Lesson 10.3.1

10-39. Answers may vary. Most reasonable answers involve some form of simulation.

10-40. a. Red lines represent confidence intervals that did NOT capture the true mean of 0, while black lines represent the confidence intervals that did capture the true mean.

b. It should be quite close to a 0.95 proportion captured after a few thousand samples.

c. They are all the same width, since each one is using the same critical value (from the confidence interval) and standard deviation of the estimate (calculated using the true sigma).

d. As n gets higher, the confidence intervals get narrower for a given confidence level. As the confidence level gets higher, the confidence intervals get wider.

10-41. a. With a sample size of 10, the proportion captured should hover around 0.92—definitely lower than set confidence level!

b. Since each confidence interval is calculated using its own sample standard deviation, the widths vary.

c. Lower sample sizes are more “wrong”—trying with $n = 3$ tends to results in capture proportions around 80% or less! But $n = 100$ comes very close to the expected 0.95.

d. If the population standard deviation is known and used, then it is always reliable. If the sample size is very large, then the normal distribution is a reasonable approximation even using sample standard deviations, but for small n it is not a reliable method without the population standard deviation.

10-42. Increasing the critical value will increase the width of all confidence intervals. Some intervals that missed will now capture the true value.

10-43. a. They all get a bit wider, and some of the red ones probably turn black.

b. It should be! It should be quite close to 95%.

10-44. a. The shapes are very similar: bell-shaped, unimodal, symmetric, and the means/medians are both near 0. However, the t-statistic has much larger spread, with a standard deviation nearly three times as large as the z-statistic and a huge range! Visually, the t-statistic has much “thicker tails.”

b. The t-distribution in this case is much closer to the normal distribution—visually, they are almost impossible to tell apart, though if you look closely you can see the tails are still a bit “fatter” in the t-distribution than the normal distribution. The variance, standard deviation, and range are all larger in the t-distribution than in the z-distribution, but much smaller than when $n = 3$.

c. The critical value will be a little bit larger for the t-distribution with $n = 9$ and even larger for the one with $n = 3$.

d. You will need one fewer degree of freedom than the original sample size for the data from Lesson 10.2.1.

10-45. a. Lowest sample size is the one with the critical value of 3.18, then 2.57, then 2.03.

b. 1.96 is the value for a normal curve, and a t-distribution with a high enough degrees of freedom could be very close to that.

c. If $n = 20$, degrees of freedom = 19. You can use the invT function on most calculators with 0.025 as the input to find a critical value of 2.093. Table B, if provided by your teacher, makes it even easier!

d. You can use the tcdf function on most calculators to find this. The probability is about 0.056.

10-46. Using an inverse normal probability density function or table: $z_1 = -2.3264$, $z_2 = 0.95393$; 7.439 = mean + (–2.3264)stDev; 13.278 = mean + (0.95393)stDev. Solving the system of equations gives mean = 11.58 cm and standard deviation = 1.78 cm.

10-47. <u>Identify</u>: One sample test of proportion where H_0: $p = 0.68$, H_A: $p \neq 0.68$ and p is the population proportion of consumers that would answer "yes". Sample evidence for H_A: $\hat{p} = 0.75$. $\alpha = 0.05$ level assumed.
<u>Check conditions</u>: Random selection, to avoid bias: "random sample" was mentioned in the problem description with regards to sampling technique. Independent trials, for an accurate σ: it must be assumed that the population of interest is larger than ten times the sample size of 100. Large counts, so the sampling distribution ≈ normal: $np = 68$ and $n(1-p) = 32$, which are both at least 10.
<u>Calculate</u>: z = 1.5, p-value = 0.1335.
<u>Conclude</u>: p-value > 0.05, therefore you cannot reject the null hypothesis and are unable to show the local consumers are behaving different than the national average.

