Lesson 8.1.3

8-19. a. Outcome 1: reject the null hypothesis, concluding that more than 1% of the nails are bent, in which case the advertisement will not run. Outcome 2: fail to reject the null hypothesis, so the advertisement is allowed to run.

b. Outcome 1: reject the null hypothesis, accepting the alternative, so the advertisement will be run. Option 2: fail to reject the null hypothesis, so the advertisement will not run.

c. The second one. The first one runs the advertisement "by default," so it could easily run even if it is not true. The second one requires that there be strong evidence of truth before it allows the ad to run.

8-20. a. Outcome 1: Reject the null hypothesis, accepting the alternative that more than 2% are unhappy, so the company loses the case! Outcome 2: fail to reject the null, so the company wins the case by default.

b. Outcome 1: reject the null, accepting the alternative that less than 2% are unhappy, so the company wins the case. Outcome 2: fail to reject the null, so the company loses the case by default.

c. The first one is gives the defendant the benefit of the doubt, which is the correct thing to do in this case. Therefore, it is the better choice.

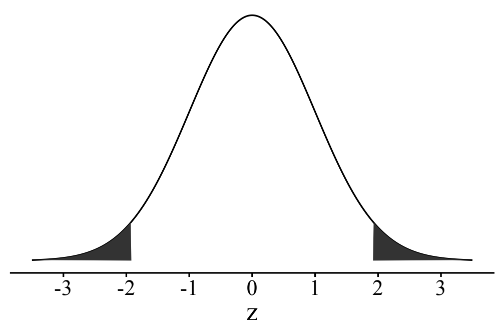
8-21. a. $p = 0.51$, where p = proportion of county voters who voted for Obama

b. The claim is a claim of equality, and the tester does not seem to have a preconception of which direction the claim might be off, so both directions of "wrongness" needs to be considered!

c. Random sample stated in problem. Expected counts of successes and failures are $np = 333$ and $n(1 - p) = 319$, both greater than 10, so the large counts condition is met so sampling distribution ≈ normal. $12000 > 652 \cdot 10$, so the independent trials condition for accurate $\sigma_{\hat{p}}$ is met.

d. $\hat{p} = 0.472$

e. The standard deviation is 0.0196. The sample proportion $\hat{p}$ is $\frac{308}{652} \approx 0.472$, with a z-score of -1.92. See diagram at right.



f. $2 \cdot 0.0275 = 0.055$

g. No. Since p-value > 0.05, he does not have sufficient evidence to reject the county's claim in this case. It is reasonably possible the county did indeed have 51% vote for Obama. He cannot conclude that the county voted with a proportion different than 51%.

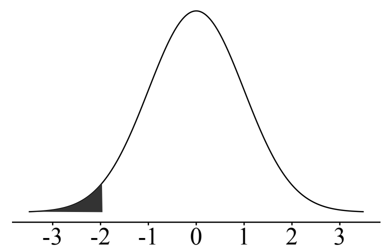
8-22. a. H_0: $p = 0.5$, H_A: $p < 0.5$. Anna should give Ms. Spotdale the benefit of the doubt, so she needs to show convincingly that Ms. Spotdale is wrong to draw a conclusion.

b. H_0: $p = 0.75$, H_A: $p < 0.75$. Jane should give the magazine number the benefit of the doubt unless she can show for sure that the proportion is less than 75% at her school.

c. H_0: $p = 0.24$, H_A: $p \neq 0.24$. In this case, either direction is equally "extreme" so it is a two-tailed test.

d. H_0: $p = 0.05$. H_A: $p < 0.05$. In this case, the new drug should not be given the benefit of the doubt; the drug company needs to show convincingly that $p < 0.05$ for approval.

8-23. Identify: This is a one-sample z-test for a population proportion. H_0: $p = 0.5$ and H_A: $p < 0.5$ where p is the population proportion of skirts out of dress code. Let $\alpha = 0.05$. Sample evidence for H_A: $\hat{p} \approx 0.404$.
Check conditions: The expected p is 0.5, so the expected successes and failures under the null are both 52, satisfying large counts so sampling distribution ≈ normal. Anna chose randomly, satisfying random sampling to avoid bias. And since she drew with replacement, the independent trials for an accurate $\sigma_{\hat{p}}$ condition is met regardless of population size.
Calculate: The mean is 0.5 and the standard deviation is $\sigma_{\hat{p}} = \sqrt{\frac{(0.5)(0.5)}{104}} \approx 0.049$ students. The test statistic is $\frac{0.404-0.5}{0.49} = -1.96$. The p-value is therefore about 0.025. See graph at right.
Conclude: Since the p-value of 0.025 is less than α of 0.05, the null hypothesis can be rejected; Anna has sufficient evidence to refute Ms. Spotdale's exaggerated claim.



8-24. The $\hat{p}$ would have been 0.548, so the sample evidence for the H_A would not really exist! If a p-value were calculated anyway, it would have been 0.836! Obviously, Anna would be unable to refute Ms. Spotdale.

8-25. a. 47.62%, 56.38%

b. Decrease the confidence or increase the sample size.

c. No. The true percentage is either in the interval or it is not. You are 95% confident that the actual percent is between the 47.62% and 56.38%. Meaning if you took repeated samples of 500, and then formed confidence intervals, 95% of them would contain the parameter.

d. $2500 \frac{\text{visitors}}{\text{day}} \cdot 7 \frac{\text{days}}{\text{week}} = 17,500 \frac{\text{visitors}}{\text{week}}$

A population of 17,500 site visitors/week is at least 10 times larger than the sample of 500 needed to satisfy the independent trials condition and ensure the formula for $\sigma_{\hat{p}}$ is accurate.

8-26. a. $r = 0.9752$, indicating a strong positive linear association between the mass versus swarm size of locusts, which is confirmed by an inspection of the residual plot where there is no discernable pattern in the residuals.

b. $R^2 = 0.9510$, meaning that 95% of the variation observed in mass is explained by a linear relationship with swarm size. In other words, knowing the swarm size associated with each mass reduces the error in predicting mass by 95% over using just the mean mass as a predictor.