Introduction to Superconducting Quantum Circuits

- Review of Quantum Mechanics for Quantum Computing -

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Keywords in Quantum Mechanics for Quantum Computing

Postulates of Quantum Mechanics

Wave Function
Schrödinger Equation
Uncertainty Principle
Ehrenfest's Theorem

Quantum Harmonic Oscillator

Canonical Quantization
Ladder Operator
Eigenenergy

Quantum Mechanical Analysis

Schrödinger Pictures
Heisenberg pictures

Quantum Mechanical Two-Level Systems

Superposition
Entanglement
Bloch Sphere
Bell State
Pauli Matrix
Rabi Oscillation
Pure State
Mixed State
Density Matrix

Introduction to Quantum Mechanics

- What is Quantum Mechanics?
  - fundamental theory that describes the behavior of nature that are quantized to discrete values
  - The development of quantum mechanics arose gradually from the black-body radiation problem in early 1900's
- Why is Quantum Mechanics Important for Studying Quantum Computing?
  - Almost everything of superconducting quantum circuits are based on quantum mechanics

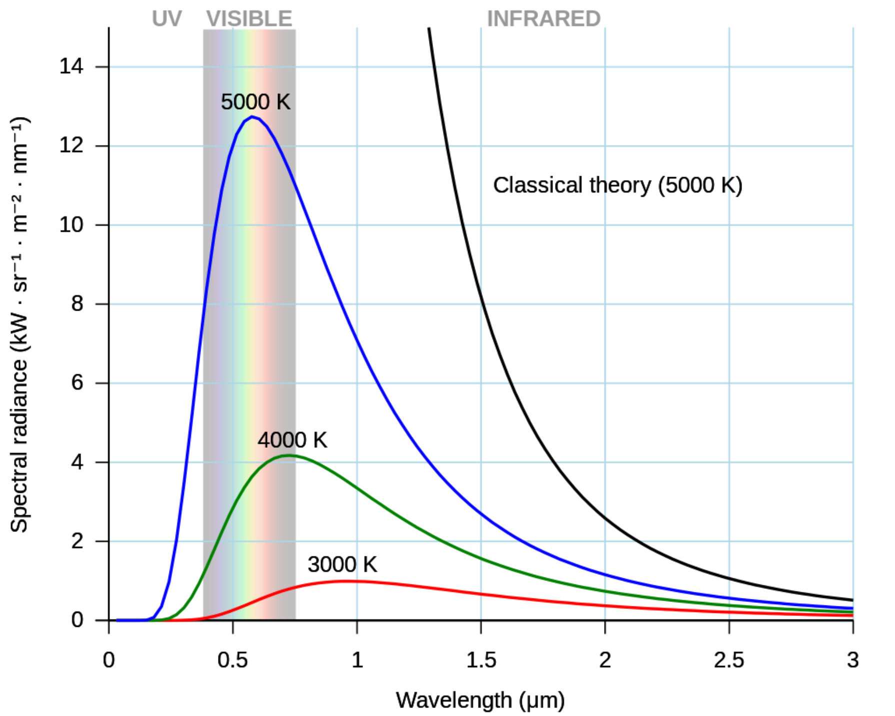

Fig. Blackbody radiation and the prediction from classical Rayleigh theory

Image from: https://en.wikipedia.org/wiki/Black-body_radiation

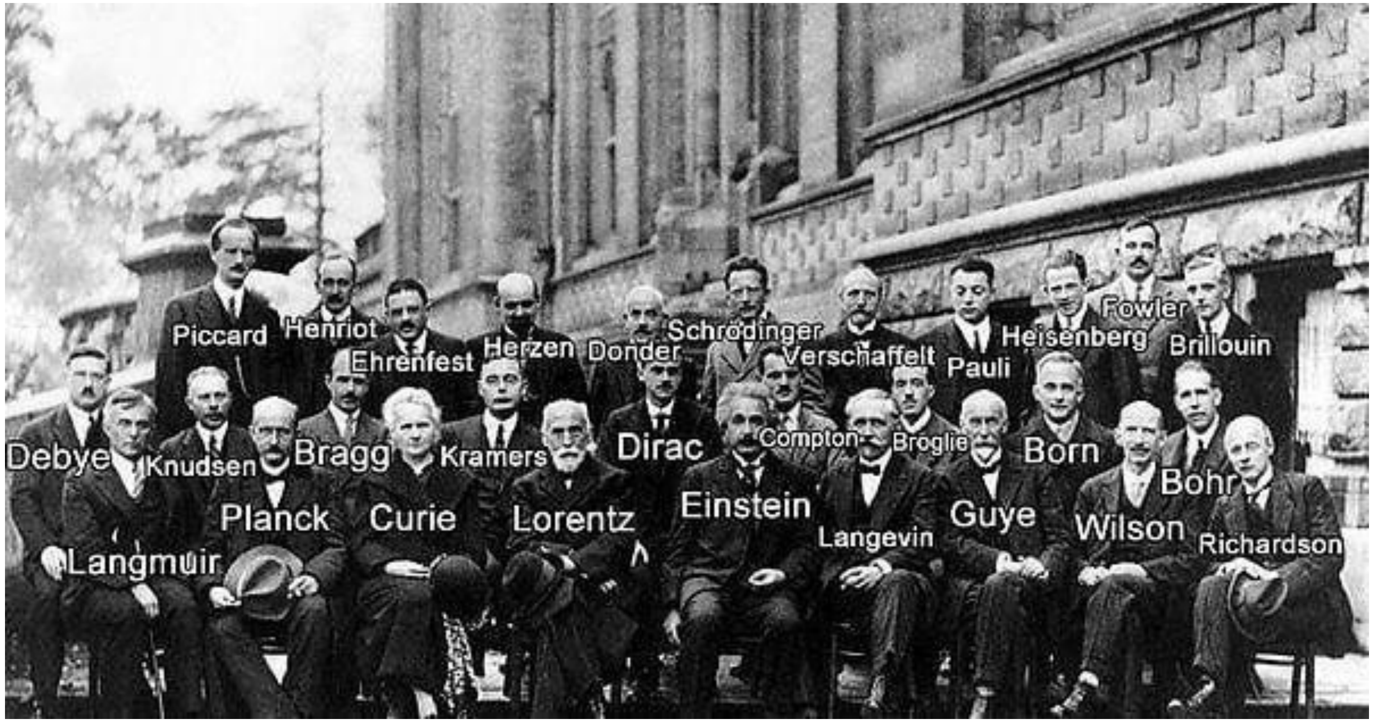

Fig. Solvay Conference in 1927

Postulates in Quantum Mechanics: Wave Function

- Definition of Wave Function
  - Mathematical description of the quantum state of an isolated quantum system
  - Common symbols for a wave function are $\psi(\boldsymbol{x}, t)$, where $\boldsymbol{x}$ and t represent position and time, respectively
  - A generalized wave function can be expressed as

$$\psi(\boldsymbol{x}, t) = Ae^{i(kx-\omega t)}$$

where
k is wave number
ω is angular frequency

- Characteristics of Wave Function
  - Probability: the square of the amplitude of a wave function $|\psi(\boldsymbol{x}, t)|^2$ is probability density of finding the particle at $\boldsymbol{x}$ and t
  - Normalization: the total probability of finding the particle must be 1 that

$$\int_{-\infty}^{\infty} |\psi(\boldsymbol{x}, t)|^2 \, dx = 1$$

  - Superposition: The superposed wave functions $\psi_1(\boldsymbol{x}, t)$ and $\psi_2(\boldsymbol{x}, t)$ can be a sum that

$$\psi(\boldsymbol{x}, t) = c_1\psi_1(\boldsymbol{x}, t) + c_2\psi_2(\boldsymbol{x}, t)$$

where c_1 and c_2 are constants

  - Expectation: The expectation value of an arbitrary operator $\hat{A}$ is defined as

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$$

Recall that… $\langle Q \rangle = \int_{-\infty}^{\infty} \psi^* Q_{operator} \psi dV$ integral over all space

Postulates in Quantum Mechanics: Schrödinger's Equation

- Definition of Schrödinger's Equation
  - Fundamental equation of motion to describe the quantum state (or wave function) of a system evolving over time
  - See E. Schrödinger, "An undulatory theory of the mechanics of atoms and molecules," *Phys. Rev.*, **28**, 1049-1070 (1926)
- Time-dependent Schrödinger's Equation
  - For a time-dependent potential $V(\boldsymbol{x}, t)$, the Schrödinger equation is:

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(x,t)\right)\psi(x,t)$$

where
$\hbar$: reduced Planck's constant ($h/(2\pi)$)
m: mass of a particle

- Time-independent Schrödinger Equation
  - For a time-independent potential $V(\boldsymbol{x})$, the wave function can be separated as $\psi(x,t) = \psi(x)e^{-iEt/\hbar}$

$$E\psi(x) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right)\psi(x)$$

where
E: total energy of the quantum system

NOTE:
Most of quantum mechanical equations that we will solve are time-independent !
(except for quantum gate analysis)

Postulates in Quantum Mechanics: Commutation Relation

- **Definition of Commutator**
  - In classical mechanics, all observables with the Poisson bracket is:

$$\{x, p\} = 1$$

  - In quantum mechanics, all observables are Hermitian operators
  - In quantum mechanics, for arbitrary two operators of $\hat{A}$ and $\hat{B}$, the commutator is:

$$\left[\hat{A}, \hat{B}\right] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

- **Characteristics of Commutation Relation**
  - Canonical commutation relation: the commutation equals to $j\hbar$ for canonical conjugate operators (proposed by Paul Dirac)
  - For example, position $\hat{x}$ and momentum $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ are canonical conjugates that $[\hat{x}, \hat{p}_x] = i\hbar$
  - Simple proof,

$$\begin{aligned}[\hat{x}, \hat{p}_x]\cancel{\psi} &= (\hat{x}\hat{p}_x - \hat{p}_x\hat{x})\psi \\ &= -i\hbar\left[x\frac{\partial}{\partial x} - \frac{\partial}{\partial x}x\right]\psi \\ &= -i\hbar\cancel{\psi}\end{aligned}$$

NOTE:
The momentum operator in quantum mechanics is defined as $p = -i\hbar\nabla$ from the generalized definition of wave function

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Postulates in Quantum Mechanics: Uncertainty Principle

- Definition of Heisenberg's Uncertainty principle
  - Fundamental concept in quantum mechanics that it is impossible to simultaneously measure canonically conjugate variables
  - Recall that position x and momentum p are canonically conjugated variables
  - The uncertainty principle regarding x and p is:

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

where
σ_x: standard deviation (uncertainty) of position x
σ_p: standard deviation (uncertainty) of momentum p

- Characteristics of Heisenberg's Uncertainty principle
  - The uncertainty principle IS NOT DUE TO measurement error, but a fundamental limit of nature
  - The uncertainty principle is applicable for all pairs of quantities that are canonically conjugated

- Uncertainty principle in Quantum Computing
  - The uncertainty principle is important concept in qubit state readout
  - The uncertainty principle is important concept to understand the squeezed coherent state

NOTE:
Proof of the uncertainty principle can be found in
https://www.phys.ufl.edu/courses/phy4604/fall18/uncertaintyproof.pdf

Postulates in Quantum Mechanics: Ehrenfest's Theorem

- Definition of Ehrenfest's Theorem
  - Fundamental concept in quantum mechanics to describe how the expectation values of quantum operators evolve over time
  - [analogy] Quantum mechanical Newton's laws of motion
  - The Ehrenfest's theorem for an arbitrary observable operator, $\hat{A}$, is defined as:

$$\frac{d}{dt}\langle\hat{A}\rangle = \frac{i}{\hbar}\langle[\widehat{H},\hat{A}]\rangle + \left\langle\frac{\partial\hat{A}}{\partial t}\right\rangle$$

where
$\langle\hat{A}\rangle$: expectation value of an observable operator $\hat{A}$
$\widehat{H}$: Hamiltonian of the quantum system

- Characteristics of Ehrenfest's Theorem
  - Bridge between classical and quantum mechanics: by showing the quantum system behave over time

$$\boxed{m\frac{d}{dt}x = p}$$

classical mechanics

$$\boxed{m\frac{d}{dt}\langle\hat{x}\rangle = \langle\hat{p}\rangle}$$

quantum mechanics

  - Expectation values: measurable quantities, as they represent the average outcome of many measurements

- Ehrenfest's Theorem in Quantum Computing
  - In superconducting quantum systems, Ehrenfest's theorem governs the evolution of observables such as phase and charge

Definition of Canonical Quantization

- Systematic transition from a classical mechanics to quantum mechanics, introduced by Paul Dirac
- Recall that “canonical” arises from the classical mechanics (see previous lecture notes)
- In classical mechanics, one can define the generalized coordinates q and momenta p
- In quantum mechanics, one can replace canonical variables with quantum operators as $q \rightarrow \hat{q},\ p \rightarrow \hat{p} = -i\hbar \frac{\partial}{\partial x}$

$$\boxed{\{q, p\} = 1} \qquad \boxed{[\hat{q}, \hat{p}] = i\hbar}$$

Poisson bracket for canonical variables (classical)

Commutation for canonical variables (quantum)

Canonical Quantization in Quantum Computing

- Superconducting circuits can be modeled by the generalized coordinates (here, flux Φ) and momenta (here, charge Q)
- By canonical quantization, the following commutation relation is confirmed:
  $[\hat{\Phi}, \hat{Q}] = i\hbar$

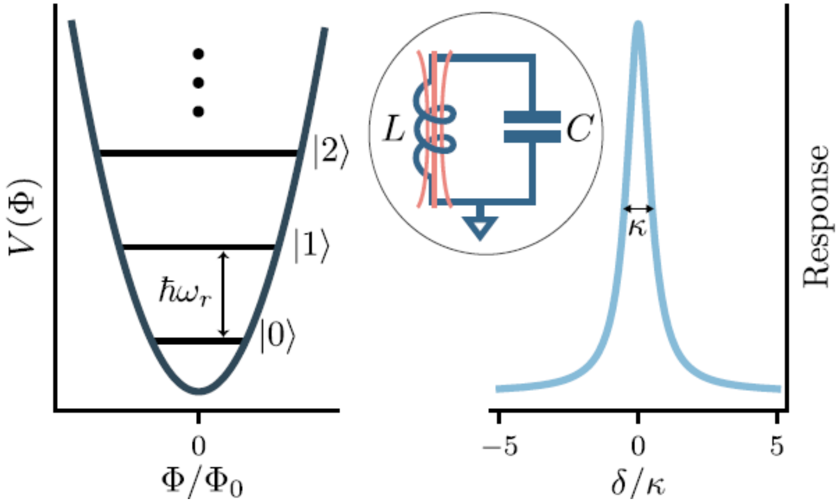


Fig. Harmonic potential of electrical LC resonator.

Image from: A Blais *et al.*, “Circuit Quantum Electrodynamics,” *Rev. Mod. Phys.*, **93** 025005 (2021).

Quantum Harmonic Oscillator: (1) Introduction

- Definition of Quantum Harmonic Oscillator
  - Quantum mechanical analog of the classical harmonic oscillator
  - The difference between consequent eigenenergy levels is always same
- Hamiltonian of Quantum Harmonic Oscillator
  - Quantum mechanical analog of the classical harmonic oscillator
  - For 1-dimensional coordinate, the Hamiltonian is expressed as:

$$\hat{H} = \underbrace{\boxed{\frac{\hat{p}^2}{2m}}}_{\text{kinetic}} + \underbrace{\boxed{\frac{1}{2}k\hat{x}^2}}_{\text{potential} \leftarrow \text{time-independent}} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

where
$\hat{p}$: momentum operator
$\hat{x}$: position operator
m: mass of a particle
k: force constant
$\omega = \sqrt{k/m}$: angular frequency

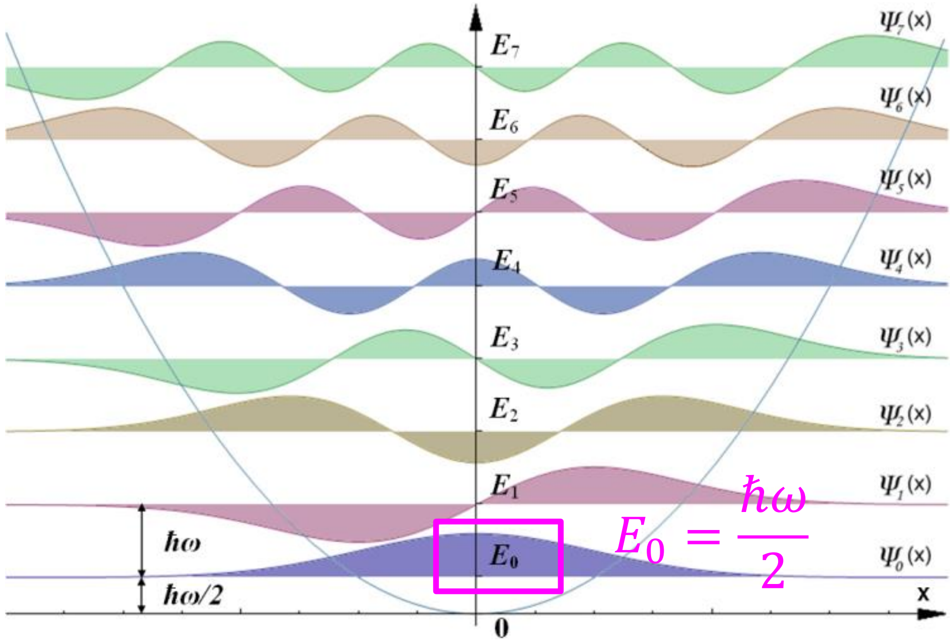


Fig. Visualized wave function of a quantum harmonic oscillator

- Schrödinger Equation of Quantum Harmonic Oscillator

$$\hat{H}|\psi\rangle = E|\psi\rangle \longleftarrow$$

NOTE:
Solving this partial derivative equation is challenging…
Instead, we will calculate the eigenenergy by novel approach

Quantum Harmonic Oscillator: (2) Ladder Operator

- Definition of Ladder Operator
  - Ladder operator was developed by Paul Dirac to calculate eigenenergy
  - There are two types of ladder operator:
    (1) lowering operator $\hat{a}$ and (2) raising operator $\hat{a}^{\dagger}$ (adjoint of $\hat{a}$)

    NOTE:
    Regarding superconducting circuits, $\hat{a}$ is often referred as ‘annihilation’ while $\hat{a}^{\dagger}$ is referred as ‘creation’
  - The ladder operators can be defined as:

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right) \text{ and } \hat{a}^{\dagger} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$$

  - From the definition, the following relations are satisfied:

$$\hat{a}^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle \text{ and } \hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

where
$|n\rangle$: eigenstate of n-th level (*the eigenstate does not need to be derived)

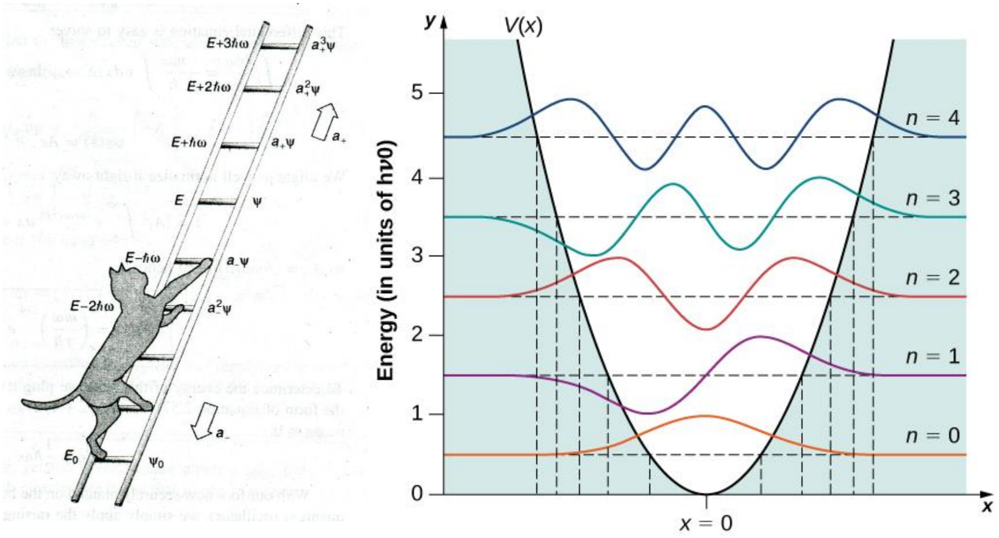


Fig. Famous illustration of Schrödinger cat’s ladder operator from David Griffiths, *Introduction to Quantum Mechanics*, 2nd edition

- Characteristics of Ladder Operator
  - Commutation: $[\hat{a}, \hat{a}^{\dagger}] = 1$
  - Number operator: $N = \hat{a}^{\dagger}\hat{a}$ (with eigenvalue of n)

$[\hat{a}, \hat{a}^{\dagger}] = 1$ and $N|n\rangle = n|n\rangle$ can be easily derived from the definition of the ladder operators (try it ☺)

Quantum Harmonic Oscillator: (3) Eigenenergy and Eigenstate

- Simplified Hamiltonian with Ladder Operators
  - Recall that the original Hamiltonian is $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$
  - Recall that the ladder operators are $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$ and $\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$
  - The Hamiltonian can be simplified as:

$$\hat{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) = \hbar\omega\left(\hat{N} + \frac{1}{2}\right)$$

where
$\hat{N} = \hat{a}^\dagger\hat{a}$: number operator

  - Thus, the n-th eigenenergy and eigenstate of the Hamiltonian are:

$$\therefore \boxed{E_n = \hbar\omega\left(n + \frac{1}{2}\right)} \text{ and } |\psi_n\rangle = |n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle$$

where
$|0\rangle$: the zeroth state (ground state)

From the calculated eigenenergy, one can know that the quantum harmonic oscillator has equidistant energy levels, as expected

Quantum Mechanical Analysis: Schrödinger's and Heisenberg's Picture

Definition of Schrödinger's Picture

- To understand quantum mechanics, there are two different ways to interpret time-evolving quantum system
- In the Schrödinger's framework, wave functions evolve with time, while operators remain constant (time-independent)

$$\hat{H}|\psi(t)\rangle = i\hbar \frac{d}{dt}|\psi(t)\rangle$$

- Schrödinger's picture focuses on the dynamics of the quantum states themselves

most of students may be more familiar with Schrodinger's picture than Heisenberg's picture…

Both frameworks shows the same results!

Definition of Heisenberg's Picture

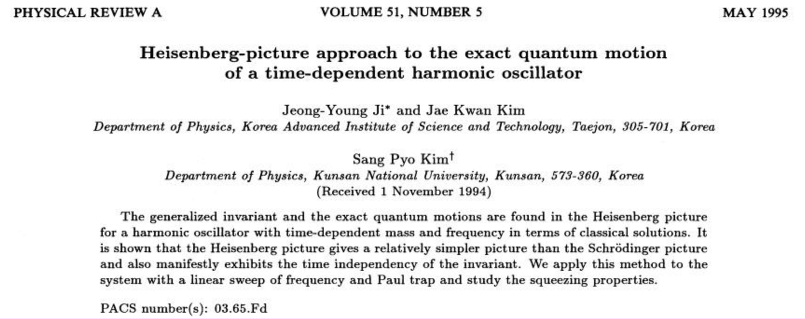
- In the Heisenberg's framework, operators evolve with time, while wave functions remain constant (time-independent)

$$\frac{d\hat{A}(t)}{dt} = \frac{i}{\hbar}\left[\hat{H}, \hat{A}(t)\right] + \left(\frac{\partial \hat{A}(t)}{\partial t}\right)$$

- If $\hat{A}$ has no explicit time dependence, $\frac{\partial \hat{A}(t)}{\partial t} = 0$ can be assumed

See Heisenberg's picture approach to a quantum harmonic oscillator in:

J-Y Ji, JK Kim, SP Kim, Phys. Rev. A, **51**, 4268-4271 (1995).



PHYSICAL REVIEW A VOLUME 51, NUMBER 5 MAY 1995

Heisenberg-picture approach to the exact quantum motion of a time-dependent harmonic oscillator

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The generalized invariant and the exact quantum motions are found in the Heisenberg picture for a harmonic oscillator with time-dependent mass and frequency in terms of classical solutions. It is shown that the Heisenberg picture gives a relatively simpler picture than the Schrödinger picture and also manifestly exhibits the time independency of the invariant. We apply this method to the system with a linear sweep of frequency and Paul trap and study the squeezing properties.

PACS number(s): 03.65.Fd

Quantum Mechanical Analysis: Schrödinger's and Heisenberg's Picture

- Simple Example: Time Evolution of Superconducting Qubit
  - Consider a superconducting qubit in the ground state $|0\rangle$ at time $t = 0$ with Hamiltonian:

$$\hat{H} = \frac{1}{2}\hbar\omega\hat{\sigma}_x = \frac{1}{2}\hbar\omega\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad |\psi(t=0)\rangle = |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

Schrödinger's picture

- The qubit state at time t is:

$$|\psi(t)\rangle = e^{-i\frac{\hat{H}t}{\hbar}}|\psi(0)\rangle$$

$$|\psi(t)\rangle = e^{-i\frac{\omega\hat{\sigma}_x t}{2}}|0\rangle$$

- Probability of measuring the qubit in $|1\rangle$ is:

$$P_{|1\rangle}(t) = |\langle 1|\psi(t)\rangle|^2 = \sin^2\left(\frac{\omega t}{2}\right)$$

Heisenberg's picture

- The operator at time t is:

where
$\hat{\sigma}_y$ and $\hat{\sigma}_z$: Pauli-Y,Z operators

$$\frac{d\hat{\sigma}_x(t)}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{\sigma}_x(t)] = 0$$ → time-independent operator

$$\left.\begin{aligned}\frac{d\hat{\sigma}_y(t)}{dt} &= \frac{i}{\hbar}[\hat{H}, \hat{\sigma}_y(t)] = \frac{i}{\hbar}\left[\frac{\hbar\omega}{2}\hat{\sigma}_x, \hat{\sigma}_y\right] = -\omega\hat{\sigma}_z\\ \frac{d\hat{\sigma}_z(t)}{dt} &= \frac{i}{\hbar}[\hat{H}, \hat{\sigma}_z(t)] = \frac{i}{\hbar}\left[\frac{\hbar\omega}{2}\hat{\sigma}_x, \hat{\sigma}_z\right] = \omega\hat{\sigma}_y\end{aligned}\right\}\text{solve these}$$

$$\therefore \hat{\sigma}_z(t) = \hat{\sigma}_z(0)\cos(\omega t) + \hat{\sigma}_y(0)\sin(\omega t)$$

$$= \begin{pmatrix}\cos(\omega t) & -i\sin(\omega t)\\ i\sin(\omega t) & \cos(\omega t)\end{pmatrix}$$

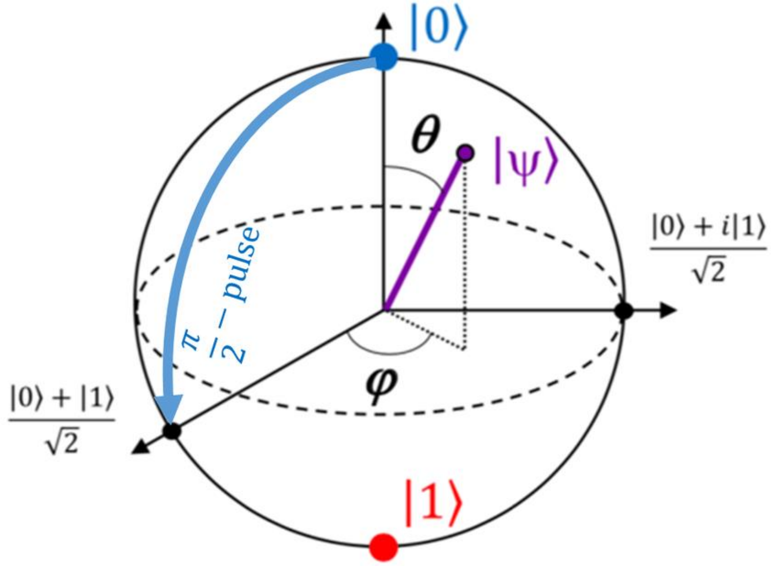
- Probability of measuring the qubit in $|1\rangle$ is:

$$P_{|1\rangle}(t) = \frac{1}{2}(1 - \langle\psi|\hat{\sigma}_z(t)|\psi\rangle) = \sin^2\left(\frac{\omega t}{2}\right)$$

Quantum Mechanical Two Level System: (1) Introduction

Definition of Two Level System

- Simplest quantum mechanical system, characterized by having only two possible energy states
- The quantum system has ground state $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and excited state $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$
- Superconducting qubit and resonator can be modelled as two level system



$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi} sin\frac{\theta}{2}|1\rangle$$

Fig. Bloch sphere of a two level system

Characteristics of Two Level System

- Superposition: the system can exist in a combination of $|0\rangle$ and $|1\rangle$ simultaneously

$$|\psi\rangle = c_1|0\rangle + c_2|1\rangle$$ where c_1 and c_2 are constant (complex)

NOTE: visualization of the superposed state is often illustrated on Bloch sphere

- Entanglement: the quantum state of each two level system of the group cannot be described independently of the state of the others

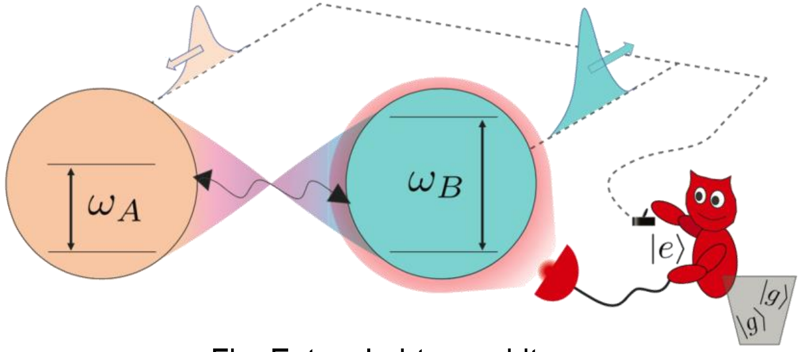


Fig. Entangled two qubits

← measurement of a qubit will collapse the quantum state of the entangled pair

https://phys.org/news/2221-04-two-qubit-powered-entanglement-local.html

Quantum Mechanical Two Level System: (1) Introduction

- Supplementary Video on Superposition

https://en.wikipedia.org/wiki/Quantum_superposition

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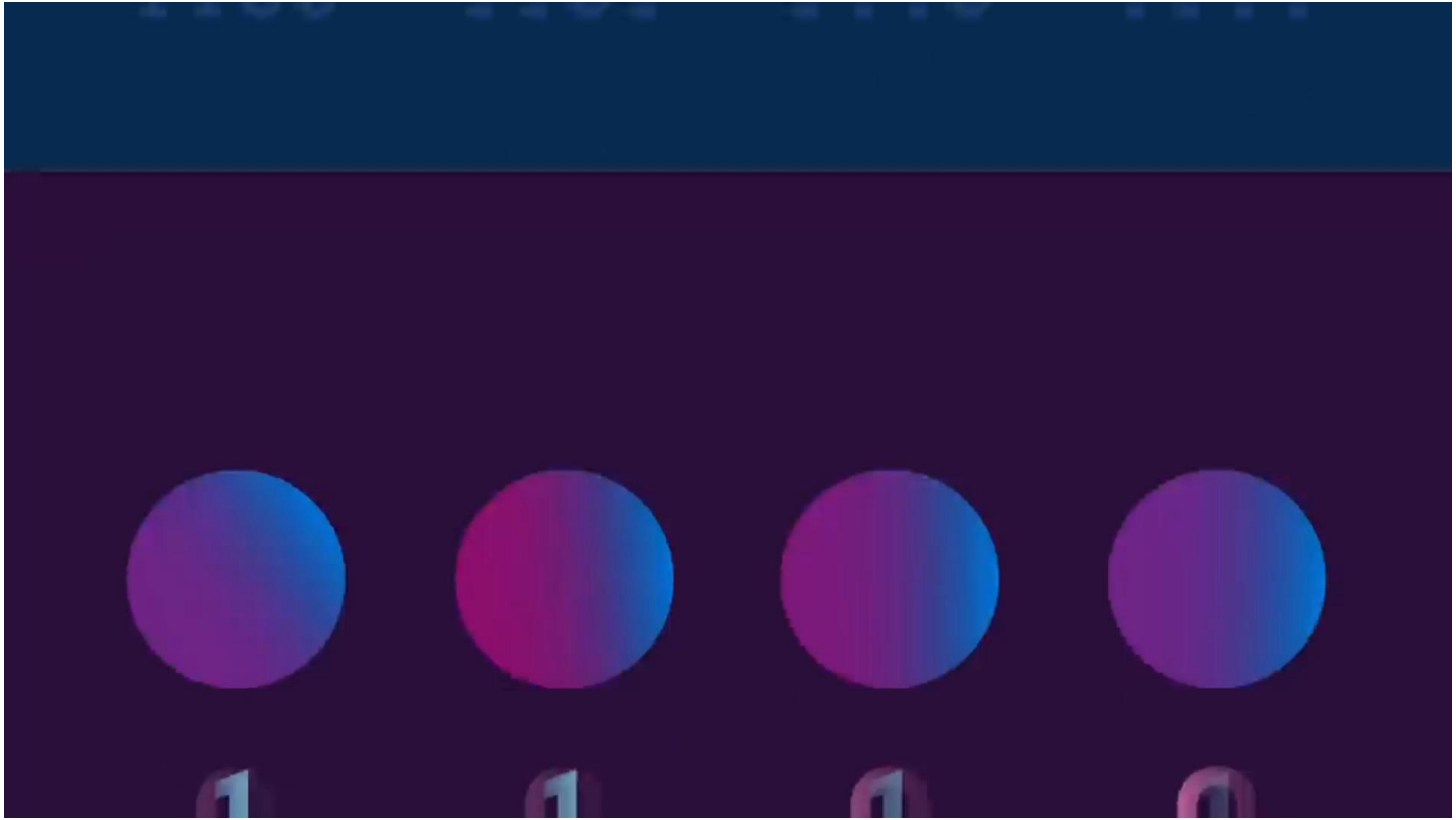
- Supplementary Video on Entanglement



https://www.youtube.com/watch?v=JhHMJCUmq28

Quantum Mechanical Two Level System: (2) Pauli Operators

Definition of Pauli Operators

- A set of three 2×2 complex matrices that are Hermitian and unitary
- Pauli X operator:
  $$\hat{\sigma}_x = \hat{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
  bit-flip operation!
- Pauli Y operator:
  $$\hat{\sigma}_y = \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
  bit & phase-flip operation!
- Pauli Z operator:
  $$\hat{\sigma}_z = \hat{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
  phase-flip operation!

Operator	Gate(s)
Pauli-X (X)	X
Pauli-Y (Y)	Y
Pauli-Z (Z)	Z

Fig. Pauli operators as quantum logic gates

Characteristics of Pauli Operators

- Hermitian: the Pauli operators are Hermitian that $\hat{\sigma} = \hat{\sigma}^\dagger$
- Unitary: the Pauli operators are unitary that $\hat{\sigma}\hat{\sigma}^\dagger = \hat{\sigma}^\dagger\hat{\sigma} = \mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Commutation Relations of Pauli Operators

- $[\hat{\sigma}_i, \hat{\sigma}_j] = 2i\epsilon_{ijk}\hat{\sigma}_k$ where ϵ_{ijk}: Levi-Civita
- $[\hat{\sigma}_x, \hat{\sigma}_y] = [\hat{\sigma}_1, \hat{\sigma}_2] = 2i\hat{\sigma}_z = 2i\hat{\sigma}_3$
- $[\hat{\sigma}_y, \hat{\sigma}_z] = [\hat{\sigma}_2, \hat{\sigma}_3] = 2i\hat{\sigma}_x = 2i\hat{\sigma}_1$
- $[\hat{\sigma}_z, \hat{\sigma}_x] = [\hat{\sigma}_3, \hat{\sigma}_1] = 2i\hat{\sigma}_y = 2i\hat{\sigma}_2$

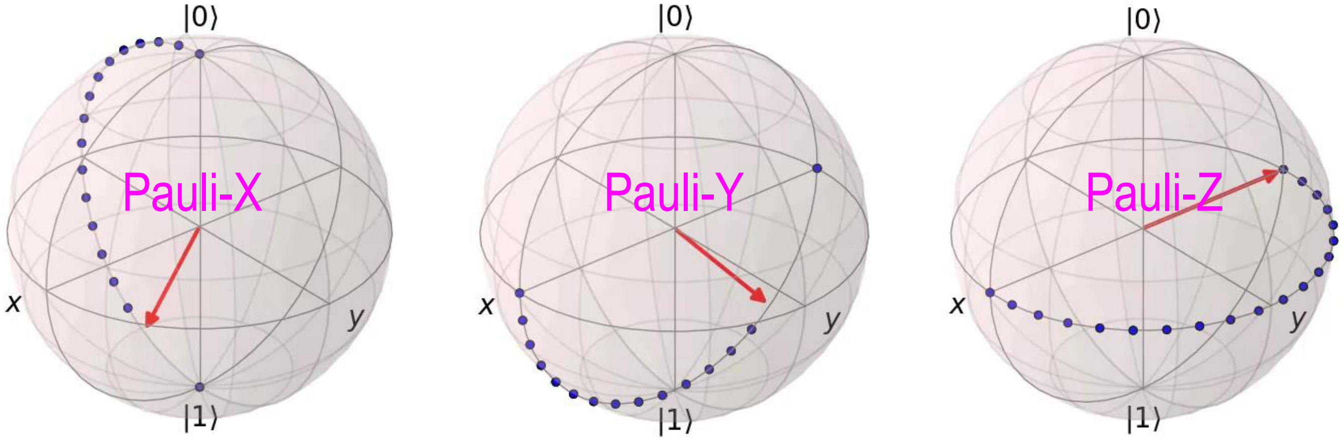


Fig. Pauli operators as quantum logic gates illustrated on Bloch sphere

Quantum Mechanical Two Level System: (3) Rabi Oscillation

Definition of Rabi Oscillation

- The coherent oscillations of a two-level system's population between its $|0\rangle$ and $|1\rangle$ when subjected to a resonant oscillating external field (for superconducting qubits, microwave pulse drive)

Characteristics of Rabi Oscillation

- Rabi frequency Ω_R: the frequency rate at which oscillation occurs
- Drive amplitude A_R: the amplitude of the applied oscillating field
- Drive time t_R: the duration of the applied oscillating field
- As the drive amplitude increases, the Rabi oscillation occurs faster

Rabi Oscillation in Quantum Computing

- Microwave pulse drive is applied to a qubit to manipulate its state

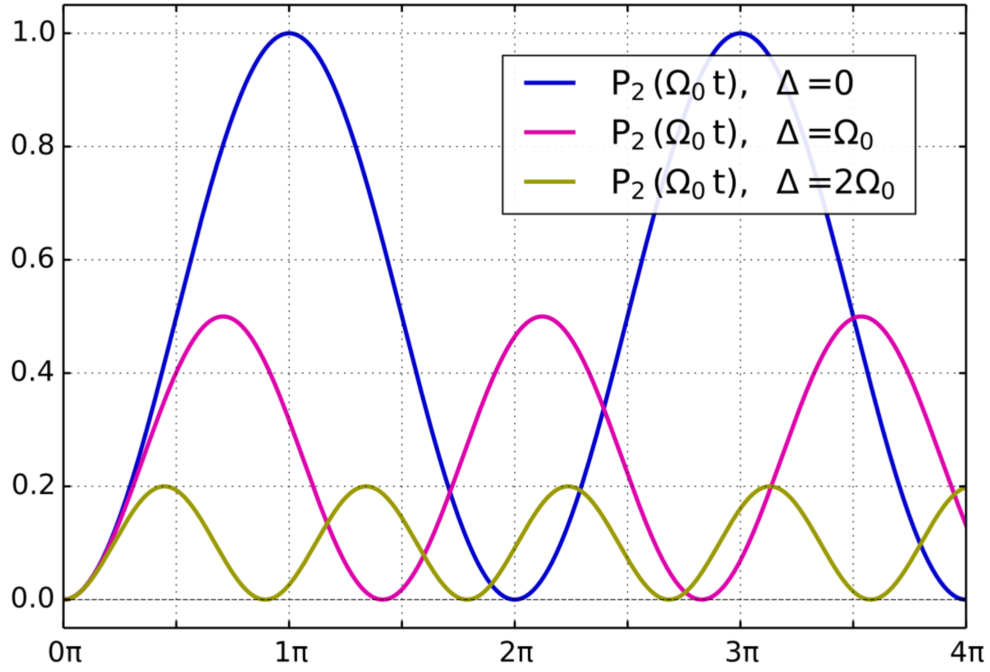
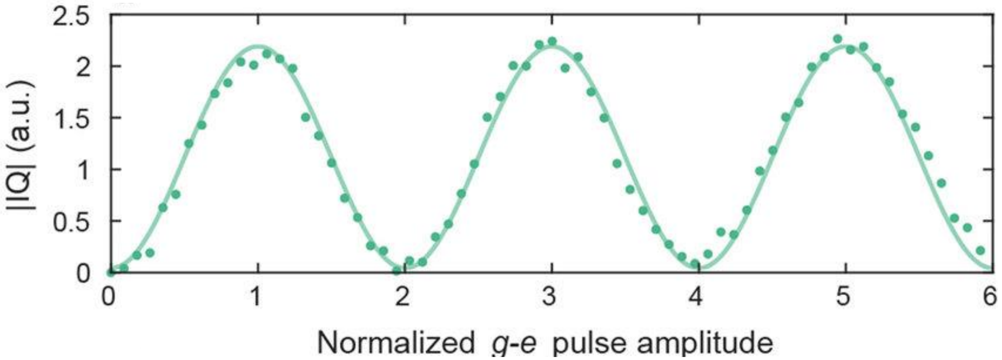


Fig. Rabi oscillations, showing the probability of a two-level system

https://en.wikipedia.org/wiki/Rabi_cycle



NOTE

The π-pulse (full transition $|0\rangle \leftrightarrow |1\rangle$) is calibrated first with t_π and A_π.

Then, the $\frac{\pi}{2}$-pulse (superposition) is adjusted by $t_{\frac{\pi}{2}} = \frac{1}{2} t_\pi$ or $A_{\frac{\pi}{2}} = \frac{1}{2} A_\pi$.

Quantum Mechanical Two Level System: (4) Pure State and Mixed State

- Definition of Pure State
  - A quantum system is in a pure state when it is described by a single wave function $|\psi_{\mathrm{pure}}\rangle$
- Characteristics of Pure State
  - Density matrix: $\rho = |\psi_{\mathrm{pure}}\rangle\langle\psi_{\mathrm{pure}}|$ with its trace $\mathrm{Tr}(\rho) = 1$
  - Purity: defined as the trace of ρ^2 that $\mathrm{Tr}(\rho^2) = 1$
- Definition of Mixed State
  - A quantum system is in a mixed state when it is described by an ensemble of pure states that $|\psi\rangle = \sum c_i |\psi_i\rangle$

where c_i: constant

- Characteristics of Mixed State
  - Density matrix: $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ with its trace $\mathrm{Tr}(\rho) = 1$
  - Purity: $\mathrm{Tr}(\rho^2) < 1$
- Examples (Show It by Yourself)
  - $\rho = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
  - $\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Answers will be disclosed…

See Also…

- Textbooks:

[1] DJ Griffiths, *Introduction to Quantum Mechanics*, Pearson Prentice Hall, 2004. * recommended

[2] SM Girvin, Basic Concepts in Quantum Information, *arXiv:1302.5842v1*, 2013.

- Open Courses:

[1] Barton Zwiebach, *Quantum Physics*, MIT OCW, 2016. [Online Available]

https://ocw.mit.edu/courses/8-04-quantum-physics-i-spring-2016/pages/video-lectures/part-1/

[2] Senthil Todadri, *Quantum Theory*, 2017 [Online Available]

https://ocw.mit.edu/courses/8-321-quantum-theory-i-fall-2017/pages/syllabus/