APPLIED SUPERCONDUCTIVITY LABORATORY

Introduction to Superconducting Quantum Circuits

- Review of Classical Mechanics for Quantum Computing -

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Lecture Overview

Week 1. Introduction to Superconducting Quantum Circuits
Week 2. Review of Mathematics and Microwave Engineering
Week 3. Review of Classical and Quantum Mechanics
Week 4. Review of Superconductivity
Week 5. Quantum Harmonic/Anharmonic Oscillators and Light-Matter Interaction
Week 6. Circuit Quantization Methods
Week 7. Parametrically Pumped Josephson Devices
Week 8. Design and Analysis of Superconducting Resonators
Week 9. Design and Analysis of Superconducting Qubits
Week 10. Design and Analysis of Single-Qubit Device: 3D Cavity
Week 11. Design and Analysis of Single-Qubit Device : 2D Chip
Week 12. Design and Analysis of Two-Qubit Device
Week 13. Design and Analysis of Josephson Parametric Amplifier
Week 14. Term Project
Week 15. Term Project

overall backgrounds, terminologies
of quantum computing

mathematical and engineering backgrounds
general superconductivity

Quantum circuit analysis

design and analysis of superconducting RF devices

Keywords in Classical Mechanics for Quantum Computing

Classical Mechanics: Newtonian

Conservative Force

Newton's Laws of Motion

Potential Energy

Kinetic Energy

Classical Mechanics: Lagrangian

Principle of Least Action

Euler-Lagrangian Equation

Classical Mechanics: Hamiltonian

Phase Space

Canonical Momentum

Legendre Transformation

Poisson Braket

Canonical Transformation

Classical Mechanics: Simple Harmonic Oscillator

Ladder Operator

Introduction to Classical Mechanics

- What is Classical Mechanics?
  - Physical theory describing the motion of objects
  - The development of classical mechanics involved substantial change in the methods and philosophy of physics
- Why is Classical Mechanics Important for Studying Quantum Computing?
  - Jargons, analogy, formula,…etc. of superconducting quantum circuits are based on quantum and classical mechanics
  - In this lecture, we will briefly review selected topics in classical mechanics

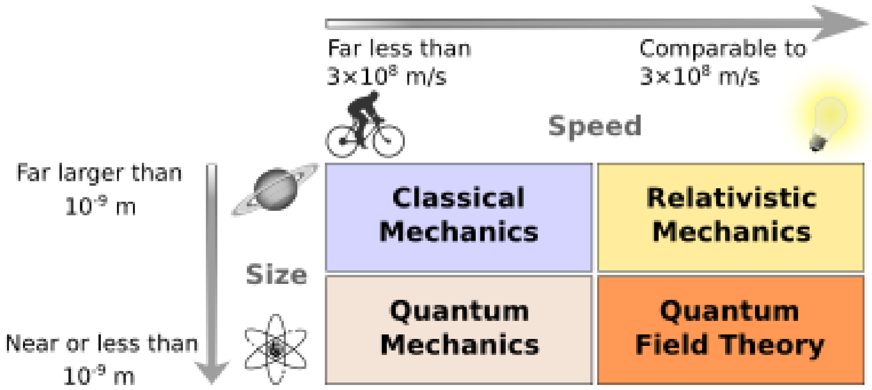
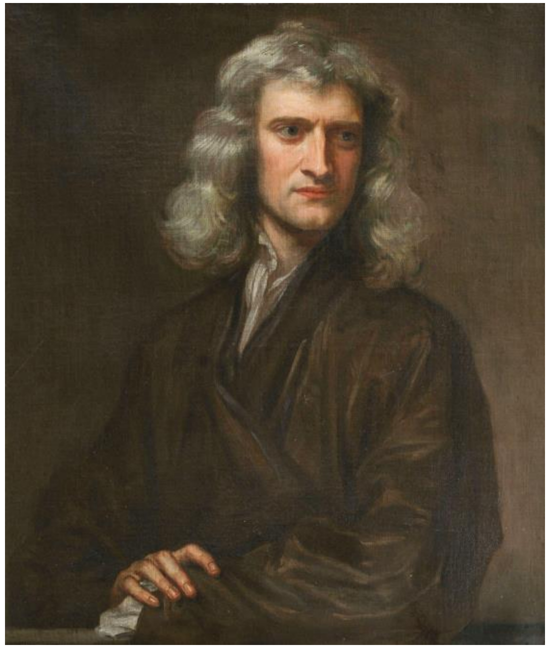
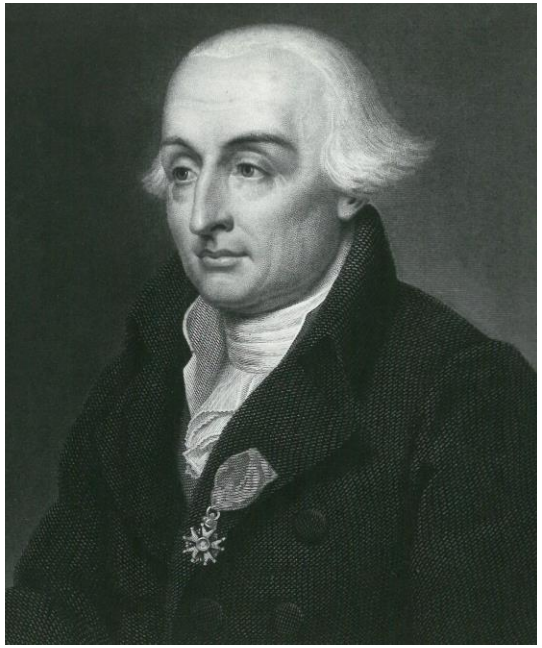


Fig. Domain of validity for classical mechanics



Isaac Newton (1642-1727)



Joseph-Louis Lagrange (1736-1810)

William Rowan Hamilton (1805-1865)

Image from: https://en.wikipedia.org/wiki/Classical_mechanics

Brief Review of Newtonian Mechanics: (1) Newton's Laws of Motion

- Newton's Laws of Motion
  - Force $\boldsymbol{F}$: a vector quantity [N]
  - Mass m: a scalar quantity [kg]
  - Acceleration $\boldsymbol{a}$: a vector quantity [m/s^2]
  - Velocity $\boldsymbol{v}$: a vector quantity [m/s]
  - Momentum $\boldsymbol{p} = m\boldsymbol{v}$: a vector quantity
  - Equation of motion: $\sum \boldsymbol{F} = m\boldsymbol{a}$

NOTE:
Electrical circuit variables (ex: charge) can be also represented by the above variables

- Concept of Conservative Force
  - The work done by conservative force depends only on the initial and final positions (independent of path)
- Examples of Conservative Force
  - Gravity, elastic spring, electrostatic force

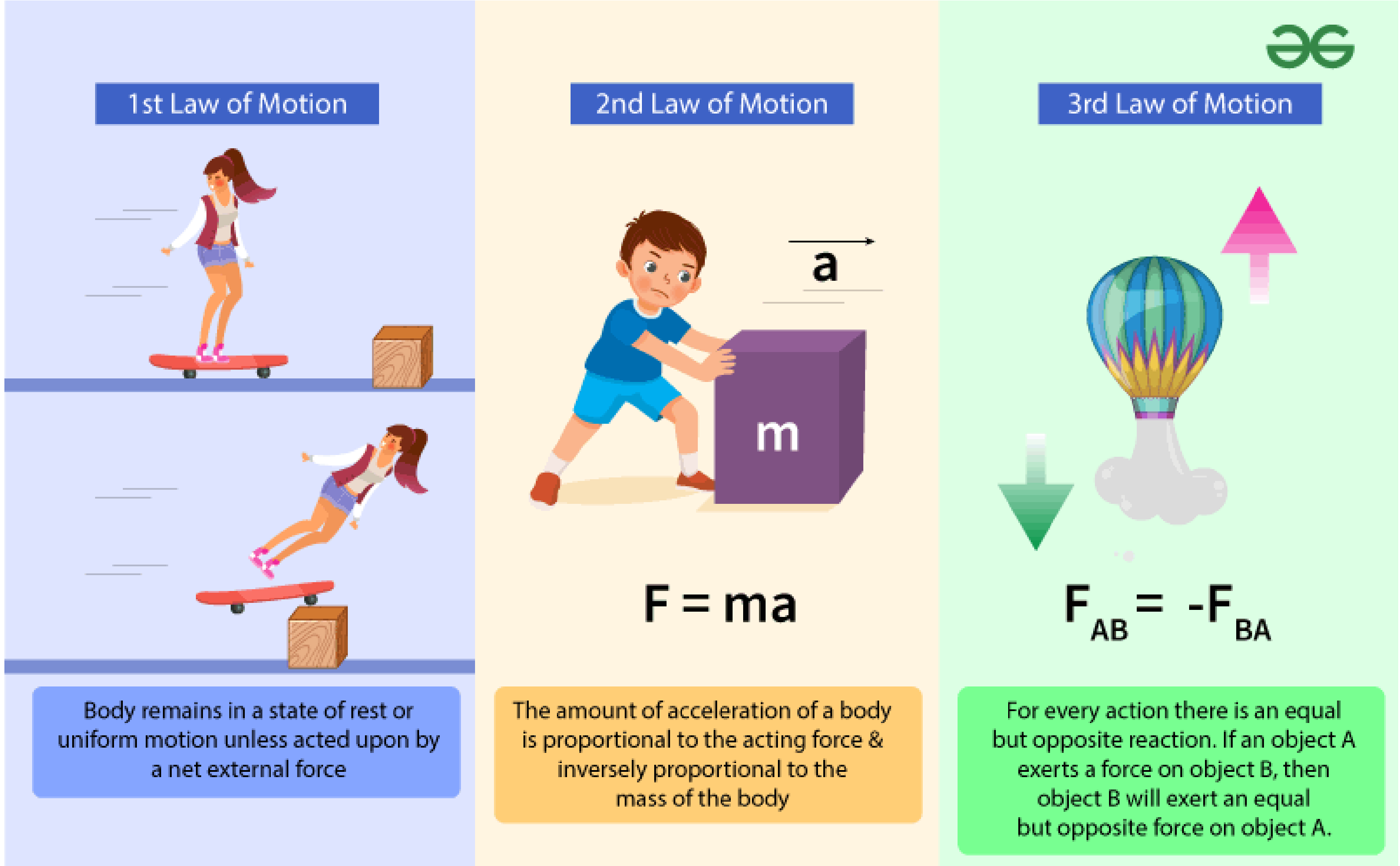
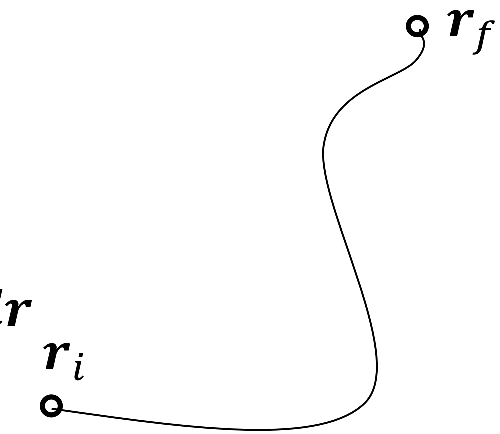


Fig. Definition, formula, and example of Newton's laws of motion.

Image from: https://www.geeksforgeeks.org/newtons-laws-of-motion/

Brief Review of Newtonian Mechanics: (2) Potential and Kinetic Energy

- Definition of Potential Energy
  - Total work done by conservative forces, defined by virtue of an object's position relative to others
- Formula of Potential Energy
  - For conservative force $\boldsymbol{F}$ and path between the initial point $\boldsymbol{r}_i$ and the final point $\boldsymbol{r}_f$,
  - The difference of the potential energy U is: $U(\boldsymbol{r}_f) - U(\boldsymbol{r}_i) = -W$ (negative work) $= -\int_{\boldsymbol{r}_i}^{\boldsymbol{r}_f} \boldsymbol{F} \cdot d\boldsymbol{r}$



negative notation (typical, but not necessary)

- Definition of Kinetic Energy
  - Energy due to the motion
- Formula of Kinetic Energy
  - For a point object (assuming that its mass exist at one point) with mass m and velocity $\boldsymbol{v}$,
  - The kinetic energy T is: $T = \frac{1}{2} m\boldsymbol{v}^2 = \frac{\boldsymbol{p}^2}{2m}$
- Law of Conservation of Energy
  - The total energy (sum of U and T) of an isolated system remains constant
  - The kinetic energy can be converted to the potential energy, vice versa

Brief Review of Newtonian Mechanics: (3) Harmonic Oscillator Example

- Definition of Harmonic Oscillator
  - Special type of periodic trigonometric oscillation towards the equilibrium point
- Properties of Harmonic Oscillator
  - Conservation of the total energy for ideal harmonic oscillators
  - In real oscillators, damping (frictional or dragging force) exists
- Example of Harmonic Oscillator: Mass on an Ideal Spring
  - In Newtonian mechanics, the restoring force $\boldsymbol{F}$ at the distance $\boldsymbol{x}$ is

$$\boldsymbol{F} = m\frac{d^2\boldsymbol{x}}{dt^2} = -k\boldsymbol{x}$$

where k: spring constant [N/m]

  - The above equation is the second-order differential equation
  - The solution for $\boldsymbol{x}$ is

$$\boldsymbol{x}(t) = c_1\cos(\omega t) + c_2\sin(\omega t)$$

where ω: angular frequency, defined as $\omega = \sqrt{\frac{k}{m}}$

  - The coefficients c_1 and c_2 can be determined by the initial conditions

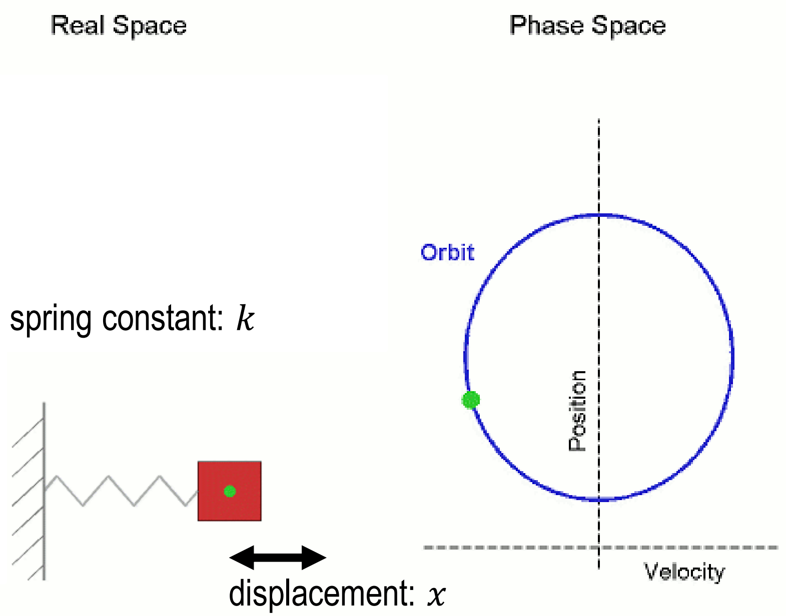


Fig. Motion of harmonic oscillator in real space and phase space.

Image from: https://en.wikipedia.org/wiki/Simple_harmonic_motion

NOTE:
Harmonic oscillator is very important concept
for quantum mechanics AND superconducting circuits

Brief Review of Lagrangian Mechanics: (1) Principles of Least Action

Philosophy of Lagrangian Mechanics

- The fundamental idea of the Lagrangian mechanics is to reformulate the equations of motion in terms of the dynamical variables that describe the degrees of freedom (no explicit force terms)
- For complex physical systems, obtaining exact force is difficult

Characteristics of Lagrangian Mechanics

- Generalized coordinates $\{q_k\}$: fully specify the motion of the system → potential energy $U(q,t)$
- Generalized velocity coordinates $\{\dot{q}_k\}$: total time derivatives of the generalized coordinates → kinetic energy $T(\dot{q},t)$
- Lagrangian $\mathcal{L}(q,\dot{q},t) = T - U$: defined as the difference between the kinetic energy T and the potential energy U
- Lagrangian is scalar quantity! While complex vector calculation is required with Newtonian mechanics

Principles of Least Action

- Path taken between two states is the one for which the action is minimized
- The action S is defined as

$$S = \int_{t_1}^{t_2} \mathcal{L}\, dt = \int_{t_1}^{t_2} T - U\, dt$$

The path $x(t)$ that makes S stationary satisfies the Euler-Lagrange equation!

example: harmonic oscillator (in previous slide)
Describe the system with q and $\dot{q}$ ONLY

- potential energy $U = \frac{1}{2}kx^2$ (here, q is x)
- kinetic energy $T = \frac{1}{2}m\dot{x}^2$
- Lagrangian $\mathcal{L} = T - U = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$

Brief Review of Lagrangian Mechanics: (2) Euler-Lagrangian Equation

- Definition of Euler-Lagrangian Equation
  - Second-order ODEs, whose solutions are stationary points of the given action

- Formula of Euler-Lagrangian Equation
  - For n-dimensional generalized coordinate vector $\boldsymbol{q} = \{q_1(t), \dots, q_n(t)\}$ and speed vector $\dot{\boldsymbol{q}} = \{\dot{q}_1(t), \dots, \dot{q}_n(t)\}$
  - The Lagrangian $\mathcal{L}$ is dependent on $\boldsymbol{q}, \dot{\boldsymbol{q}}, t$ that $\mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)$ can be expressed
  - The Euler-Lagrangian equations are defined as

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = 0$$

where $i = 1, \dots, n$

Derivation of the Euler-Lagrangian equation is beyond the scope of this lecture (also, not necessary)
See Fowles, *Analytical Mechanics,* (2004) for the proof

example: harmonic oscillator

- $\mathcal{L} = T - U = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$
- The Euler-Lagrangian equation is
- $\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = -kx - \frac{d}{dt}(m\dot{x})$
  $= -kx - m\ddot{x} = 0$
- $\therefore kx + m\ddot{x} = 0$ (equation of motion)
- same result from Newtonian mechanics

- Lagrangian in Quantum Computing
  - The Lagrangian will be utilized as the fundamental analysis tool for superconducting quantum circuits
  - To analyze superconducting qubits and resonators, deriving the Lagrangian is a key procedure

Brief Review of Lagrangian Mechanics: (3) Simple Pendulum Example

Simple Pendulum Problem using Newtonian Mechanics

- The total force $\boldsymbol{F}$ on the object of mass m is

$$\boldsymbol{F} = m\boldsymbol{a} = -mg\sin\theta(t)$$

where g: gravity [m/s^2]

NOTE:
acceleration along the tangential axis (red line in the figure) ONLY

- Relation between the tangential axis (red) and the angle θ is

$$\boldsymbol{s} = l\boldsymbol{\theta},$$

$$\boldsymbol{a} = \frac{d^2\boldsymbol{s}}{dt^2} = l\frac{d^2\boldsymbol{\theta}}{dt^2},$$

where l: length of pendulum [m]

- Thus, the equation of motion is $\therefore\ l\frac{d^2\theta}{dt^2} + g\sin\theta = 0$

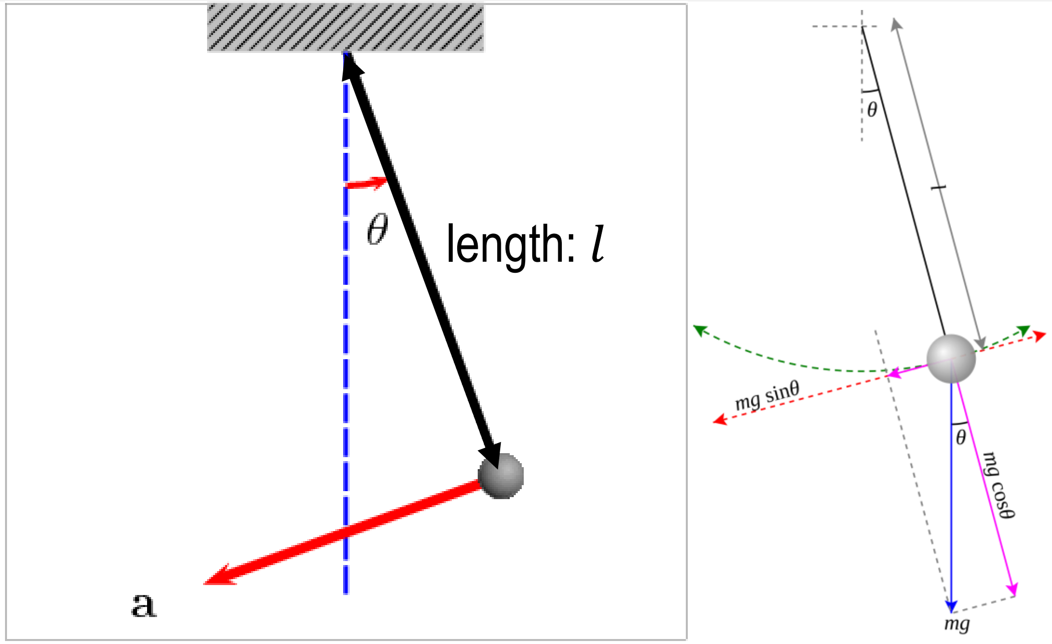


Fig. Motion of pendulum and force diagram under the gravity.

Image from: https://en.wikipedia.org/wiki/Pendulum_(mechanics)

Simple Pendulum Problem using Lagrangian Mechanics

- Assuming the coordinates with respect to θ only, the kinetic energy, potential energy, and Lagrangian are

$$T = \frac{1}{2}ml^2\dot{\theta}^2 \text{ and } U = -mgl\cos\theta \Rightarrow \text{Lagrangian is } \mathcal{L}(\theta,\dot{\theta},t) = T - U = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta$$

- From the Euler-Lagrangian equation,

$$\frac{\partial\mathcal{L}}{\partial\theta} - \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right) = -mgl\sin\theta - ml^2\ddot{\theta} = 0 \Rightarrow \therefore\ l\frac{d^2\theta}{dt^2} + g\sin\theta = 0$$

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Brief Review of Hamiltonian Mechanics: Introduction

- Philosophy of Hamiltonian Mechanics
  - Recall that Lagrangian mechanics allows us to find the equations of motion in terms of generalized coordinates and velocities
  - Extend the method using the canonical momenta (we will learn about this in the next slide) instead of generalized velocities

- Characteristics of Hamiltonian Mechanics
  - The Hamiltonian, denoted by $\mathcal{H}$, is the sum of kinetic energy T and potential energy U

$$\mathcal{H} = T + U$$

Recall that Lagrangian is defined as $\mathcal{L} = T - U$

  - If the generalized coordinates for the system are time-independent, the Hamiltonian is also time-independent (conserved)
  - The Hamiltonian is also scalar

- Summary of Newtonian, Lagrangian, and Hamiltonian Mechanics
  - Newtonian: directly based on forces and accelerations → intuitive approach but cumbersome for complex systems
  - Lagrangian: difference between kinetic and potential energy → simplified problems with generalized coordinates
  - Hamiltonian: total energy of the system → deeper insight into the conservation laws and symmetries of the system

Brief Review of Hamiltonian Mechanics: (1) Canonical Conjugate Momentum

Definition of Canonical Conjugate Momentum

- Canonical coordinates: can describe a physical system with generalized position quantities $\{q_i\}$ and velocity quantities $\{\dot{q}_i\}$
- Conjugate momentum: partial derivative of the Lagrangian, with respect to the generalized velocity

Formula of Canonical Conjugate Momentum

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

for n-dimensional generalized coordinates, there will be n canonical conjugate momentum

Example of Canonical Conjugate Momentum for Harmonic Oscillator

- Coordinate q: x (and recall that the Lagrangian is $\mathcal{L}(x, \dot{x}) = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} kx^2$)
- Momentum p: $\frac{\partial \mathcal{L}}{\partial \dot{x}} = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$

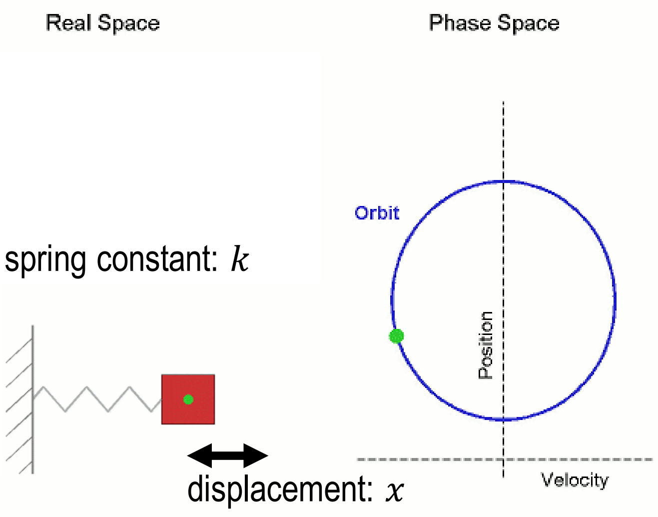

Fig. Motion of harmonic oscillator

Canonical Conjugate Momentum in Quantum Computing

- Assume a superconducting LC parallel circuit
- From the electrical circuit theory, flux $\Phi(t) = \int_{t_0}^{t} V(t')dt'$ and $Q(t) = CV(t)$

where t_0: reference time

- Flux Φ = coordinate and charge Q = canonical conjugate momentum

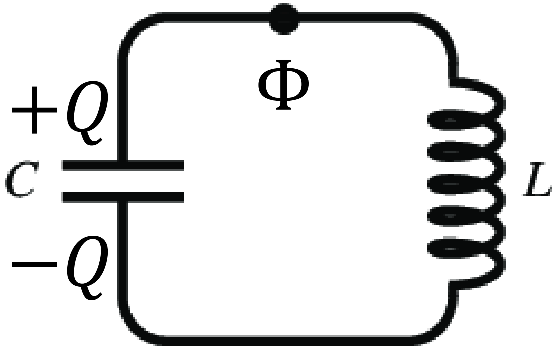

Fig. Parallel LC circuit

NOTE: Quantum mechanical analysis of electrical circuits will be introduced later

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Brief Review of Hamiltonian Mechanics: (2) Phase Space

Definition of Phase Space

- A multidimensional space in which all possible states of a physical system can be represented
- The phase space can be defined by the generalized coordinates q_i and their canonical conjugate momenta p_i

Characteristics of Phase Space

- For n-dimensional generalized coordinates, the phase space has n-canonical coordinates and n-canonical momenta
- If the Lagrangian does not depend on the k^{th} coordinate variable q_k, q_k is cyclic canonical variable
- Without q_k dependence (q_k is cyclic), the following relations are satisfied

$$\frac{\partial \mathcal{L}}{\partial q_k} = 0 \quad \text{and} \quad p_k = \frac{\partial \mathcal{L}}{\partial \dot{q}_k} = \text{constant}$$

Example of Phase Space: Harmonic Oscillator

- The Lagrangian is $\mathcal{L} = T - U = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} kx^2$
- Generalized coordinate = $\{x\}$ and canonical conjugate momentum = $\{m\dot{x}\}$
- The corresponding phase space has $\{x, m\dot{x}\}$ canonical variables

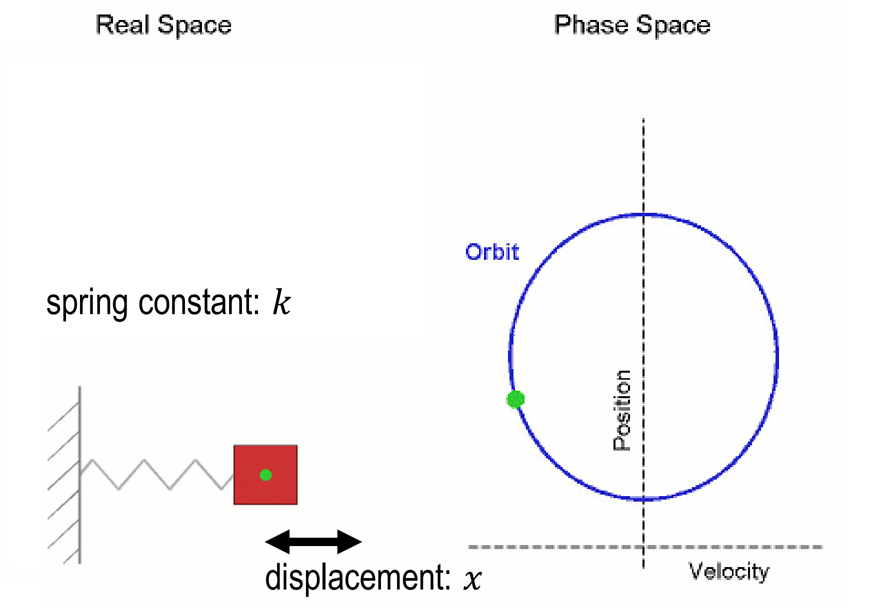

Fig. Motion of harmonic oscillator in real space and phase space.

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Brief Review of Hamiltonian Mechanics: (3) Legendre Transformation

- **Definition of Legendre Transformation**
  - Transformation of the Lagrangian $\mathcal{L}$ into the Hamiltonian $\mathcal{H}$

- **Formula of Legendre Transformation**
  - For n-generalized coordinates q_i and the corresponding momenta p_i, the Hamiltonian $\mathcal{H}$ can be obtained as follows

Recall that $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$

$$\mathcal{H}(q_1, \dots, q_n, p_1, \dots, p_n, t) = \sum_{i=1} p_i \dot{q}_i - \mathcal{L}(q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n, t)$$

- **Characteristics of Legendre Transformation**

$$\frac{\partial \mathcal{H}}{\partial q_i} = -\frac{\partial \mathcal{L}}{\partial q_i} \text{ and } \frac{\partial \mathcal{H}}{\partial t} = -\frac{\partial \mathcal{L}}{\partial t}$$

$$-\dot{p}_i = \frac{\partial \mathcal{H}}{\partial q_i} \text{ and } \dot{q}_i = \frac{\partial \mathcal{H}}{\partial p_i}$$

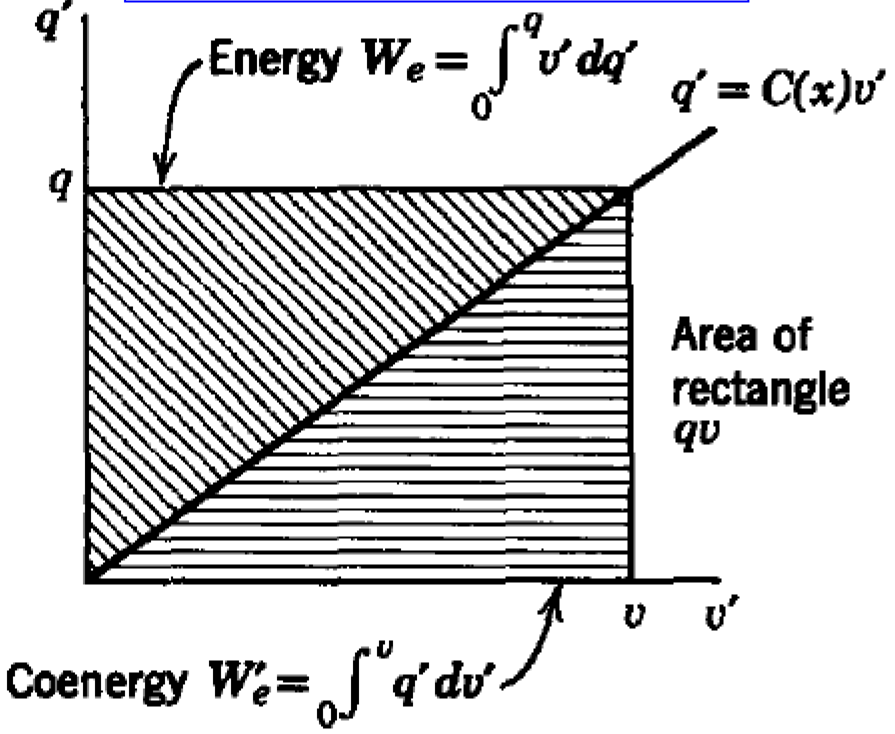
You can derive these relations from the Legendre transformation

- **Legendre Transformation in Quantum Computing**
  - Legendre transformation from the Lagrangian of superconducting quantum circuit to derive the circuit's Hamiltonian

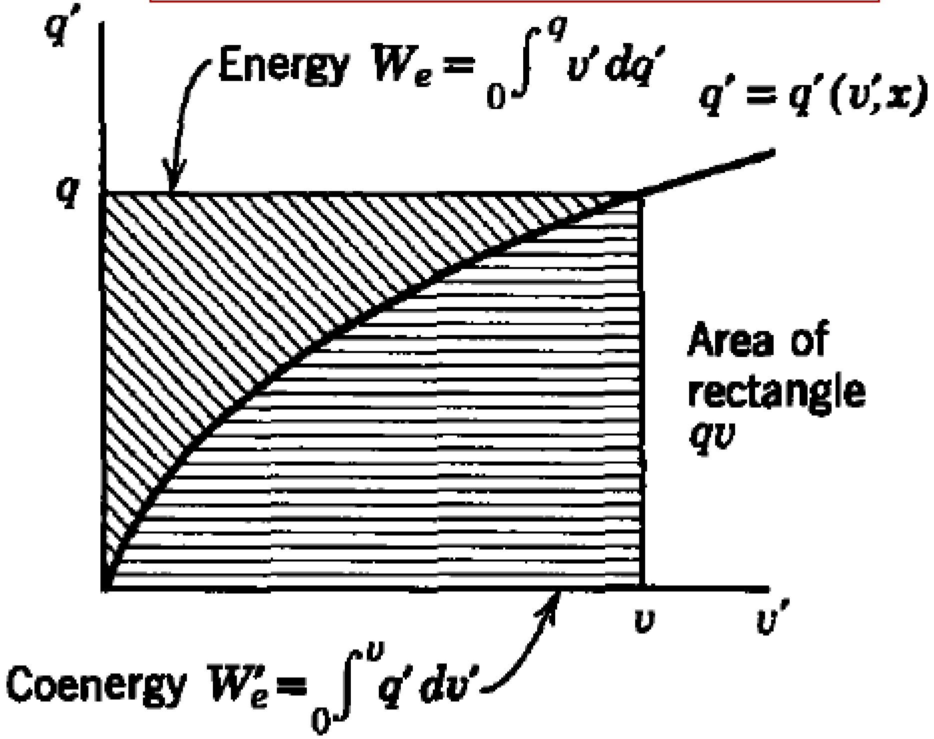
Similar Technique in Electrical Engineering: Coenergy Method

- Comparison of Energy W_e and Coenergy W_e' between Linear and Nonlinear Systems
  - By definition, $W_e + W_e'$ is equal to qv in electric system and λi in magnetic system
  - Only in the *linear* system, $W_e = W_e'$; if a system is *non-linear*, $W_e \neq W_e'$
  - Even if a system is linear, the mathematical expression of W_e is *completely different* from that of W_e'

For Electrically Linear System



For Electrically Non-linear System



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Brief Review of Hamiltonian Mechanics: (4) Poisson Bracket

- Definition of Poisson Bracket
  - An operator that measures the infinitesimal change of one observable quantity with respect to another in phase space
- Formula of Poisson Bracket
  - For two functions $f(q,p)$ and $g(q,p)$, the Poisson bracket is defined as

$$\{f,g\} = \sum_i \left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i} \right)$$

where for $i = 1, \dots, n$,
q_i and p_i are the generalized coordinates and momenta

- Characteristics of Poisson Bracket
  - Anti-symmetry: $\{f,g\} = -\{g,f\}$
  - Linearity: $\{af + bg,\ h\} = a\{f,h\} + b\{g,h\}$
  - Leibniz Rule: $\{q_i, p_j\} = \delta_{ij}$

where a and b: constants
where δ_{ij}: Kronecker-delta function

- Poisson Bracket and Hamiltonian Dynamics
  - For any function $f(q,p)$ in phase space, its time evolution is

$$\frac{df}{dt} = \{f, \mathcal{H}\} + \frac{\partial f}{\partial t}$$

NOTE: The Poisson bracket is the classical analogy of the commutator in quantum mechanics

Brief Review of Hamiltonian Mechanics: (5) Canonical Transformation

Definition of Canonical Transformation

- A generalized coordinate transformation that changes the generalized coordinates q_i and conjugate momenta p_i in $\mathcal{H}$ to a new set of coordinates Q_i and momenta P_i in $\mathcal{H}'$, which satisfy canonical relations
- For generating function F (mapping function), the transformed Hamiltonian is

$$\mathcal{H}'(Q_1, \dots, Q_n, P_1, \dots, P_n) = \mathcal{H}(q_1, \dots, q_n, p_1, \dots, p_n) + \frac{\partial F}{\partial t}$$

Formula of Canonical Transformation

- Type 1: $F_1(q, Q, t)$

$$p_i = -\frac{\partial F_1}{\partial q_i}, P_i = -\frac{\partial F_1}{\partial Q_i}$$

Trivial example of $F_1 = \sum q_i Q_i$ where $Q_i = p_i$ and $P_i = -q_i$

- Type 2: $F_2(q, P, t) - \sum Q_i P_i$

$$p_i = \frac{\partial F_2}{\partial q_i}, Q_i = \frac{\partial F_2}{\partial P_i}$$

Trivial example of $F_2 = \sum q_i P_i$ where $Q_i = q_i$ and $P_i = p_i$

- Type 3: $F_3(p, Q, t) + \sum q_i p_i$

$$q_i = -\frac{\partial F_3}{\partial p_i}, P_i = -\frac{\partial F_1}{\partial Q_i}$$

Trivial example of $F_3 = \sum p_i Q_i$ where $Q_i = -q_i$ and $P_i = -p_i$

- Type 4: $F_4(p, P, t) + \sum q_i p_i - \sum Q_i P_i$

$$q_i = -\frac{\partial F_4}{\partial p_i}, Q_i = \frac{\partial F_4}{\partial P_i}$$

Trivial example of $F_4 = \sum p_i P_i$ where $Q_i = p_i$ and $P_i = -q_i$

Example of Canonical Transformation

- Kinetic energy T, potential energy U, and Lagrangian $\mathcal{L}$ of the system are

$$T = \frac{1}{2}m\dot{x}^2, U = \frac{1}{2}kx^2, \mathcal{L}(x,\dot{x}) = T - U = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

- Canonical momentum p and Hamiltonian $\mathcal{H}$ of the system are

$$p = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x} \quad \mathcal{H} = p\dot{x} - \mathcal{L},$$

$$\therefore \mathcal{H}(x,p) = \frac{kx^2}{2} + \frac{p^2}{2m} = \frac{m\omega^2x^2}{2} + \frac{p^2}{2m}$$

where $\omega \equiv \sqrt{\frac{k}{m}}$ oscillator's eigenfrequency

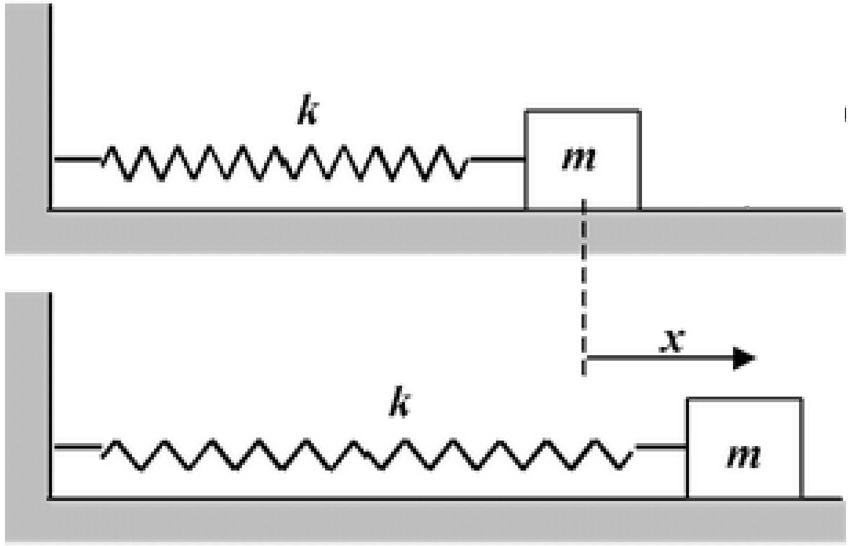


Fig. Motion of mass on a spring (harmonic oscillator)

- Let the generating function $F_1(q,Q) = \frac{m\omega q^2}{2}\cot Q$,

$$p = \frac{\partial F_1(q,Q)}{\partial q_i} = m\omega q \cot Q,$$

$$P = -\frac{\partial F_1(q,Q)}{\partial Q} = \frac{m}{2}\frac{\omega q^2}{\sin^2 Q},$$

$$q = \sqrt{\frac{2P}{m\omega}}\sin Q,$$

$$p = \sqrt{2m\omega P}\cos Q,$$

$$\mathcal{H}'(P,Q) = \omega P(\cos^2 Q + \sin^2 Q)$$

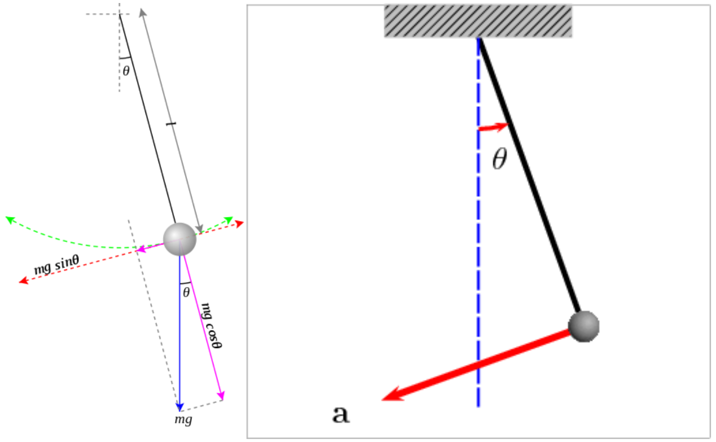
$$\boxed{\therefore \mathcal{H}' = \omega P}$$

Implies that Q is a cyclic coordinate for $\mathcal{H}'$ and the Hamiltonian means the total energy

NOTE: The above generating function will be utilized to solve quantum mechanical harmonic oscillator

Brief Review of Classical Mechanics: Simple Pendulum Example

Newtonian Mechanics



Newton's Force equation

- Equation of motion: $F = ma$

$$\therefore F = -mg\sin\theta$$

- From force vector analysis,

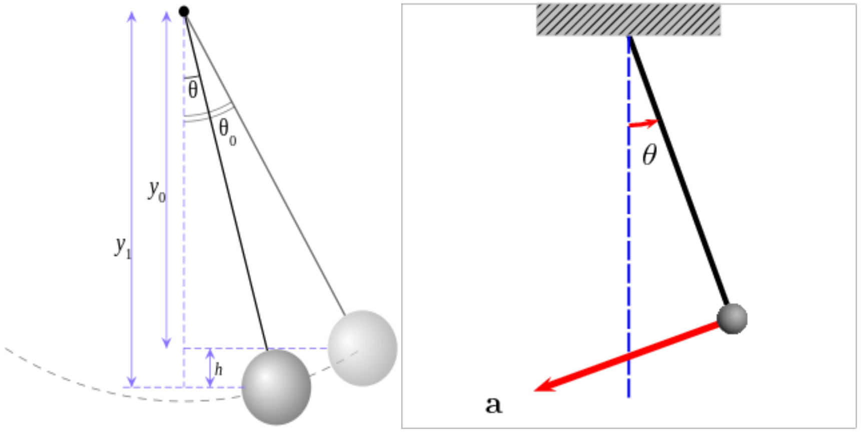
$$s = l\theta,$$

$$v = \frac{ds}{dt},$$

$$a = \frac{d^2s}{dt^2} = l\ddot{\theta}$$

$$\therefore l\ddot{\theta} = -g\sin\theta$$

Lagrangian Mechanics



Generalized coordinate θ and velocity $\dot{\theta}$

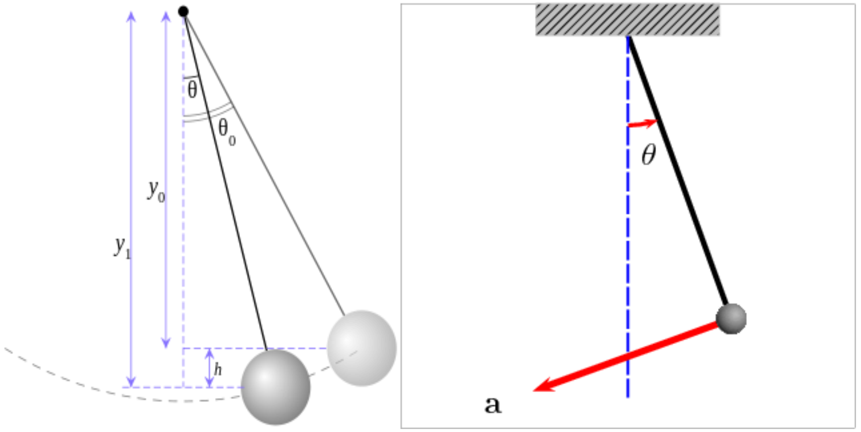
- Potential energy: $U = -mgl\cos\theta$
- Kinetic energy: $T = \frac{1}{2}m\left(l\dot{\theta}\right)^2$
- Lagrangian: $\mathcal{L} = T - U$

$$\mathcal{L} = \frac{1}{2}m\left(l\dot{\theta}\right)^2 + mgl\cos\theta$$

- The Euler-Lagrangian equation: $\frac{\partial\mathcal{L}}{\partial\theta} - \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right) = 0$

$$\therefore g\sin\theta + l\ddot{\theta} = 0$$

Hamiltonian Mechanics



From the Lagrangian,

- Canonical conjugate momentum: $p = \frac{\partial\mathcal{L}}{\partial\dot{q}} = ml^2\dot{\theta}$
- Legendre transformation: $\mathcal{H} = \sum_{i=1} p_i\dot{q}_i - \mathcal{L}$

$$\mathcal{H} = ml^2\dot{\theta}^2 - \mathcal{L}$$

$$= \frac{1}{2}m\left(l\dot{\theta}\right)^2 - mgl\cos\theta$$

- From the Hamiltonian's characteristic: $-\dot{p} = \frac{\partial\mathcal{H}}{\partial q}$

$$-ml^2\ddot{\theta} = \frac{\partial\mathcal{H}}{\partial\theta}$$

$$\therefore g\sin\theta + l\ddot{\theta} = 0$$

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See Also…

- Textbooks:

[1] Grant R. Fowles and George L. Cassiday, *Analytical Mechanics*, Cengage Learning, 2004.

* recommended

- Open Courses:

[1] Iain W. Stewart, *Advanced Classical Mechanics*, MIT OCW, 2014. [Online Available]
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