Chapter 2

Matrix description of wave propagating and polarization

2.1 Electromagnetic waves 电磁波

●Complex representation of EM waves 电磁波的复振幅表示

A general solution to the Helmholtz equation can be written as

$$\tilde{\mathbf{F}}(\mathbf{r}) = \tilde{\mathbf{F}}_{+}e^{-j\mathbf{k}\cdot\mathbf{r}} + \tilde{\mathbf{F}}_{-}e^{j\mathbf{k}\cdot\mathbf{r}}$$

F+ represents a plane wave propagating in the direction k, while F- represents propagating in the opposite direction.

From the calculation, one can find that k, E(D in anisotropic materials) and H are from an orthogonal set. (Note P13)

●EM waves in waveguides 波导中电磁波的模态解

We can suggest a forward-propagating solution of the Maxwell eqs for the waveguide modes:

$$\begin{cases} \mathbf{E}(x,y,z) = [\mathbf{e_T}(x,y) + e_z(x,y)\mathbf{1_z}]e^{-j\beta z} \\ \mathbf{H}(x,y,z) = [\mathbf{h_T}(x,y) + h_z(x,y)\mathbf{1_z}]e^{-j\beta z} \end{cases}$$

β is the propagation constant. Lossless means beta is purely real. 注意，波导中模式沿着 z 方向传播 (isotropic z-invariant dielectric waveguide)，对于电场表达式的 TM 波部分，电场沿着传播方向，则某一 z 位置场的复振幅只与其在 xy 平面上的位置有关 与方向无关。对 Te 波部分，电场沿着传播方向没有分量，故 e_T 是 xy 平面上的向量。

For a forward propagating mode the field is :（电场/磁场的四个分量）

$$a[\boldsymbol{e}_T, e_Z\mathbf{1}_Z, \boldsymbol{h}_T, h_z\mathbf{1}_Z]$$

There can be a corresponding backward mode with fields:

$$b[\boldsymbol{e}_T, -e_z\mathbf{1}_Z, -\boldsymbol{h}_T, h_z\mathbf{1}_Z]$$

波导模式的正交性质：Note P14, Slide P11

2.2 Matrix description of wave propagation in linear systems 矩阵表示

●Black box circuit description 黑盒描述

We'll look at the circuit as a black box, which exchanges energy with the utside through several physical outlets that can be optical waveguides or mere free space electromagnetic beams. Note P15 Slide P13

●Scattering matrix description 散射矩阵

As the ports are considered to be lossless waveguides, the orthonormality relation for the fields can be written as

$$\frac{1}{2}\iint \mathbf{e}_i \times \mathbf{h}_{i'}^* \cdot \mathbf{1}_z dS = \delta_{ii'}$$

Integrated over the surface S, the power absorbed in the circuit is obtained by adding the net powers entering the various ports

$$P = \sum(|a_i|^2 - |b_i|^2) = \mathbf{A}^\dagger\mathbf{A} - \mathbf{B}^\dagger\mathbf{B}$$

With A the column vector of incident field amplitudes and B the column vector with the amplitudes of the outgoing fields. 转置共轭：complex conjugate of the transpose.

Since we assumed the N-port circuit to be linear, the relation between A and B can be described by an NxN matrix S:

$$\mathrm{B} = \mathrm{SA}$$

The diagonal term Sii is the reflection coeff. of mode i, while Sij are the transmission coefficient from mode i towards mode j. Slide P15 Note P15

● General properties of S matrices 散射矩阵的性质

1. lossless circuits:

   P=0 → S is unitary. ($S^+S=1$)

2. lossy circuits:

   P>0 → $\sum_{j=1}^{N} \left|S_{jk}\right|^2 \leq 1$

3. reciprocal circus

   if a circuit is made out of materials with symmetrical constitutive parameters, the circuit is reciprocal, which means the transmission between port j and k does not depend on the propagation sense. 即使电路在几何上是不对称的，互易关系任然可以满足。注意这里的对称本构关系几乎适用于所有材料（OM 中从玻印亭矢量出发，证明过 susceptibility tensor 的对称结构，条件是 nonmagnetic，nondispersive materials.）特别的，对于对称的结构，散射矩阵的对角元(diagonal elements) 应该都是相同的.

●Applications (Beam splitter, optical power divider and plane waves)

1. beam splitter

   Consider the directional coupler. If 2 dielectric waveguides are brought closely together, light can couple between bothe waveguides. Assume no reflections, reciprocal and lossless, we have

$$\mathbf{S}=\begin{bmatrix} 0 & 0 & a & b \\ 0 & 0 & c & d \\ a & c & 0 & 0 \\ b & d & 0 & 0 \end{bmatrix}$$

With |a|=|d|, |b|=|c| (unitary). In the case of symmetry, further conclusions can be derived. (Slide p19, Note p19)

2. Optical power divider

   The ideal optical power dividers are lossy circuits even if they're made with lossless dielectric waveguides. The reason is the power is converted into radiating modes, which are not included in this scattering matrix description. (Slide p19, Note p20)

3. Plane waves 略

- **Transfer matrix description 传输矩阵描述**

The ports of the circuit are divided into ports considered as input ports and output ports.

$$\begin{bmatrix} \boldsymbol{b}_l \\ \boldsymbol{a}_l \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} \\ \boldsymbol{T}_{21} & \boldsymbol{T}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_r \\ \boldsymbol{b}_r \end{bmatrix}$$

As both the scattering matrix description and the transfer matrix description relate to the same parameters using a different set of input and output parameters, both descriptions are strongly related.

设想对每个 port 都有输入/输出信号 a/b；散射矩阵 S 描述了所有的输出信号振幅和所有的输入信号振幅之间的关系，而传输矩阵 T 描述了所有的左边端口的输入输出信号与右边端口的输入输出信号之间的关系。所以，如果我们在散射矩阵描述中也把端口分成左右两边，输入/输出信号振幅关系应该写成：

$$\begin{bmatrix} \mathbf{b}_l \\ \mathbf{b}_r \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{a}_l \\ \mathbf{a}_r \end{bmatrix}$$

可以推导出两个矩阵之间的关系。Note P22, Slide P24

2.3 Matrix description of light polarization 光偏振的矩阵表示

Polarization refers to the time dependence of the electric field vector E(r,t) at a point r in space. The orientation of the electric wavevector changes in time at a certain point r. The monochromatic electric field vector is described by:

$$\mathbf{E}(\mathbf{r},t) = E_x(t)\mathbf{1}_x + E_y(t)\mathbf{1}_y + E_z(t)\mathbf{1}_z$$

After some calculations, the electric field can finally be written as

$$\mathbf{E}(\mathbf{r},t) = \mathbf{v}_1'\cos(\omega t') + \mathbf{v}_2'\sin(\omega t') = \mathbf{E}_{x'}\cos(\omega t') + \mathbf{E}_{y'}\sin(\omega t')$$

Notice that linear polarizations have no handedness. Slide P27

●Jones Vector

The electric field component of a plane monochromatic wave, propagating in the positive z direction of a right handed Cartesian coordinate system is described by

$$\mathbf{E}(z,t) = E_x\cos\left(\omega t - \frac{2\pi}{\lambda}z + \phi_x\right)\mathbf{1}_x + E_y\cos\left(\omega t - \frac{2\pi}{\lambda}z + \phi_y\right)\mathbf{1}_y$$

Choose the reference plane at z=0 the plane wave can be represented by a Jones vector. Note P25 Slide P33

$$\boldsymbol{E} = \begin{bmatrix} E_x e^{j\phi_x} \\ E_y e^{j\phi_y} \end{bmatrix}$$

●Poincar´e sphere 庞加莱球面

The polarisation can be represented as a point on a sphere, with the parameters determined by its longitude and latitude.

●Quasi-monochromatic wave 准单色平面波和斯托克斯参数

Consider a quasi-monochromatic wave with both field srength and phase become slowly varying functions in time

$$\mathbf{E}(z,t) = E_x(t)\cos\left(\omega t - \frac{2\pi}{\lambda}z + \phi_x(t)\right)\mathbf{x} + E_y(t)\cos\left(\omega t - \frac{2\pi}{\lambda}z + \phi_y(t)\right)\mathbf{y}$$

Then the Jones vector will become a function of time (+SVEA). For a completely polarised light, it should have (振幅比和相位差都要统一)

$$\frac{E_x(t)}{E_y(t)} = \text{ constant}$$

$$\phi_x(t) - \phi_y(t) = \text{ constant}$$

Also, the parameters can be described by Stokes parameters, which stand for total intensity of the wave, tendency towards x or y linear polarisation(x/y 方向线性极化趋势), tendency towards 45°linear polarisation & tendency towards circular polarisation.
(更多内容参阅 Slide P39, Note P29) 部分偏振光可以分解为非偏振光+完全偏振光;

●Jones Matrix 琼斯矩阵

Consider a uniform monochromatic plane wave that is incident onto a non-depolarizing optical system. 考虑平面单色波入射到“非去偏振”光学系统, which is defined as an optical system in which the ‘degree of polarization’ of the input polarization of the output beam is larger than or equal to that of the input beam. 对于完全偏振入射波，输出也是完

全偏振的。

1. Quarter wave plate 四分之一波片

   The difference between the optical thickness for ordinary/extra-ordinary direction is a quarter wavelength. Phase retardation Gamma=pi/2, thus a linear polarization can be changed to a circular polarization. (注意，要同时考虑入射偏振光的两个正交分量在单轴晶体中的两个折射率，因为教材中例子是 azimuth angle 为 45°, 所以可以分解为沿着 x 和 y 方向的两个偏振分量，要求这两个偏振分量的振幅不变，仅要求相位延迟为 pi/2 就可以合称为一个圆偏振。i.e., The orthogonal polarizations are parallel to the principal axes of the birefringent material. 反之如果 azimuth angle 不为 45°那么可能很难得到想要的偏振态。)

2. Optical activity 旋光材料

   Consider an optical system that rotates an arvitrary linear polarization stste over an angle. (pure rotation, 注意这个和半波片的区别，半波片是相位延迟片，可以把线偏振旋转两倍的 azimuth angle，在材料中的偏振态不是线偏振。但是在旋光材料中是 continuously rotating the linear polarisation, determined by the specific rotary power.)

   Note P31 Slide P44

●Graphical representation of polarization change 偏振变化的图形表示

Note P33 Slide P45

●Scattering matrix and transfer matrix description including polarization 偏振的散射/传播矩阵描述 Note P34

Chapter 3 Thin Films

3.1 Basics of interference

Any detector will detect the energy density associated with the total electric field. The intensity of the wave is defined as the time avergage of U.

If two fields are present, the time-averaged energy density is the result of the total field, which is the superposition of E1 and E2.

$$\begin{aligned} I = I(\mathbf{r},t) &= \langle U(\mathbf{r},t)\rangle \\ &= \epsilon\langle(\mathbf{E}_1(\mathbf{r},t)+\mathbf{E}_2(\mathbf{r},t))\cdot(\mathbf{E}_1(\mathbf{r},t)+\mathbf{E}_2(\mathbf{r},t))\rangle \\ &= \epsilon\langle\mathbf{E}_1(\mathbf{r},t)\cdot\mathbf{E}_1(\mathbf{r},t)+\mathbf{E}_2(\mathbf{r},t)\cdot\mathbf{E}_2(\mathbf{r},t)+2\mathbf{E}_1(\mathbf{r},t)\cdot\mathbf{E}_2(\mathbf{r},t)\rangle \\ &= I_1+I_2+2I_{12} \end{aligned}$$

The cross-term gives rise to interference phenomena. The corss-term is a dot-product of 2 vectors and is zero only when the 2 vectors have orthogonal directions. Thus, orthogonal polarizations do not interfere. Slide P4, Note P41

●interference of 2 plane waves 两个平面单色波的干涉

Consider 2 monochromatic plane waves with frequencies, wave vectors and with colinear polarisation.

$$\begin{aligned} \mathbf{E}_1(\mathbf{r},t) &= \mathbf{A}_1\cos\left(\omega_1 t-\mathbf{k}_1\cdot\mathbf{r}+\phi_1(t)\right) \\ \mathbf{E}_2(\mathbf{r},t) &= \mathbf{A}_2\cos\left(\omega_2 t-\mathbf{k}_2\cdot\mathbf{r}+\phi_2(t)\right) \end{aligned}$$

A is a complex vector that lies in the xy plane.

$$\mathbf{A} = A_x e^{j\phi_x(t)}\mathbf{e}_x + A_y e^{j\phi_y(t)}\mathbf{e}_y$$

注意，由于线偏振 A 的两个相位因子相等；提取公因式到 E 中可以写成给出的形式。或者也可以考虑为 quasi-monochromatic wave 的表述形式。

Calculate the time averaged energy, we have

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} < \cos\left[(\omega_1-\omega_2)t-(\boldsymbol{k}_1-\boldsymbol{k}_2)\cdot\boldsymbol{r}+\phi_1-\phi_2\right] >$$

If the 2 frequencies are different, since the averaging time is relatively long, there will be no interference. (incoherent light). Otherwise the interferce pattern will depend on the initial intensity and phase difference. Note P43 Slide P9

●Multi-beam interference 多光束干涉

The calculated results can be found in the syllabus or the slides. In this section, we will look into 2 special cases that are regularly encountered in thin film applications.

1. Interference of M plane waves with equal amplitudes and constant phase difference. 有限个等振幅等相位差的平面波干涉

$$E_m = A_m e^{j\omega t}$$

$$A_m = \sqrt{I_0} e^{[j(m-1)\delta]}, m = 1,2, \dots, M.$$

 Then we can calculate the total amplitude with the sum of these complex amplitudes and the intensity distribution Note P46, Slide P12

为什么这里的 x 轴是相位差：相位差隐含了干涉图样在空间中的分布信息。假定相位差是一个固定值，多光束同时打到空间的一点上其辐照度也是一定值。然而，干涉图样的呈现是由于空间各点有不同的相位差，故真实干涉图样是如左图所示的图样经过某些变形/坐标变换后的结果。

2. Interference of an infinite number of waves with progressively declining amplitude and equal phase difference 无限个振幅逐渐下降、等相位差波的干涉

 Consider an infinite number of plane waves with exponentially decreasing amplitude coming together. The changing ratio of complex amplitute is h, which contains the phase difference and the decreasing ratio at the same time.

 h is used to define the finesse. When the finesse is large (|h|~1), the intensity function is sharply peaked ans vice versa. Also FWHM is reverse proportional to the modulus of h.

Slide 13, Note P46

3.2 Transfer Matrix Formulation for Multilayer System

In this section we will build the matrix T by regarding the multilayer stack as a cascaded system of interfaces and layers, each with their own transfer matrix. We get the complete transfer matrix T by multiplying the individual transfer matrices.

As we noted before, it's sufficient to treat the problem for 2 orthogonal polarizations. The easiest way is to slove th eproblem once for the TM-polarization (s-wave) with the electric field parallel to the interfaces of the stack and once for the TM-polarization (p-wave) with the magnetic field paraller to the interfaces of the stack. 注意：在处理相关偏振/极化问题时，国内教材好像没有所谓 TM 波与 TE 波的叫法，一般称作 TM（s）极化与 TE（p）极化；TM 极化指平面波的入射面与电场分量平行，TE 则是垂直。（这也解释了光为什么是横波：只有两种横向的偏振模式）

此外，在提到波的偏振/极化时还有圆偏振线偏振等概念，要注意偏振（除了线偏振）是电场分量方向随时间改变的现象。因此离了线偏振就无谓什么 TM、TE 极化了。

最后，似乎该课程中偏振态是就平面波而言的，虽然球面波等也可以定义偏振态(自

行查阅参考文献)。

In each layer, all contributions to the forward propagating wave have the same direction and thus form on plane wave, as well as the backward propagating wave. The total field can be described as (电场只有 xy 分量，参考图片为 TM 波)

$$\begin{aligned} E(x,y,z) &= A_F e^{-j(k_{x,i}x+k_z z)} + A_B e^{-j(-k_{x,i}x+k_z z)} \\ &= A_F e^{-jk_{x,i}x} e^{-jk_z z} + A_B e^{+jk_{x,i}x} e^{-jk_z z} \\ &= E_F(x) e^{-jk_z z} + E_B(x) e^{-jk_z z} \end{aligned}$$

However, k_x can be a complex number because of 1)lossy medium, 2)total internal reflection (ecanescent wave)

●matrix formalism 矩阵形式

Consider the interface between layer i and layer j for a given z.

$$\begin{bmatrix} E_F(x_i^+) \\ E_B(x_i^-) \end{bmatrix} = \begin{bmatrix} t_{ij} & r_{ji} \\ r_{ij} & t_{ji} \end{bmatrix} \begin{bmatrix} E_F(x_i^-) \\ E_B(x_i^+) \end{bmatrix}$$

Fresnel coefficients can be calculated using the boundary conditions. Note P52

TE 极化：

$$r_{ij} = \frac{E_B(x_i^-)}{E_F(x_i^-)} = \frac{n_i\cos\,\theta_i - n_j\cos\,\theta_j}{n_i\cos\,\theta_i + n_j\cos\,\theta_j}$$

$$t_{ij} = \frac{E_F(x_i^+)}{E_F(x_i^-)} = 1 + r_{ij} = \frac{2n_i\cos\,\theta_i}{n_i\cos\,\theta_i + n_j\cos\,\theta_j}$$

TM 极化：

$$r_{ij} = \frac{E_B(x_i^-)}{E_F(x_i^-)} = \frac{n_j\cos\,\theta_i - n_i\cos\,\theta_j}{n_j\cos\,\theta_i + n_i\cos\,\theta_j}$$

$$t_{ij} = \frac{E_F(x_i^+)}{E_F(x_i^-)} = \frac{n_i}{n_j}\left(1 + r_{ij}\right) = \frac{2n_i\cos\,\theta_i}{n_j\cos\,\theta_i + n_i\cos\,\theta_j}$$

然后再把 x+，x-改写一下放到同一边，传输矩阵变成 Note P51 的形式

Also consider the phase transfer matrix,

$$\mathbf{T}_i = \begin{bmatrix} e^{j\Phi_i} & 0 \\ 0 & e^{-j\Phi_i} \end{bmatrix}$$

Then the full transfer matrix will be

$$\boldsymbol{T}_{0N} = \boldsymbol{T}_{01}\boldsymbol{T}_1\boldsymbol{T}_{12}\boldsymbol{T}_2 \dots \boldsymbol{T}_{N-1}\boldsymbol{T}_{(N-1)N}$$

Power transmittion: Note P54, Slide P24

3.4 Applications of thin films

●Fabry-Perot etalon 法布里玻罗腔 Note P56, **Slide P29**

1.transmission 最大时为 resonance: all transmitted contributions interfere constructivelly.

2.沿着反射方向向上走，结果看起来很像无限多振幅逐渐减弱的等相位差光束互相干涉的条纹，这是易于理解的。

3.精细度可以通过 power reflection R 来定义，同样地，精细度越高，the sharper are the transmission peaks.

4.Another important dimensionless quantity is the quality factor (Q factor) defined by resonance anglular frequency over FWHM. It’s also a measurement method of sharpness.

5.Also, define group dealy (群延迟)，which measures the propagation time of a signal through the stucture,

6.事实上，由于材料折射率的限制，很难得到很高的反射率；解决办法一般是在法布里玻罗腔两侧镀一层金属膜，作用是反射时增加额外相移（吸收）Note P59, Slide P32

●AR Coatings (anti-refelction) 增透膜 Note P62, Slide P34

容易想到分别来自涂层前后被反射的两束光 interfere destructively 的时候，图层的光学厚度应该是四分之一波长（在假设下两个反射都有半波损，不考虑）。在此基础上，还要决定涂层+图层前后介质组成的三层结构中，折射率应该满足什么样的关系。通过矩阵得出 n2=sqrt(n1*n3)

注意 n2 在 n1, n3 之间时，无论镀层厚度如何都能得到增透膜，看 r 公式即可。我们之前同时要求 d 和 n 满足这样的关系只是为了好算+能得出 0 反射解?? （因为 e^pi=-1）

●High-reflection coating 增反膜/布拉格反射器 Note P62, Slide P37

注意，此时的光学厚度仍然是四分之一波长，然而由于垂直入射的半波损失，将产生相长干涉！→ Δn 大，带宽（bandwidth）宽。

Chapter 4 Fourier Optics

4.1 Basic principles of scalar diffraction theory

Diffraction can be defined as "any deviation of a light ray from rectilinear propagation, with is not caused by reflection nor reflection."

●Helmholtz-Kirchhoff integral theorem 赫姆霍兹-基尔霍夫积分定理

This formula allows for the field at an arbitrary poing P0 to be expressed in terms of its boundary values on any closed surface surrounding that point. Slide P19, Note P71

● Diffraction through a flat screen—Rayleigh-Sommerfeld diffraction formulas 平板上一小孔的衍射——瑞利索墨菲衍射公式 Slide P29, Note P74

$$U(P_0) = \iint_{\sigma} h(P_0, P_1) U(P_1) ds_1$$

$$h(P_0, P_1) = \frac{-1}{j\lambda} \cos\left(\mathbf{n}, \mathbf{r}_1 - \mathbf{r}_0\right) \frac{e^{-jk|\mathbf{r}_1 - \mathbf{r}_0|}}{|\mathbf{r}_1 - \mathbf{r}_0|}$$

注意，光瞳在面 x1,y1 上，观察平面是 x0, y0.

h is a weighting factor that is applied to the field U(P1) in order to synthesize(合成) the field in P0. They show that the field is a superposition of spherical waves starting from each point in the aperture, each with an appropriate amplitude and obliquity factor.

4.2 Fresnel and Fraunhofer diffraction

● Fresnel diffraction formula 菲涅尔衍射

When z is sufficiently large,

$$|\mathbf{r}_1 - \mathbf{r}_0| = \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$
$$\cong z\left[1 + \frac{1}{2}\left(\frac{x_1 - x_0}{z}\right)^2 + \frac{1}{2}\left(\frac{y_1 - y_0}{z}\right)^2\right]$$

$$U(x_0, y_0) = \frac{-e^{-jkz}}{j\lambda z} \iint_{-\infty}^{+\infty} U(x_1, y_1) e^{-\frac{jk}{2z}[(x_1 - x_0)^2 + (y_1 - y_0)^2]} dx_1 dy_1$$

It shows that the field in the observation plane is the 2-D Fourier transform of the field in the object plane. Slide P34,35; Note P76

This result is valid close to the aperture, “near-field approximation”, one sometimes speaks of the Fresnel diffraction regime. However, this approximation is not valid too close to the aperture.

● **Fraunhofer diffraction formula** 夫琅禾费衍射

The quadratic phase term can also be neglected.
It shows that the field in the image plane is the Fourier transform of the field in the aperture.

Fraunhofer region: $z \gg \frac{k(x_1^2+y_1^2)_{max}}{2}$

●**Apertures** 各种光瞳函数和夫琅禾费衍射

1. Rectangular aperture 矩形光瞳 Slide P39，Note P77
   矩形光瞳的夫琅禾费（远场）衍射图样是 sinc 函数平方的乘积
2. Circular aperture 圆形光瞳 Slide P40，Note P79
   圆形光瞳的夫琅禾费衍射图样是艾里斑（一阶贝塞尔函数）
3. Gaussian beam 高斯光束型 Slide P41，Note P81
   高斯光束在远场还是高斯光束分布

● **Fresnel diffraction at a square aperture** 正方形光瞳的菲涅尔衍射

经过一系列计算后可以得到辐照度分布。Slide P55，Note P82

Cornu sprial/Euler sprial: the behavior of Fresnel integrals can be illustrated by Euler sprials, which means the irradiance is proprtional to the length of a line segment connecting 2 points on the sprial. → the irradiance oscillates strongly.

● **The angular spectrum of plane waves** 平面波的角谱

Note P85，Slide P44-47

Fourier decomposition of U(x1,y1) is the expansion in plane waves (propagating + evanescent).

总结：Slide P61

4.3 Fourier transforming properties of optical systems

● **A thin spherical lens** 单个薄透镜

传播函数：Slide P64；只考虑相位变化

厚度计算：Slide P65；傍轴近似 Slide P66；

凸透镜 convex lens/凹透镜 concave lens 符号约定和具体传播函数、光场：Slide P68-70

When a plane unit-amplitude wave incident perpendicular on the lens, under the paraxial approximation, we can conclude that: the transmitting-spherical wave front converges towards a point on the z-axis at a distance f from the lens. (这里是纯几何+波动光学推导的，不显式涉及傅里叶变换，也不需要，因为就是单纯的汇聚/发散球面波)

● Mask plate in front of a lens 单个薄透镜前方近距离放置一个掩膜

We can immdiately obtain the field after the lens. When this field propagates further along the z-axis, the field at a distance z can be calculated with the fresnel diffraction formula.
Field in the pocal plane: Slide P73, Note P92
This means that the field in the back focal plane is proportional to the 2D Fourier transform of the transmittance function t0(x,y) of the mask. The fourier transform relation is not an exact one because of the quadratic phase factor which is not constant in the focal phane. However, this factor vanishes when calculating the power distribution in the focal plane.
➔ Without a lens the far field is found at large distance, but with the lens the far field is found in the lens focal plane.

● Mask at a distance of a lens 单个薄透镜前方一定距离放置一个掩膜

The field in the back focal plane is the exact Fourier transform of transmittance At0(x,y).
Slide P77 ,Note P93

● Optical convolution processor 光学卷积处理器

2D convolution：

$$\left|\iint_{-\infty}^{+\infty} g(\xi,\eta)h(x-\xi,y-\eta)d\xi d\eta\right|$$

1. The input function g is realized as a mask with transmittance function g(x1,y1) and placed in the plane P1, which is the first focal plane of a lens L1.
2. It is normally illuminated with a monochromatic plane wave.
3. In the second focal plane P2 of this lens on obtains a Fourier transform of g.
4. In this plane P2 one put a second mask with specfic transmittance function t(x2,y2)=k2H, then the field in the focal plane P3 of the second lens can be found as the convolution of h&g

Slide P78-82, Note P94

● Correlator & Spatial Filters 自行查看 Slide P84 85

4.4 Resolving power of an optical system

In real-life lens systems, lenses have finite diameter D and aberrations.

● The point spread function of a diffraction-limited system

We first start with considering an object composed of one single point, and an optical system with one single aberration-free thin lens. 考虑单个点对无像差透镜的成像

The field response in the image plane to this point source h(xi,yi; xo,yo) is called **point spread function (PSF)**. The intensity response is the square of its modulus.

1. with perfect coherent illumination
   obiect can be considered to consist of an infinite set of point sources with a given amplitude and phase relationship. With Rayleigh-Sommerfield formula, we can obain the field distribution & intensity distribution in the object plane.
2. with incoherent illumination
   the point source at position (xo,yo) has a real intensity, then the image has an intensity distribution with a normalised factor kappa.
   Slide P91+92，Note P96

接下来我们考虑相干照明:；

考虑傍轴球面波入射，透镜后面的光场可以写成：

$$U_{l'}(x_{l'},y_{l'};x_o,y_o)=U_l(x_{l'},y_{l'};x_o,y_o)P(x_{l'},y_{l'})\exp\left(j\frac{k}{2f}\left[x_{l'}^2+y_{l'}^2\right]\right)$$

就是透镜前光场*透镜相位因子*光瞳函数。光瞳函数就是光瞳内为 1，光瞳外为 0（因为透镜的尺寸有限）。结合几何光学中物距、相距倒数相加等于焦距倒数，以及横向放大率 lateral magnification 的定义，我们可以写出点扩散函数的形式。Note P97； Slide P96

This implies that the point spread function is nothing else than the Fraunhofer diffraction pattern(=fourier transform) of the exit pupil P(x,y) cuntered on the image corrdinates xi=Mxo and yi=Myo.

Further more, we see that the spread function only depends on the relative position with respect to the ideal geometrical image point. If the point source in the object plane moves around, the resulting image will only shift and will not change its intensity distribution.

(space-invariant) Slide P97

Also, one can found that the actual image of a diffraction limited system, is the convolution of the perfect image and a function h’ (which is the inverse Fourier transform of the pupil function) 空间域上真实图像 Ui 是理想图像 Uo 和出瞳傅里叶逆变换的卷积 Note P98

● Resolution ability of an optical system—Rayleigh criterion 瑞利判据

The definition of resolution is the minimal distance between 2 object points such that they can still be discerned(被分辨) in the image,

When the exit pupil is circular, the point spread function is the 2D Fourier transform of the aperture function which is a Bessel function. (Airy pattern)

Rayleigh criterion states that 2 points are jjust resolved when the center of one of the airy patternns coincides with the first zero of the other one. Slide P103, Note P98

阿贝极限 abbe diffraction limit: 传统光学显微镜分辨极限大约是半波长（200nm 左右）

● Optical transfer function 光学传递函数

将视角从空间域转到频域，可以用光学传递函数来描述光学系统的分辨能力。

分别对相干、非相干的真实图像做傅里叶变换得到频域。

注意，空间域上真实图像为 h 和理想图像的卷积，而卷积的傅里叶变换等于各自傅里叶变换的乘积，故：

1. <u>相干情况下</u>

   定义 H(fx,fy)：点源相应函数 h'的傅里叶变换，即 CTF (coherent transfer functiom)
2. <u>非相干情况下</u>

   定义 O(fx,fy)：kappa*h'^2 的傅里叶变换（从辐照度角度出发），为 OTF(optical tranfer function). OTP 的绝对值是 MTF(modulation transfer function)，OTF 的相位是相位传递函数(phase transfer function)
3. 如何联系相干/非相干情况；OTF 是 CTF 的自相关 (auto-correlation)
4. 如何与光瞳函数联系：Note P99，Slide P107

   CTF 是光瞳函数的标量版本（换一下变量除以放大率即可）

   那么通过光瞳函数得到 CTF，再计算自相关就可以得到 OTF，从而得到 MTF. 无像差系统的 OTF 总是正实数，因为他是 the area of overlap after translation Slide P111
5. 正方形出瞳、圆形出瞳、矩形出瞳的相关公式：Slide P112-114，Note P100，101
6. 截止频率 cut-off frequencies：(非相干)2fc 以上，MTF 为零，this means that the intensity distribution in the image plane cannt contain spatial frequency components beyond 2fc, i.e., the frequencies beyond 2fc cannot be resolved in the image plane.

➔ Same as the Rayleigh criterion.

7. 计算截止频率：MTF 为零时显然 OTF 为零，此时 CTF 也为零。所以找到 rect 或者 circ 函数为零的边界就是截止频率。可以发现在相干/非相干情况下系统的分辨能

力是不太一样的。Note P102

● Measuring the MTF 测量 MTF

The MTF of an optical system can be measured by incoherently illuminating a mask on which a sinusodial transmittance pattern is written. This pattern has only one single sptial frequency. One then measure the amplitute of the image, this gives 1 point of the MTF. Repeating this procedure for different spatial frequencies finally gives the complete MTF. One may have to continue with some detailed work such as normalisation.

*注意定义，OTF 本来就是衡量非相干情形下物面辐照度分布到相面辐照度分布的函数，所以这么测是完全没有问题的。

● Systems with aberrations 相差系统的光学传递函数

写广义(generalized)光瞳函数，OTF 会变成复数。然而相差是对称时，OTF 仍然是实的，但却有可能是负数，导致 MTF 也是负数→条纹反向

Chapter 5 Dielectric waveguides

5.1 Introduction 略 Note P107, Slide P1-22

5.2 Modes of Optical Waveguides

● Materials 用于制造波导的材料

可能需要有印象的内容：Note P110，Slide P28

1.InGaAsP/InP， Indium gallium arsenide phosphid，砷化铟镓

Low losses at 1.3/1.55 micro meters → telecommunication

高折射率，制造紧凑型波导（compact waveguides），尺寸在 1 micrometer 左右

一般的单模波导尺寸在几个 micrometers

2. GaAlAs / GaAs 砷化镓， Gallium arsenide

用于短波长有源元件（active components for short wavelengths）, useful for short-range communication, CD-players, etc.

3. Silicon-on-Insulator

Silicon layer bonded onto Silica (SiO2).

有两种类型，一个是“fiber matched”，典型尺寸在 7 微米；另一个是“high-contrast”，典型尺寸 500nm.

● Modes of optical waveguides 波导中的模式

The Helmholtz equation for every component of the field vector in a dielectric waveguide & the boundary conditions are

$$\nabla^2\Psi(\mathbf{r}) + k_0^2 n^2(\mathbf{r})\Psi(\mathbf{r}) = 0$$

$$n \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$$
$$n \times (\mathbf{H}_1 - \mathbf{H}_2) = 0$$
$$n \cdot (\varepsilon_1\mathbf{E}_1 - \varepsilon_2\mathbf{E}_2) = 0$$
$$n \cdot (\mathbf{H}_1 - \mathbf{H}_2) = 0$$

Which means the normal electric displacement field is coutinuous at an interface.

*To solve Maxwells eqs for an arbitrary dielectric structure requires the solution of a complex set of partial differential eqs and a typical example of approach is the effective index method.

● Modes of longitudinally invariant dielectric waveguide structures 纵向结构不变的波导中的传播模式

An eigenmode of the waveguide structure is a propagating or evanescent wave of which the transversal shape does not change during propagation.

An eigenmode propagating in the positive z direction:

$$\mathbf{E}(\mathbf{r}) = \mathbf{e}(x,y)e^{-j\beta z}$$
$$\mathbf{H}(\mathbf{r}) = \mathbf{h}(x,y)e^{-j\beta z}$$

With beta is the propagation constant, and the effective index is defined as, 相当于介质内的平均波矢 :

$$n_{eff} = \frac{\beta}{k_0}$$

Effective dielectric constant: $\varepsilon_{eff} = n_{eff}^2$

可以证明 e(x,y), h(x,y)都是一个本征值问题的解. Slide P40，Note P113

在简化问题之前，我们可以先探索一下这种波导的一些性质：

1. 有效介电常数比波导中材料最大的介电常数还大时，不存在本征模（传输截面不变的模）
2. 导模是一组分立的本征值，其有效介电常数介于 cladding 和 max 之间.（几何上看，这些模在波导芯层中全内反射，在包层中不振荡，呈倏逝波。 一个波导有可能不支持任何 guiding modes）
3. Radiating mode 是连续的本征值，至少在波导的一侧显出振荡行为。导模和辐射模是一组完备基，即波导中的任何一个场都可以展开成这两种模的线性和

● Slab waveguide 更简化的情况： 平板波导中的传播模式

这时可以把 y 分量忽略，分离变量后把 E、H 再次代入 Maxwell 方程组，得到两组本征方程，分别对应 TE 模式和 TM 模式。

$$\text{TE: } \frac{\mathrm{d}^2 e_y(x)}{\mathrm{d}x^2} + k_0^2 n^2(x) e_y(x) = \beta^2 e_y(x)$$

$$\text{TM: } \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\mathrm{k}_0^2 \mathrm{n}^2(\mathrm{x})}\frac{\mathrm{d}h_y(x)}{\mathrm{d}x}\right) + h_y(x) = \frac{1}{k_0^2 n^2(x)}\beta^2 h_y(x)$$

只分析 TE 模式，解和边界条件参考 Slide P53，Note P117 .

发现电场的解和需要满足的边界条件主要与传播常数, 波导的折射率 n1、n2、n3 和真空波矢（入射波频率）有关。With each eigenvalue an eigenmode can be associated. The dispersion relation for the different eigenmodes is resented by the ω-β diagram. 可以使用几个 normalized parameters 表示波导中的色散关系。一般的，频率越高，有效折射率越大；同时也和波导层之间折射率之差有关系。Slide P56，Note P119

利用导模是分立的这一特点和之前得到的公式与边界条件示意图，可以计算出波导中导模的数量：Slide P57，对对称波导，公式可以简化为

$$M = 1 + \mathrm{Int}\left[\frac{\nu}{\pi}\right]$$

● **Effective-index method** 有效指数法：回到二维波导的情况

For many waveguides with larger lateral dimensions and low—contrasted vertical index, the modes of the waveguide often will show a quasi-TE or quasi-TM behaviour and can be approcimately described by the scalar Helmholtz equation. The effective index method gives an approximate solution to this equation.

有效指数法任然从标量赫姆霍兹方程出发，将波函数分离变量为 z 方向和 xy 平面，找到 xy 平面波函数满足的方程。接下来，做一个 SVEA 假设（误差来源），即

$$\psi(x,y) = F(x,y)G(y)$$

With F(x,y) slowly varying in y. (相当于又分离了一次变量). 代入波动方程，得到一个关于 F、G 的二阶 PDE。We now apply a technique closely resembling the classical technique of separation of variables, the only difference is that F(x,y) shows a weak dependence on y and that we have to introduce an y-dependent separation variable n_eff(y). 再分离了一次变量, 但又没有完全分，因为下面又引入了 y 方向上的折射率函数，大概是 F(x,y)=F(x)*n_eff(y)。有效指数法的基本方程是

$$\frac{1}{F}\frac{\partial^2 F}{\partial x^2} + k_0^2 n^2(x,y) = k_0^2 n_{\mathrm{eff}}^2(y)$$

$$\frac{1}{G}\frac{d^2 G}{dy^2} - \beta^2 = -k_0^2 n_{eff}^2(y)$$

接下来的步骤是：

1. 对所有的 y=y0，解第一个方程。此时可以近似为平板波导直接求解。可以发现 n_eff*k0 其实就是传播常数。
2. 知道了 n_eff 在所有 y 上的分布之后，解第二个方程。此时波导就像一个躺下的平板波导，可以解出 TE 或 TM 模式。

Note P121，Slide P63

● **Plasmon waveguide 等离子体波导**

We will discuss the waveguiding properties of an interface between a semi-infinite metal with a complex permittivity and a semi-infinite dielectric. We assume d→0, we can conclude that TE mode doesn’t exist.

Slide P75.76 ,Note P123 略

5.3 Propagation through dielectric waveguide structures

在这些结构中 transversal cross section varies (不是本征模)，大多数时候需要完整的数值处理。

● **Mode expansion 本征模展开**

数学中学过了。本质就是把任意场展开成 discrete guided modes+continuous radiating modes。由于 eigrnmodes 的正交性，展开系数可以求出。Slide P80-83，Note P128

● **Couple mode theory 耦合模理论**

In a regular z-independent waveguide, wigenmodes are orthonormal and propagate in an independent way. In some situations the waveguide structure can be seen as a small perturbation. In this case, the modes will no longer be decuopled.

$$\begin{aligned}\psi(x,z) &= \sum_{i=0}^{N-1} C_i(z)\psi_i(x)\mathrm{e}^{-j\beta_i z} \\ &= \sum_{i=0}^{N-1} X_i(z)\psi_i(x) \text{ with } X_i(z) = C_i(z)\mathrm{e}^{-j\beta_i z}\end{aligned}$$

描述两个本征模耦合的项是 z-dependent 的 Ci(z)（解耦的时候场的表达式是模式的线性和，即系数是常数）。下面只考虑两个主导模的耦合。

假如两个模是解耦的，那么 Xi(z) 是简单的 z 方向的相位因子。但在不解耦的情况下，我们要引入 z-dependent 的耦合系数（coupling coefficients）可以探索出耦合系数的某些性质，且在初始时只有一个模被激发的条件下可以解出耦合项。

Slide P91，Note P131

When there is no phase mismatch, i.e,the propagating constants for both modes are the same, full coupling (synchronous coupling) occurs.

● **Supermodes 另一种计算方法：超模态理论，以 directional coupler 为例**

Consider again a dielectric coupler consisting of 1 indentical monomodal waveguides, both

with fundamental mode Φ0. We can also look at the structure as a whole as being a single waveguide. In this waveguide a symmetrical and antisymmetrical mode can propage.
在一根波导中激发模时，可以把总的模的线性叠加系数认为都是 1/2，接下来可以写出传播 L 后的总模式。

$$\Psi(x, z=L) = c_+ e^{-j\beta_+ L}\left[\phi_+ + \phi_- e^{+j(\beta_+ - \beta_-)L}\right]$$

指数项为-1 时，所有的能量都在第二个波导中。由此可以定义波导的 coupling length & coupling coefficient Slide P97 Note P133
波导是非对称时也有相同定义，只是此时能量不能完全耦合进第二根波导。
可以定义 phase mismatch Δ

● The beam propagation method (BPM)

略 Slide P109

5.4 Optical components

● Loss in a straight waveguide 直波导中的损耗

吸收损耗（主要是材料）；
散射损耗 scattering loss： spatial refractive index variations & roughness of the interfaces
辐射损耗 radiation loss：模式泄露 Slide P119 Note P138

● Bent waveguides

减小环半径，模式会向环外缘移动：“Whispering Gallery” modes Slide P128,Note P142

● 其余略，见 ppt 和教材

注意：Y junction 反着放时会丢掉一半的功率。However, when the waveguides are monomodal, this part of the power will radiate.

Chapter 6 Periodic Structures

6.2 Diffraction at surface gratings

An approximate theory we will use in this section is the so called transmission theory. This theory assumes that the scalar field immediately behind the grating can be obtained by simply multiplying the incident field with a transmission function. This means that the transmission theory relates the incident and transmitted field locally, opposed to the integral relations of Fresnel and Fraunhofer diffraction.

Therefore, transmission theory only applies when the thickness of the periodic media is sufficiently small.

● Transmission of surface gratings 周期台阶型光栅

The transmission function can be written as

$$\psi(x,0^+) = t(x)\psi(x,0^-)$$

$$\sum_{n=1}^{N} t_1(x-x_n) x_n = (n-1)\Lambda$$

With the period of the grating and N the number of grating periods.

对第一个方程做傅里叶变换，目的是得到透射频谱。注意乘积的傅里叶变换等于各自傅里叶变换的卷积。那么透射函数 t(x)的傅里叶变换 T(fx)可以写成每一个周期内投射函数傅里叶变换与一个周期相关函数的乘积。对台阶形的例子来说，

$$\mathrm{T}(f_x) = T_1(f_x) \cdot e^{j(N-1)\delta/2} \cdot \frac{\sin\ N\delta/2}{\sin\ \delta/2}$$

$$\delta = -2\pi f_x \Lambda$$

通过振幅透过谱，容易得到强度透过谱。

假设平面波垂直入射，可以分析得到谱函数的一些特征。

The function T(fx) consists of a slowly varying envelop T1(fx), miltiplied with the periodic function. The latter has sharp periodic peaks with side lobes (侧瓣) in between. Each peak is called a diffraction order.

垂直入射时，m 级衍射的衍射角，$\sin\ \theta_m = \pm\frac{m\lambda}{\Lambda}$ (bragg condition/grating equation)

一般的倾斜入射时，$\sin\ \theta_m - \sin\ \theta_{in} = \pm\frac{m\lambda}{\Lambda}$ Note P162，Slide P12

当 N 很大时，very sharp diffraction orders and vice versa.

● Grating with N slits: 简化情况：n 个狭缝的光栅

此时的透过率函数是阶梯型的。可以计算出传输函数和传输场（结果就是传输函数）Slide P14

● Blazed gratings 锯齿形光栅

改写 transmission function inside one period

$$t_1(x) = e^{+j\frac{2\pi}{\lambda}(n_2-n_1)xd/\Lambda}e^{-j\frac{2\pi d}{\lambda}n_2}$$

通过相似的操作可以得到透过率函数 with N periods.

$$|T(f_x)|^2 = \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)} \cdot \frac{\sin^2(\pi(f_x - f_0)\Lambda)}{\left(\pi(f_x - f_0)\right)^2}$$

注意比较锯齿形光栅和之前的 surface grating，we find that transmittance is only significant when fx lies around 1/Lambda. So this is diffraction to only 1 diffraction order.

有趣的一点是，平面波经过 blazed grating 之后波前变成了锯齿形。When blazing condition is fulfilled, 那么这个波前和倾斜平面波的波前一样。那么通过光栅的衍射就变成了折射。Slide P19， Note P163

● Grating spectrometer 光栅光谱仪 (monochromator 单色仪)

Monochrometer consists of slits. In the transmitted optical field there will be a well defined relation between the angle theta and the wavelengths, and resolution is larger for higher order of diffraction.

1. <u>Resolution</u> Slide P24, Note P167

   Rayleigh criterion states that smallest delta lanbda for which a maximum of lambda1 coincides with the first minimum of lambda2 for the same order of diffraction.

   The resolution can be very large when a large grating with lots of periods is used.

2. <u>Free spectral range</u>

   Frequency (wavelength) range over which the spectrometer functions unambiguously. E.g, order m at lambda1 coincides with order m+1 at lambda2 then |lambda1-lambda2| will be the free spectral range of the specrometer.

● Czerny-Turner monochromator 车尔尼-特纳光谱仪

其实就是入射光通过狭缝入射到镜子上，镜子聚焦这些光线到倾斜的锯齿形光栅，通过光栅衍射、反射出的光线只有一个单色光能从出口输出。

6.3 Bragg condition and k-vector diagram

● k-vector diagram

The Bragg condition is a generalization of Snells law. It relates the k-vector of the incident plane wave and the k-vectors of the diffraction orders.

使用微扰法来推导布拉格条件。折射率和波函数都可以使用微扰近似。对于周期性结构的倒格子矢量（reciprocal lattice K-vector），**K 空间里的周期是 2pi/Lambda**，可以推出在傅里叶空间中新场的方程，最后得到

$$\boldsymbol{k}'_{m,n,l} = \boldsymbol{K}_{m,n,l} + \boldsymbol{k}_{in}$$

It's a relation between direction of incident and scattered waves. 同样地可以写出微扰场和微扰场的展开系数 Note P171，Slide P38.

这个关系可以用 k-vector diagram 来表示

1. Around k_in we draw a sphere with radius k0n0
2. The k-vectors of the diffraction orders are found by adding integer multiples of Kx,Ky and Kz, until again the suifave of the sphere with radius k0n0 is reached.
3. All k-vectors originate from the same point. Note P172

● 1D periodic layered medium & surface gratings 周期性层状结构和表面光栅

层状结构：Slide P40-41

表面光栅：注意虚线的存在是因为结构只在 x 方向上周期性，对 y 方向没有限制

两种不同折射率介质表面的 k-diagram：Slide P47

光栅波导表面的 k-diagram：Slide P49

6.4 Floquet-Bloch theorem and Photonic bandgap

● Floquet-Bloch theorem 佛罗恺-布洛赫理论

To investigate the propagation behavior in periodic structures, we will consider the modes of these structures. A general solution of the Maxwell's eqs is given by the Floquet-Bloch theorem and the modes of the periodic medium are called Floquet-Bloch modes.

Floquet-Bloch theorem states that: Slide P52; Note P178

When n(x) is a periodic function with period Lambda,

$$\psi(x) = e^{-jkx} u_k(x)$$

k is function of k0, uk strictly periodic with period Lambda.

Omega 是真空中频率；解出方程后介质中呈周期的 k 与 Omega 有一关系，在图中呈

现能带 bandgap；
Due to the symmetry behavior, for a given k-value, different omega are found. The forbidden zone is called the bandgap. → a source with a frequency inside the forbidden zone will not emit electromagnetic power. → frequency range in which lighe cannot propagate in a photonic crystal. Slide P60 Note P181

6.5 Periodically layered media

传输矩阵法 Slide P75
耦合模理论 Slide P79 coupling length Slide P86 Note P190
Floquet-Bloch Slide P90

6.6 Acousto-optical diffraction

When there is an optical beam present, this optical beam wll feel the scoustical wave by its effect on the refractive index of the material. This willl change the wavefront of the optical wave, which is called acousto-optical diffraction

- **Raman-Nath Acousto-Optic diffraction** Slide P103 Note P197

When the d is sufficient small, with a critical thickness. (Raman-Nath → Bragg regime)

6.7 Holography 全息术

We can first write the complex amplitudes of the object and reference wave at the photo-sensitive material. The total intensity on the photo-sensitive material is therefore with I0 the intensity of the reference wave.
The amplitude and phase imformation of the object wave is coded under the form of the repective amplitude and phase modulation of a periodic carrier with spatial frequency.
radiofrequent electrical signal on a piezo-electric material
As the hologram has the character of a quasi-periodic grating, diffraction effects will occur in transmission. For reconstructing the object wave, the developed hologram is illuminated with the reference wave again, but now the object is no longer there.
The third term and actually the most important term is, disregarding a constant term, the complex amplitude of the object wave and therefore forms a virtual image of the object on its original place behind the hologram.
In other words the pseudoscopic image features depth reversal.