Indian Institute of Technology Guwahati
Mathematical Statistics (MA212M)
Problem Set 11

1. Let $X_1, X_2, \ldots, X_n$ be a random sample form a $N(\mu, \sigma^2)$ distribution, where σ is known.
   (a) Find MP level α test for $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$, where $\mu_1 < \mu_0$.
   (b) Find UMP level α test for $H_0 : \mu = \mu_0$ against $H_1 : \mu < \mu_0$.

2. Let $\phi(\cdot)$ be a most powerful level α test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. Then show that $\beta(\theta_0) \leq \beta(\theta_1)$, where $\beta(\cdot)$ is the power function of the most powerful test.

3. Let X_1 and X_2 be a random sample of size two from a probability density function $f(x)$, $x \in \mathbb{R}$. Consider the following two functions

$$f_0(x) = \frac{3}{64}x^2 I_{(0,\,4)}(x) \qquad \text{and} \qquad f_1(x) = \frac{3}{16}\sqrt{x} I_{(0,\,4)}(x).$$

   Determine the most powerful level α test for testing $H_0 : f(x) = f_0(x)$ against $H_1 : f(x) = f_1(x)$.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $\mathrm{P}(\lambda)$, where $\lambda > 0$. Find the most powerful level α test for $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$.

5. Suppose that $X_1, X_2, \ldots, X_n$ are *i.i.d.* random variables form a exponential probability density function

$$f(x;\, \theta) = \theta^{-1} e^{-x/\theta} I_{(0,\,\infty)}(x),$$

   where $\theta > 0$ is assumed unknown. With preassigned $\alpha \in (0,\, 1)$, derive a level α likelihood ratio test for $H_0 : \theta = \theta_0 (> 0)$ against $H_1 : \theta \neq \theta_0$.