Introduction to Superconducting Quantum Circuits

- Review of Quantum Harmonic/Anharmonic Oscillators -

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Lecture Overview

Week 1. Introduction to Superconducting Quantum Circuits	overall backgrounds, terminologies of quantum computing
Week 2. Review of Mathematics and Microwave Engineering Week 3. Review of Classical and Quantum Mechanics Week 4. Review of Superconductivity	mathematical and engineering backgrounds general superconductivity
Week 5. Quantum Harmonic/Anharmonic Oscillators and Light-Matter Interaction Week 6. Circuit Quantization Methods Week 7. Parametrically Pumped Josephson Devices	Quantum circuit analysis
Week 8. Design and Analysis of Superconducting Resonators Week 9. Design and Analysis of Superconducting Qubits Week 10. Design and Analysis of Single-Qubit Device: 3D Cavity Week 11. Design and Analysis of Single-Qubit Device : 2D Chip Week 12. Design and Analysis of Two-Qubit Device Week 13. Design and Analysis of Josephson Parametric Amplifier	design and analysis of superconducting RF devices
Week 14. Term Project	
Week 15. Term Project	

Keywords in Quantum Harmonic/Anharmonic Oscillators

<u>Superconducting Resonator: Quantum Harmonic Oscillator</u>

Generalized Coordinates	Node and Branch
Canonical Quantization	Zero-point fluctuation
Eigenenergy	

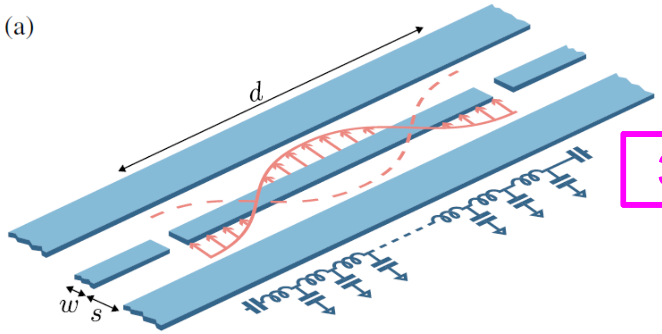
<u>Superconducting Qubit: Quantum Anharmonic Oscillator</u>

Cooper Pair Box	Transmon Qubit
Tunable Transmon Qubit	Flux Qubit
Transmon Regime	Weakly Anharmonic Qubit
Josephson Energy	Charging Energy

Introduction to Superconducting Resonator

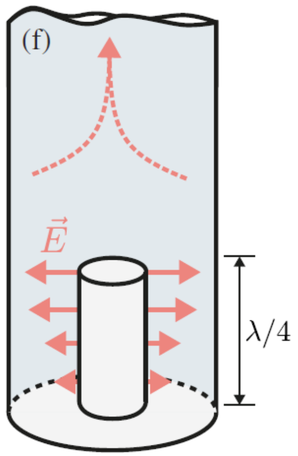
- **What Is Superconducting Resonator?**
  - A component designed to store and confine electromagnetic energy at specific microwave frequencies with minimal loss
  - Made from superconducting materials, it exhibits extremely high quality factors Q

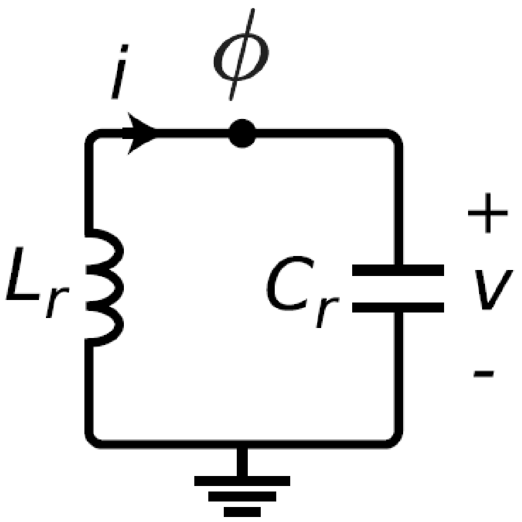
- **Equivalent Circuit Diagram for a Superconducting Resonator**
  - Whether a resonator is 3D cavity or planar layout, an equivalent circuit diagram is a parallel LC circuit



Planar coplanar-waveguide (CPW) resonator

3D cavity resonator





Q. How to describe a superconducting resonator with quantum mechanics?

A. Set the generalized coordinates and perform canonical quantization!

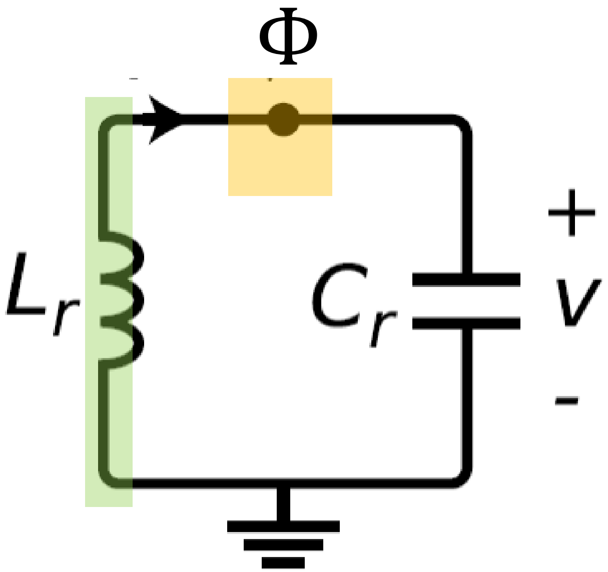
Image from A. Blais, AL Grimsmo, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021).

Superconducting Resonator as a Quantum Harmonic Oscillator (1/3)

Generalized Coordinates and Canonical Conjugate Momentum of a Superconducting Resonator

- Recall the standard methods to obtain the canonical conjugate momentum from the Lagrangian
- Let the position q, the velocity $\dot{q}$, the kinetic energy T and the potential energy U, the Lagrangian is $\mathcal{L}(q, \dot{q}, t) = T - U$
- The canonical conjugate momentum is $p = \frac{\partial \mathcal{L}}{\partial \dot{q}}$

Node: a point between two or mode elements



Branch: a portion of the circuit and its two terminal nodes

Table. Analogy between electrical circuit and quantum mechanics.

Electrical circuit	Mechanics
Circuit node flux $\Phi = \int_{-\infty}^{t} V(t')dt'$	Position x
Circuit node voltage $V = \frac{d\Phi}{dt}$	Velocity v
Circuit node charge $Q = CV = C\frac{d\Phi}{dt}$	(canonical conjugate) Momentum p
Inductive energy $E_L = \frac{1}{2}LI^2 = \frac{\Phi^2}{2L}$	Potential energy U
Capacitive energy $E_c = \frac{1}{2}CV^2 = \frac{1}{2}C\dot{\Phi}^2$	Kinetic energy T

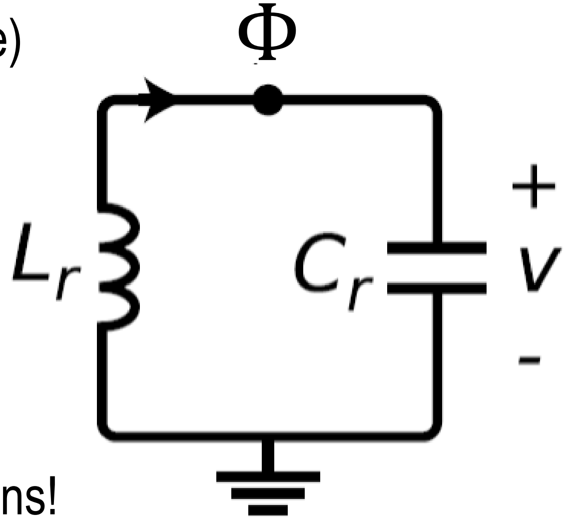
From electrical circuit variables, we can choose *node flux* as *position* and *node voltage* as *velocity* to perform the standard canonical quantization method

Image from P Krantz *et al.*, “A quantum engineer’s guide to superconducting qubits,” *Appl. Phys. Rev.*, **6,** 021318 (2019).

Superconducting Resonator as a Quantum Harmonic Oscillator (2/3)

- **Lagrangian of a Superconducting Resonator**
  - For a parallel LC circuit, we can define a node flux Φ as a generalized position coordinate
  - Then, a node voltage V can be assumed to be a generalized velocity (as shown in previous slide)
  - The Lagrangian of the LC circuit can be derived as

$$\mathcal{L}(q,\dot{q},t) = \mathcal{L}(\Phi,\dot{\Phi},t) = T - U$$
$$= E_c - E_L = \frac{1}{2}C\dot{\Phi}^2 - \frac{\Phi^2}{2L}$$



  - From the Lagrangian, we can define a canonical conjugate momentum as

$$p = \frac{\partial\mathcal{L}(\Phi,\dot{\Phi},t)}{\partial\dot{\Phi}} = C\dot{\Phi} = CV = Q$$ ← in electrical circuit relations!

- **Hamiltonian of a Superconducting Resonator**
  - Recall that the Hamiltonian can be derived by Legendre transform as $\mathcal{H} = p\dot{q} - \mathcal{L}$
  - Thus, the Hamiltonian of the LC circuit can be derived as

$$\mathcal{H} = \dot{\Phi}Q - \mathcal{L}$$
$$= \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

classical

replacing classical variables with quantum operators

(canonical commutation relation, Dirac method)

$$\mathcal{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

quantum

Superconducting Resonator as a Quantum Harmonic Oscillator (3/3)

Superconducting Resonator as a Quantum Harmonic Oscillator

- The Hamiltonian $\mathcal{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$ of a resonator (parallel *LC* circuit) is similar to that of a harmonic oscillator
- Using the definition of ladder operators, $\hat{a}$ and $\hat{a}^\dagger$

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right) \text{ and } \hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$$

m: mass (for LC circuit, $m = C$)
ω: resonant frequency (for LC circuit, $\omega_r = \frac{1}{\sqrt{L_r C_r}}$)
$\hat{a}$: annihilation operator (lowering operator)
$\hat{a}^\dagger$: creation operator (raising operator)
$Z_r = \sqrt{\frac{L_r}{C_r}}$: characteristic impedance

- The Hamiltonian of a resonator can be simplified as

$$\hat{\Phi} = \sqrt{\frac{\hbar Z_r}{2}}(\hat{a}^\dagger + \hat{a}) = \Phi_{\mathrm{ZPF}}(\hat{a}^\dagger + \hat{a})$$

$$\hat{Q} = i\sqrt{\frac{\hbar}{2Z_r}}(\hat{a}^\dagger - \hat{a}) = Q_{\mathrm{ZPF}}(\hat{a}^\dagger - \hat{a})$$

$$\boxed{\therefore \mathcal{H} = \hbar\omega_r\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)}$$

Note that
$[\hat{\Phi}, \hat{Q}] = \hat{\Phi}\hat{Q} - \hat{Q}\hat{\Phi} = i\hbar$

n^{th} eigenenergy is $\hbar\omega_r\left(n + \frac{1}{2}\right)$
The energy spacing is equidistant

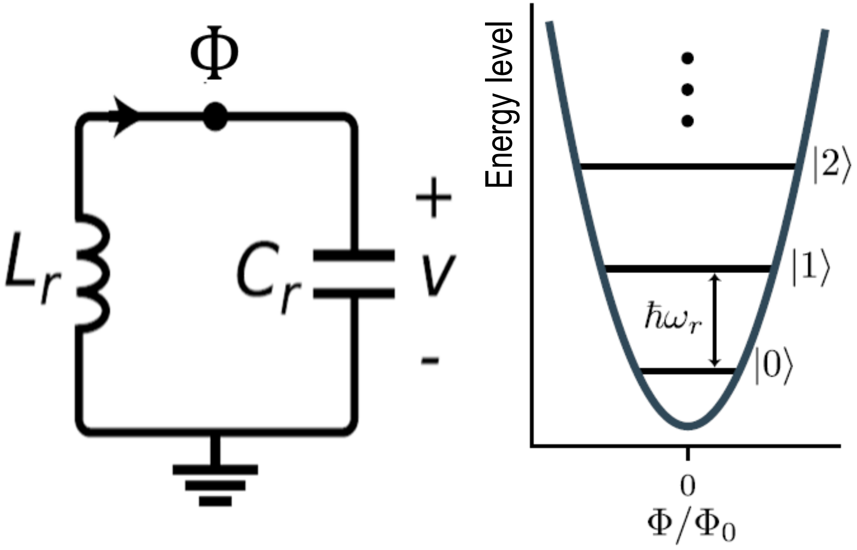


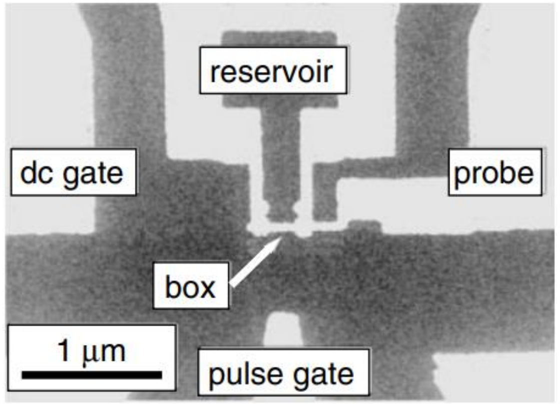
Fig. Circuit diagram and harmonic energy level vs flux of the circuit.

Image from A. Blais, AL Grimsmo, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021).

Introduction to Superconducting Qubit

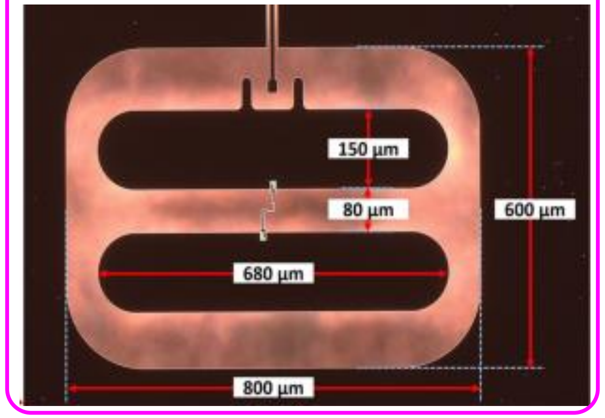
- What Is Superconducting Qubit?
  - A superconducting qubit is a quantum two-level system implemented using Josephson junctions
  - The first superconducting qubit was realized as Cooper pair box *see Y. Nakamura *et al*., *Nature* **398**, 786-788 (1999).
- Key characteristics of Superconducting Qubits
  - Superconductivity: low coherence loss and macroscopic quantum object
  - Josephson junction: core component of a qubit to provide the nonlinearity needed to define discrete energy levels
- Types of Superconducting Qubits

Cooper pair box (CPB)



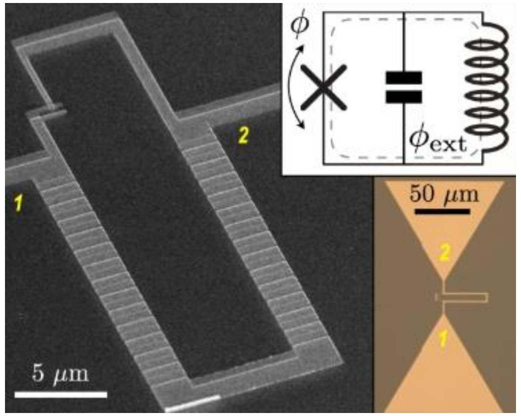
Y. Nakamura *et al*., *Nature* **398** , 786-788 (1999)

Transmon



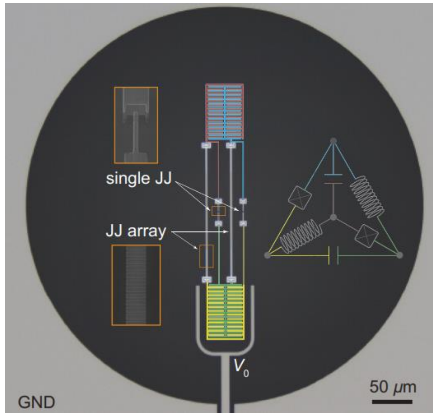
C Wang *et al*., *npj Quantum Inf.*, **8**, 3 (2022)

Fluxonium



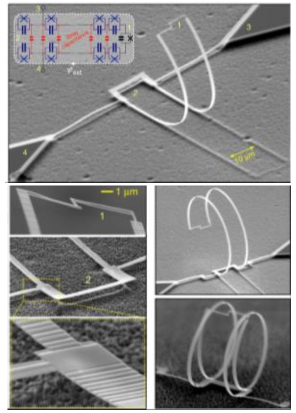
N Cottet *et al*., *Nat. Commun.*, **12**, 6383 (2021)

$0 - \pi$ qubit



A Gyenis *et al*., *PRX Quantum*, **2,** 010339 (2021)

Blochnium



IV Pechenezhskiy *et al*., *Nature*, **585**, 368-371 (2020)

Transmon qubit is well-established qubit architecture and one of the most popular candidate to realize a quantum computer

Selected Milestones in Transmon-Based Research (1/4)

■ Development of Cooper-pair Box (1999)

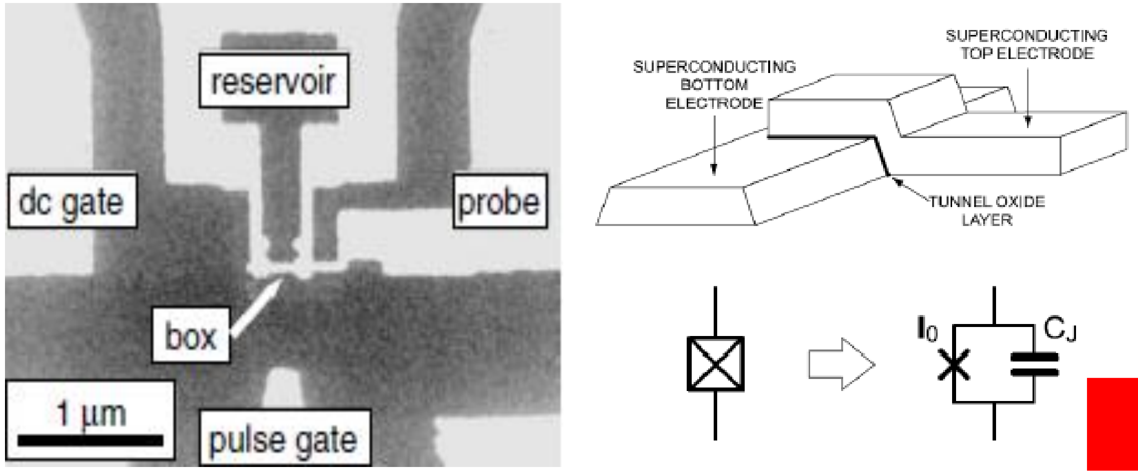


Fig. The first realization of Cooper-pair box and its components

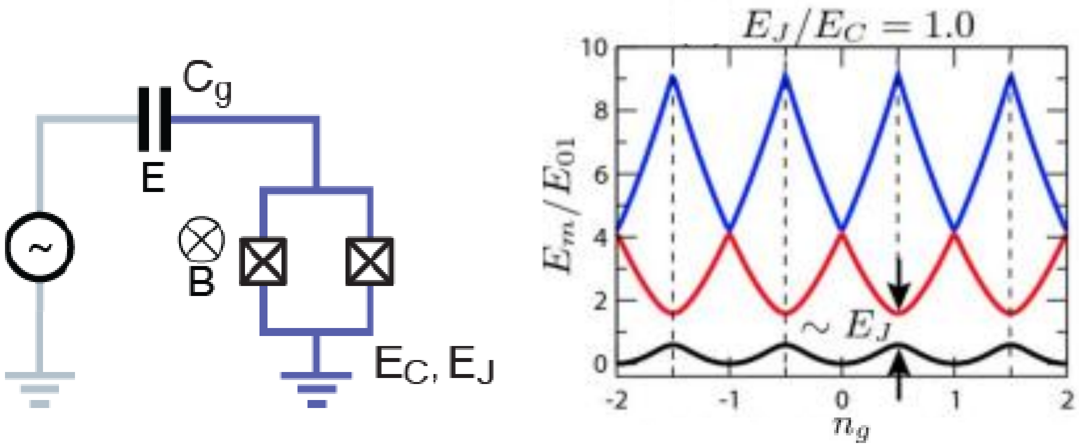


Fig. Circuit diagram and energy level diagram of a typical Cooper-pair box

■ Development of Transmon Qubit (2007)

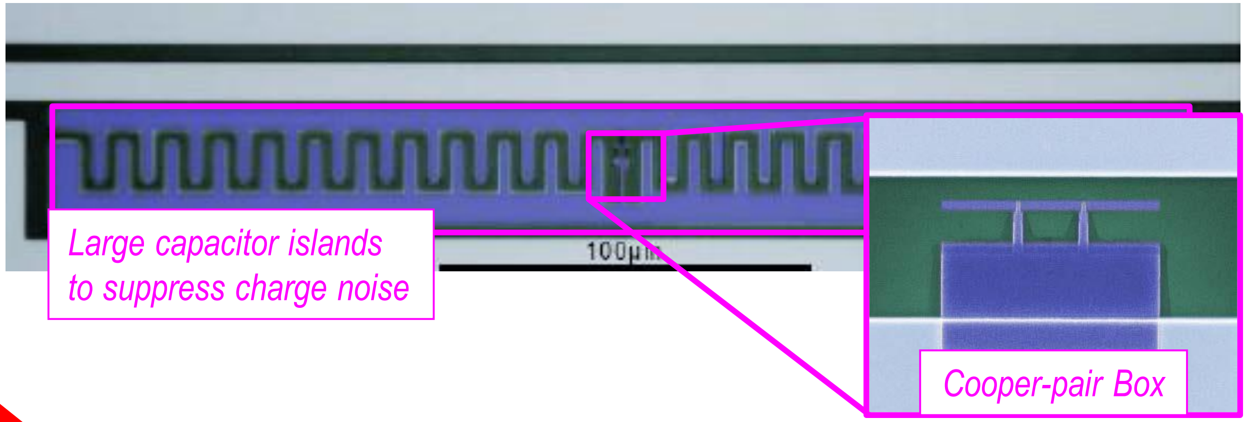


Fig. An optical image of a transmon qubit

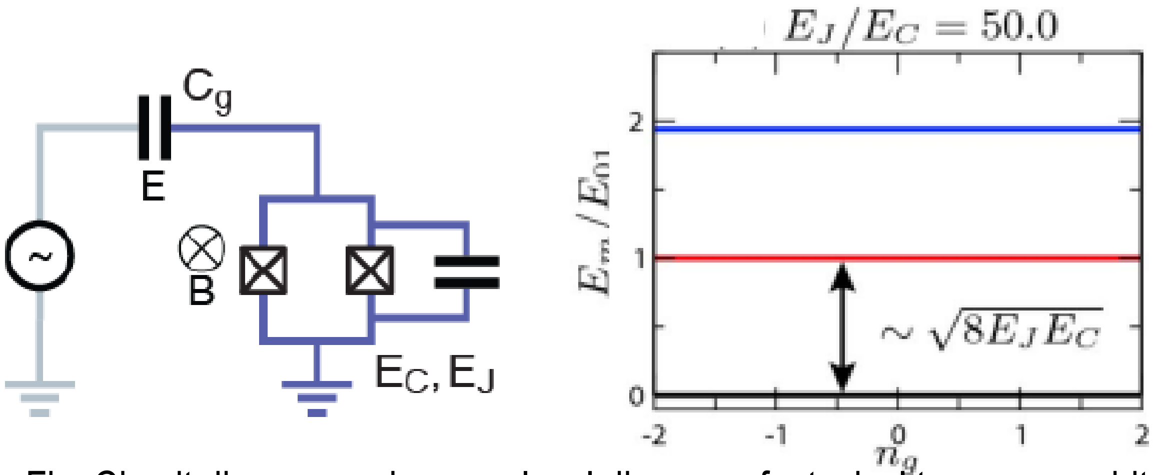
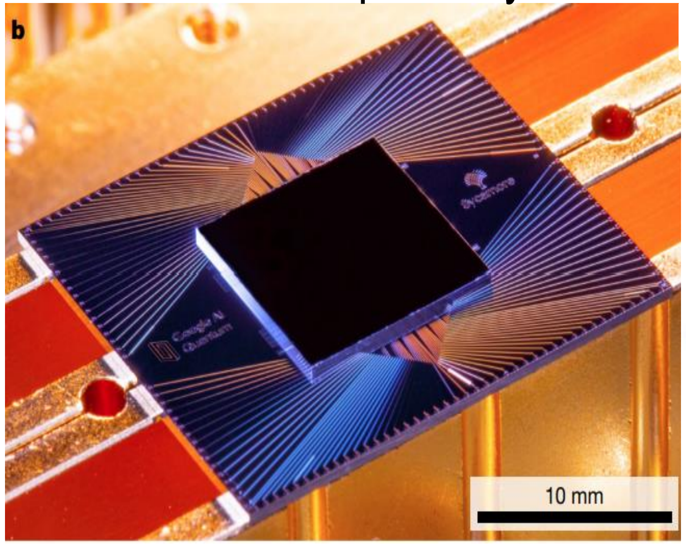


Fig. Circuit diagram and energy level diagram of a typical transmon qubit

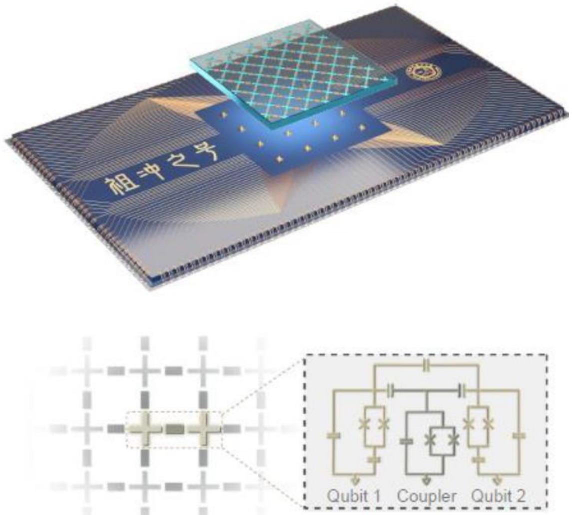
Y. Nakamura, Y.A. Pashkin, J.S. Tsai, "Coherent control of macroscopic quantum states in a single-Cooper-pair box," *Nature* **398** , 786-788 (1999)
J. Koch, *et al.*, "Charge-insensitive qubit design derived from the Cooper pair box," *Phys. Rev. A*, **76**, 042319 (2007).

- Demonstration of Quantum Supremacy with Superconducting Qubits (2019)
  - Quantum supremacy: no classical computer can solve in any feasible amount of time



< Google, 53-qubit >

Claimed their 200 seconds work will take at least 10,000 years with supercomputer



< Hefei, 66-qubit >

Claimed their 6000 seconds work will take at least 8 years with supercomputer

However, Google and Hefei performed "RANDOM circuits sampling" for benchmarking, which has nothing to do with solving practical problems…

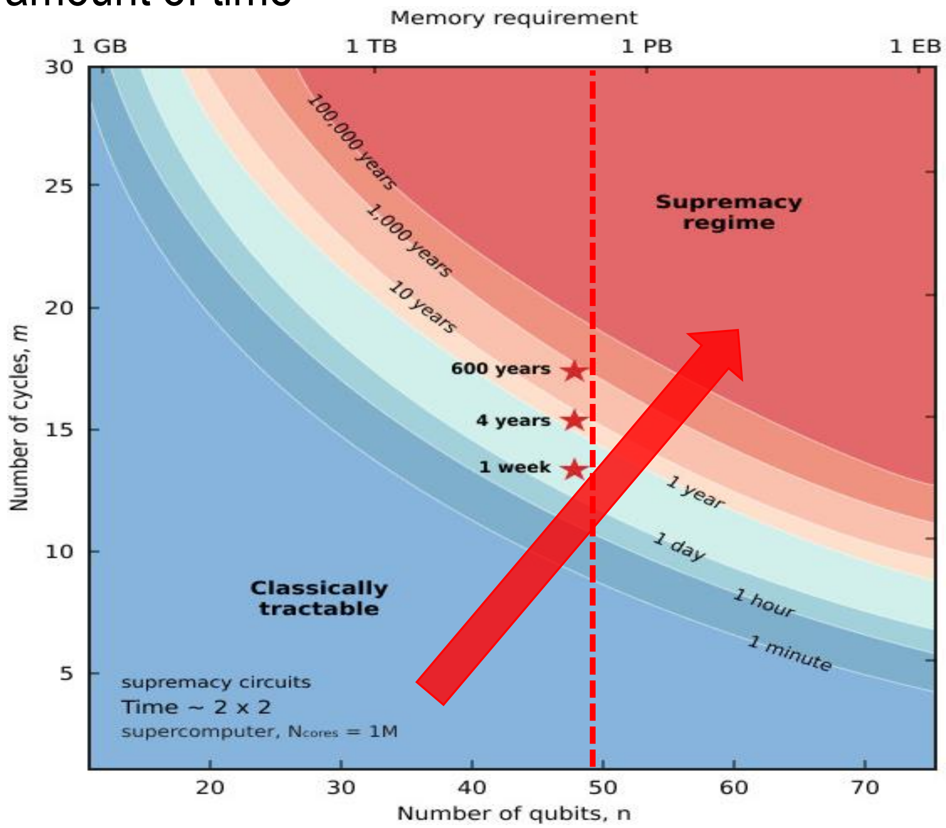


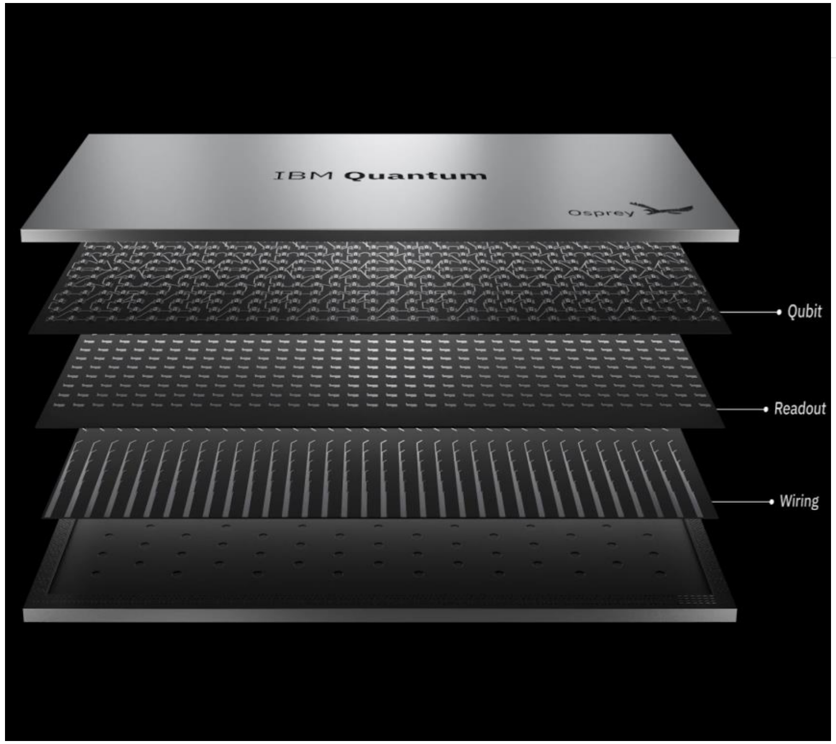
Fig. Scaling of the computational cost of cross entropy benchmarking with classical supercomputer.

F. Arute, *et al.*, "Quantum supremacy using a programmable superconducting processor," *Nature*, **574**, 505–510 (2019).
Y. Wu, *et al.*, "Strong quantum computational advantage using a superconducting quantum processor," *Phys. Rev. Lett.*, **127** 180501 (2021).

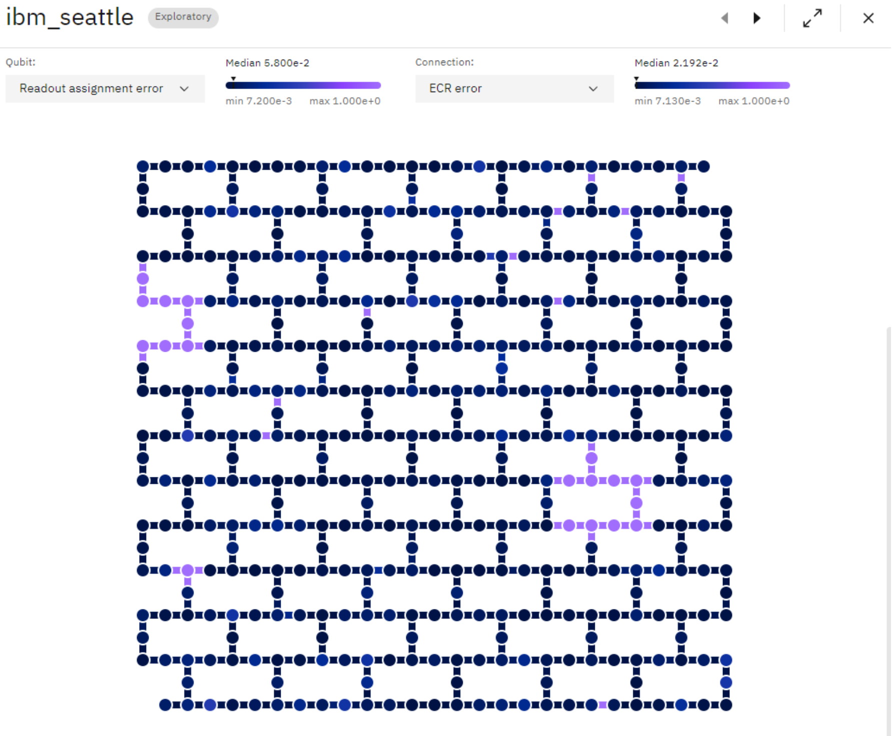
Selected Milestones in Transmon-Based Research (3/4)

- IBM Quantum Computer Breaks the 100-Qubit Barrier (2021)
  - Following the first quantum computer with more than 100-qubit back in 2021: “IBM 127-Q”

(a)



(b)



(c)

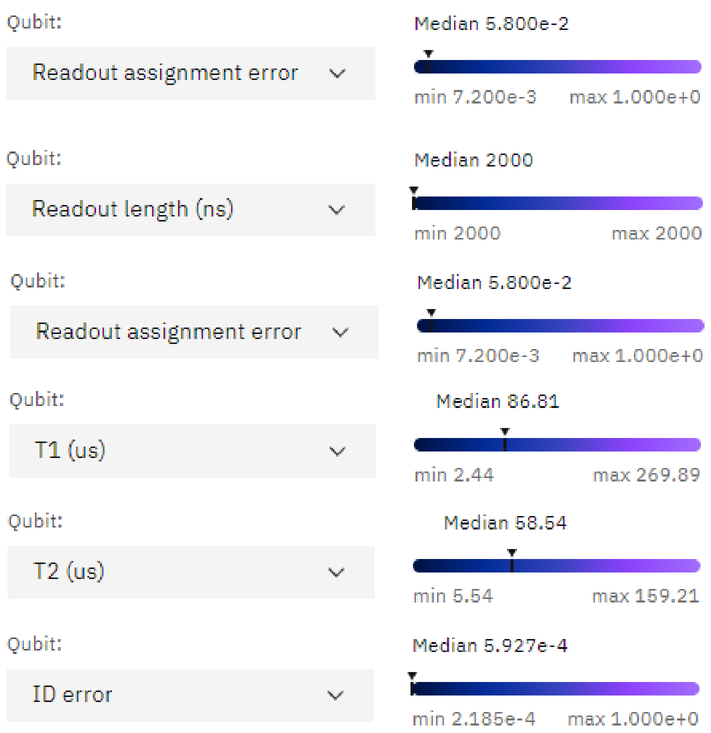
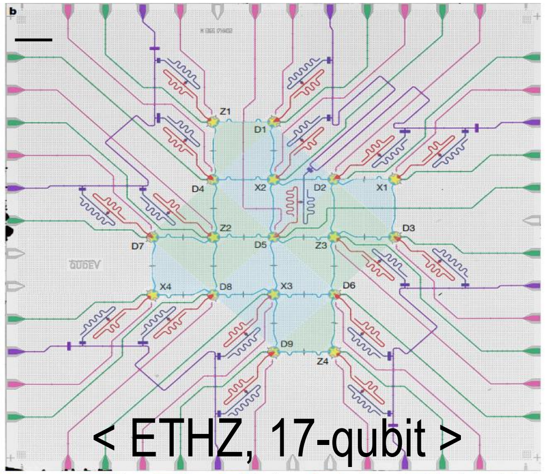


Fig. (a) Rendered image of IBM 433-Q compute; (b) connectivity of IBM 433-Q; (c) key parameters. [*access date: 2023/06/20]

J. Chow, O. Dial, J. Gambetta, “IBM Quantum breaks the 100-qubit processor barrier,” *IBM Research Blog*, 2021.
https://quantum-computing.ibm.com/services/resources?tab=systems&system=ibm_seattle

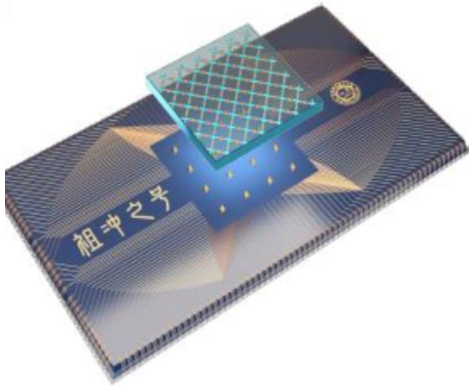
Selected Milestones in Transmon-Based Research (4/4)

- Demonstration of Quantum Error Correction with Superconducting Qubits (2022~2024)
  - The first demonstration of quantum error correction by increasing the size of the error correction protocol



< ETHZ, 17-qubit >

< Google, 72-qubit >



< Hefei, 66-qubit >

ETHZ: distance-3 code
Google: distance-3 code
Google: distance-5 code
Hefei: distance-3 code

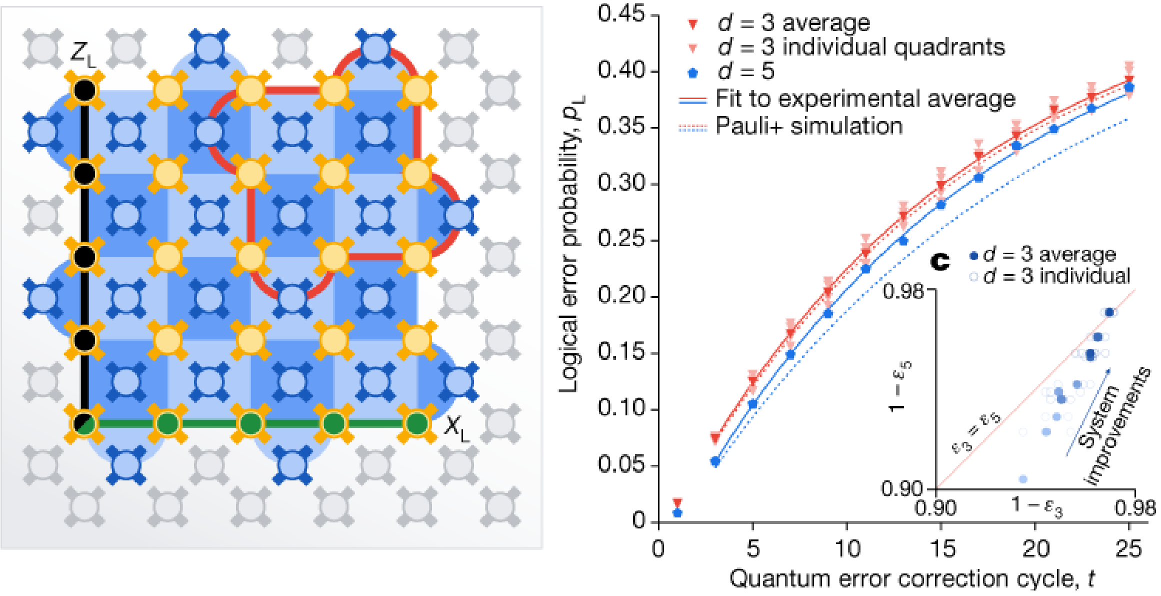


Fig. A schematic of the Google's quantum computer and the logical error rate for different surface code protocol size.

Currently available quantum computers are noisy, necessitating the development of quantum error correction!

S. Krinner, *et al.*, "Realizing repeated quantum error correction in a distance-three surface code," *Nature*, **605**, 669-674 (2022).
Y Zhao, *et al.*, "Realization of an error-correcting surface code with superconducting qubits," *Phys. Rev. Lett.*, **129**, 030501 (2022).
Google Quantum AI, "Suppressing quantum errors by scaling a surface code logical qubit," *Nature*, **614**, 676-681, (2023).

Progress of Superconducting Qubits and Quantum Computers

- Performance Metrics of Currently Available Quantum Computers
  - State-of-the-art superconducting quantum computers: Noisy Intermediate Scale Quantum (NISQ)

FTQC: Fault-tolerant Quantum Computer

NISQ: Noisy Intermediate Scale Quantum

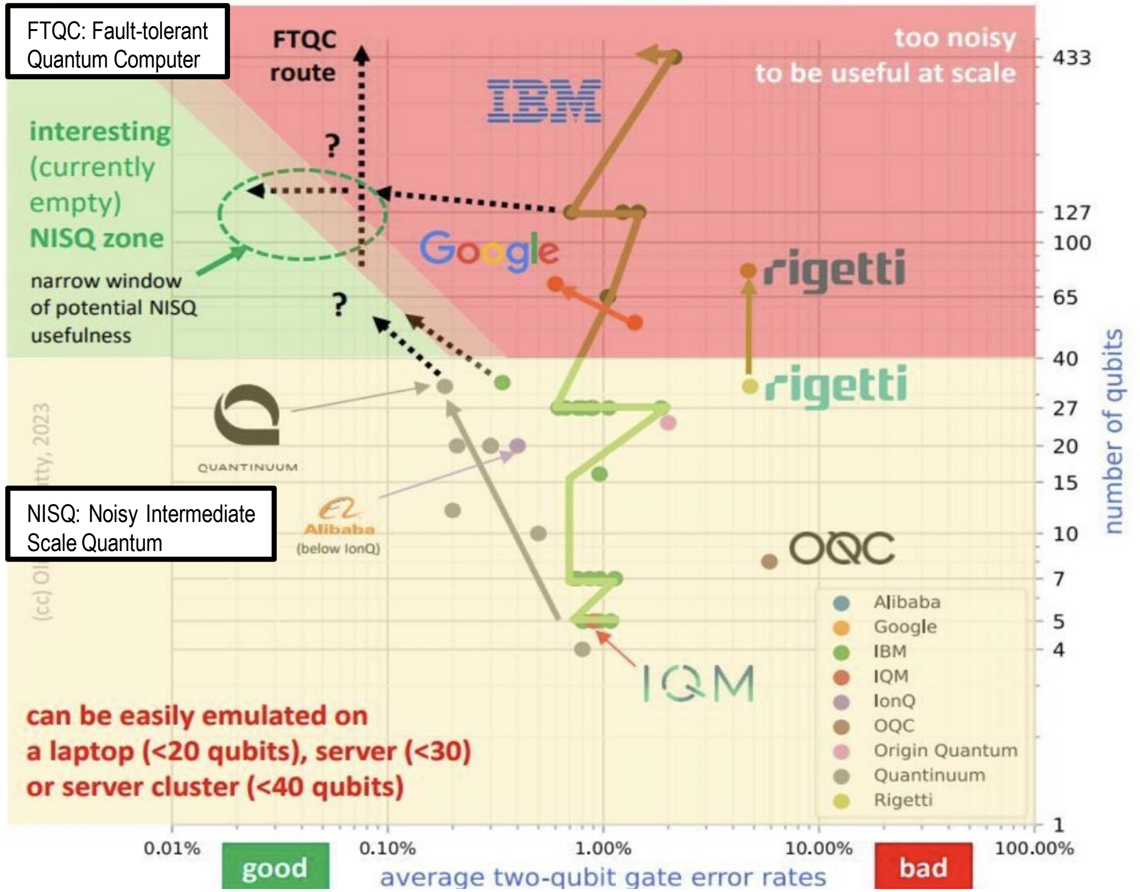


Fig. Progress of quantum computers with two-qubit gate error and the number of qubits

Unofficial record of State-of-the-art T_1 > 1 ms (2023)

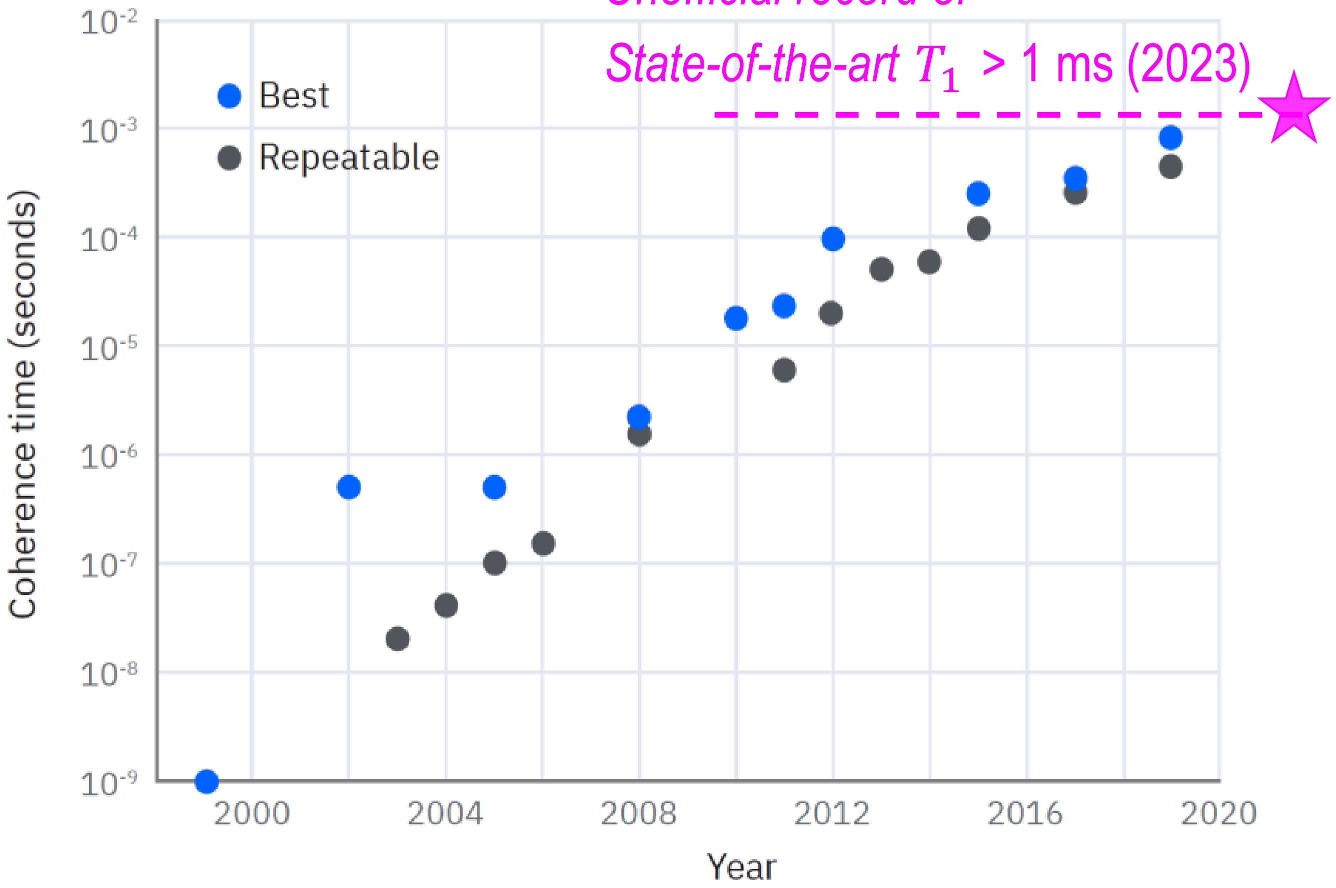


Fig. Historical improvement in coherence time of superconducting qubits

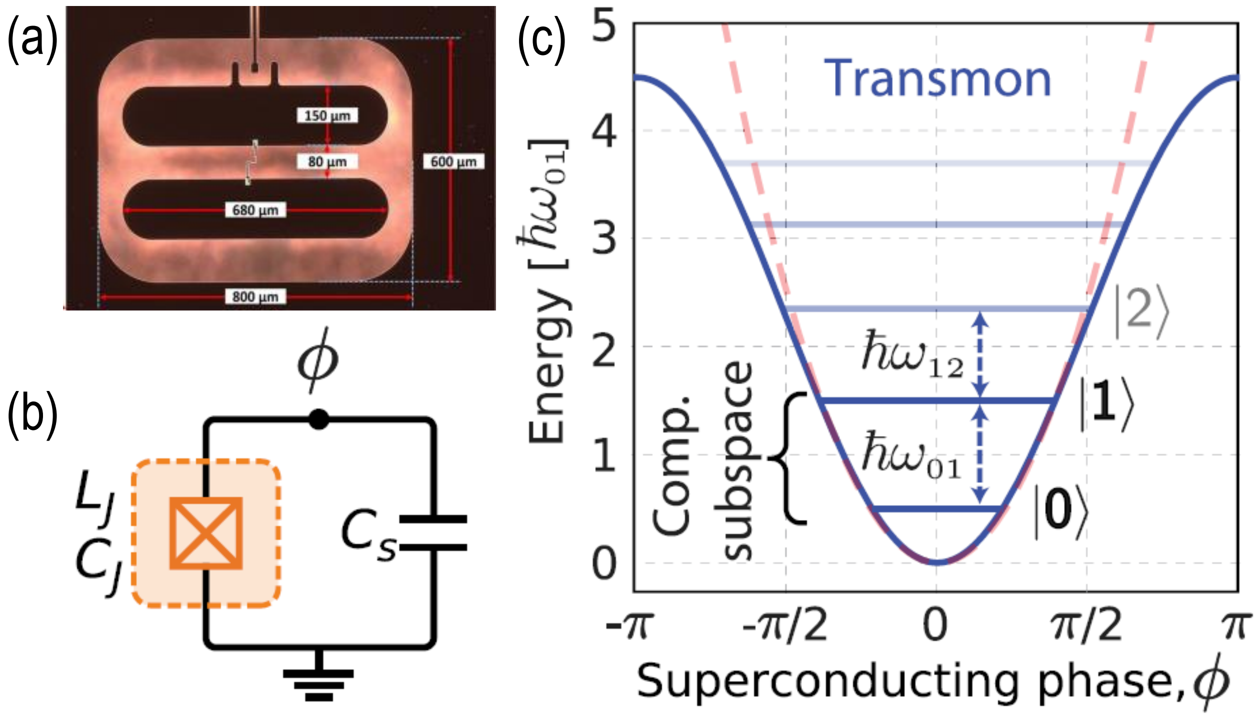
O. Ezratty, "Where are we heading with NISQ?," 2023. [online]. Available: arXiv:2305.09518, 2023.
D. Gil, W.M.J. Green, "The Future of Computing: Bits + Neurons + Qubits," in *IEEE Int. Solid-State Circuits Conf.*, (2020)

Introduction to Superconducting Transmon Qubit

- What Is Superconducting Transmon Qubit?
  - A transmon qubit minimizes sensitivity to charge noise by a large shunt capacitance connected to a Cooper pair box
  - Due to the nonlinearity of a Josephson junction, a transmon qubit is anharmonic oscillator

- Strengths of Transmon Qubits
  - Reduced noise sensitivity and long coherence times
  - Scalability and reproducibility
  - Well-established control and readout schemes

- Weaknesses of Transmon Qubits
  - Dielectric two-level system loss
  - Large footprint size
  - Weakly anharmonic energy level diagram



There are many other types of qubit as shown in previous slide!
(in this lecture, we focus on the transmon qubit only)

Fig. (a) Example of a transmon qubit. (b) Equivalent circuit diagram. (c) Energy level of a transmon qubit as a function of superconducting phase. The two lowest energy levels can be effectively isolated from others, forming a computational subspace of $|0\rangle$ and $|1\rangle$.

Image from P Krantz *et al.*, "A quantum engineer's guide to superconducting qubits," *Appl. Phys. Rev.*, **6,** 021318 (2019). and C Wang *et al.*, *npj Quantum Inf.*, **8**, 3 (2022).

Transmon Qubit as a Quantum Anharmonic Oscillator (1/3)

Nonlinear Josephson Junction Equations

- Recall that from the current-phase relations,

$$I = I_c \sin(\phi) \qquad V = \frac{\hbar}{2e}\frac{d\phi}{dt}$$

I_c: critical current of a Josephson junction
ϕ: phase difference in a Josephson junction

- The potential Josephson energy E_J of a Josephson junction can be expressed as

$$E_J = \frac{1}{L_J}\left(\frac{\Phi_0}{2\pi}\right)^2 = I_c\left(\frac{\Phi_0}{2\pi}\right)$$

L_J: Josephson junction inductance
$\Phi_0 = \frac{h}{2e}$: quantum of flux

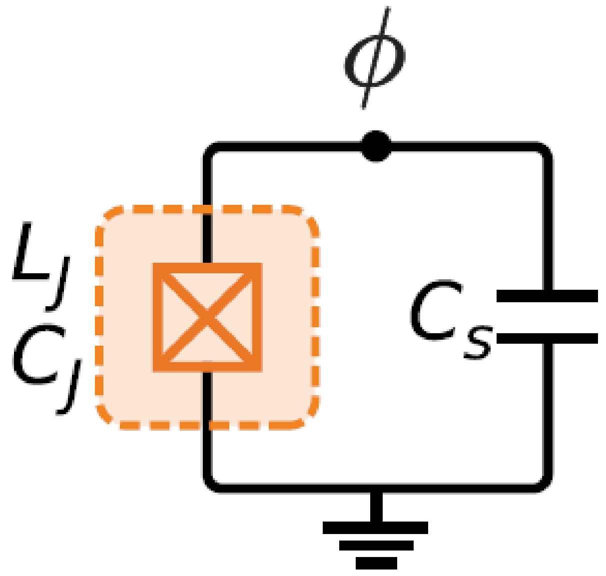


Fig. Equivalent circuit diagram. Here, Josephson inductance and capacitance are represented by L_J and C_J. The shunt capacitance is denoted as C_s

Hamiltonian of a Transmon Qubit

- Replace the total capacitance with charging energy
  ($E_c = e^2/2C_\Sigma$, where $C_\Sigma = C_J + C_s$: total capacitance)
- The Hamiltonian can be expressed as

$$\therefore \mathcal{H} = 4E_c\left(\hat{n} - n_g\right)^2 - E_J \cos\hat{\phi}$$

n_g: offset charge in a Josephson junction

The Hamiltonian of a transmon is same as that of a CPB…
What's the difference?

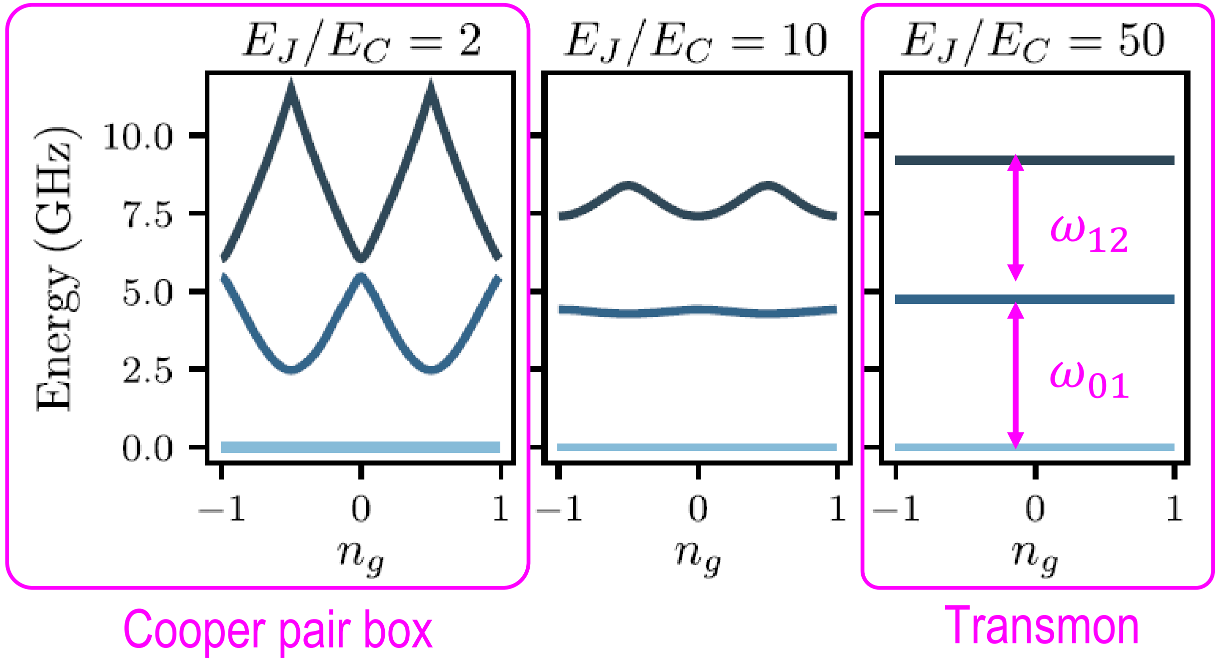
Image from P Krantz *et al.*, "A quantum engineer's guide to superconducting qubits," *Appl. Phys. Rev.*, **6,** 021318 (2019)

Transmon Qubit as a Quantum Anharmonic Oscillator (2/3)

From Cooper Pair Box to Transmon Qubit

- A typical Cooper pair box exhibits $E_J/E_c \sim 1$ ratio
- In contrast, a transmon qubit exhibits $E_J/E_c \gg 1$ ratio
- Physical meaning: a transmon has large shunt capacitance
- As a result, a transmon qubit suppresses charge noise!

Transmon qubits exhibit almost constant energy levels with respect to charge offset !



Characteristic Parameters of Transmon Qubits

- Qubit frequency ω_{01}: energy difference when $|1\rangle$ and $|0\rangle$ (* by multiplying Planck's constant h, frequency → energy unit)
- For simplicity, $\omega_{01} \approx \sqrt{8E_JE_c} - E_c$
- Qubit anharmonicity $\alpha = \omega_{12} - \omega_{01}$: energy difference between $|2\rangle \rightarrow |1\rangle$ transition and $|1\rangle \rightarrow |0\rangle$ transition
- For simplicity,$\alpha \approx -E_c$

Since the anharmonicity ($\alpha = \omega_{12} - \omega_{01}$) is not large, transmon qubits are *weakly anharmonic* qubit!

Image from A. Blais, AL Grimsmo, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021).

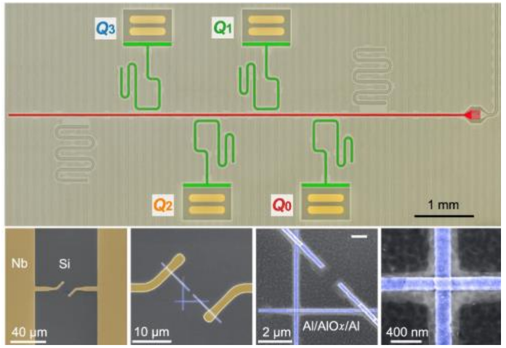
Transmon Qubit as a Quantum Anharmonic Oscillator (3/3)

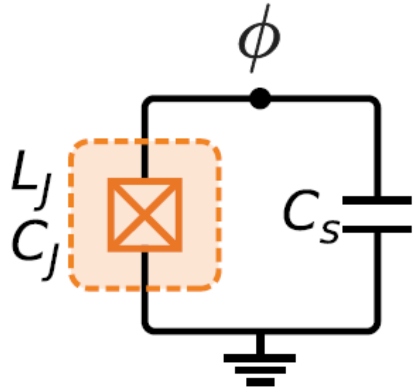
Fixed-Frequency Transmon Qubits

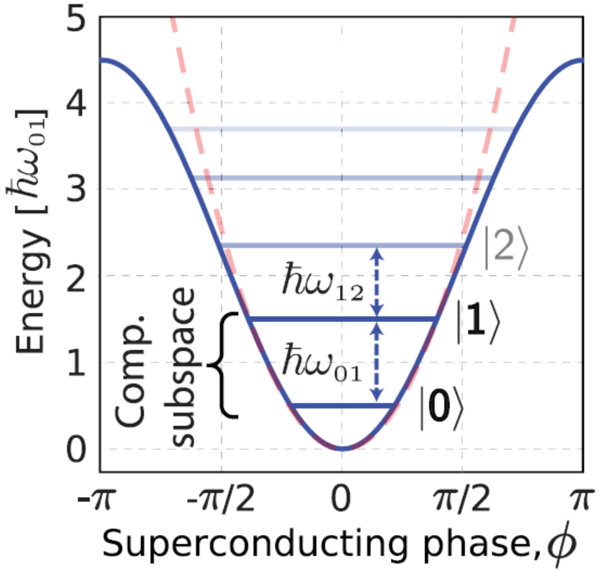
- A single Josephson junction shunted to a capacitor
- Qubit frequency can not be tuned but good T_1 and T_2
- Recently, post-processing techniques to adjust the qubit frequency have developed (ex: laser annealing process)

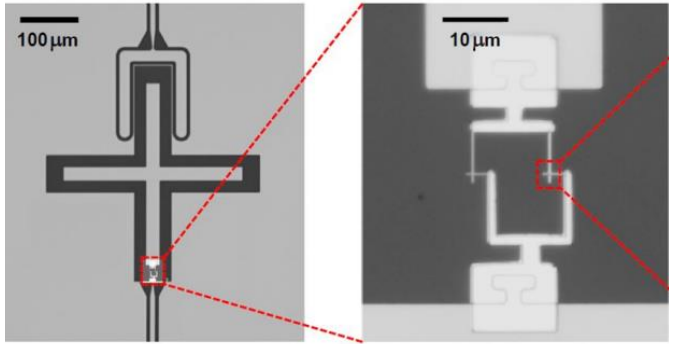
Tunable-Frequency Transmon Qubits

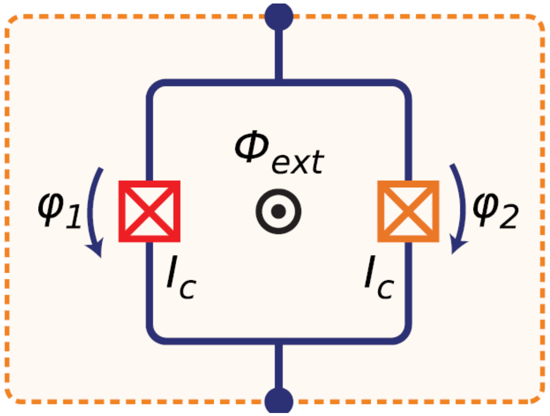
- A SQUID (two junctions in parallel) shunted to a capacitor
- Qubit frequency can be tuned by applying magnetic flux
- However, sensitive magnetic flux noise → bad T_1 and T_2

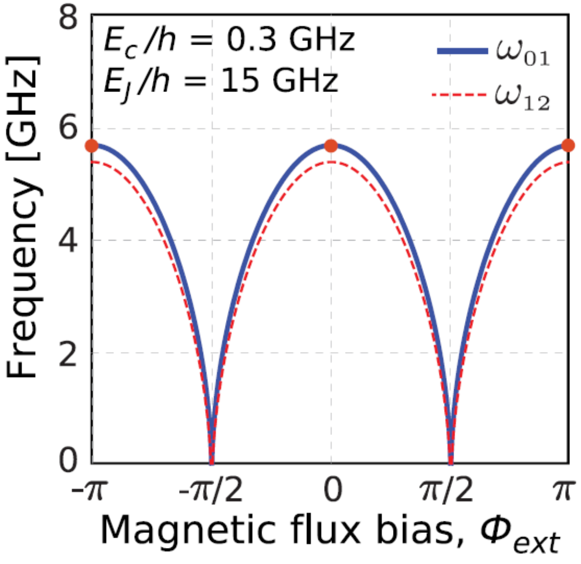












Images from P Krantz *et al.*, *Appl. Phys. Rev.*, **6,** 021318 (2019) and C Wang *et al.*, *npj Quantum Inf.*, **8**, 3 (2022)

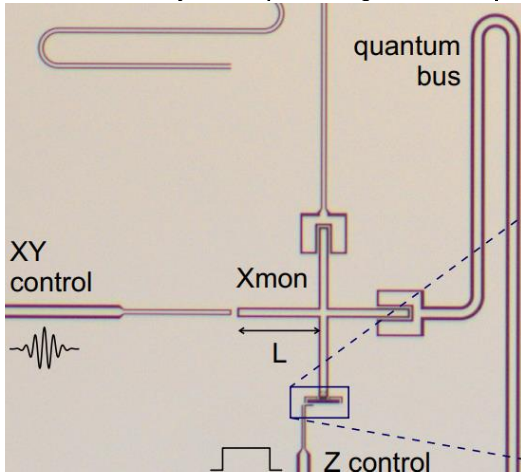
Practical Guidelines to Design Transmon Qubits

- **Typical Choice for Josephson Junction Parameters**
  - Critical current ~ 30 nA ↔ junction inductance ~ 10nH ↔ junction energy ~ 16 GHz (assuming $h = 1$ for simplicity)
- **Typical Choice for Shunt Capacitance**
  - capacitance ~ 60 fF ↔ charging energy ~ 300 MHz (assuming $h = 1$ for simplicity)
- **Resulting Qubit Parameters**
  - Qubit frequency ω_{01} ~ 6 GHz and qubit anharmonicity α ~ -330 MHz
- **Various Design Examples of Transmon Qubits**

You will design your own transmon qubits in lab sessions

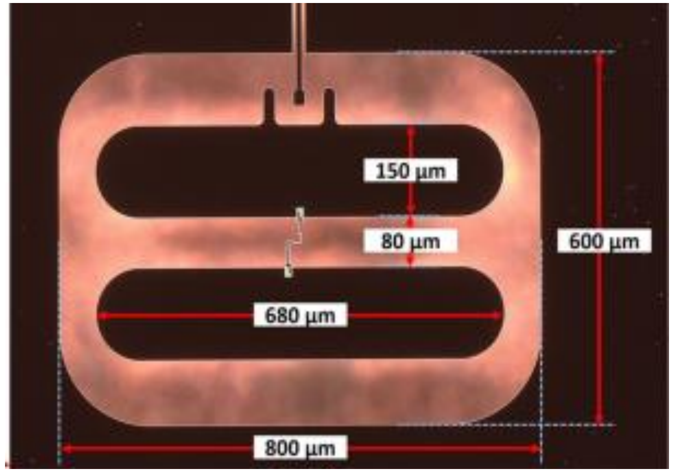
Popular choices

Xmon type (Google-like)



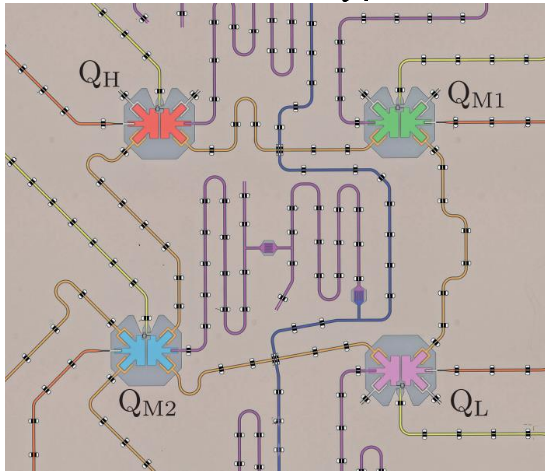
R Barends *et al.*, *Phys. Rev. Lett.*, **111**, 080502 (2013)

Double pad type (IBM-like)



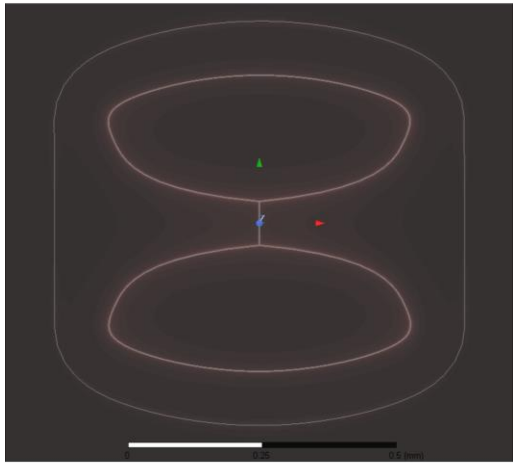
C Wang *et al.*, *npj Quantum Inf.*, **8**, 3 (2022)

Starmon type



V. Negîrneac *et al.*, *Phys. Rev. Lett.*, **126**, 220502 (2021)

Optimized pad type



S Eun *et al.*, *J. Phys. D*, . **56**, 505306 (2023).

See Also…

Review Papers:

[1] A. Blais, AL Grimsmo, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021).

[2] P Krantz *et al.*, "A quantum engineer's guide to superconducting qubits," *Appl. Phys. Rev.*, **6**, 021318 (2019).

[3] U Vool, M Devoret, "Introduction to quantum electromagnetic circuits," *Int. J. Circ. Theor. Appl.*, **45**, 897-934 (2017).

[4] J Koch *et al.*, "Charge-insensitive qubit design derived from the Cooper pair box," *Phys. Rev. A*, **76**, 042319 (2007). ← transmon qubit proposal

[5] SM Girvin, Circuit QED: Superconducting Qubits Coupled to Microwave Photons, Les Houches, Oxford University Press

Open Courses:

[1] Qiskit Global Summer School (2020): *for circuit quantization, see videos #16~ #21 (but every video is very useful)

https://youtube.com/playlist?list=PLOFEBzvs-VvrXTMy5Y2IqmSaUjfnhvBHR&si=xE-4OjWXYSxf5gY6

[2] Rob Schoelkopf: Experimental Platforms: Superconducting Circuit

https://youtu.be/nYcNQL6pS0o?si=K4JhxXM-rQGeC3aj

[3] Lecture Notes by Prof. Steven M Girvin (Yale University)

https://girvin.sites.yale.edu/lectures