
  T dot(P a, P b) { return (conj(a)*b).f; }
  T cross(P a, P b) { return (conj(a)*b).s; }
  T cross(P p, P a, P b) { return cross(a-p,b-p); }
  P rotate(P a, T b) { return a*P(cos(b),sin(b)); }

  P reflect(P p, P a, P b) { return a+conj((p-a)/(b-a))*(b
    ↪-a); }
  P foot(P p, P a, P b) { return (p+reflect(p,a,b))/(T)2;
    ↪}
  bool onSeg(P p, P a, P b) { return cross(a,b,p) == 0 &&
    ↪dot(p-a,p-b) <= 0; }
};

using namespace Point;
```

AngleCmp.h

Description: sorts points according to atan2

"Point.h" 5 lines

```
template<class T> int half(pair<T,T> x) { return mp(x.s,x.
  ↪f) > mp((T)0,(T)0); }
bool angleCmp(P a, P b) {
  int A = half(a), B = half(b);
  return A == B ? cross(a,b) > 0 : A < B;
}
```

LineDist.h

Description: computes distance between P and line AB

"Point.h" 1 lines

```
T lineDist(P p, P a, P b) { return abs(cross(p,a,b))/abs(a
  ↪-b); }
```

SegDist.h

Description: computes distance between P and line segment AB

"lineDist.h" 5 lines

```
T segDist(P p, P a, P b) {
  if (dot(p-a,b-a) <= 0) return abs(p-a);
  if (dot(p-b,a-b) <= 0) return abs(p-b);
  return lineDist(p,a,b);
}
```

LineIntersect.h

Description: computes the intersection point(s) of lines AB, CD; returns -1,0,0 if infinitely many, 0,0,0 if none, 1,x if x is the unique point

"Point.h" 8 lines

```
P extension(P a, P b, P c, P d) {
  T x = cross(a,b,c), y = cross(a,b,d);
  return (d*x-c*y)/(x-y);
}
pair<int,P> lineIntersect(P a, P b, P c, P d) {
  if (cross(b-a,d-c) == 0) return {-(cross(a,c,d) == 0),P(0,0)};
  return {1,extension(a,b,c,d)};
}
```

SegIntersect.h

Description: computes the intersection point(s) of line segments AB, CD

"Point.h" 11 lines

```
vP segIntersect(P a, P b, P c, P d) {
  T x = cross(a,b,c), y = cross(a,b,d);
  T X = cross(c,d,a), Y = cross(c,d,b);
  if (sgn(x)*sgn(y) < 0 && sgn(X)*sgn(Y) < 0) return {(d*x-c*y)/(x-y)};
  set<P> s;
  if (onSeg(a,c,d)) s.insert(a);
  if (onSeg(b,c,d)) s.insert(b);
  if (onSeg(c,a,b)) s.insert(c);
  if (onSeg(d,a,b)) s.insert(d);
  return {all(s)};
}
```

7.2 Polygons

Area.h

Description: computes area + the center of mass of a polygon with constant mass per unit area

Time: $\mathcal{O}(N)$

"Point.h" 16 lines

```
T area(const vP& v) {
  T area = 0;
  F0R(i,sz(v)) {
    int j = (i+1)%sz(v); T a = cross(v[i],v[j]);
    area += a;
  }
  return std::abs(area)/2;
}
P centroid(const vP& v) {
  P cen(0,0); T area = 0; // 2*signed area
  F0R(i,sz(v)) {
    int j = (i+1)%sz(v); T a = cross(v[i],v[j]);
    cen += a*(v[i]+v[j]); area += a;
  }
  return cen/area/(T)3;
}
```

InPoly.h

Description: tests whether a point is inside, on, or outside the perimeter of any polygon

Time: $\mathcal{O}(N)$

"Point.h" 10 lines

```
string inPoly(const vP& p, P z) {
  int n = sz(p), ans = 0;
  F0R(i,n) {
    P x = p[i], y = p[(i+1)%n];
    if (onSeg(z,x,y)) return "on";
    if (x.s > y.s) swap(x,y);
    if (x.s <= z.s && y.s > z.s && cross(z,x,y) > 0) ans ^= 1;
  }
  return ans ? "in" : "out";
}
```

ConvexHull.h

Description: Top-bottom convex hull

Time: $\mathcal{O}(N \log N)$

48 lines

```
struct convexHull {
    set<pair<ld,ld>> dupChecker;
    vector<pair<ld,ld>> points;
    vector<pair<ld,ld>> dn, up, hull;

    convexHull() {}
    bool cw(pd o, pd a, pd b) {
        return ((a.f-o.f)*(b.s-o.s)-(a.s-o.s)*(b.f-o.f) <= 0);
    }
    void addPoint(pair<ld,ld> p) {
        if(dupChecker.count(p)) return;
        points.pb(p);
        dupChecker.insert(p);
    }
    void addPoint(ld x, ld y) {
        addPoint(mp(x,y));
    }
    void build() {
        sort(points.begin(), points.end());
        if(sz(points) < 3) {
            for(pair<ld,ld> p: points) {
                dn.pb(p);
                hull.pb(p);
            }
            M00d(i, sz(points)) {
                up.pb(points[i]);
            }
        } else {
            for(int i = 0; i < (int)points.size(); i++) {
                while(dn.size() >= 2 && cw(dn[dn.size()-2], dn[dn.size()-1], points[i])) {
                    dn.erase(dn.end()-1);
                }
                dn.push_back(points[i]);
            }
            for(int i = (int)points.size()-1; i >= 0; i--) {
                while(up.size() >= 2 && cw(up[up.size()-2], up[up.size()-1], points[i])) {
                    up.erase(up.end()-1);
                }
                up.push_back(points[i]);
            }
            sort(dn.begin(), dn.end());
            sort(up.begin(), up.end());

            for(int i = 0; i < up.size()-1; i++) hull.pb(up[i]);
            for(int i = sz(dn)-1; i > 0; i--) hull.pb(dn[i]);
        }
    }
};
```

PolyDiameter.h

Description: computes longest distance between two points in P

Time: $\mathcal{O}(N)$ given convex hull

"ConvexHull.h" 10 lines

```
ld diameter(vP P) { // rotating calipers
  P = hull(P);
  int n = sz(P), ind = 1; ld ans = 0;
  F0R(i,n)
    for (int j = (i+1)%n;;ind = (ind+1)%n) {
      ckmax(ans,abs(P[i]-P[ind]));
      if (cross(P[j]-P[i],P[(ind+1)%n]-P[ind]) <= 0) break;
    }
  return ans;
}
```

7.3 Circles

Circles.h

Description: misc operations with two circles

"Point.h" 46 lines

```
typedef pair<P,T> circ;
bool on(circ x, P y) { return abs(y-x.f) == x.s; }
bool in(circ x, P y) {  return abs(y-x.f) <= x.s; }
T arcLength(circ x, P a, P b) {
  P d = (a-x.f)/(b-x.f);
  return x.s*acos(d.f);
}

P intersectPoint(circ x, circ y, int t = 0) { // assumes intersection points exist
  T d = abs(x.f-y.f); // distance between centers
  T theta = acos((x.s*x.s+d*d-y.s*y.s)/(2*x.s*d)); // law of cosines
  P tmp = (y.f-x.f)/d*x.s;
  return x.f+tmp*dir(t == 0 ? theta : -theta);
```

```
}
T intersectArea(circ x, circ y) { // not thoroughly tested
  T d = abs(x.f-y.f), a = x.s, b = y.s; if (a < b) swap(a,
    ↪b);
  if (d >= a+b) return 0;
  if (d <= a-b) return PI*b*b;
  auto ca = (a*a+d*d-b*b)/(2*a*d), cb = (b*b+d*d-a*a)/(2*b
    ↪*d);
  auto s = (a+b+d)/2, h = 2*sqrt(s*(s-a)*(s-b)*(s-d))/d;
  return a*a*acos(ca)+b*b*acos(cb)-d*h;
}

P tangent(P x, circ y, int t = 0) {
  y.s = abs(y.s); // abs needed because internal calls y.s
    ↪ < 0
  if (y.s == 0) return y.f;
  T d = abs(x-y.f);
  P a = pow(y.s/d,2)*(x-y.f)+y.f;
  P b = sqrt(d*d-y.s*y.s)/d*y.s*unit(x-y.f)*dir(PI/2);
  return t == 0 ? a+b : a-b;
}
vector<pair<P,P>> external(circ x, circ y) { // external
  ↪tangents
  vector<pair<P,P>> v;
  if (x.s == y.s) {
    P tmp = unit(x.f-y.f)*x.s*dir(PI/2);
    v.pb(mp(x.f+tmp,y.f+tmp));
    v.pb(mp(x.f-tmp,y.f-tmp));
  } else {
    P p = (y.s*x.f-x.s*y.f)/(y.s-x.s);
    F0R(i,2) v.pb({tangent(p,x,i),tangent(p,y,i)});
  }
  return v;
}
vector<pair<P,P>> internal(circ x, circ y) { // internal
  ↪tangents
  x.s *= -1; return external(x,y);
}
```

Circumcenter.h

Description: returns {circumcenter,circumradius}

"Point.h" 5 lines

```
pair<P,T> ccCenter(P a, P b, P c) {
  b -= a; c -= a;
  P res = b*c*(conj(c)-conj(b))/(b*conj(c)-conj(b)*c);
  return {a+res,abs(res)};
}
```

MinEnclosingCircle.h

Description: computes minimum enclosing circle

Time: expected $\mathcal{O}(N)$

"Circumcenter.h" 13 lines

```
pair<P, T> mec(vP ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0]; T r = 0, EPS = 1 + 1e-8;
  F0R(i,sz(ps)) if (abs(o-ps[i]) > r*EPS) {
    o = ps[i], r = 0;
    F0R(j,i) if (abs(o-ps[j]) > r*EPS) {
      o = (ps[i]+ps[j])/2, r = abs(o-ps[i]);
      F0R(k,j) if (abs(o-ps[k]) > r*EPS)
        tie(o,r) = ccCenter(ps[i],ps[j],ps[k]);
    }
  }
  return {o,r};
}
```

7.4 Misc

ClosestPair.h

Description: line sweep to find two closest points

Time: $\mathcal{O}(N \log N)$

21 lines

```
using namespace Point;

pair<P,P> solve(vP v) {
  pair<ld,pair<P,P>> bes; bes.f = INF;
  set<P> S; int ind = 0;

  sort(all(v));
  F0R(i,sz(v)) {
    if (i && v[i] == v[i-1]) return {v[i],v[i]};
    for (; v[i].f-v[ind].f >= bes.f; ++ind)
      S.erase({v[ind].s,v[ind].f});
    for (auto it = S.ub({v[i].s-bes.f,INF});
      it != end(S) && it->f < v[i].s+bes.f; ++it) {
      P t = {it->s,it->f};
      ckmin(bes,{abs(t-v[i]),{t,v[i]}});
    }
    S.insert({v[i].s,v[i].f});
  }

  return bes.s;
}
```

DelaunayFast.h

Description: Delaunay Triangulation, concyclic points are OK (but not all collinear)

Time: $\mathcal{O}(N \log N)$

"Point.h" 94 lines

```
typedef ll T;

typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other
  ↪point

struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
};

// test if p is in the circumcircle
bool circ(P p, P a, P b, P c) {
  ll ar = cross(a,b,c); assert(ar); if (ar < 0) swap(a,b);
  lll p2 = norm(p), A = norm(a)-p2,
    B = norm(b)-p2, C = norm(c)-p2;
  return cross(p,a,b)*C+cross(p,b,c)*A+cross(p,c,a)*B > 0;
}

Q makeEdge(P orig, P dest) {
  Q q[] = {new Quad{0,0,0,orig}, new Quad{0,0,0,arb},
      new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
  F0R(i,4) q[i]->o = q[-i & 3], q[i]->rot = q[(i+1) & 3];
  return *q;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back
      ↪());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = cross(s[0], s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r()
      ↪};
  }

#define H(e) e->F(), e->p
#define valid(e) (cross(e->F(),H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((cross(B->p,H(A)) < 0 && (A = A->next())) ||
      (cross(A->p,H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
```

```
  return {ra, rb};
}

vector<array<P,3>> triangulate(vector<P> pts) {
  sort(all(pts)); assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};

  Q e = rec(pts).f; vector<Q> q = {e};
  int qi = 0;
  while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->
    ↪p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;

  vector<array<P,3>> ret;
  FOR(i,sz(pts)/3) ret.pb({pts[3*i],pts[3*i+1],pts[3*i
      ↪+2]});
  return ret;
}
```

7.5 3D

Point3D.h

Description: Basic 3D Geometry

45 lines

```
typedef ld T;

namespace Point3D {
  typedef array<T,3> P3;
  typedef vector<P3> vP3;

  T norm(const P3& x) {
    T sum = 0; FOR(i,sz(x)) sum += x[i]*x[i];
    return sum;
  }
  T abs(const P3& x) { return sqrt(norm(x)); }

  P3& operator+=(P3& l, const P3& r) { FOR(i,3) l[i] += r[
      ↪i]; return l; }
  P3& operator-=(P3& l, const P3& r) { FOR(i,3) l[i] -= r[
      ↪i]; return l; }
  P3& operator*=(P3& l, const T& r) { FOR(i,3) l[i] *= r;
      ↪return l; }
  P3& operator/=(P3& l, const T& r) { FOR(i,3) l[i] /= r;
      ↪return l; }

  P3 operator+(P3 l, const P3& r) { return l += r; }
  P3 operator-(P3 l, const P3& r) { return l -= r; }
  P3 operator*(P3 l, const T& r) { return l *= r; }
  P3 operator*(const T& r, const P3& l) { return l*r; }
  P3 operator/(P3 l, const T& r) { return l /= r; }

  T dot(const P3& a, const P3& b) {
    T sum = 0; FOR(i,3) sum += a[i]*b[i];
    return sum;
  }
  P3 cross(const P3& a, const P3& b) {
    return {a[1]*b[2]-a[2]*b[1],
        a[2]*b[0]-a[0]*b[2],
        a[0]*b[1]-a[1]*b[0]};
  }

  bool isMult(const P3& a, const P3& b) {
    auto c = cross(a,b);
    FOR(i,sz(c)) if (c[i] != 0) return 0;
    return 1;
  }
  bool collinear(const P3& a, const P3& b, const P3& c) {
      ↪return isMult(b-a,c-a); }
  bool coplanar(const P3& a, const P3& b, const P3& c,
      ↪const P3& d) {
    return isMult(cross(b-a,c-a),cross(b-a,d-a));
  }
}

using namespace Point3D;
```

Hull3D.h

Description: 3D Convex Hull + Polyedron Volume

Time: $\mathcal{O}\left(N^2\right)$

"Point3D.h" 48 lines

```
struct ED {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vP3& A) {
  assert(sz(A) >= 4);
  vector<vector<ED>> E(sz(A), vector<ED>(sz(A), {-1, -1}))
      ↪;
  #define E(x,y) E[f.x][f.y]
  vector<F> FS; // faces
  auto mf = [&](int i, int j, int k, int l) { // make face
    P3 q = cross(A[j]-A[i],A[k]-A[i]);
    if (dot(q,A[l]) > dot(q,A[i])) q *= -1; // make sure q
        ↪ points outward
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.pb(f);
  };
  FOR(i,4) FOR(j,i+1,4) FOR(k,j+1,4) mf(i, j, k, 6-i-j-k);

  FOR(i,4,sz(A)) {
    FOR(j,sz(FS)) {
      F f = FS[j];
      if (dot(f.q,A[i]) > dot(f.q,A[f.a])) { // face is
          ↪visible, remove edges
        E(a,b).rem(f.c), E(a,c).rem(f.b), E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    FOR(j,sz(FS)) { // add faces with new point
      F f = FS[j];
      #define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.
          ↪b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  trav(it, FS) if (dot(cross(A[it.b]-A[it.a],A[it.c]-A[it.
      ↪a]),it.q) <= 0)
    swap(it.c, it.b);
  return FS;
} // computes hull where no four are coplanar

T signedPolyVolume(const vP3& p, const vector<F>& trilist)
    ↪ {
  T v = 0;
  trav(i,trilist) v += dot(cross(p[i.a],p[i.b]),p[i.c]);
  return v/6;
}
```

Strings (8)

8.1 Lightweight

KMP.h

Description: `f[i]` equals the length of the longest proper suffix of the i-th prefix of s that is a prefix of s

Time: $\mathcal{O}(N)$

15 lines

```
vi kmp(string s) {
  int N = sz(s); vi f(N+1); f[0] = -1;
  FOR(i,1,N+1) {
    f[i] = f[i-1];
    while (f[i] != -1 && s[f[i]] != s[i-1]) f[i] = f[f[i
        ↪]];
    f[i] ++;
  }
  return f;
}

vi getOc(string a, string b) { // find occurrences of a in
    ↪ b
  vi f = kmp(a+"@"+b), ret;
  FOR(i,sz(a),sz(b)+1) if (f[i+sz(a)+1] == sz(a)) ret.pb(i
      ↪-sz(a));
  return ret;
}
```

Z.h

Description: for each index i, computes the the maximum len such that `s.substr(0,len) == s.substr(i,len)`

Time: $\mathcal{O}(N)$

19 lines

```
vi z(string s) {
  int N = sz(s); s += '#';
  vi ans(N); ans[0] = N;
  int L = 1, R = 0;
  FOR(i,1,N) {
    if (i <= R) ans[i] = min(R-i+1,ans[i-L]);
```

```
    while (s[i+ans[i]] == s[ans[i]]) ans[i] ++;
    if (i+ans[i]-1 > R) L = i, R = i+ans[i]-1;
  }
  return ans;
}

vi getPrefix(string a, string b) { // find prefixes of a
   ↪in b
  vi t = z(a+b), T(sz(b));
  FOR(i,sz(T)) T[i] = min(t[i+sz(a)],sz(a));
  return T;
}

// pr(z("abcababcabcaba"),getPrefix("abcab","
   ↪uwetrabcerabcab"));
```

Manacher.h

Description: Calculates length of largest palindrome centered at each character of string
Time: $\mathcal{O}(N)$ 18 lines

```
vi manacher(string s) {
  string s1 = "@";
  trav(c,s) s1 += c, s1 += "#";
  s1[sz(s1)-1] = '&';

  vi ans(sz(s1)-1);
  int lo = 0, hi = 0;
  FOR(i,1,sz(s1)-1) {
    if (i != 1) ans[i] = min(hi-i,ans[hi-i+lo]);
    while (s1[i-ans[i]-1] == s1[i+ans[i]+1]) ans[i] ++;
    if (i+ans[i] > hi) lo = i-ans[i], hi = i+ans[i];
  }

  ans.erase(begin(ans));
  FOR(i,sz(ans)) if ((i&1) == (ans[i]&1)) ans[i] ++; //
     ↪adjust lengths
  return ans;
}
// ps(manacher("abacaba"))
```

MinRotation.h

Description: minimum rotation of string
Time: $\mathcal{O}(N)$ 8 lines

```
int minRotation(string s) {
  int a = 0, N = sz(s); s += s;
  FOR(b,N) FOR(i,N) { // a is current best rotation found
     ↪up to b-1
    if (a+i == b || s[a+i] < s[b+i]) { b += max(0, i-1);
       ↪break; } // b to b+i-1 can't be better than a to
       ↪a+i-1
    if (s[a+i] > s[b+i]) { a = b; break; } // new best
       ↪found
  }
  return a;
}
```

LyndonFactorization.h

Description: A string is "simple" if it is strictly smaller than any of its own nontrivial suffixes. The Lyndon factorization of the string s is a factorization $s = w_1 w_2 \dots w_k$ where all strings w_i are simple and $w_1 \geq w_2 \geq \dots \geq w_k$
Time: $\mathcal{O}(N)$ 20 lines

```
vector<string> duval(const string& s) {
  int n = sz(s); vector<string> factors;
  for (int i = 0; i < n; ) {
    int j = i + 1, k = i;
    for (; j < n && s[k] <= s[j]; j++) {
      if (s[k] < s[j]) k = i;
      else k ++;
    }
    for (; i <= k; i += j-k) factors.pb(s.substr(i, j-k));
  }
  return factors;
}

int minRotation(string s) { // get min index i such that
   ↪cyclic shift starting at i is min rotation
  int n = sz(s); s += s;
  auto d = duval(s); int ind = 0, ans = 0;
  while (ans+sz(d[ind]) < n) ans += sz(d[ind++]);
  while (ind && d[ind] == d[ind-1]) ans -= sz(d[ind--]);
  return ans;
}
```

RabinKarp.h

Description: generates hash values of any substring in $O(1)$, equal strings have same hash value
Time: $\mathcal{O}(N)$ build, $\mathcal{O}(1)$ get hash value of a substring 25 lines

```
template<int SZ> struct rabinKarp {
    const ll mods[3] = {1000000007, 999119999,
       ↪1000992299};
    ll p[3][SZ];
    ll h[3][SZ];
    const ll base = 1000696969;
    rabinKarp() {}
    void build(string a) {
        M00(i, 3) {
            p[i][0] = 1;
            h[i][0] = (int)a[0];
            M00(j, 1, (int)a.length()) {
                p[i][j] = (p[i][j-1] * mods[i]) % base;
                h[i][j] = (h[i][j-1] * mods[i] + (int)a[j
                   ↪]) % base;
            }
        }
    }
    tuple<ll, ll, ll> hsh(int a, int b) {
        if(a == 0) return make_tuple(h[0][b], h[1][b], h
           ↪[2][b]);
        tuple<ll, ll, ll> ans;
        get<0>(ans) = (((h[0][b] - h[0][a-1]*p[0][b-a+1])
           ↪% base) + base) % base;
        get<1>(ans) = (((h[1][b] - h[1][a-1]*p[1][b-a+1])
           ↪% base) + base) % base;
        get<2>(ans) = (((h[2][b] - h[2][a-1]*p[2][b-a+1])
           ↪% base) + base) % base;
        return ans;
    }
};
```

Trie.h

Description: trie 25 lines

```
struct tnode {
    char c;
    bool used;
    tnode* next[26];
    tnode() {
        c = ' ';
        used = 0;
        M00(i, 26) next[i] = nullptr;
    }
};

tnode* root;

void addToTrie(string s) {
    tnode* cur = root;
    for(char ch: s) {
        int idx = ch - 'a';
        if(cur->next[idx] == nullptr) {
            cur->next[idx] = new tnode();
        }
        cur = cur->next[idx];
        cur->c = ch;
    }
    cur->used = 1;
}
```

8.2 Suffix Structures

ACfixed.h

Description: for each prefix, stores link to max length suffix which is also a prefix
Time: $\mathcal{O}(N\sum)$ 36 lines

```
struct ACfixed { // fixed alphabet
  struct node {
    array<int,26> to;
    int link;
  };
  vector<node> d;
  ACfixed() { d.eb(); }

  int add(string s) { // add word
    int v = 0;
    trav(C,s) {
      int c = C-'a';
      if (!d[v].to[c]) {
        d[v].to[c] = sz(d);
        d.eb();
      }
      v = d[v].to[c];
    }
```

```
    return v;
  }

  void init() { // generate links
    d[0].link = -1;
    queue<int> q; q.push(0);
    while (sz(q)) {
      int v = q.front(); q.pop();
      FOR(c,26) {
        int u = d[v].to[c]; if (!u) continue;
        d[u].link = d[v].link == -1 ? 0 : d[d[v].link].to[
            ↪c];
        q.push(u);
      }
      if (v) FOR(c,26) if (!d[v].to[c])
        d[v].to[c] = d[d[v].link].to[c];
    }
  }
};
```

PalTree.h

Description: palindromic tree, computes number of occurrences of each palindrome within string

Time: $\mathcal{O}(N\sum)$ 25 lines

```
template<int SZ> struct PalTree {
  static const int sigma = 26;
  int s[SZ], len[SZ], link[SZ], to[SZ][sigma], oc[SZ];
  int n, last, sz;
  PalTree() { s[n++] = -1; link[0] = 1; len[1] = -1; sz =
      ↪2; }

  int getLink(int v) {
    while (s[n-len[v]-2] != s[n-1]) v = link[v];
    return v;
  }
  void addChar(int c) {
    s[n++] = c;
    last = getLink(last);
    if (!to[last][c]) {
      len[sz] = len[last]+2;
      link[sz] = to[getLink(link[last])][c];
      to[last][c] = sz++;
    }
    last = to[last][c]; oc[last] ++;
  }
  void numOc() {
    vpi v; FOR(i,2,sz) v.pb({len[i],i});
    sort(rall(v)); trav(a,v) oc[link[a.s]] += oc[a.s];
  }
};
```

SuffixArray.h

Description: ?

Time: $\mathcal{O}(N\log N)$ 43 lines

```
template<int SZ> struct suffixArray {
    const static int LGSZ = 33-__builtin_clz(SZ-1);
    pair<pi, int> tup[SZ];
    int sortIndex[LGSZ][SZ];
    int res[SZ];
    int len;
    suffixArray(string s) {
        this->len = (int)s.length();
        M00(i, len) tup[i] = MP(MP((int)s[i], -1), i);
        sort(tup, tup+len);
        int temp = 0;
        tup[0].F.F = 0;
        M00(i, 1, len) {
            if(s[tup[i].S] != s[tup[i-1].S]) temp++;
            tup[i].F.F = temp;
        }
        M00(i, len) sortIndex[0][tup[i].S] = tup[i].F.F;
        M00(i, 1, LGSZ) {
            M00(j, len) tup[j] = MP(MP(sortIndex[i-1][j],
                ↪(j+(1<<(i-1))<len)?sortIndex[i-1][j+(1<<(
                ↪i-1))]:-1), j);
            sort(tup, tup+len);
            int temp2 = 0;
            sortIndex[i][tup[0].S] = 0;
            M00(j, 1, len) {
                if(tup[j-1].F != tup[j].F) temp2++;
                sortIndex[i][tup[j].S] = temp2;
            }
        }
        M00(i, len) res[sortIndex[LGSZ-1][i]] = i;
    }
    int LCP(int x, int y) {
        if(x == y) return len - x;
        int ans = 0;
        M00d(i, LGSZ) {
            if(x >= len || y >= len) break;
            if(sortIndex[i][x] == sortIndex[i][y]) {
                x += (1<<i);
                y += (1<<i);
                ans += (1<<i);
            }
        }
        return ans;
    }
};
```

ReverseBW.h

Description: The Burrows-Wheeler Transform appends # to a string, sorts the rotations of the string in increasing order, and constructs a new string that contains the last character of each rotation. This function reverses the transform.

Time: $\mathcal{O}(N\log N)$ 8 lines

```
string reverseBW(string s) {
  vi nex(sz(s));
  vector<pair<char,int>> v; FOR(i,sz(s)) v.pb({s[i],i});
  sort(all(v)); FOR(i,sz(v)) nex[i] = v[i].s;
  int cur = nex[0]; string ret;
  for (; cur;cur = nex[cur]) ret += v[cur].f;
  return ret;
}
```

SuffixAutomaton.h

Description: constructs minimal DFA that recognizes all suffixes of a string

Time: $\mathcal{O}(N\log\sum)$ 73 lines

```
struct SuffixAutomaton {
  struct state {
    int len = 0, firstPos = -1, link = -1;
    bool isClone = 0;
    map<char, int> next;
    vi invLink;
  };

  vector<state> st;
  int last = 0;
  void extend(char c) {
    int cur = sz(st); st.eb();
    st[cur].len = st[last].len+1, st[cur].firstPos = st[
        ↪cur].len-1;
    int p = last;
    while (p != -1 && !st[p].next.count(c)) {
      st[p].next[c] = cur;
      p = st[p].link;
    }
    if (p == -1) {
      st[cur].link = 0;
    } else {
      int q = st[p].next[c];
      if (st[p].len+1 == st[q].len) {
        st[cur].link = q;
      } else {
        int clone = sz(st); st.pb(st[q]);
        st[clone].len = st[p].len+1, st[clone].isClone =
            ↪1;
        while (p != -1 && st[p].next[c] == q) {
          st[p].next[c] = clone;
          p = st[p].link;
        }
        st[q].link = st[cur].link = clone;
      }
    }
    last = cur;
  }
  void init(string s) {
    st.eb(); trav(x,s) extend(x);
    FOR(v,1,sz(st)) st[st[v].link].invLink.pb(v);
  }

  // APPLICATIONS
  void getAllOccur(vi& oc, int v) {
    if (!st[v].isClone) oc.pb(st[v].firstPos);
    trav(u,st[v].invLink) getAllOccur(oc,u);
  }
  vi allOccur(string s) {
    int cur = 0;
    trav(x,s) {
      if (!st[cur].next.count(x)) return {};
      cur = st[cur].next[x];
    }
    vi oc; getAllOccur(oc,cur); trav(t,oc) t += 1-sz(s);
```

```
    sort(all(oc)); return oc;
  }

  vl distinct;
  ll getDistinct(int x) {
    if (distinct[x]) return distinct[x];
    distinct[x] = 1;
    trav(y,st[x].next) distinct[x] += getDistinct(y.s);
    return distinct[x];
  }
  ll numDistinct() { // # of distinct substrings,
     ↪including empty
    distinct.rsz(sz(st));
    return getDistinct(0);
  }
  ll numDistinct2() { // another way to get # of distinct
     ↪substrings
    ll ans = 1;
    FOR(i,1,sz(st)) ans += st[i].len-st[st[i].link].len;
    return ans;
  }
};
```

SuffixTree.h

Description: Ukkonen's algorithm for suffix tree
Time: $\mathcal{O}(N \log \sum)$

61 lines

```
struct SuffixTree {
  string s; int node, pos;
  struct state {
    int fpos, len, link = -1;
    map<char,int> to;
    state(int fpos, int len) : fpos(fpos), len(len) {}
  };
  vector<state> st;
  int makeNode(int pos, int len) {
    st.pb(state(pos,len)); return sz(st)-1;
  }
  void goEdge() {
    while (pos > 1 && pos > st[st[node].to[s[sz(s)-pos]]].
       ↪len) {
      node = st[node].to[s[sz(s)-pos]];
      pos -= st[node].len;
    }
  }
  void extend(char c) {
    s += c; pos ++; int last = 0;
    while (pos) {
      goEdge();
      char edge = s[sz(s)-pos];
      int& v = st[node].to[edge];
      char t = s[st[v].fpos+pos-1];
      if (v == 0) {
        v = makeNode(sz(s)-pos,MOD);
        st[last].link = node; last = 0;
      } else if (t == c) {
        st[last].link = node;
        return;
      } else {
        int u = makeNode(st[v].fpos,pos-1);
        st[u].to[c] = makeNode(sz(s)-1,MOD); st[u].to[t] =
           ↪ v;
        st[v].fpos += pos-1; st[v].len -= pos-1;
        v = u; st[last].link = u; last = u;
      }
      if (node == 0) pos --;
      else node = st[node].link;
    }
  }
  void init(string _s) {
    makeNode(0,MOD); node = pos = 0;
    trav(c,_s) extend(c);
  }
  bool isSubstr(string _x) {
    string x; int node = 0, pos = 0;
    trav(c,_x) {
      x += c; pos ++;
      while (pos > 1 && pos > st[st[node].to[x[sz(x)-pos
         ↪]]].len) {
        node = st[node].to[x[sz(x)-pos]];
        pos -= st[node].len;
      }
      char edge = x[sz(x)-pos];
      if (pos == 1 && !st[node].to.count(edge)) return 0;
      int& v = st[node].to[edge];
      char t = s[st[v].fpos+pos-1];
      if (c != t) return 0;
    }
    return 1;
  }
};
```

8.3 Misc

TandemRepeats.h

Description: Main-Lorentz algorithm, finds all (x, y) such that `s.substr(x,y-1) == s.substr(x+y,y-1)`
Time: $\mathcal{O}(N \log N)$

"Z.h" 54 lines

```
struct StringRepeat {
  string S;
  vector<array<int,3>> al;
  // (t[0],t[1],t[2]) -> there is a repeating substring
     ↪starting at x
  // with length t[0]/2 for all t[1] <= x <= t[2]

  vector<array<int,3>> solveLeft(string s, int m) {
    vector<array<int,3>> v;

    vi v2 = getPrefix(string(s.begin()+m+1,s.end()),string
       ↪(s.begin(),s.begin()+m+1));
    string V = string(s.begin(),s.begin()+m+2); reverse(
       ↪all(V)); vi v1 = z(V); reverse(all(v1));

    FOR(i,m+1) if (v1[i]+v2[i] >= m+2-i) {
      int lo = max(1,m+2-i-v2[i]), hi = min(v1[i],m+1-i);
      lo = i-lo+1, hi = i-hi+1; swap(lo,hi);
      v.pb({2*(m+1-i),lo,hi});
    }
    return v;
  }

  void divi(int l, int r) {
    if (l == r) return;
    int m = (l+r)/2; divi(l,m); divi(m+1,r);

    string t = string(S.begin()+l,S.begin()+r+1);
    m = (sz(t)-1)/2;
    auto a = solveLeft(t,m);
    reverse(all(t));
    auto b = solveLeft(t,sz(t)-2-m);

    trav(x,a) al.pb({x[0],x[1]+l,x[2]+l});
    trav(x,b) {
      int ad = r-x[0]+1;
      al.pb({x[0],ad-x[2],ad-x[1]});
    }
  }

  void init(string _S) {
    S = _S; divi(0,sz(S)-1);
  }

  vi genLen() { // min length of repeating substring
     ↪starting at each index
    priority_queue<pi,vpi,greater<pi>> m; m.push({MOD,MOD
       ↪});
    vpi ins[sz(S)]; trav(a,al) ins[a[1]].pb({a[0],a[2]});
    vi len(sz(S));
    FOR(i,sz(S)) {
      trav(j,ins[i]) m.push(j);
      while (m.top().s < i) m.pop();
      len[i] = m.top().f;
    }
    return len;
  }
};
```