Lesson 9.1.3

9-21. Identify: Chi-squared goodness of fit test. H_0: The outcomes on the die each occur with $\frac{1}{6}$ probability. H_A: At least one outcome differs from the expected frequency. Sample evidence for H_A: observed frequencies do not match expected frequencies. $\alpha = 0.05$.
Check conditions: It can be assumed that this data represents a SRS of all possible rolls. Independent trials and large counts, so the sampling distribution is ≈ chi-squared. Each outcome is independent. Additionally, all expected counts are 100, well above five.
Calculate: χ^2 = sum(0.09, 0.49, 0.01, 0.16, 0.64, 0.81) = 2.2 with a p-value of 0.8208 (df = 5).
Conclude: Because the p-value is large (much higher than 5%), students do not have sufficient evidence to reject H_0. It cannot be concluded that the die is unfair.

9-22. Identify: Chi-squared goodness of fit test. H_0: The traits occur in the expected 9:3:3:1 ratio. H_A: At least one trait differs from the expected frequency. Sample evidence for H_A: observed frequencies do not match expected frequencies. $\alpha = 0.05$.
Check conditions: Random sample, to avoid bias. It can be assumed that this data represents a SRS of seeds. Independent trials and large counts, so the sampling distribution is ≈ chi-squared. Each outcome is independent. All expected counts (144, 48, 48, 16) are above 5.
Calculate: χ^2 = sum(0.6944, 0.1875, 1.6875, 1) = 3.5694 with a p-value of 0.3119 (df = 3).
Conclude: Because the p-value is large (much higher than 5%), students do not have sufficient evidence to reject H_0. It cannot be concluded that the observed traits differ significantly from the expected frequencies predicted by Mendelian genetics.

9-23. a. Identify: Chi-squared goodness of fit test. H_0: The current city population remains in the same proportion as the historical census. H_A: At least one continent now differs in its representation in this city. Sample evidence for H_A: Observed frequencies do not match expected frequencies. $\alpha = 0.05$.
Check conditions: Random sample, to avid bias. It is stated that this sample is a SRS. Independent trials and large counts, so the sampling distribution is ≈ chi-squared. The individuals can be considered independent because 500 is much less than 10% of the population of a "large city." All expected counts (65, 60, 140, 45, 175, 15) are above 5.
Calculate: χ^2 = sum(12.0615, 0.15, 0.4571, 0.0222, 1.4629, 0.2667) = 14.4204 with a p-value of 0.0131 (df = 5).
Conclude: Because the p-value is less than 5%, students have sufficient evidence to reject H_0. It be concluded that there has been a shift in the representation of at least one continent.

b. The continent of Asia had a contribution of 12.0615.

9-24. Identify: Chi-squared goodness of fit test. H_0: The digits in the decimal representation of $\sqrt{2}$ each occur with approximately a $\frac{1}{10}$ relative frequency. H_A: At least one digit differs from the expected frequency. Sample evidence for H_A: observed frequencies do not match expected frequencies. $\alpha = 0.05$.
Check conditions: Random sample, to avoid bias. Any bias found is evidence for H_A. Independent trials and large counts, so the sampling distribution is ≈ chi-squared. It can be assumed that these digits are all independent of one another. All expected counts (1000 each) are well above 5.
Calculate: χ^2 = sum(2.304, 0.025, 0.016, 0.4, 0.256, 0.001, 1.024, 1.296, 0.729, 0.361) = 6.4120 with a p-value of 0.6981 (df = 9).
Conclude: Because the p-value is high (much higher than 5%) we, do not have sufficient evidence to reject H_0. It cannot be concluded that the digits of $\sqrt{2}$ differ from the uniform distribution.

9-25. a. Expected value: \$3.802, standard deviation: \$5.425

b. \$599,173.60

c. Identify: Chi-squared goodness of fit test. H_0: The state is producing tickets at the advertised ratios. H_A: The state is not producing tickets at the advertised ratios. Sample evidence for H_A: observed frequencies do not match expected frequencies. $\alpha = 0.05$.
Check conditions: The sample was stated to be randomly gathered. Independent trials and large counts, so the sampling distribution is ≈ chi-squared. It is assumed that there were more than 500(10) tickets available. The sample size of 500 was large enough that all expected counts but one were larger than 5.
Calculate: χ^2 = 19.9975, df = 4, p-value = 0.0005.
Conclude: Because the p-value is so small (less than 5%), reject H_0. The evidence indicates that the state is not producing the tickets at the advertised ratios.

9-26. a. Identify: One-sample confidence interval of proportions.
Check conditions: Random, to avoid bias: SRS assumed, independent trials for an accurate σ: 297(10) < lake fish population assumed, large counts, so the sampling distribution is ≈ normal: np = 14 and 14 > 10.
Calculate: SE = 0.0123, find z* = 1.96, ME = 1.96(0.01213) = 0.0241, confidence interval = 0.0471 ± 0.0241, {0.0230, 0.0712}.
Conclude: We are 95% confident that the interval from 0.0230 to 0.0712 contains the true population proportion of tagged fish in the lake.

b. $\frac{150}{0.0230} = 6512$, $\frac{150}{0.0712} = 2106$. The interval (2106 to 6512) seems too wide to be of much use.

c. Increase the sample size of the fish and/or lower the confidence level.

9-27. See frequency table below. Multiply row by column to get numbers for events A, B, C, and D. Car alarms will sound in events A or D. That would be 99 + 8205 = 8304 alarms per year. Of those alarms only the 99 from event A are actual break-ins. If you hear a car alarm in Madelinton the probability the car is being broken into is just $\frac{99}{8304} \approx 0.0119$.

	100 are broken into	820,500 are *not* broken into
Alarm is correct 0.99	(A) 99	(B) 812,295
Alarm is *not* correct 0.01	(C) 1	(D) 8205 *false alarms*

