Introduction to Superconducting Quantum Circuits

- Light-Matter Interaction in Circuit Quantum Electrodynamics -

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Lecture Overview

Week 1. Introduction to Superconducting Quantum Circuits
Week 2. Review of Mathematics and Microwave Engineering
Week 3. Review of Classical and Quantum Mechanics
Week 4. Review of Superconductivity
Week 5. Quantum Harmonic/Anharmonic Oscillators and Light-Matter Interaction
Week 6. Circuit Quantization Methods
Week 7. Parametrically Pumped Josephson Devices
Week 8. Design and Analysis of Superconducting Resonators
Week 9. Design and Analysis of Superconducting Qubits
Week 10. Design and Analysis of Single-Qubit Device: 3D Cavity
Week 11. Design and Analysis of Single-Qubit Device : 2D Chip
Week 12. Design and Analysis of Two-Qubit Device
Week 13. Design and Analysis of Josephson Parametric Amplifier
Week 14. Term Project
Week 15. Term Project

overall backgrounds, terminologies of quantum computing

mathematical and engineering backgrounds
general superconductivity

Quantum circuit analysis

design and analysis of superconducting RF devices

Keywords in Light-Matter Interaction in Circuit Quantum Electrodynamics

Qubit-Resonator Interaction

Jaynes-Cummings model	Coupling Strength	Dressed Frequencies
Lamb Shift	AC Stark Shift	Dispersive Regime
Rotating Wave Approximation	Schrieffer-Wolff Transformation	Pauli Operators
Self-Kerr Frequency	Cross-Kerr Frequency	Dispersive Shift Frequency
Spontaneous Emission Loss (Purcell Loss)	Decay Rate	Signal-to-Noise Ratio

Introduction to Circuit Quantum Electrodynamics (circuit QED)

What Is Circuit QED ?

- A field of studying the fundamental interaction between light (photon) and matter (resonator or qubit)
- For superconducting circuits, the artificial atom is realized as superconducting qubits, such as Cooper-pair box or transmon…
- Fundamental theory for the circuit QED: Jaynes-Cummings model from quantum optics

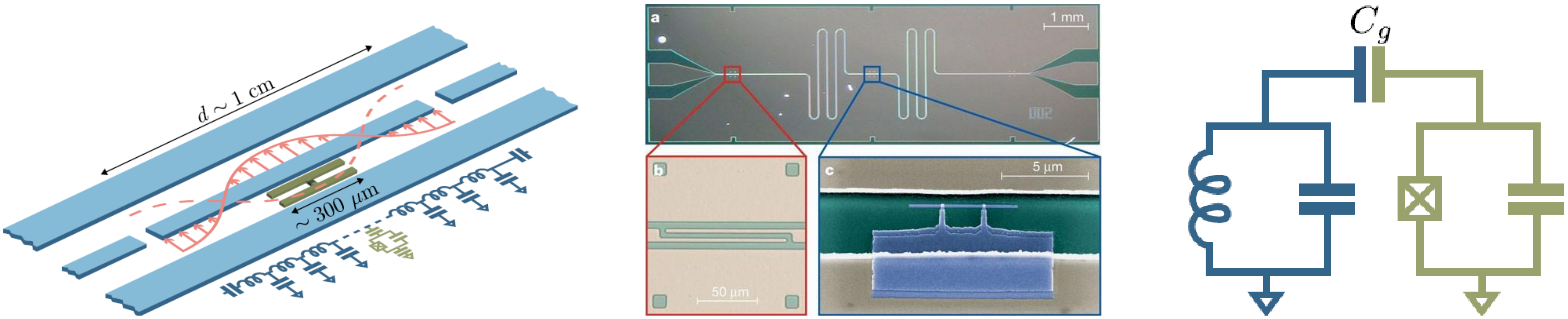


Fig. Schematic representation, example micrographs, and lumped-element circuit representation of a planar superconducting circuit. A superconducting transmon qubit is capactively coupled to a coplanar waveguide resonator via a coupling capacitor. In the lumped-element circuit representation, the input/output ports are neglected.

Image from A. Blais, AL Grimsmo, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021).
Image from A. Wallraff *et al*., "Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics," *Nature*, **431**, 162-167 (2004).

Introduction to Jaynes-Cummings Model: (1/4) Hamiltonian

- What is the Jaynes-Cummings Model?
  - A theoretical quantum mechanical model to describe the system of a two-level atom interacting with a quantized electromagnetic field of a cavity
- Mathematical Formulation for the Jaynes-Cummings Model
  - The full Hamiltonian $\hat{\mathcal{H}}_{\mathrm{full}}$ that describes the system can be expressed as

$$\hat{\mathcal{H}}_{\mathrm{full}} = \hat{\mathcal{H}}_{\mathrm{atom}} + \hat{\mathcal{H}}_{\mathrm{cavity}} + \hat{\mathcal{H}}_{\mathrm{interaction}}$$
$$= \hbar\omega_a \frac{\hat{\sigma}_z}{2} + \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar g\left(\hat{a}^\dagger \sigma^- + \sigma^+ \hat{a}\right)$$

ω_a: transition frequency of an atom

g: coupling strength between the atom-photon

ω_c: resonant frequency of a cavity

$\hat{a}^\dagger$ $(\hat{a})$: creation and annihilation of a photon

$\hat{\sigma}^+ = |e\rangle\langle g| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$: raising operator of an atom

$\hat{\sigma}^- = |g\rangle\langle e| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$: lowering operator of an atom

$\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g| = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$: inverse operator of an atom

From the Jaynes-Cummings model, we can predict the eigenstates and the eigenenergies of the system ! (+ dynamics of the system too)

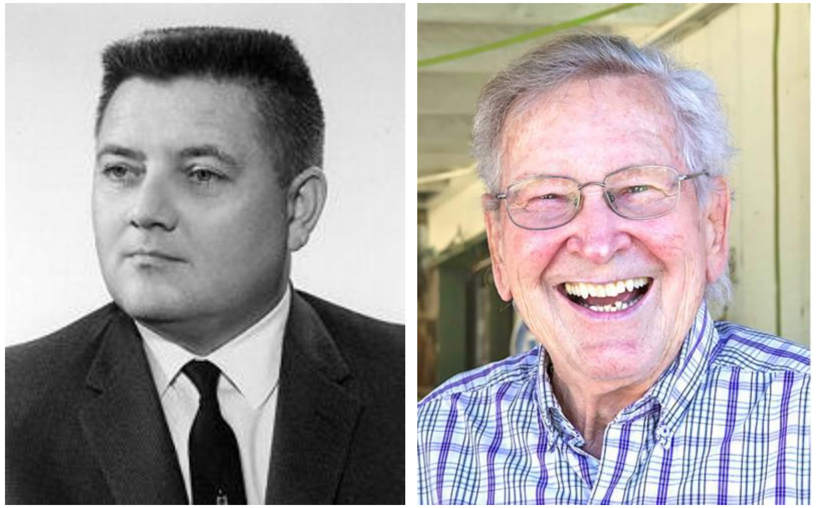


Fig. Picture of Edwin T Jaynes (1922-1960) and Frederick Cummings (1931-2019).

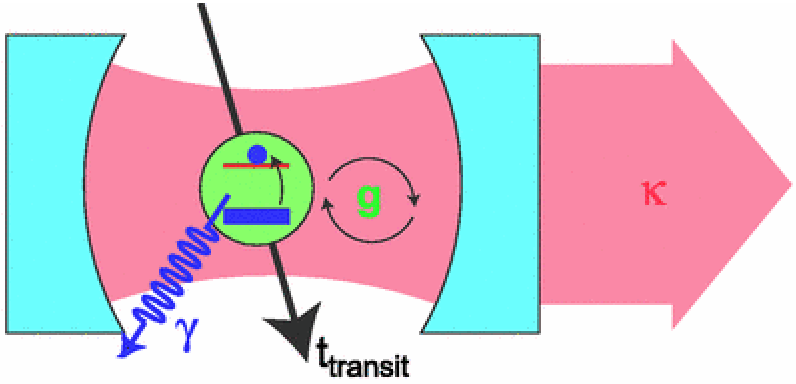


Fig. Cavity QED system, comprising a single-mode of the electromagnetic field in a cavity and a two-level system.

Image from https://en.wikipedia.org

Image from A Blais *et al.*, “Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation,” *Phys. Rev. A*, **69**, 062320 (2004).

Introduction to Jaynes-Cummings Model: (2/4) Eigenstates

- Eigenstates (Dressed States) of the Jaynes-Cummings Model
  - In the absence of loss, the dressed states can be exactly obtained through the diagonalization of the full Hamiltonian
  - For $|a, c\rangle$ eigenstate, where $a \in \{-, +\}$ and $c \in \{0,1,2 \ldots\}$, the excited dressed states can be expressed as:

$$|+, n\rangle = \cos\theta_n\,|\downarrow, n\rangle + \sin\theta_n|\uparrow, n+1\rangle$$
$$|-, n\rangle = -\sin\theta_n\,|\downarrow, n\rangle + \cos\theta_n|\uparrow, n+1\rangle$$

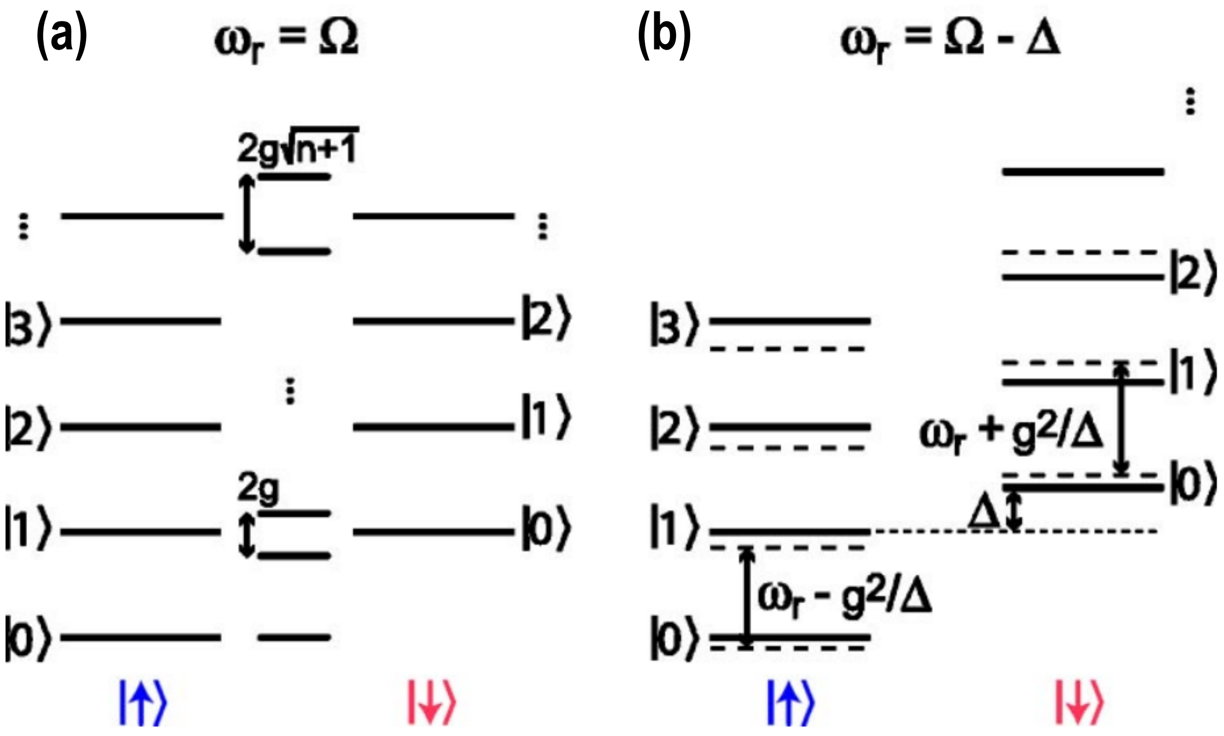
while the ground state is $|\uparrow, 0\rangle$ and $\theta_n = \frac{1}{2}\tan^{-1}\left(\frac{2g\sqrt{n+1}}{\Delta}\right)$

- Eigenenergies of the Jaynes-Cummings Model
  - The eigenenergies can be expressed as:

$$E_{+,n} = \hbar(n+1)\omega_c + \frac{\hbar}{2}\sqrt{4g^2(n+1) + \Delta^2}$$
$$E_{-,n} = \hbar(n+1)\omega_c - \frac{\hbar}{2}\sqrt{4g^2(n+1) + \Delta^2}$$

$\Delta = \omega_a - \omega_c$: detuning frequency

For the case of zero detuning, the atom and the cavity exchanges the photon via vacuum Rabi oscillation



when the atom is in $|e\rangle$ when the atom is in $|g\rangle$

Fig. (a) Energy spectrum of the system when the atom and the cavity are perfectly degenerated (zero detuning, $\Delta = 0$) The left and right levels are the uncoupled energy spectra, while the levels at the center are the energy spectra of the dressed states (b) Energy spectrum when the atom and the cavity are far-detuned.

Image from A Blais *et al.*, "Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation," *Phys. Rev. A*, **69**, 062320 (2004).

Introduction to Jaynes-Cummings Model: (3/4) Expected Transmission Spectrum

- Degenerated Atom and Cavity System
  - When the detuning frequency $\Delta = \omega_a - \omega_c$ is zero
  - The splitting transmission spectrum by $2g$ is observed

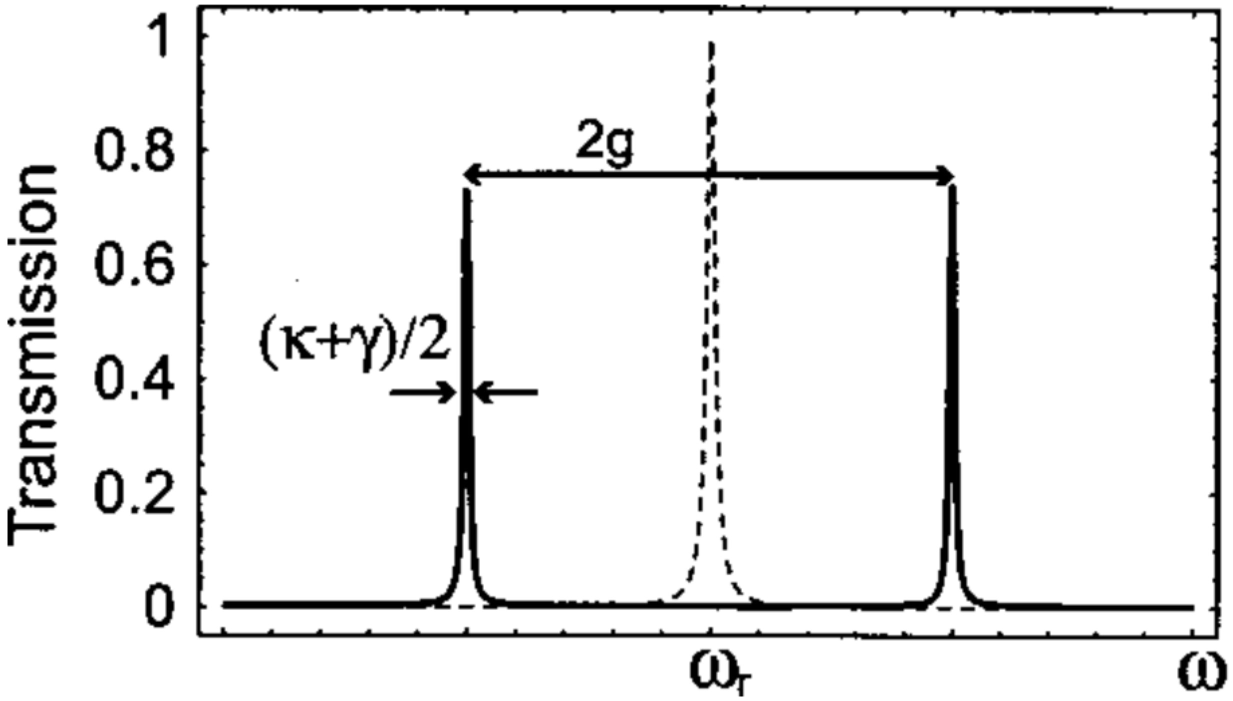


Fig. Expected transmission spectrum of the resonator in the absence (dashed line) and presence (solid line) of a superconducting atom biased at its degeneracy point. Since the system is fully hybridized with half atom and half photon, the linewidth is the average value of the photon decay rate κ and qubit decay rate γ.

- Far-Detuned Atom and Cavity System
  - When $|\Delta| \gg g$ (also known as the dispersive regime !)
  - Atom state dependent transmission spectrum is observed

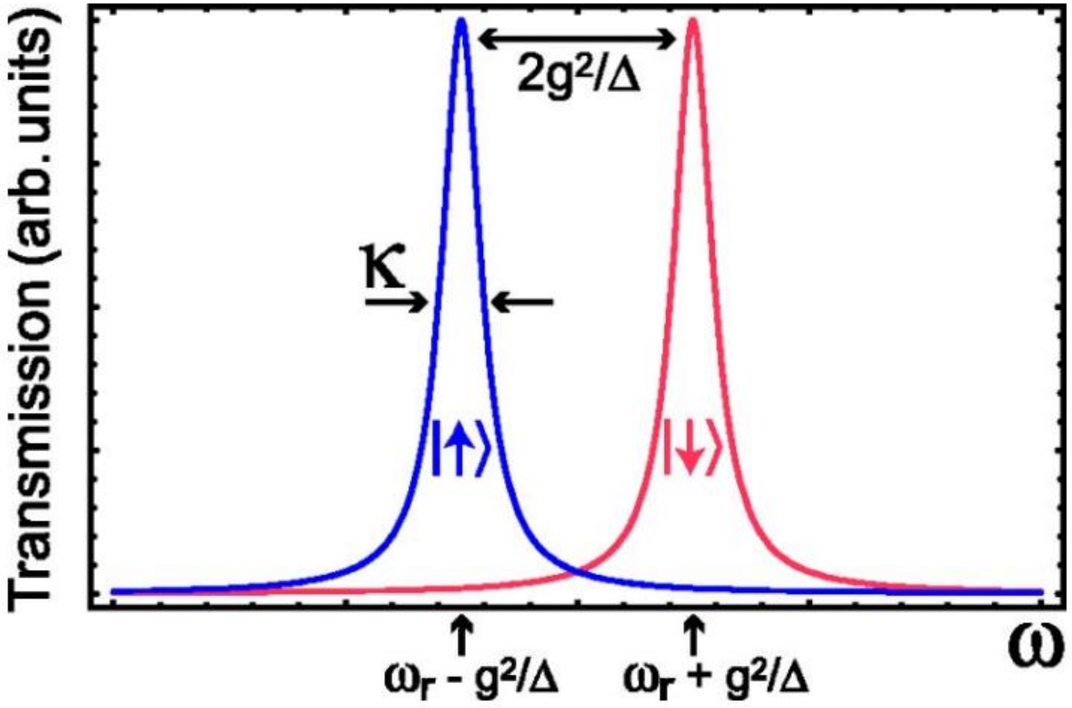


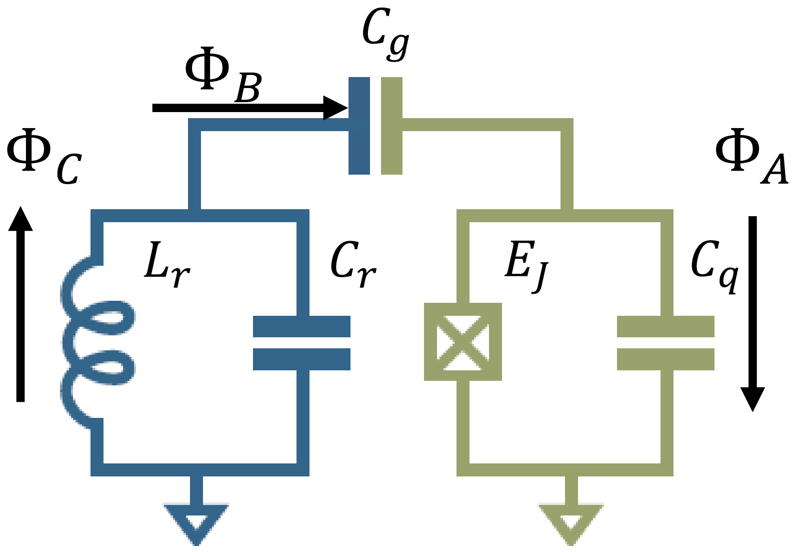
Fig. Transmission spectrum of the cavity, which is "pulled" by an amount the dispersive shift frequency $\chi \approx \frac{g^2}{\Delta^2}$, depending on the state of the atom (red: qubit in the excited state, blue: qubit in the ground state). This phenomenon is utilized to perform the quantum nondemolition readout of a qubit.

Image from A Blais *et al.*, "Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation," *Phys. Rev. A*, **69**, 062320 (2004).

Introduction to Jaynes-Cummings Model: (4/4) Superconducting Qubit

- Jaynes-Cummings Model for a Superconducting Qubit Coupled to a Resonator
  - By associating the flux of each node, generalized coordinate for the circuit can be established with Kirchhoff's voltage and current laws
  - The Lagrangian of the system can be expressed as

$$\mathcal{L} = \underbrace{\frac{C_q}{2}\dot{\Phi}_A^2 + \frac{C_g}{2}\left(\dot{\Phi}_A + \dot{\Phi}_B\right)^2 + E_J \cos\left(\left(\frac{2\pi}{\Phi_0}\right)\Phi_A\right)}_{\mathcal{L}\text{ of a qubit}} + \underbrace{\frac{C_r}{2}\dot{\Phi}_C^2 - \frac{1}{2L_r}\Phi_C}_{\mathcal{L}\text{ of a resonator}}$$



  - The Hamiltonian of the system can be derived as

$$\mathcal{H} = \frac{C_r + C_g}{2\left(C_q C_g + C_r C_g + C_q C_r\right)^2} Q_A^2 + \frac{C_g}{\left(C_q C_g + C_r C_g + C_q C_r\right)^2} Q_A Q_c - E_J \cos\left(\left(\frac{2\pi}{\Phi_0}\right)\Phi_A\right) + \frac{C_q + C_g}{2\left(C_q C_g + C_r C_g + C_q C_r\right)^2} Q_c^2 + \frac{\Phi_C}{2L}$$

  - By introducing the number and ladder operators, the Hamiltonian can be simplified through the two-level approximation as

$$\mathcal{H}_{\mathrm{JC}} \approx \frac{\hbar\omega_q}{2}\hat{\sigma}_z + \hbar\omega_r \hat{a}^\dagger \hat{a} + \hbar g\left(\hat{a}^\dagger \sigma^- + \sigma^+ \hat{a}\right)$$

See Appendix A of A. Blais *et al.*, *Rev. Mod. Phys.*, **93**, 025005 (2021) for the detailed derivation of the Hamiltonian by the standard canonical quantization process

$\omega_q \approx \sqrt{8E_J E_c} - E_c$: qubit frequency

$\omega_r = 1/\sqrt{L_r C_r}$: resonator frequency

$g = \omega_r \frac{C_g}{C_q}\left(\frac{E_J}{2E_c}\right)^{\frac{1}{4}}\sqrt{\frac{\pi Z_r}{R_k}}$: coupling strength

$Z_r = \sqrt{\frac{L_r}{C_r}}$: characteristic Z of resonator

$R_k = \frac{h}{e^2} \approx 25.8\ \mathrm{k}\Omega$: resistance quantum

$E_c = \frac{e^2}{2C_q}$: charging energy

Image from A. Blais et al., "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021)

The Hamiltonian of the Qubit-Resonator System in Other Representations

Hamiltonian (Bare-Mode Basis Representation)

- Bare-mode basis representation: the components are represented individually in their uncoupled (bare-mode) basis, while the coupling between the components is treated as a perturbation to the bare-mode Hamiltonian
- Using the creation and annihilation operators, the bare-mode Hamiltonian is

$$\mathcal{H}_{\text{bare}} = \underbrace{\hbar\omega_r \hat{a}^\dagger \hat{a}}_{\text{resonator part}} + \underbrace{\hbar\omega_q \hat{b}^\dagger \hat{b} - \frac{E_c}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}}_{\text{qubit part}} \underbrace{- \hbar g\left(\hat{b}^\dagger - \hat{b}\right)\left(\hat{a}^\dagger - \hat{a}\right)}_{\text{coupling part}}$$

Hamiltonian (Rotating-Wave Approximation)

- Rotating-wave approximation: simplification of the Hamiltonian in the experimentally relevant situation where $\omega_r, \omega_q \gg g$
- $\hat{b}^\dagger\hat{a}^\dagger$ and $\hat{b}\hat{a}$ terms can be neglected, since their eigenfrequencies are related to $\omega_q + \omega_r$
- Using the creation and annihilation operators, the Hamiltonian after the approximation is

$$\mathcal{H}_{\text{RWA}} = \underbrace{\hbar\omega_r \hat{a}^\dagger \hat{a}}_{\text{resonator part}} + \underbrace{\hbar\omega_q \hat{b}^\dagger \hat{b} - \frac{E_c}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}}_{\text{qubit part}} + \underbrace{\hbar g\left(\hat{b}^\dagger \hat{a} + \hat{b}\hat{a}^\dagger\right)}_{\text{coupling part}}$$

NOTE: here, a qubit is not two-level approximated

$\hat{a}^\dagger$ $(\hat{a})$: creation and annihilation of a photon in a resonator
$\hat{b}^\dagger$ $(\hat{b})$: creation and annihilation of a photon in a qubit

The Hamiltonian of the Qubit-Resonator System in the Dispersive Regime

- Approximated Hamiltonian in the Dispersive Regime
  - Dispersive regime: The detuning frequency is sufficiently large that $|\Delta| = |\omega_q - \omega_r| \gg g$
  - Schrieffer-Wolff transformation: a unitary transformation to determine a diagonalized Hamiltonian in the dressed-state basis (often referred to as the normal-mode basis) from the Hamiltonian in the bare-mode basis

$$\mathcal{H}_{\mathrm{disp}} = \underbrace{\hbar\omega_r \hat{a}^\dagger \hat{a}}_{\text{resonator part}} + \underbrace{\hbar\omega_q \hat{b}^\dagger \hat{b} + \frac{\hbar\alpha_q}{2}\hat{b}^\dagger \hat{b}^\dagger \hat{b}\hat{b}}_{\text{qubit part}} + \underbrace{\hbar\chi \hat{a}^\dagger \hat{a}\hat{b}^\dagger \hat{b}}_{\text{coupling part}}$$

$\alpha_q \approx -E_c$: anharmonicity of the qubit

  - The Hamiltonian in the dispersive regime $\mathcal{H}_{\mathrm{disp}}$ can be simplified after two-level approximation as

$$\mathcal{H}_{\mathrm{disp,TLS}} \approx \frac{\hbar\omega_q'}{2}\hat{\sigma}_z + \hbar\omega_r' \hat{a}^\dagger \hat{a} + \hbar\chi\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)\hat{\sigma}_z$$

NOTE: here, a qubit is assumed to be a transmon qubit (weakly anharmonic qubit)!
In addition, this expression is not an exact expression but an approximation

$\chi \approx \frac{g^2}{\Delta}$: dispersive shift frequency

$\omega_q' \approx \omega_q + \frac{g^2}{\Delta}$: dressed qubit frequency

$\omega_r' \approx \omega_r - \frac{g^2}{\Delta}$: dressed resonator frequency

- Characteristics of the Hamiltonian in the Dispersive Regime
  - The Hamiltonian allows us to intuitively identify phenomena that can be observed experimentally
  - For example, the coupling strength g between a fixed-frequency transmon qubit and a resonator is difficult to be measured
  - However, the dispersive shift frequency χ is directly obtained from the qubit readout experiments

Characteristics of Dispersive Qubit-Resonator System: (1) AC-Stark / Lamb Shifts

- Definitions of AC-Stark Shift and Lamb shift
  - AC-stark shift: the shift in the energy levels of a qubit when it interacts with a microwave drive
  - Lamb shift: caused by interactions between the virtual photons created through vacuum energy fluctuations

- AC-Stark Shift and Lamb shift in Superconducting Qubit-Resonator System
  - AC-stark shift: the qubit frequency is linearly shifted to the photon number (drive amplitude)
  - Lamb shift: the qubit frequency is shifted due to the interaction with the quantized field modes of the resonator

- Hamiltonian Analysis
  - From the dispersive Hamiltonian expression, the AC-stark shift and the Lamb shift can be easily identified

$$\mathcal{H}_{\mathrm{disp,TLS}} \approx \frac{\hbar\omega_q}{2}\hat{\sigma}_z + \hbar\omega_r \hat{a}^\dagger\hat{a} + \hbar\chi\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)\hat{\sigma}_z$$

$$= \frac{\hbar}{2}\left(\omega_q \underbrace{+\,\chi}_{\text{Lamb shift}} \underbrace{+\,2\chi\hat{a}^\dagger\hat{a}}_{\text{AC-stark shift}}\right)\hat{\sigma}_z + \hbar\omega_r\hat{a}^\dagger\hat{a}$$

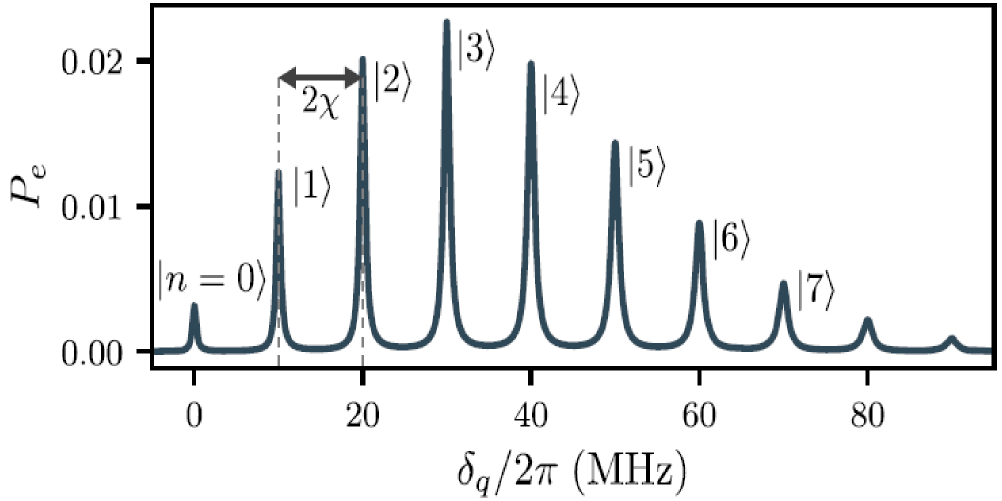


Fig. Example of AC-stark shift in a qubit-resonator system. Excited state population as a function of the qubit drive frequency. strong dispersive limit with $\chi/2\pi = 5$ MHz. The resolved peaks correspond to different cavity photon numbers $|n\rangle$.

Image from A. Blais *et al.*, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021)

Qubit-Resonator System: (2) Self- and Cross-Kerr Shift Frequencies

- Approximated Hamiltonian by the Bogoliubov Transformation
  - Bogoliubov transformation: another way to diagonalize the Hamiltonian in the bare mode (we used Schrieffer-Wolff before)
  - Key concept: rewrite the Hamiltonian as the sum of a linear and nonlinear part and exactly diagonalize them
  - Self-Kerr shift: nonlinear effect where the frequency of a mode shifts due to the presence of photons
  - Cross-Kerr shift: shift of one mode caused by photons in another mode due to nonlinear interaction

- Characteristics of Superconducting Qubit-Resonator System
  - Self-Kerr shift: superconducting qubits, as well as resonators, have anharmonicity due to the presence of the nonlinear component (e.g., Josephson junction). However, resonators usually have very small anharmonicity compared to qubits
  - Cross-Kerr shift: twice of the dispersive shift ($\chi \approx \frac{g^2}{\Delta}$)

$$\mathcal{H}_{\mathrm{disp}} = \underbrace{\hbar\omega_r \hat{a}^\dagger \hat{a} + \frac{\hbar\alpha_r}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}}_{\text{resonator part}} + \underbrace{\hbar\omega_q \hat{b}^\dagger \hat{b} + \frac{\hbar\alpha_q}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}}_{\text{qubit part}} + \underbrace{\hbar\chi\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}_{\text{coupling part}}$$

$\alpha_q \approx -E_c$: self-Kerr (anharmonicity) of the qubit

$\alpha_r \approx -E_c\left(\frac{g}{\Delta}\right)^4$: self-Kerr (anharmonicity) of the resonator

$\chi \approx -2\frac{g^2 E_c}{\Delta(\Delta - E_c)}$: cross-Kerr between a qubit and a resonator

NOTE:
Circuit quantization analysis, such as black-box quantization or energy-participation-ratio, yield the Hamiltonian in this form

Qubit-Resonator System: (3) Purcell Effect and Purcell Loss

- Requirements for Fast Measurement of Superconducting Qubits
  - Purcell loss: a qubit in the excited state emits amount of energy (photon) to its surrounding environment
  - Decay rate $\kappa_r = \omega_r / Q_r$ ➔ ↓ Q_r equals to the large decay rate (fast measurement protocol is possible)
  - However, the Purcell loss limited T_1 of a qubit is $T_{1,\text{Purcell}} \approx \frac{1}{\kappa_r}\left(\frac{g}{\Delta}\right)^2$… A Purcell filter (kind of bandpass filter in microwave engineering) can engineer the Purcell loss at ω_q to achieve fast measurement of a qubit while preserving its T_1

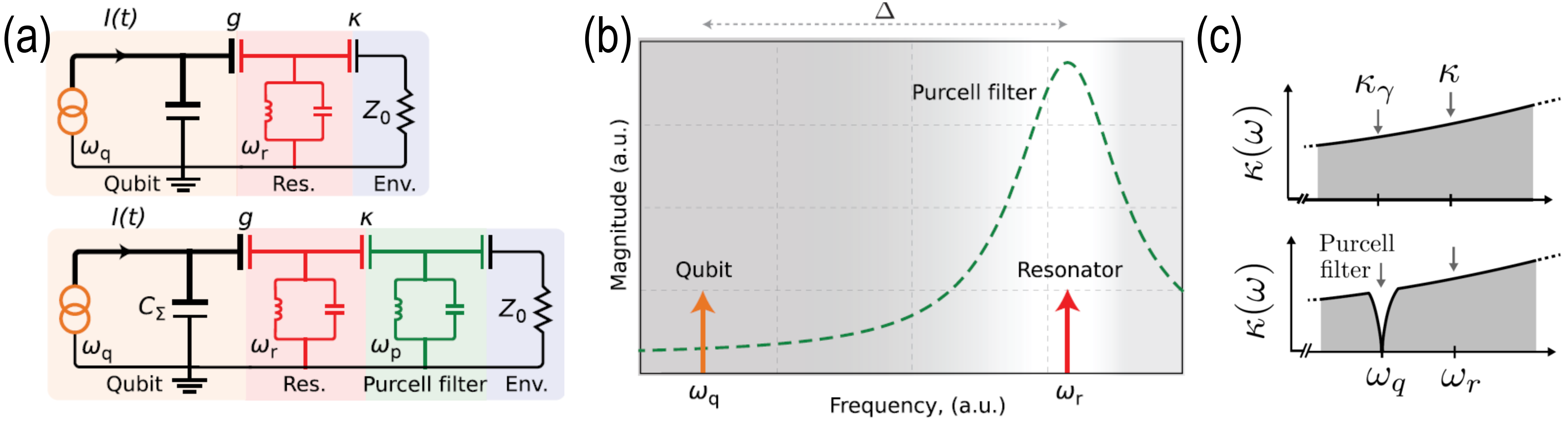


Fig. (a) Circuit diagrams of a typical dispersive readout system of a qubit, a readout resonator, and the external environment with $Z_0 = 50\ \Omega$. A Purcell-filter (green) is added to the circuit, providing protection for the qubit, while allowing the resonator field to decay fast in the environment. (b) Transmission spectrum of a Purcell filter (dashed green), centered around the resonator frequency (red arrow), whereas the qubit frequency (orange arrow) is far detuned.(c) Adding a Purcell filter reduces the cavity density of states at the qubit frequency and therefore suppresses Purcell decay.

Image from P Krantz *et al.*, "A quantum engineer's guide to superconducting qubits", *Appl. Phys. Rev.* **6**, 021318 (2019)
Image from J Gambetta *et al.*, "Quantum trajectory approach to circuit QED: Quantum jumps and the Zeno effect," *Phys. Rev. A*, **77**, 012112 (2008)

Qubit-Resonator System: (4) Signal-to-Noise Ratio of Qubit Measurement

- Requirements for High-Fidelity Measurement of Superconducting Qubits
  - From the dispersive Hamiltonian, ↑ χ yields large shifts of the resonator depending on the qubit state…However, is it always desirable to have ↑ χ to achieve better qubit measurement fidelity ?
  - By solving the master equation, the signal-to-noise ratio (SNR) depends on the ratio between the decay rate κ and the dispersive shift χ that $\kappa/2\chi$ yields the best SNR value !

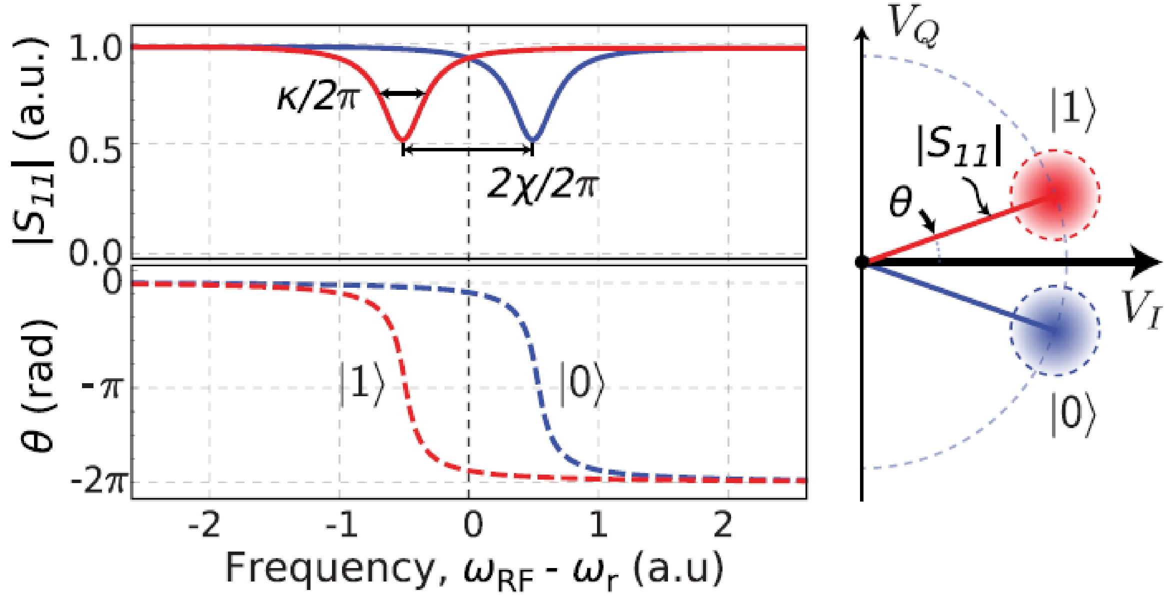


Fig. Reflected magnitude $|S_{11}|$ and phase θ response of the resonator with linewidth κ, when the qubit is in its ground state $|0\rangle$ (blue) and excited state $|1\rangle$. (red). The best state discrimination is obtained when probing the resonator in-between the two resonances, as dashed in the left figure.

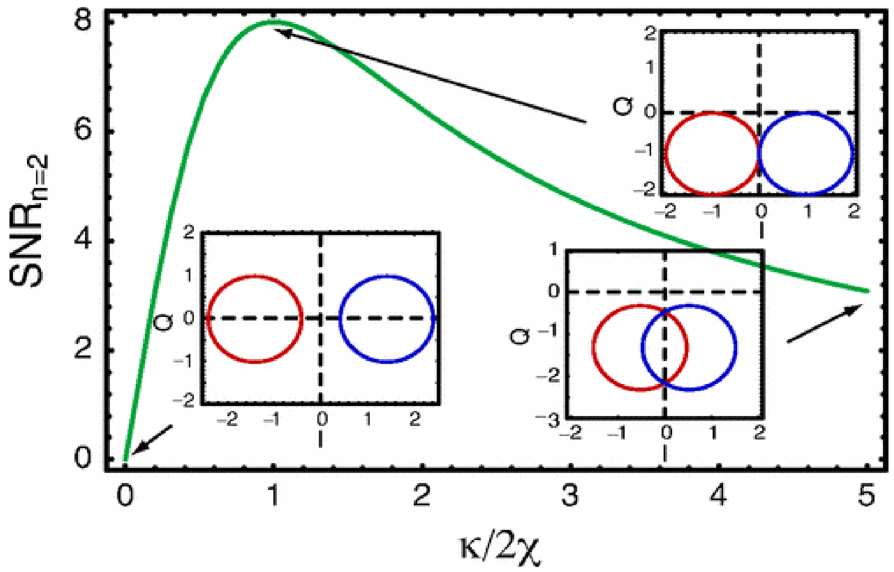
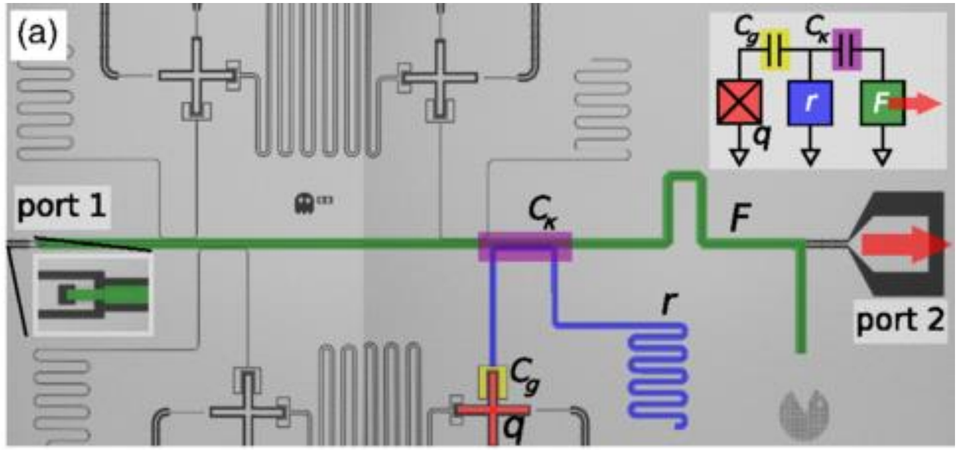


Fig. The signal-to-noise ratio (SNR) for a fixed number of photons in the resonator ($n = 2$). The three insets show the phase space representation of the resonator state for $\kappa = 0.01\chi$, $\kappa = 2\chi$, and $\kappa = 5\chi$, respectively. The best SNR occurs at $\kappa = 2\chi$.

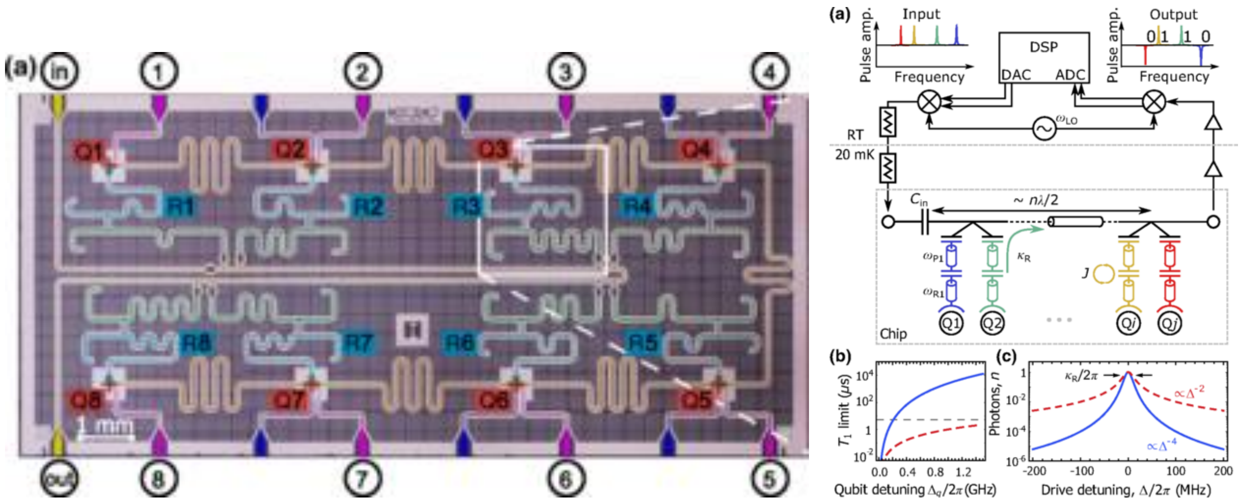
Image from P Krantz *et al.*, "A quantum engineer's guide to superconducting qubits", *Appl. Phys. Rev.* **6**, 021318 (2019)
Image from J Gambetta *et al.*, "Quantum trajectory approach to circuit QED: Quantum jumps and the Zeno effect," *Phys. Rev. A*, **77**, 012112 (2008)

Selected Previous Studies on Fast, High-Fidelity Qubit Measurement System

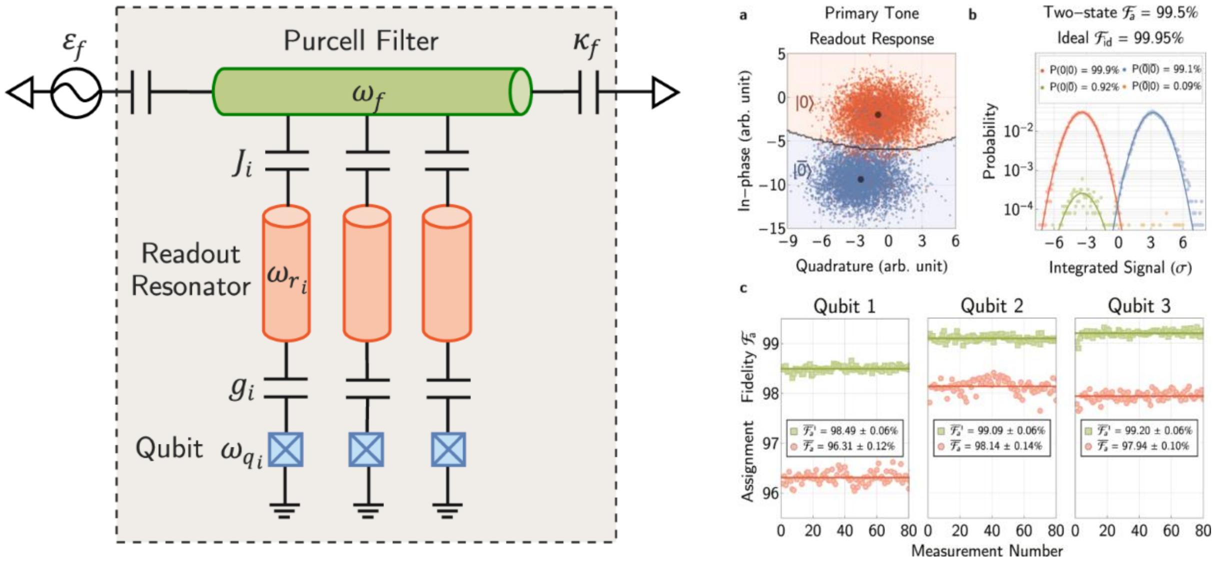
- E Jeffrey *et al.*, *Phys. Rev. Lett.* **112**, 190504 (2014)



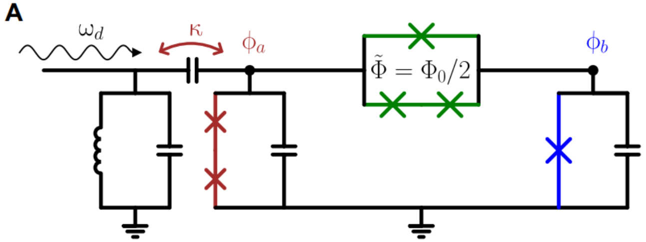
- J Heinsoo *et al.*, *Phys. Rev. Appl.*, **10**, 034040 (2018)



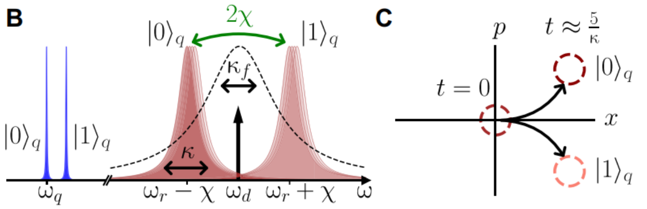
- L Chen *et al.*, *npj Quantum Inf.* **9**, 26 (2023)



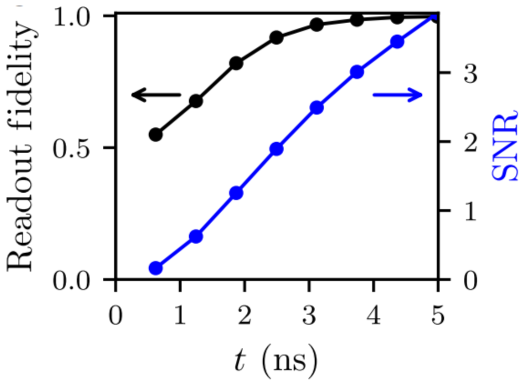
- Y Ye *et al.*, *Sci. Adv.*, **10**, eado9094 (2024)



$$U = -2E_{Ja}\cos\frac{\phi_a}{2} - E_{Jb}\cos\phi_b - 2E_J\cos\left(\frac{\phi_a-\phi_b}{2}\right) + \alpha E_J\cos(\phi_a-\phi_b)$$



Qubit initial state	Readout time (ns)	Readout fidelity	QND fidelity
|0⟩	5	99.71%	99.96%
|1⟩	5	99.73%	99.96%



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Practical Guidelines for Qubit-Resonator System Design (as of 2024)

- Typical Choice for Qubit Parameters
  - Qubit frequency ω_q~ 5 GHz and qubit anharmonicity α ~ -300 MHz

- Typical Choice for Readout Resonator Parameters
  - Resonator frequency ω_r~ 7 GHz and decay rate $\kappa = \omega_r/Q_r$ between 1 MHz ~ 20 MHz (depending on the readout speed)

- Typical Choice for Qubit-Resonator Coupler Parameters
  - Detuning frequency between the qubit and the resonator $\Delta = \omega_q - \omega_r$ between 1 GHz to 2 GHz in its magnitude !
  - Coupling strength g between 50 MHz ~ 300 MHz
  - Dispersive shift frequency χ between 1 MHz ~ 40 MHz (depending on κ_r and target SNR value)

- Typical Choice for Purcell Filter Parameters
  - Center frequency of the passband: same as the frequency of the readout resonator
  - Bandwidth of the passband: 50 MHz ~ 1 GHz (depending on the filter topology and footprint size)

See Also…

Review Papers:

[1] A. Blais, AL Grimsmo, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys*., **93**, 025005 (2021).

[2] P Krantz *et al*., "A quantum engineer's guide to superconducting qubits," *Appl. Phys. Rev*., **6**, 021318 (2019).

[3] U Vool, M Devoret, "Introduction to quantum electromagnetic circuits," *Int. J. Circ. Theor. Appl*., **45**, 897-934 (2017).

[4] J Koch *et al*., "Charge-insensitive qubit design derived from the Cooper pair box," *Phys. Rev. A*, **76**, 042319 (2007). ← transmon qubit proposal

[5] SM Girvin, Circuit QED: Superconducting Qubits Coupled to Microwave Photons, Les Houches, Oxford University Press

Open Courses:

[1] Qiskit Global Summer School (2020): *for circuit quantization, see videos #16~ #21 (but every video is very useful)

https://youtube.com/playlist?list=PLOFEBzvs-VvrXTMy5Y2IqmSaUjfnhvBHR&si=xE-4OjWXYSxf5gY6

[2] Rob Schoelkopf: Experimental Platforms: Superconducting Circuit

https://youtu.be/nYcNQL6pS0o?si=K4JhxXM-rQGeC3aj

[3] Lecture Notes by Prof. Steven M Girvin (Yale University)

https://girvin.sites.yale.edu/lectures