Lesson 6.1.3

6-21. a. X: {1, 2}, $P(X)$: { $\frac{1}{2}$, $\frac{1}{2}$ }, $E(X) = 1.5$, $\text{Var}(X) = 0.25$, $\text{sd}(X) = 0.50$

b. $X - X$ represents the difference between two independent observations of X. Flip a coin once. Flip it again. Find the difference.

c. Expected mean is zero, expected standard deviation is 0.707. A sample response might look like, "Our sample representing the distribution of $X - X$ is mostly symmetric, centered about a mean of 0.08 with a standard deviation of 0.68."

d. X: {–1, 0, 0, 1}, $P(X)$: { $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$ }, $E(X) = 0$, $\text{Var}(X) = 0.50$, $\text{sd}(X) = 0.707$

e. The sample statistics should be reasonably close. If there were 400 observations, they would be even closer to the theoretical values.

f. The expected value is zero, but the variability of X allows independent observations of $X - X$ to differ from observation to observation.

6-22.

Probability Distribution for $X - X$		First Spin 2 | 0.3	4 | 0.2	5 | 0.5
Second Spin	2 | 0.3	0 | 0.09	2 | 0.06	3 | 0.15
	4 | 0.2	–2 | 0.06	**0 | 0.04**	**1 | 0.1**
	5 | 0.5	**–3 | 0.15**	**–1 | 0.1**	**0 | 0.25**

The mean, $E(X - X)$, is $E(X) - E(X) = 0$. The variance, $\text{Var}(X - X)$, is $\text{Var}(X) + \text{Var}(X) = 3.38$. The standard deviation for $X - X$ is $\sqrt{\text{Var}(X) + \text{Var}(X)} = 1.838$.

6-23. a. $E(C) = 1.5$, $\text{Var}(C) = 0.25$, $\text{sd}(C) = 0.5$

b. If $C - Y$ is greater than zero, then C is greater than Y.

c. See table below. $E(C - Y) = 1.5$

		Y										
		–5 $\frac{1}{36}$	**–4** $\frac{2}{36}$	**–3** $\frac{3}{36}$	**–2** $\frac{4}{36}$	**–1** $\frac{5}{36}$	**0** $\frac{6}{36}$	**1** $\frac{5}{36}$	**2** $\frac{4}{36}$	**3** $\frac{3}{36}$	**4** $\frac{2}{36}$	**5** $\frac{1}{36}$
C	**1** $\frac{1}{2}$	**6** $\frac{1}{72}$	**5** $\frac{2}{72}$	**4** $\frac{3}{72}$	**3** $\frac{4}{72}$	**2** $\frac{5}{72}$	**1** $\frac{6}{72}$	**0** $\frac{5}{72}$	**–1** $\frac{4}{72}$	**–2** $\frac{3}{72}$	**–3** $\frac{2}{72}$	**–4** $\frac{1}{72}$
	2 $\frac{1}{2}$	**7** $\frac{1}{72}$	**6** $\frac{2}{72}$	**5** $\frac{3}{72}$	**4** $\frac{4}{72}$	**3** $\frac{5}{72}$	**2** $\frac{6}{72}$	**1** $\frac{5}{72}$	**0** $\frac{4}{72}$	**–1** $\frac{3}{72}$	**–2** $\frac{2}{72}$	**–3** $\frac{1}{72}$

d. $P(C - Y > 0) = \frac{47}{72} = 0.6528$

6-24. a. The change will have no effect on the correlation coefficient r.

b. For each additional day of life, an insect is predicted to gain 0.06 grams.

c. Using the linear equation, the mass would be predicted to be 0.95 grams. This prediction should not be trusted because the scientists' model is based on the first week of life. 10 days is extrapolation.

6-25. $\mu_{Y-X} = 1.4 - 1.8 = -0.4$ lbs, $\sigma_{Y-X} = \sqrt{0.25^2 + 0.35^2}$, approximately normally distributed: $P(Y - X > 0) = 0.1762$

6-26. a. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

b. $P(X = 2) = \frac{3}{8}$

6-27. a. $\mu_X = 5.05$, $\sigma_X \approx 2.269$, $\mu_Y = 5.02$, $\sigma_Y \approx 1.0294$

b. $\mu_Z = 11.1$, $\sigma_Z = \sqrt{10.295} \approx 3.2086$

c. $\mu_Z = 11.1$, $\sigma_Z = 2(2.269) \approx 4.538$

d. 0.03

6-28. a. Following the value for 0.5 on the y-axis over to the graph, it looks like a about 117 minutes.

b. Moving vertically upward from 140 until hitting the curve, it looks like it hits around 0.90.

c. The distribution is right skewed. With the "steepness" all in the earlier part of the domain, the median (117 min) is less than the middle of the domain (135 min).