

**Indian Institute of Technology Guwahati**  
**Mathematical Statistics (MA212M)**  
**Problem Set 09**

1. Let  $X_1, X_2, \dots, X_n$  be a random sample from  $U(\theta_1, \theta_2)$ ,  $-\infty < \theta_1 < \theta_2 < \infty$ . Find moment estimator of  $(\theta_1, \theta_2)$ .
2. A sample  $(X_1, \dots, X_{10})$  is drawn from a distribution with a probability density function

$$\frac{1}{2} \left( \frac{1}{\theta} e^{-x/\theta} + \frac{1}{10} e^{-x/10} \right), \quad 0 < x < \infty.$$

The sum of all 10 observations equals 150. Estimate  $\theta$  by the method of moments.

3. Let  $X_1, \dots, X_n$  be a random sample from the probability density function

$$f(x; \theta) = \frac{1}{2} e^{-|x-\theta|} \quad \text{for } x \in \mathbb{R},$$

where  $\theta \in \mathbb{R}$ . Find the maximum likelihood estimator of  $\theta$ .

4. Estimate the unknown parameter  $\theta$  from a sample  $\{3, 3, 3, 3, 3, 7, 7, 7\}$  drawn from a discrete distribution with the probability mass function

$$P(3) = \theta, \quad P(7) = 1 - \theta.$$

Obtain MME and MLE of  $\theta$ .

5. Let  $X_1, \dots, X_n$  be a random sample from the probability mass function

$$f(x; \theta) = \begin{cases} \frac{1-\theta}{2} & \text{if } x = 1 \\ \frac{1}{2} & \text{if } x = 2 \\ \frac{\theta}{2} & \text{if } x = 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the maximum likelihood estimator of  $\theta \in (0, 1)$ .

6. Let  $X_1, X_2, \dots, X_n$  be a sample from probability mass function

$$P(X = k) = \begin{cases} \frac{1}{N} & \text{if } k = 1, 2, \dots, N \\ 0 & \text{otherwise,} \end{cases}$$

where  $N$  is a positive integer. Find the maximum likelihood estimator of  $N$ .

7. Suppose we want to estimate the number of fishes in a pond. The following procedure is followed to perform the estimation. First  $M$  fishes are caught from the pond, tagged and returned to the pond. Next  $n$  fishes are caught at random out of which  $x$  fishes are tagged. Based on this data, find the maximum likelihood estimator of the total number of fishes present in the pond. Hint: You should not differentiate the likelihood function with respect to  $N$ , as  $N$  is integer valued and  $N \geq \max\{M, n\}$ .
8. Let  $X_1, X_2, \dots, X_n$  be a random sample on the lifetime of an integrated circuit. Let the lifetime of the integrated circuit has the probability density function

$$f(x, \theta) = \begin{cases} 2\lambda x e^{-\lambda x^2} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Let  $\tau > 0$  be a known time, and  $X$  be the number of integrated circuits that fail before  $\tau$ . Find the maximum likelihood estimator of the variance of  $X$ .