information theory exam 2025 0528 8:30-12:30

Question 1. Erasure distortion

distribution of X: $P(X=1) = P(X=0) = \frac{1}{2}$ quantize to $\tilde{X}$

distortion measure: $d(x, \hat{x}) = \begin{cases} 0 & (x = \hat{x}) \\ 1 & (\hat{x} = e) \\ \infty & (x=1, \hat{x}=0 \ \& \ x=0, \hat{x}=1) \end{cases}$

Hint: define $P(e|0) = \alpha_0$, $P(e|1) = \alpha_1$

① Show that to reach rate-distribution function, channel should be symmetric, i.e., $\alpha_0 = \alpha_1 = \alpha$

② Calculate distortion D in terms of $P(\tilde{X}|X)$
Determine the condition of $P(\tilde{X}|X)$ for finite distortion D.
rewrite D in terms of α.

③ Compute rate distortion function $R(D)$.
determine D_{max}. explain.

Q_2: (grouping property in Ex.1)

$X = \begin{cases} X_1, & P(X=X_1) = \alpha \\ X_2, & P(X=X_2) = (1-\alpha) \end{cases}$

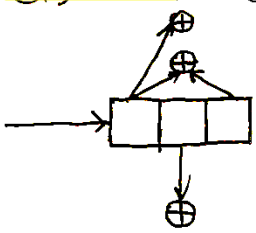
$X_1 = \{1, 2, \dots, m\}$ with probability mass function $P_1(\cdot)$ & $P_2(\cdot)$

$X_2 = \{m+1, \dots, n\}$

① Calculate entropy $H(X)$ in terms of $H(X_1)$, $H(X_2)$, α

② Show that $2^{H(X)} \leq 2^{H(X_1)} + 2^{H(X_2)}$ by maximize over α.
interpret.

Q3: shift register for convolutional code



① Determine: $g_i^{(j)}$. $G(x)$ "polynomial matrix" G "generator matrix"

② Draw trellis diagram

③ minimal distance d_{free} (don't compute transfer function)

④ Calculate the probability of Viterbi Algorithm doesn't give out a correct code word.

⑤ Is this convolutional code suitable for generating a trellis code modulation.

Q4: True or False:

1. $f(\cdot)$ deterministic
   - ① $H(X|f(Y)) \geq H(X|Y)$
   - ② $H(f(x)|Y) \geq H(X|Y)$
2. Consider a 3-error-correcting RS code block length $n=15$ in $GF(2^4)$
   - ① $g(x)$ only has roots: $\alpha^0, \alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6$
   - ② $S_0=\alpha^5$, $S_2=\alpha^2$, $S_4=\alpha^8$, then $S_1=\alpha^{10}$, $S_3=\alpha^5$, $S_5=\alpha^4$
3. Markov Chain: $X \mapsto Y \mapsto Z$
   - ① $I(X;Z) \leq \log |\mathcal{Y}|_c$; $|\mathcal{Y}|_c$: number of elements in support
   - ② Capacity C_{XZ} is lower bounded by C_{XY}, C_{YZ}
   - ③ $C_{XZ} \leq \log |\mathcal{Y}|_c$
4. They have the same distortion: