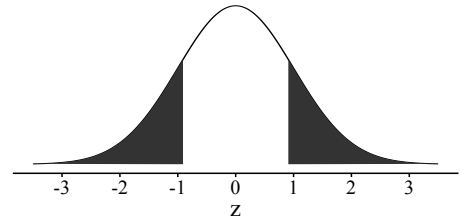


## Lesson 8.3.3

- 8-57. a. A two-proportion test, or a test on the difference of proportions.  $H_0: p_F - p_G = 0$ ,  $H_A: p_F - p_G \neq 0$ , where  $p_F$  and  $p_G$  are the population proportion of Faulder and Griswold field goals respectively. Sample evidence for  $H_A$ :  $\frac{105}{120} - \frac{100}{120} = 0.0417$ ,  $\alpha = 0.05$ .
- b. The number of successes and failures are 100, 20, 105, and 15, all greater than 10, satisfying the large counts condition so sampling distribution  $\approx$  normal. Each kick is independent of the others, as stated, satisfying independent trials for an accurate standard error. Neither coach is getting better or worse it seems reasonable to treat the kicks as a random, representative sample. And it seems likely that the two coaches' kicks have no effect on each other, so the samples are independent.
- c. The standard error is found by pooling the proportions to get  $\hat{p} = \frac{205}{240} \approx 0.854$ . Using this for both proportions yields  $SE_{\hat{p}_F - \hat{p}_G} \approx 0.046$ .  $z \approx 0.91$ ,  $p$ -value is 0.36. See diagram at right.
- d. Since the  $p$ -value of 0.36 is much higher than  $\alpha$  of 0.05, you fail to reject the null hypothesis. It is possible that there is no difference in the coaches' skill levels.



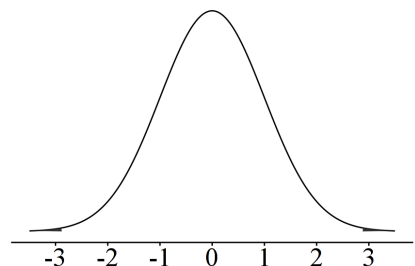
- 8-58.  $SE_{\hat{p}_F - \hat{p}_G} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = \sqrt{\frac{0.833(0.167)}{120} + \frac{0.875(0.125)}{120}} \approx 0.046$ . Confidence interval for the difference is  $0.042 \pm 0.09$ . Since the confidence interval contains 0, you cannot reject the claim that the coaches have the same skill level.

- 8-59. Identify: This is a two-proportion z-test with  $\alpha = 0.10$ . Let  $p_1$  = proportion of tall African elephants and  $p_2$  = proportion of tall Asian elephants.  $H_0: p_1 - p_2 = 0$ .  $H_A: p_1 - p_2 \neq 0$ . Sample evidence for  $H_A$ :  $\frac{173}{250} - \frac{112}{200} = 0.132$ .

Check conditions: These are clearly independent sample proportions. The problem statement assures randomness, so no bias. The success and failure counts are 173, 77, 112, and 88—all greater than 10, satisfying large counts for normal sampling distribution. Assume the elephant populations are larger than 2500 African elephants and 2000 Asian elephants so independent trials is satisfied for accurate standard error.

Calculate: The test statistic is  $z = 2.89$ ,  $p$ -value is 0.004. See diagram at right.

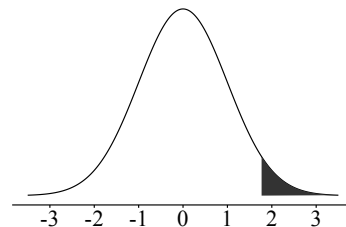
Conclude: since the  $p$ -value of 0.004 is less than  $\alpha = 0.10$ , you will reject the null hypothesis and conclude the proportion of  $\geq 10$  feet tall Asian elephants differs from the proportion of  $\geq 10$  feet tall African elephants.



- 8-60. Check conditions: The sample is mentioned as random. The actual counts of successes and failures are 159 and 41, both greater than 10.  $\hat{p} = \frac{159}{200} = 0.795$ . The medicine's behavior can reasonably be treated as independent. The standard error of the sampling distribution is 0.029, so the confidence interval is (0.739, 0.851).

Conclude: Since there are values below 0.74 in the confidence interval, this does not provide evidence that the new drug is more effective.

- 8-61.  $H_0: p = 0.74$ .  $H_A: p > 0.74$ . Sample evidence for  $H_A$ :  $\frac{159}{200} = 0.795$ . z-score: 1.77,  $p$ -value = 0.038. Since the  $p$ -value is less than  $\alpha = 0.05$ , you can reject the null hypothesis and conclude the new drug IS significantly more effective than the old one!



- 8-62. a. Yes. The  $p$ -value would have been doubled, giving a  $p$ -value of 0.076, higher than  $\alpha$ .
- b. Create a 90% confidence interval. That way, one of the two tails has 5% of the space in it, to match up with the 0.05 probability that is covered by the rejection region of the hypothesis test.
- c. The new confidence interval is (0.748, 0.842), so 0.74 is not in the interval and you can accept the hypothesis that the new drug is better. Confirmed!
- d.  $\alpha = 0.025$ . The  $p$ -value of 0.038 would have been higher than  $\alpha$  in that case, so no rejection would occur, matching the 95% confidence interval.
- 8-63. a. 80% confidence interval
- b. 98% confidence interval
- c. 99% confidence interval
- d. two-tailed test,  $\alpha = 0.15$  OR one-tailed test,  $\alpha = 0.075$
- e. two-tailed test,  $\alpha = 0.001$  OR one-tailed test,  $\alpha = 0.0005$

8-64. Identify: Given:  $\hat{p} = 0.5188$ ,  $x = 235$ ,  $n = 453$ , significance = 0.04. Perform a one proportion z-test where  $p$  is: The proportion of people having blue eye coloring.

$H_0: p = 0.58$ ,  $H_A: p \neq 0.58$ , Sample evidence for  $H_A$ :  $\hat{p} = 0.5188$ .

Check conditions: Random selection, to avoid bias: a sampling technique using randomness was mentioned in the problem description. Independent trials, for an accurate

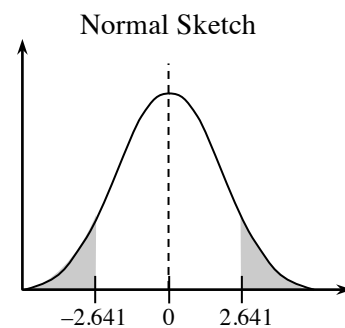
$\sigma$ : it must be assumed that the population of interest is larger than ten times the sample size of 453. Large counts, so the sampling distribution  $\approx$  normal:

$np = 263$ , and  $n(1 - p) = 190$ , which are both at least 10.

Calculate:  $SE = \sqrt{\frac{0.58(1-0.58)}{453}} \approx 0.0232$  people,

$z = \frac{0.5188-0.58}{0.0232} \approx -2.641$ ,  $p\text{-value} = 2 \cdot \text{normalcdf}(2.641, 1E99, 0, 1) = 0.0083$ . See diagram at right.

Conclude: Because the  $p$ -value is less than the significance level of the test, Malik can reject the null hypothesis and validate Imani's claim that "the proportion of people having blue eye coloring is not equal to 0.58" with 96% confidence.



8-65. To simulate each trip you could use a calculator or random number table to produce random digits in groups of four from 0 to 9. randInt(0, 9, 4). Consider each digit to be a leg of his journey with zeros and ones to represent delays. Simulated journeys would look like {5 8 1 1}, {4 9 6 0}, {7 6 7 3}, etc., with each "leg" having a 20% chance of a "delay." Continue generating simulated trips (about 30 to 50) until you feel you have an accurate proportion of missed cruise ships out of total trip attempts. In this example, 5 missed cruise ships out of 28 trips is  $\frac{5}{28} = 0.179$ . The theoretical probability is 0.1808 but simulated probabilities  $\approx 0.07$  to 0.30.

(5 7 5 4)	ok	(2 7 2 1)	ok	(6 9 9 2)	ok	(7 9 8 9)	ok
(9 8 8 3)	ok	(9 0 7 9)	ok	(8 2 7 5)	ok	(6 9 6 3)	ok
(0 1 2 9)	miss	(7 0 9 4)	ok	(8 7 8 0)	ok	(8 2 0 9)	ok
(8 5 0 3)	ok	(8 2 7 3)	ok	(5 1 1 1)	miss	(5 7 4 8)	ok
(6 0 4 5)	ok	(3 7 9 9)	ok	(1 0 0 0)	miss	(9 9 0 0)	ok
(8 3 9 9)	ok	(4 5 7 9)	ok	(6 0 1 6)	miss	(6 1 2 0)	ok
(8 3 0 4)	ok	(1 5 0 4)	miss	(8 4 7 0)	ok	(9 1 5 8)	ok