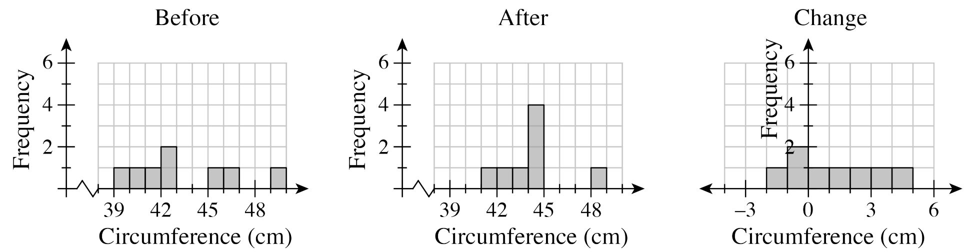
Lesson 11.1.2

11-8. a. See bold values in table below.

Circ. before (cm)	45	39.5	42.0	46.0	41.5	40.0	42.0	49.5
Circ. after (cm)	44	43.5	44.0	44.0	44.5	41.0	42.0	48.5
Change	**–1**	**4**	**2**	**–2**	**3**	**1**	**0**	**–1**

b. See diagrams below. The "before" and "after" rows both have high outliers.

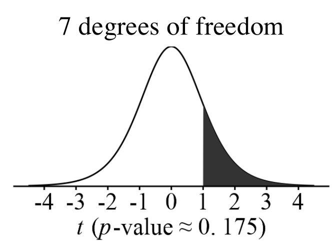


c. From the body builder who went from 39.5 to 43.5 cm.

d. Since this is not a true experiment with a control group, cause and effect cannot be established. There could always be an uncontrolled confounding variable. It is possible that their normal workout routine alone could be responsible for any changes.

e. Random selection condition is stated in the problem. Assuming there are more than 80 mid-tier body builders, independent trials is met. And the large sample condition is met well enough since the sample *differences* have no major skew or outliers (it is almost a uniform distribution).

f. <u>Identify</u>: hypotheses: H_0: $\mu_D = 0$; H_A: $\mu_D > 0$. μ_D is the population mean of the difference in bicep size. Let $\alpha = 0.05$.
<u>Check conditions</u>: Conditions are already checked.
<u>Calculate</u>: $\bar{D} = 0.75$ and $s_D \approx 2.1213$ so the t-statistic is $t = \frac{0.75-0}{\frac{2.1213}{\sqrt{8}}} \approx 1.00$ with 7 degrees of freedom.
$P(t > 1.00)$ with 7 degrees of freedom is ≈ 0.175. See diagram at right.
<u>Conclude</u>: Since the p-value is greater than the α of 0.05, the GunShow company does not have enough evidence to honestly claim that their protein shake is associated with an increase in bicep circumference—it is quite possible that the average difference seen is simply due to random variation!



11-9. a. The lack of randomization makes confounding variables more likely, making it so Lilly could not conclude cause and effect as designed. For example, perhaps everybody will be a little bit better the second day simply due to practice. She can fix it by giving half of the participants the placebo on day 1 and caffeine on day 2 and the other half the caffeine on day 1 and the placebo on day 2.

b. No. Since her volunteers were not randomly chosen from the entire population of humans, its results cannot extend to that population. However, the results can be extended to "populations like the volunteers."

d. The standard deviation of the difference is not necessarily connected to the standard deviation of each individual value. It has to be calculated directly from the data. However, the mean difference *is* equal to the difference of the means.

e. The 95% confidence interval for the difference is 1.18 to 5.62 seconds. This means you can be 95% confident that it took longer to type under the placebo, so the caffeine increased typing speed! However, the effect is small: a one to six second savings out of an average of 330+ seconds is not very much.

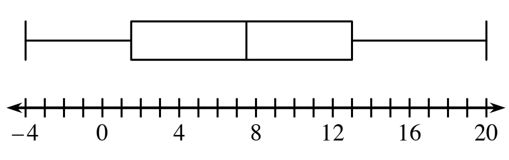
11-10. The answer given here is for a testing method, but a confidence interval method that shows 0 is not in the interval for the difference is also acceptable, as long as the four-step process is followed.

<u>Identify</u>: this is a paired t-test for the mean difference. Hypotheses: H_0: $\mu_D = 0$, H_A: $\mu_D \neq 0$. Let $\alpha = 0.05$. Where μ_D is the population mean difference in jellybean number estimations. Sample evidence for H_A: $\bar{x}_D = 7.5$.

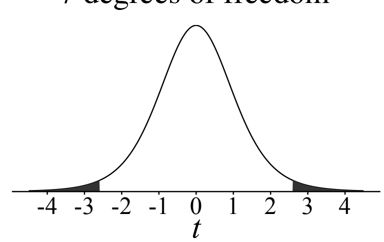
<u>Check conditions</u>: Random selection, to avoid bias—satisfied by the randomness of his purchase. Independent trials, for and accurate σ—lots more than 80 types of jars. Large samples, so the sampling distribution is normal—a plot must be made *of the differences*. A boxplot, histogram, or normal probability plot will all be considered acceptable, with appropriate commentary. Using the boxplot shown below left, it is sufficient to note that there are no outliers and minimal skew. A histogram should have similar commentary. For a normal probability plot, a statement of approximate linearity is adequate.

<u>Calculate</u>: The mean of the differences is 7.5 jellybeans, with a standard deviation of 8.18 jellybeans. The test statistic is $t = \frac{7.5}{\sqrt{8.18}} \approx 2.59$. The p-value for the two-tailed t-test with 7 degrees of freedom, as shown in the figure below right, is 0.036.

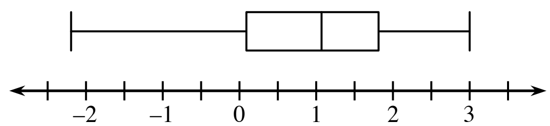
<u>Conclude</u>: Since the p-value < α of 0.05, you can conclude that Mark's two methods estimate a different number of jellybeans, at least with jars like those he used for his test. He should probably decide which one is closer to the true values!



7 degrees of freedom



11-11. The answer given here is for a testing method, but a confidence interval method that shows 0 is not in the interval for the difference is also acceptable, as long as the four-step process is followed. <u>Identify</u>: This is a paired t-test for the mean difference. Hypotheses: Let $\alpha = 0.05$. H_0: $\mu_D = 0$, H_A: $\mu_D \neq 0$, where μ_D is the population mean time difference in minutes between name and no-name problems. Sample evidence for H_A: $\bar{x}_D = 0.927$. <u>Check conditions</u>: Random—randomly selected 10 of each type of problem in each chapter. Independent trials—lots of homework problems. Large samples, so the sampling distribution is normal—since there are only 11 averages being subtracted, a plot must be made *of the differences*. A boxplot, histogram, or normal probability plot will all be considered acceptable, with appropriate commentary. Using the boxplot at right, it is sufficient to note that there are no outliers and minimal skew. A histogram should have similar commentary. For a normal probability plot, a statement of approximate linearity is adequate.

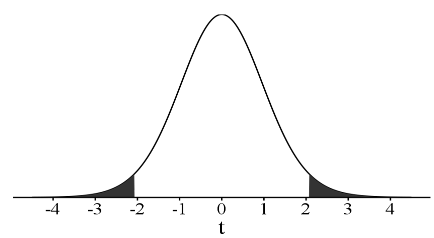


<u>Calculate</u>: The mean of the differences is 0.927, with a standard deviation of 1.366. The test statistic is $t = \frac{0.927}{1.366} \approx 2.25$. The p-value for the two-tailed t-test with 10 degrees of freedom is 0.0481.
<u>Conclude</u>: Since the p-value < α of 0.05, students can conclude that statistics problems with names tend to take longer than those without names.

11-12. a. $\frac{4}{23} \approx 17.39\%$

b. <u>Identify</u>: σ is unknown so use a one-sample t-interval.
<u>Check conditions</u>: Random selection, to avoid bias—given in problem. Independent trials, for an accurate σ—assumed that Anna has more than $n10 = 230$ friends. Large sample, so the sampling distribution is ≈ normal—the sample size is not larger than 30, however the t-distribution can be used if there is no evidence of strong outliers or skewing in the sample. The stem-and-leaf plot of the sample data shows neither.

22 degrees of freedom



<u>Calculate</u>: $\bar{x} = \$28.91$, $s = \$11.32$ g, df = 22, $t^* = 2.074$, $\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$.

$28.91 \pm (2.069)\frac{11.32}{\sqrt{23}} = 28.91 \pm 4.884$ $(24.02 < \mu < 33.79)$.
<u>Conclude</u>: We are 95% confident that the true average amount of money Anna's friends have is between \$24.02 and \$33.81.

11-13. a. $r = 0.9915$, indicating a strong positive linear association between the length versus mass of king cobras, which is confirmed by an inspection of the residual plot where there is no discernable pattern in the residuals.

b. $R^2 = 0.9830$, meaning that 98% of the variation observed in length is explained by a linear relationship with mass In other words, knowing the mass associated with each length reduces the error in predicting length by 98% over using just the mean length as a predictor.

c. When making predictions about king cobra lengths by using mass, one can expect to typically be off by 6.50 cm.