

**Indian Institute of Technology Guwahati**  
**Mathematical Statistics (MA212M)**  
**Problem Set 10**

1. A sample of size 25 from a normal population with variance 81 produced a sample mean 81.2. Find a 95% confidence interval for the mean. Ans: [77.67, 84.73].
2. A sample of size 25 from a normal population produced a sample mean 81.2 and sample variance 81. Find a 95% confidence interval for the mean. Ans: [77.48, 84.96].
3. Let  $\bar{X}$  be the mean of a random sample of size  $n$  from  $N(\mu, 16)$ . Find the smallest sample size  $n$  such that  $(\bar{X} - 1, \bar{X} + 1)$  is a 90% confidence interval for  $\mu$ . Ans: 44.
4. Let  $X_1, X_2, \dots, X_m$  and  $Y_1, Y_2, \dots, Y_n$  be independent random samples from  $N(\mu_1, \sigma^2)$  and  $N(\mu_2, \sigma^2)$ , respectively. Find a  $100(1 - \alpha)\%$  confidence interval for  $\mu_1 - \mu_2$  when (a)  $\sigma$  is known, and (b)  $\sigma$  is unknown.
5. Two independent samples, each of size 7, from normal populations with common unknown variance  $\sigma^2$  produced sample means 4.8 and 5.4 and sample variances 8.38 and 7.62, respectively. Find a 95% confidence interval for the difference between the means of samples 1 and 2. Ans: [-3.89, 2.69].
6. Let  $X_1, X_2, \dots, X_n$  be identically and independently distributed random variables with common probability density function

$$f(x; \theta) = e^{-(x-\theta)} I_{(\theta, \infty)}(x).$$

Find a pivot and construct  $100(1 - \alpha)\%$  lower, upper, and symmetric confidence interval. Also find the smallest length  $100(1 - \alpha)\%$  confidence interval for  $\theta$  based on the pivot.

7. For a sample of size one from the population

$$f(x; \theta) = \frac{2}{\theta^2}(\theta - x) I_{(0, \theta)}(x).$$

Find the  $100(1 - \alpha)\%$  symmetric confidence interval for  $\theta$  using pivotal technique.