Mid Term test Photonics

Indicative time: <2.5 hours

Q1. Consider two thin lenses with focal length f and –f respectively. They are positioned at a distance f from each other. Where are the focal points of the resulting lens system? Where are the principal planes? What is the focal distance? Can you do real imaging with a magnification of +1 or -1 with this system?

Q2. Assume a plate with refractive index of 2 and a thickness of 10 micrometers surrounded by air on both sides. Visible light is surface-normal incident on this Fabry-Perot etalon.

a. how many resonance peaks will you find in transmission in the 400-700nm wavelength range?

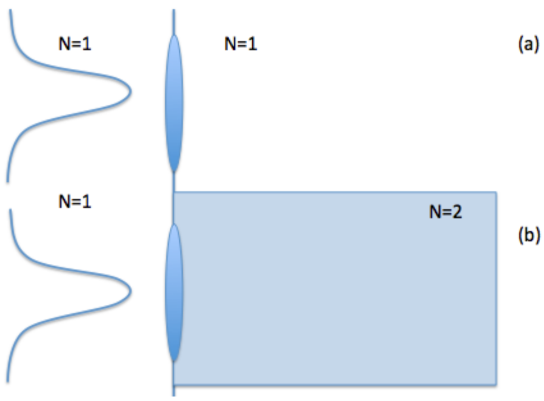
b. how do these resonances shift when the angle of incidence is changed by 10 degrees (assume TE-polarized light)?

c. do you expect only a change in resonance wavelengths, or will the spectrum also change in another way? If so, how?

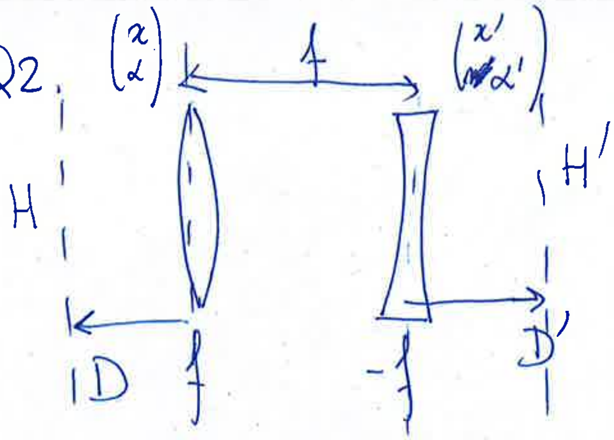
Q3. Consider a slab waveguide with a core with a refractive index of 3 and claddings at both sides with a refractive index of 1. The thickness of the slab is d and the wavelength is 1 micrometer.

a. For which thickness will the waveguide start to guide a second TE-mode?
b. Make an estimate for the propagation constant of the lowest order TE-mode for this thickness.

Q4. Consider the two cases below where a gaussian beam with wavelength λ and 1/e half-width w_0 is focused by a symmetrical biconvex lens with refractive index n=2 and radius of curvature R. In case (a) the medium on the right side of the lens is air, in case (b) the medium on the right side of the lens is made of the same material as the lens. How does the location, spot size and depth of focus change from case (a) to case (b)? You can assume that the phase front of the gaussian beam is flat right in front of the lens and that the lens is large enough to capture (almost) the complete Gaussian beam.



Q2.



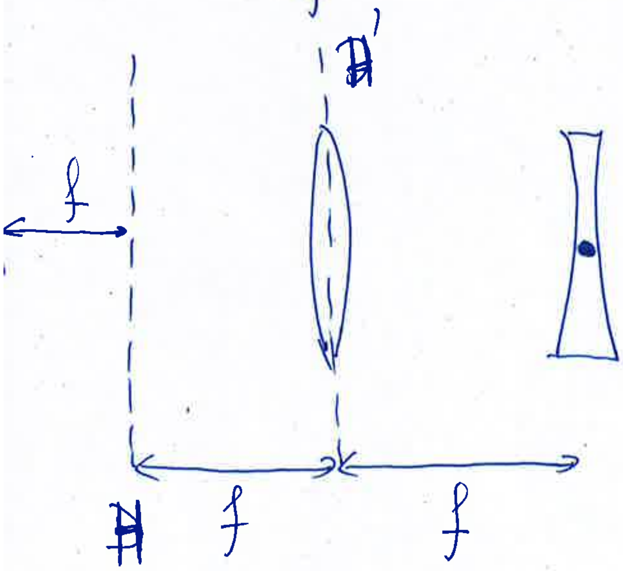
$$\begin{pmatrix} x' \\ \alpha' \end{pmatrix} = M_{system} \begin{pmatrix} x \\ \alpha \end{pmatrix}$$

$$M_{system} = \begin{bmatrix} 1 & 0 \\ +1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 0 & f \\ -1/f & 2 \end{bmatrix}$$

$$D = \frac{(1 - M_{22})}{M_{21}} = \frac{-1}{-1/f} = f$$

$$D' = \frac{(1 - M_{11})}{M_{21}} = \frac{1}{-1/f} = -f$$

focal length : f



real imaging with magnification +1 or -1

$$M'_{system} = \begin{bmatrix} 1 & B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & f \\ -1/f & 2 \end{bmatrix} \begin{bmatrix} 1 & V \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & f \\ -1/f & 2 - V/f \end{bmatrix} = \begin{bmatrix} -B/f & f + B(2 - V/f) \\ -1/f & 2 - V/f \end{bmatrix}$$

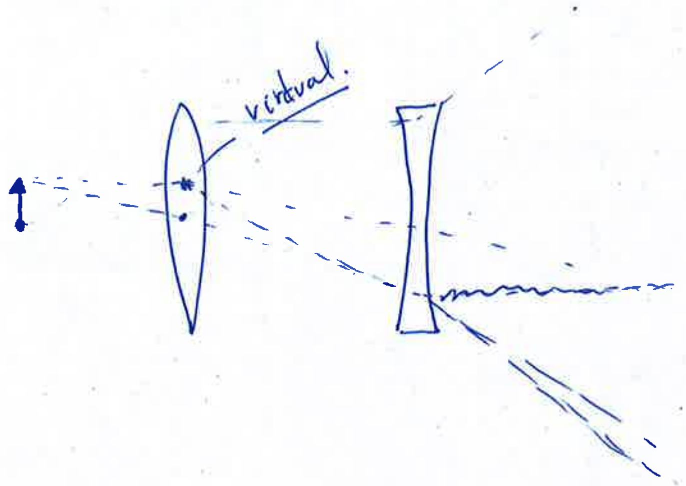
imaging: $\begin{pmatrix} x' \\ \alpha' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x \\ \alpha \end{pmatrix} \rightarrow M_{12} = 0$

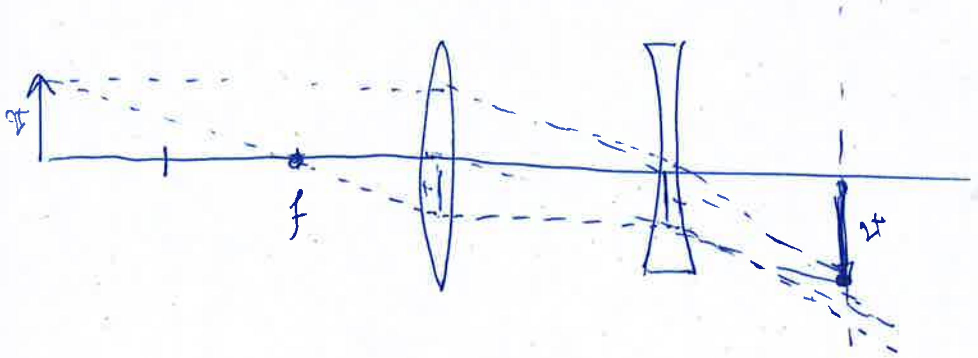
magn. ± 1. $\qquad M_{11} \doteq \pm 1$.

$$f + B\left(2 - V/f\right) = 0 \qquad \Rightarrow f \mp f \underbrace{\left(2 - V/f\right)}_{= \pm 1} = 0$$

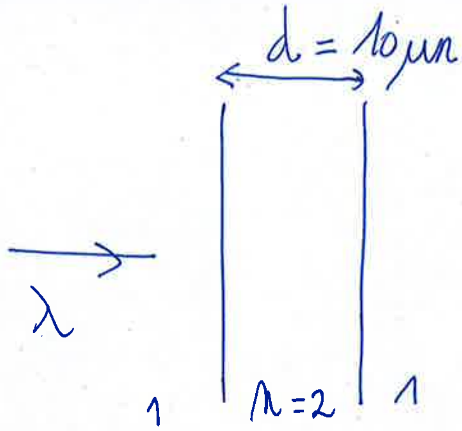
$$-B/f = \pm 1 \rightarrow B = \mp f$$

$$\Rightarrow V = \begin{cases} f \rightarrow \text{virtual.} \\ 3f \rightarrow \text{real} \end{cases}$$





Q3.



$\lambda \rightarrow 400 - 700\,nm$

a) <u>approach 1.</u> roundtrip phase in etalon : $2m\pi$ → resonance

$$2d \cdot \frac{2\pi}{\lambda/n} = 2m\pi \rightarrow m = \frac{2nd}{\lambda}$$

find closest integer m for $\lambda = 400nm$ / $\lambda = 700nm$
$\hookrightarrow m_1$ $\hookrightarrow m_2$

resonances : $m_2 - m_1 + 1 = 43$.

<u>approach 2</u> : free spectral range of FP etalon :

$$FSR = \frac{\lambda^2}{2nd} \quad (\lambda \approx 550nm)$$

resonances : $\dfrac{700nm - 400nm}{FSR} = 40$.

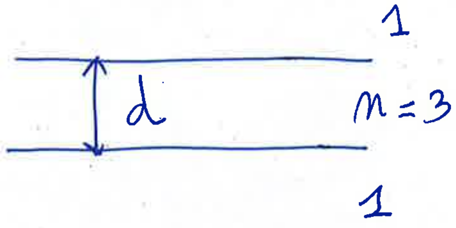
b) round trip phase : $\frac{2\pi}{\lambda} n d \cos\theta$; if $|\theta| > 0 \rightarrow \cos\theta < 1$

→ to obtain resonances shift to shorter λ's
$(\phi = 2m\pi)$

c) since Fresnel reflectivity increases for oblique incidence
→ higher finesse

Q4.

a)



$$V = \frac{2\pi}{\lambda} d \sqrt{3^2 - 1^2} < \pi \text{ for single mode}$$

$$\lambda = 1\mu m \Rightarrow d = \frac{1}{2\sqrt{8}} \mu m = \frac{1}{4\sqrt{2}} \mu m$$

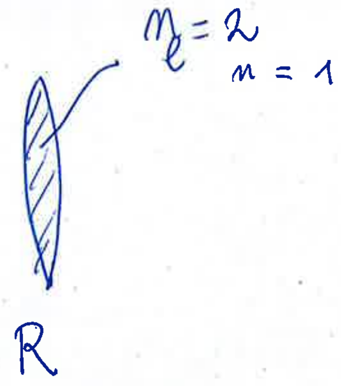
b) $b \approx 0{,}6$ for $V = \pi$ (asymmetry parameter $= 0$)

$$b = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2} \quad (n_2 = 1 ; n_1 = 3) \rightsquigarrow n_{eff}$$

$$\beta = \frac{2\pi}{\lambda} \cdot n_{eff}.$$

Q5.

a) $n = 1$



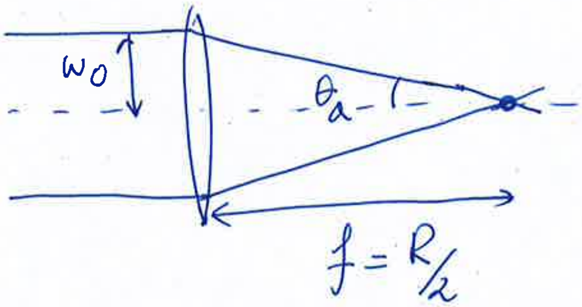
b) $n = 1$

location, spotsize, DOF?

a)

$$P_{thin} = \frac{n_\ell - 1}{R} + \frac{n_\ell - 1}{R} = \frac{2}{R}$$

$$\boxed{f = \frac{1}{P_{thin}} = \frac{R}{2}}$$



$$\theta_a = \frac{w_0}{f} = \frac{2w_0}{R} \quad ; \quad \theta_a = \frac{\lambda}{\pi w_{f,a}} \rightarrow \boxed{w_{f,a} = \frac{\lambda R}{\pi . 2w_0}}$$

$$\boxed{DOF = 2.\ \text{Rayleigh range} = k . w_{f,a}^2 = \frac{2\pi}{\lambda} . \left(\frac{\lambda R}{\pi . 2w_0}\right)^2}$$

b) $P_{thin} = \dfrac{n'}{f'} = \dfrac{2-1}{R} = \dfrac{1}{R} \rightarrow f' = 2R$

$$\theta_f = \frac{w_0}{f'} = \frac{w_0}{2R} \quad ; \quad \theta_B = \frac{\lambda/2}{\pi\, w_{f,b}} \rightarrow \boxed{w_{f,b} = \frac{\lambda \cdot 2R}{2\pi\, w_0}}$$

$$\boxed{DOF = 2 \cdot RR = \frac{2\pi}{\lambda} \cdot 2 \cdot \left[w_{f,b}\right]^2 = 2 \cdot \frac{2\pi}{\lambda}\left(\frac{2\lambda R}{2\pi\, w_0}\right)^2}$$