## Lesson 7.2.3

7-51. a. Sample proportion, $\hat{p}$.

b. Use the standard normal distribution to find the number of standard deviations away from the center you need to go to capture the confidence level.

c. $SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

7-52. a. Random: random selection given. Independent trials: the population of all clovers is much higher than $10 \cdot 250$. Large counts: there are $n\hat{p} = 14$ successes and $n(1 - \hat{p}) = 236$ failures, both greater than 10.

b. $0.056 \pm 0.019$. Students are 80% confident that the true proportion of four leaf clovers falls between 0.037 and 0.075. This means if students repeated this process the same way they would capture the true proportion 80% of the time.

c. $0.056 \pm 0.024$

d. $0.056 \pm 0.029$ or $(0.027 < p < 0.085)$

e. $0.056 \pm 0.037$ or $(0.019 < p < 0.093)$

f. The confidence intervals get wider because the z* increases.

7-53. a. The interval is $0.56 \pm 0.012$. The margin of error is half the size of the original margin of error.

b. You have to multiply the sample size by four, because the equation has a division by the square root of $n$. Thus multiplying $n$ by 4 decreases the standard error by 2.

7-55. The equation for the margin of error is (critical value) · (standard error). For a 99% confidence level, the critical value is 2.58, so the equation is $0.01 = 2.576\sqrt{\frac{0.056(0.944)}{n}}$ (the 0.056 is her current best estimate for her new $\hat{p}$). Solving for $n$ yields $n = 3507.9$, so she will need 3508 clovers. That is a lot of green.

7-56. a. A value of 0.5 gives the highest value. This can be proved by realizing that the square root and sample size can be ignored, so you are simply trying to maximize $p(1-p)$ or $p - p^2$. The vertex, which is the maximum value of this parabola, occurs at $p = \frac{1}{2}$, a fact easily seen by constructing a table or graph.

b. $0.04 = 1.96\sqrt{\frac{0.5(0.5)}{n}}$ can be solved to yield $n = 600.2$, so at least 601 people.

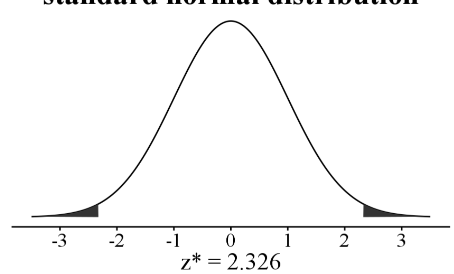
c. The new margin of error is 0.028, which is $\frac{0.04}{\sqrt{2}}$. Doubling the sample size decreases the margin of error by the square root of 2.

d. Without changing confidence, they would need a sample size of about $600 \cdot 16 = 9600$ people to get such a small margin of error, which would be very difficult to gather.

7-57. The equation for the margin of error is (critical value) · (standard error). For a 90% confidence level, the critical value is 1.645. Answers will vary based on $\hat{p}$. Using a 75/25 split, you can set up the equation: $0.01 = 1.645\sqrt{\frac{0.25(0.75)}{n}}$. Solving for $n$ yields $n = 5073.8$, you would need 5074 shots!

7-58. Identify: This is a 90% confidence interval for $p$, the proportion of all U.S. citizens who know what a "Sooner" is.
Check conditions: Random, to avoid bias: SRS given in problem. Independent trials, for an accurate $\sigma$: 588 citizens · 10 < U.S. population. Large counts, so the sampling distribution ≈ normal: $n\hat{p}$ = successes = 172 citizens, $n(1-\hat{p})$ = failures = 416 citizens, both >10.
Calculate: $\hat{p} = \frac{172}{588} \approx 0.2925$, SE ≈ 0.01877 citizens, $z^* = 2.326$,
margin of error = 2.326(0.01877) = 0.04365 citizens, confidence interval = 0.2925 ± 0.04365, {0.2489, 0.3362}. See diagram at right.
Conclude: I am 98% confident that the interval from 0.249 to 0.336 contains the true population proportion of U.S. citizens who know what a "Sooner" is.

**standard normal distribution**

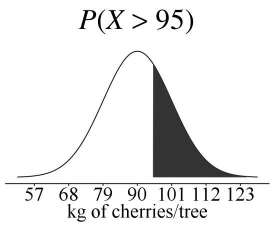


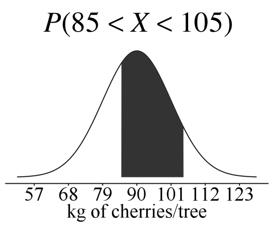
7-59. a. $0.93(–\$15) + 0.04(\$100) + 0.02(\$200) + 0.01(\$500) = –\$0.95$.

b. If Anna gets at least one non-white ball out of 6 attempts she will win some money. $P(\text{at least one non-white}) = P(1 – \text{all white}) = 1 – 0.93^6 = 0.353$.

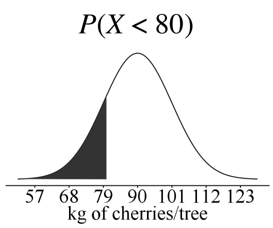
7-60. a. $P(X > 95) = \text{normalcdf}(95, 10\text{^}99, 90, 11) \approx 0.325$. See diagram below.




b. $P(85 < X < 105) = \text{normalcdf}(85, 105, 90, 11) \approx 0.589$. See diagram below.




c. $P(X < 80) = \text{normalcdf}(–10\text{^}99, 80, 90, 11) \approx 0.182$ or the 18th percentile. See diagram below.




7-61. a. Neither the large counts nor the independent trials claims are met by Summer's scenario.

b. The mean value is 0.16. If Summer expands out by 0.12 from her mean in both directions, she will capture almost exactly 90% of the data, so a good estimated confidence interval is $0.16 \pm 0.12$, or 0.04 to 0.28.