

Introduction to Superconducting Quantum Circuits

- Review of Quantum Harmonic/Anharmonic Oscillators -

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Keywords in Quantum Harmonic/Anharmonic Oscillators

Superconducting Resonator: Quantum Harmonic Oscillator

Generalized Coordinates	Node and Branch
Canonical Quantization	Zero-point fluctuation
Eigenenergy	

Superconducting Qubit: Quantum Anharmonic Oscillator

Cooper Pair Box	Transmon Qubit
Tunable Transmon Qubit	Flux Qubit
Transmon Regime	Weakly Anharmonic Qubit
Josephson Energy	Charging Energy

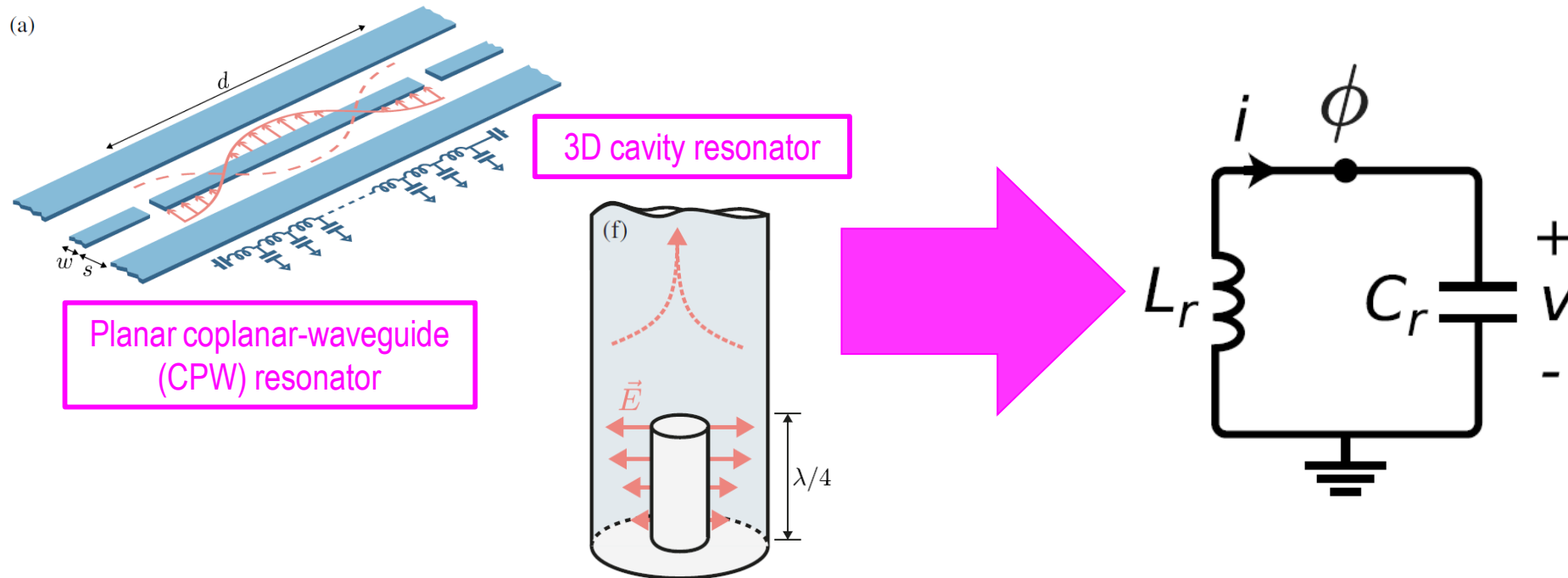
Introduction to Superconducting Resonator

■ What Is Superconducting Resonator?

- A component designed to store and confine electromagnetic energy at specific microwave frequencies with minimal loss
- Made from superconducting materials, it exhibits extremely high quality factors Q

■ Equivalent Circuit Diagram for a Superconducting Resonator

- Whether a resonator is 3D cavity or planar layout, an equivalent circuit diagram is a **parallel LC circuit**



Q. How to describe a superconducting resonator with quantum mechanics?

A. Set the generalized coordinates and perform canonical quantization!

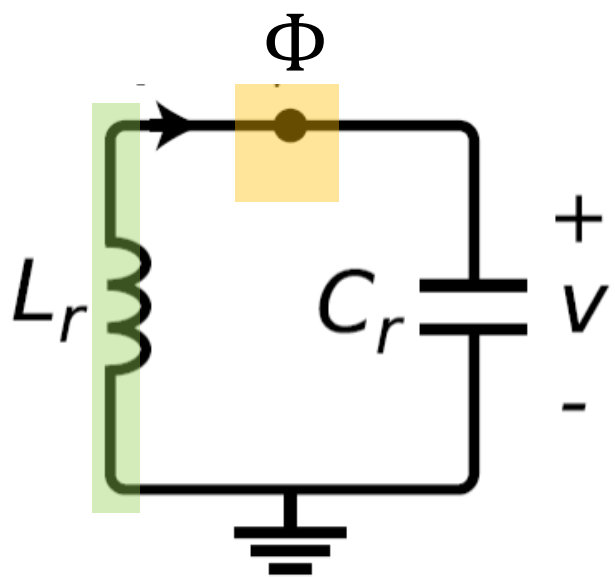
Image from A. Blais, AL Grimsco, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021).

Superconducting Resonator as a Quantum Harmonic Oscillator (1/3)

- Generalized Coordinates and Canonical Conjugate Momentum of a Superconducting Resonator
 - Recall the standard methods to obtain the canonical conjugate momentum from the Lagrangian
 - Let the position q , the velocity \dot{q} , the kinetic energy T and the potential energy U , the Lagrangian is $\mathcal{L}(q, \dot{q}, t) = T - U$
 - The canonical conjugate momentum is $p = \frac{\partial \mathcal{L}}{\partial \dot{q}}$

Node: a point between two or more elements

Table. Analogy between electrical circuit and quantum mechanics.



Electrical circuit	Mechanics
Circuit node flux $\Phi = \int_{-\infty}^t V(t')dt'$	Position x
Circuit node voltage $V = \frac{d\Phi}{dt}$	Velocity v
Circuit node charge $Q = CV = C \frac{d\Phi}{dt}$	(canonical conjugate) Momentum p
Inductive energy $E_L = \frac{1}{2}LI^2 = \frac{\Phi^2}{2L}$	Potential energy U
Capacitive energy $E_c = \frac{1}{2}CV^2 = \frac{1}{2}C\dot{\Phi}^2$	Kinetic energy T

Branch: a portion of the circuit and its two terminal nodes

From electrical circuit variables, we can choose node flux as position and node voltage as velocity to perform the standard canonical quantization method

Image from P Krantz et al., "A quantum engineer's guide to superconducting qubits," Appl. Phys. Rev., 6, 021318 (2019).

Superconducting Resonator as a Quantum Harmonic Oscillator (2/3)

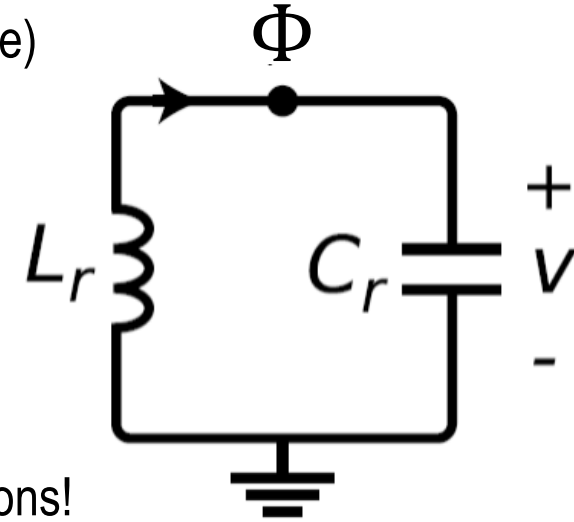
■ Lagrangian of a Superconducting Resonator

- For a parallel LC circuit, we can define a node flux Φ as a generalized position coordinate
- Then, a node voltage V can be assumed to be a generalized velocity (as shown in previous slide)
- The Lagrangian of the LC circuit can be derived as

$$\begin{aligned}\mathcal{L}(q, \dot{q}, t) &= \mathcal{L}(\Phi, \dot{\Phi}, t) = T - U \\ &= E_c - E_L = \frac{1}{2} C \dot{\Phi}^2 - \frac{\Phi^2}{2L}\end{aligned}$$

- From the Lagrangian, we can define a canonical conjugate momentum as

$$p = \frac{\partial \mathcal{L}(\Phi, \dot{\Phi}, t)}{\partial \dot{\Phi}} = C \dot{\Phi} = CV = Q \leftarrow \text{in electrical circuit relations!}$$



■ Hamiltonian of a Superconducting Resonator

- Recall that the Hamiltonian can be derived by Legendre transform as $\mathcal{H} = p\dot{q} - \mathcal{L}$
- Thus, the Hamiltonian of the LC circuit can be derived as

$$\begin{aligned}\mathcal{H} &= \dot{\Phi}Q - \mathcal{L} \\ &= \frac{Q^2}{2C} + \frac{\Phi^2}{2L}\end{aligned}$$

classical

replacing classical variables with quantum operators

(canonical commutation relation, Dirac method)

$$\mathcal{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

quantum

Superconducting Resonator as a Quantum Harmonic Oscillator (3/3)

■ Superconducting Resonator as a Quantum Harmonic Oscillator

- The Hamiltonian $\mathcal{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$ of a resonator (parallel LC circuit) is similar to that of a harmonic oscillator
- Using the definition of ladder operators, \hat{a} and \hat{a}^\dagger

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \left(\hat{x} + \frac{i}{m\omega} \hat{p} \right) \text{ and } \hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \left(\hat{x} - \frac{i}{m\omega} \hat{p} \right)$$

- The Hamiltonian of a resonator can be simplified as

$$\hat{\Phi} = \sqrt{\frac{\hbar Z_r}{2}} (\hat{a}^\dagger + \hat{a}) = \Phi_{\text{ZPF}} (\hat{a}^\dagger + \hat{a})$$

$$\hat{Q} = i \sqrt{\frac{\hbar}{2Z_r}} (\hat{a}^\dagger - \hat{a}) = Q_{\text{ZPF}} (\hat{a}^\dagger - \hat{a})$$

Note that

$$[\hat{\Phi}, \hat{Q}] = \hat{\Phi}\hat{Q} - \hat{Q}\hat{\Phi} = i\hbar$$

$$\therefore \mathcal{H} = \hbar\omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

n^{th} eigenenergy is $\hbar\omega_r \left(n + \frac{1}{2} \right)$
The energy spacing is equidistant

m : mass (for LC circuit, $m = C$)
 ω : resonant frequency (for LC circuit, $\omega_r = \frac{1}{\sqrt{L_r C_r}}$)
 \hat{a} : annihilation operator (lowering operator)
 \hat{a}^\dagger : creation operator (raising operator)
 $Z_r = \sqrt{\frac{L_r}{C_r}}$: characteristic impedance

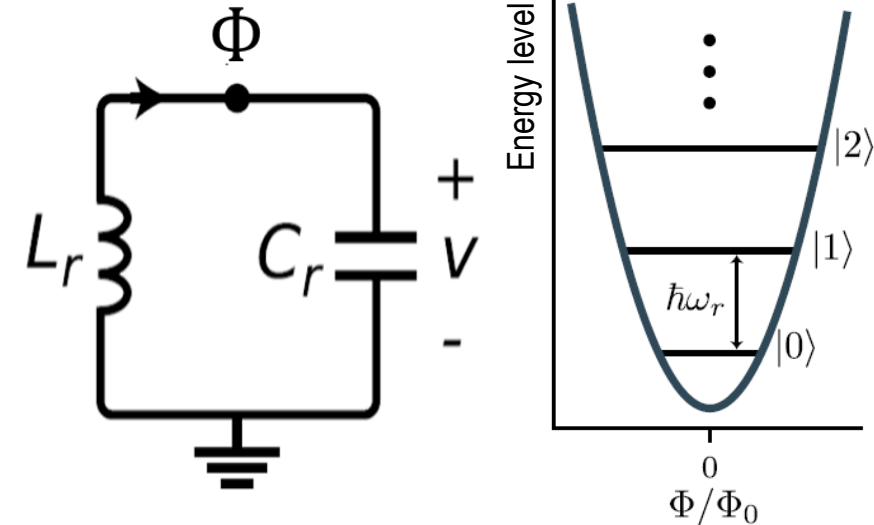


Fig. Circuit diagram and harmonic energy level vs flux of the circuit.

Image from A. Blais, AL Grimsco, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021).

Introduction to Superconducting Qubit

■ What Is Superconducting Qubit?

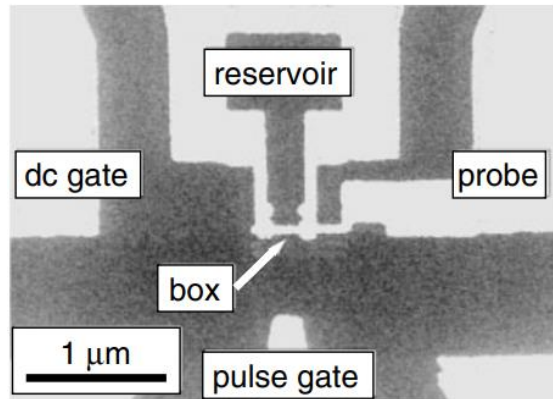
- A superconducting qubit is a quantum two-level system implemented using Josephson junctions
- The first superconducting qubit was realized as Cooper pair box *see Y. Nakamura *et al.*, *Nature* **398**, 786-788 (1999).

■ Key characteristics of Superconducting Qubits

- Superconductivity: low coherence loss and macroscopic quantum object
- Josephson junction: core component of a qubit to provide the nonlinearity needed to define discrete energy levels

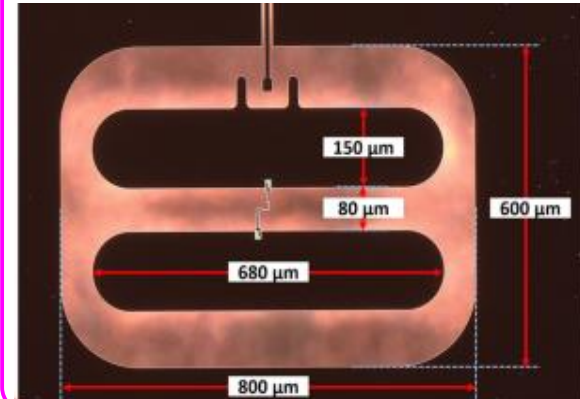
■ Types of Superconducting Qubits

Cooper pair box (CPB)



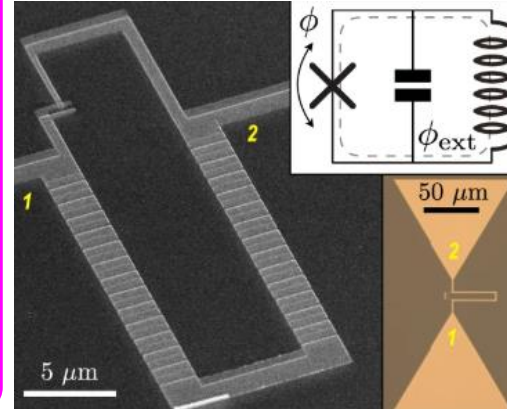
Y. Nakamura *et al.*, *Nature* **398**, 786-788 (1999)

Transmon



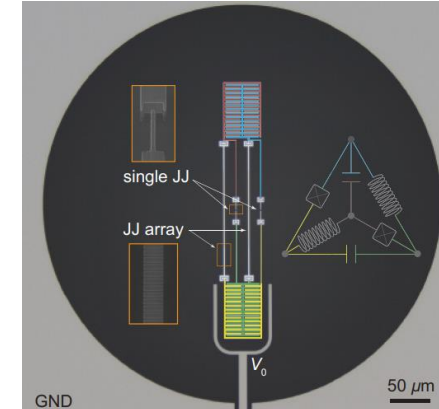
C Wang *et al.*, *npj Quantum Inf.*, **8**, 3 (2022)

Fluxonium



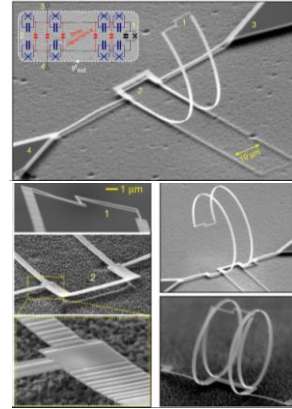
N Cottet *et al.*, *Nat. Commun.*, **12**, 6383 (2021)

0 — π qubit



A Gyenis *et al.*, *PRX Quantum*, **2**, 010339 (2021)

Blochnium



IV Pechenezhskiy *et al.*, *Nature*, **585**, 368-371 (2020)

Transmon qubit is well-established qubit architecture and one of the most popular candidate to realize a quantum computer

Selected Milestones in Transmon-Based Research (1/4)

Development of Cooper-pair Box (1999)

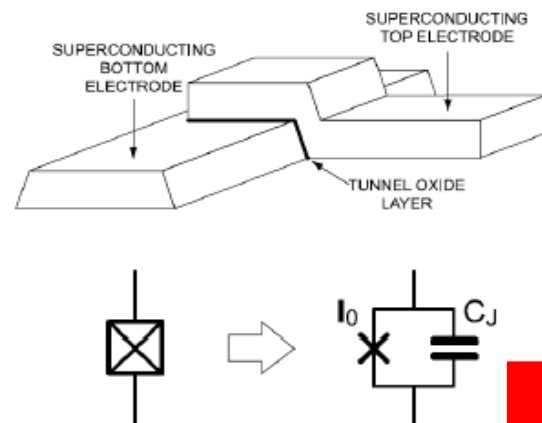
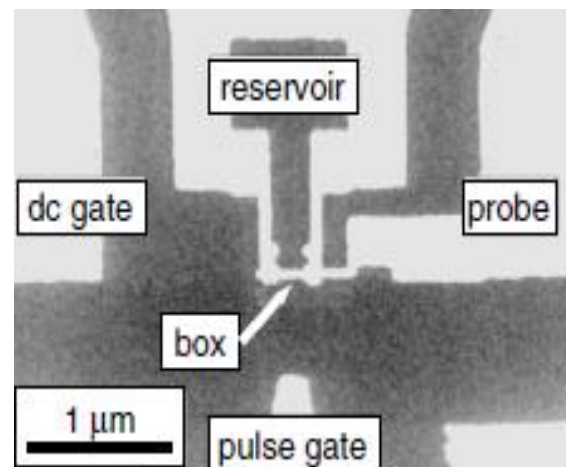


Fig. The first realization of Cooper-pair box and its components

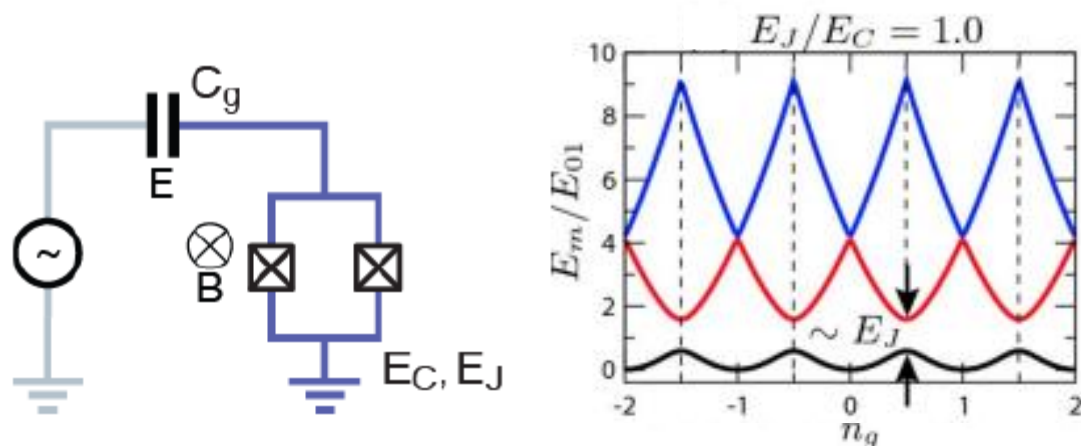


Fig. Circuit diagram and energy level diagram of a typical Cooper-pair box

Y. Nakamura, Y.A. Pashkin, J.S. Tsai, "Coherent control of macroscopic quantum states in a single-Cooper-pair box," *Nature* **398**, 786-788 (1999)

J. Koch, *et al.*, "Charge-insensitive qubit design derived from the Cooper pair box," *Phys. Rev. A*, **76**, 042319 (2007).

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Development of Transmon Qubit (2007)

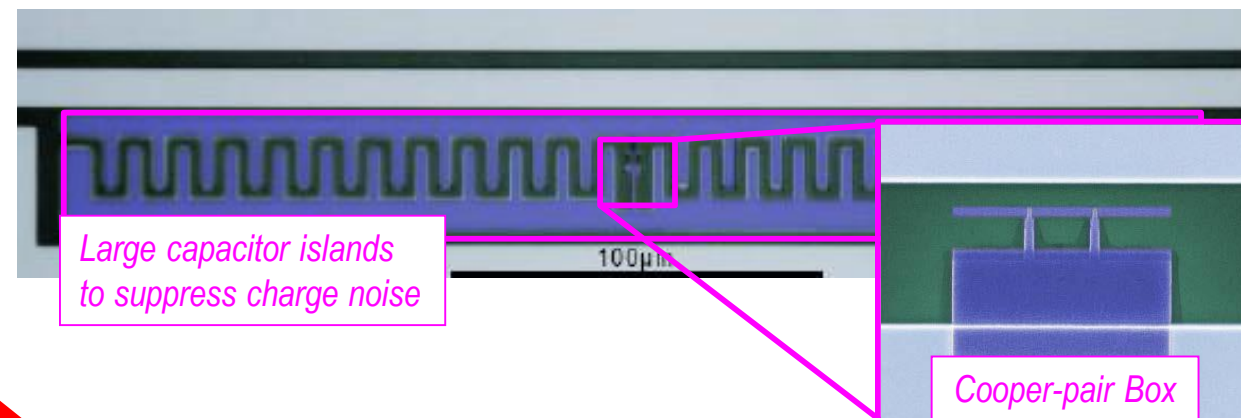


Fig. An optical image of a transmon qubit

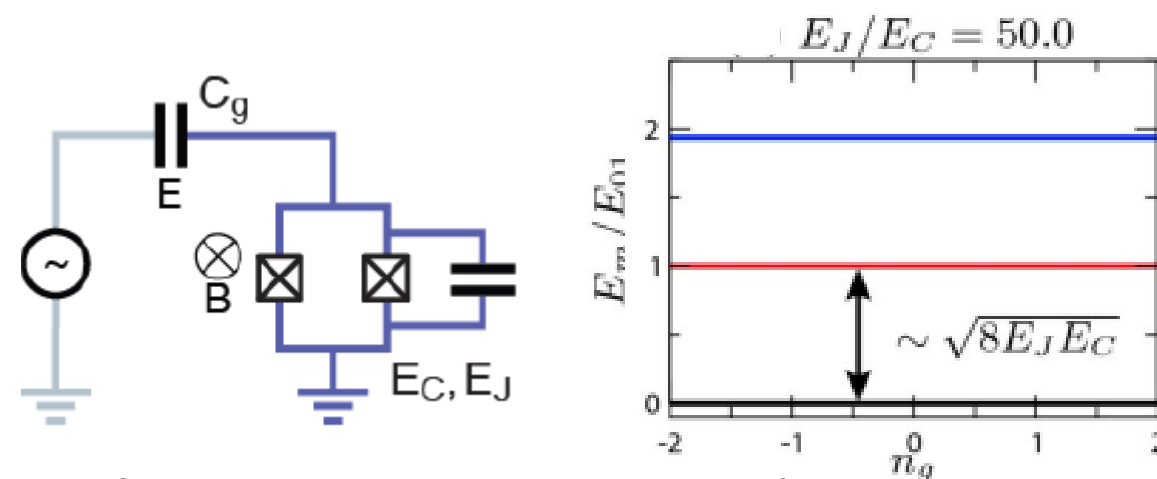


Fig. Circuit diagram and energy level diagram of a typical transmon qubit

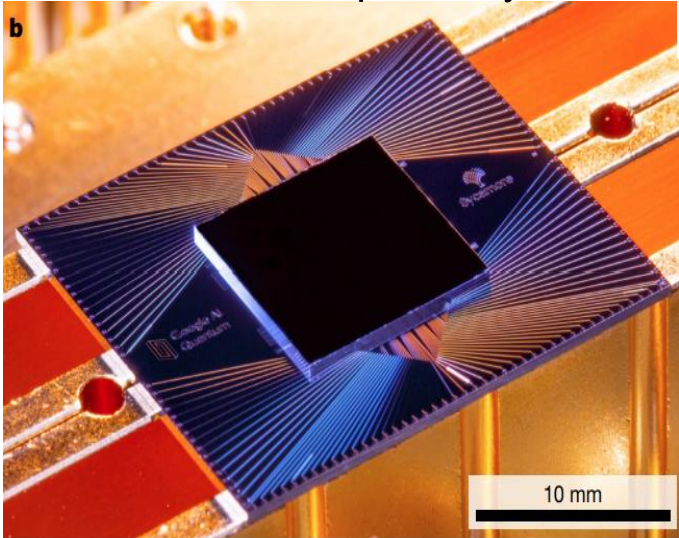
- Review of Quantum Harmonic/Anharmonic Oscillators for Quantum Computing -

ASL Quantum Lecture Meeting, Seoul, Republic of Korea, 2024/11/27

Selected Milestones in Transmon-Based Research (2/4)

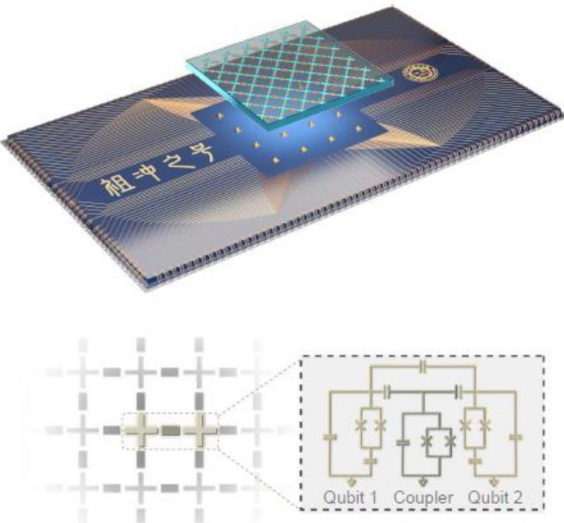
■ Demonstration of Quantum Supremacy with Superconducting Qubits (2019)

□ Quantum supremacy: no classical computer can solve in any feasible amount of time



< Google, 53-qubit >

Claimed their 200 seconds work will take at least 10,000 years with supercomputer



< Hefei, 66-qubit >

Claimed their 6000 seconds work will take at least 8 years with supercomputer

However, Google and Hefei performed “RANDOM circuits sampling” for benchmarking, which has nothing to do with solving practical problems...

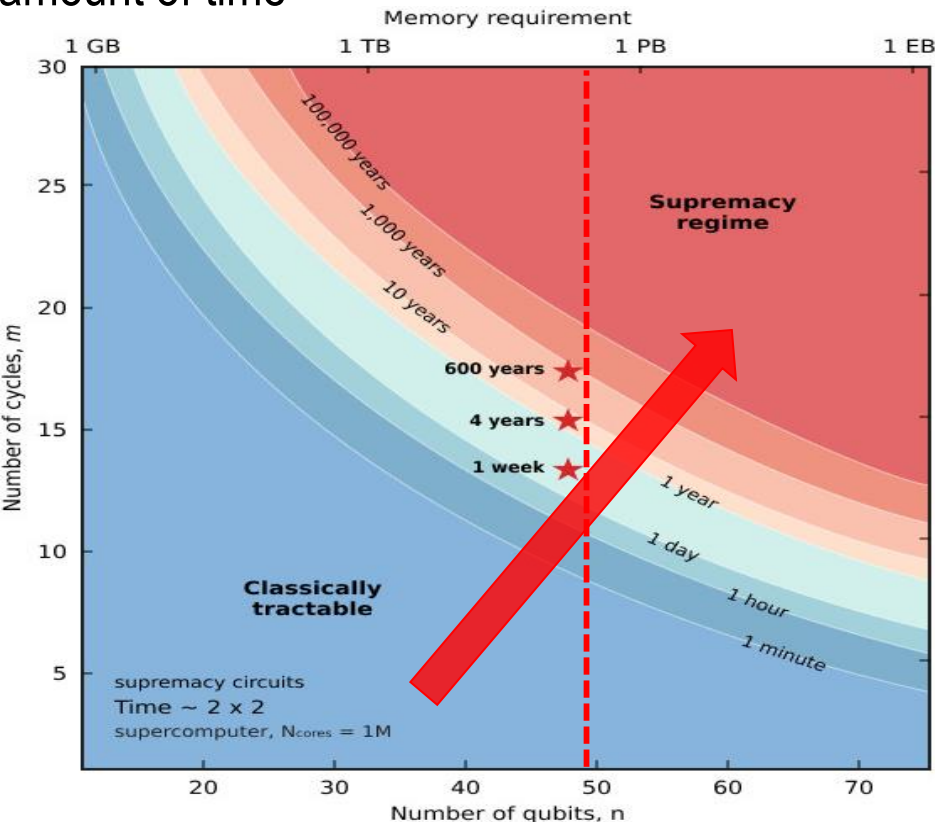


Fig. Scaling of the computational cost of cross entropy benchmarking with classical supercomputer.

F. Arute, *et al.*, “Quantum supremacy using a programmable superconducting processor,” *Nature*, **574**, 505–510 (2019).
Y. Wu, *et al.*, “Strong quantum computational advantage using a superconducting quantum processor,” *Phys. Rev. Lett.*, **127** 180501 (2021).

Selected Milestones in Transmon-Based Research (3/4)

- IBM Quantum Computer Breaks the 100-Qubit Barrier (2021)
 - Following the first quantum computer with more than 100-qubit back in 2021: “IBM 127-Q”

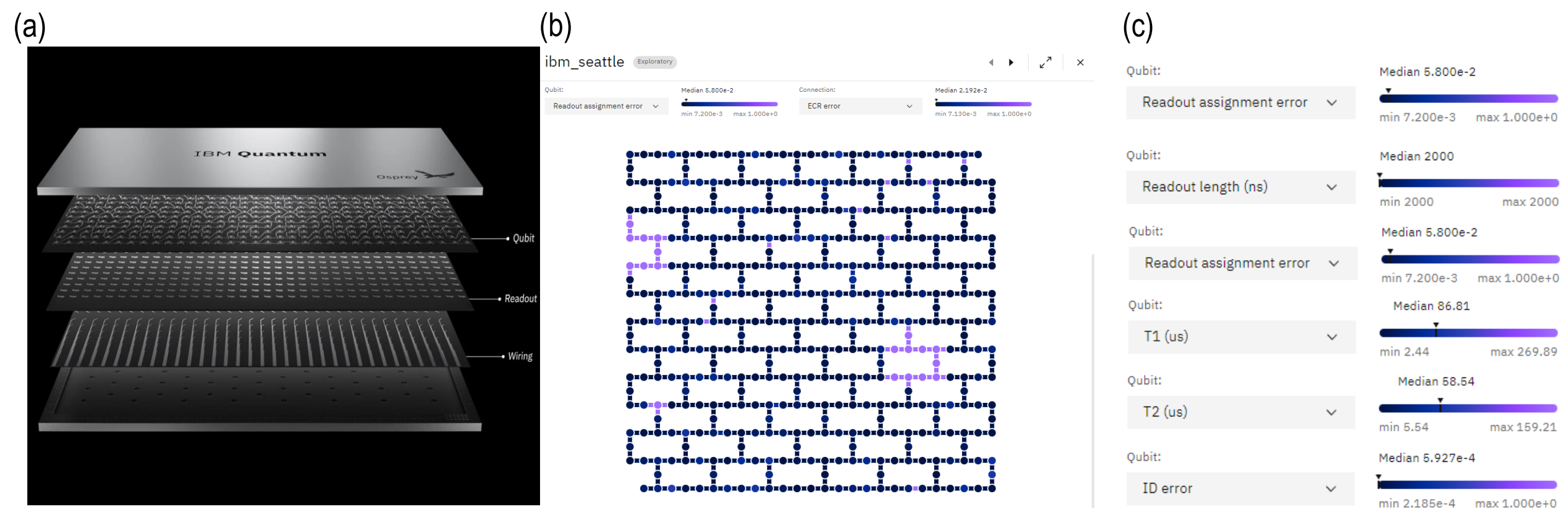
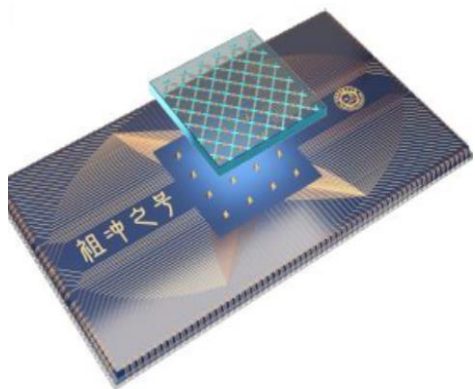
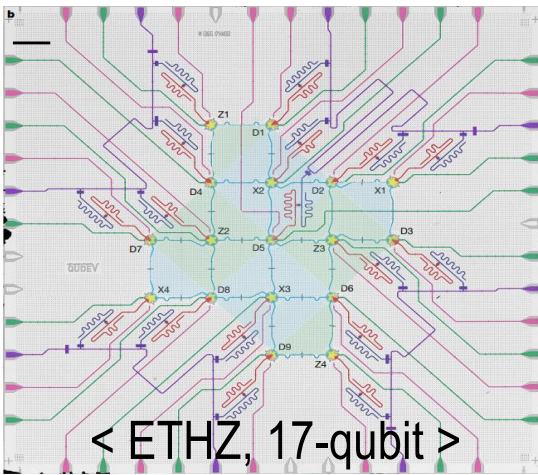


Fig. (a) Rendered image of IBM 433-Q compute; (b) connectivity of IBM 433-Q; (c) key parameters. [*access date: 2023/06/20]

J. Chow, O. Dial, J. Gambetta, “IBM Quantum breaks the 100-qubit processor barrier,” *IBM Research Blog*, 2021.
https://quantum-computing.ibm.com/services/resources?tab=systems&system=ibm_seattle

Selected Milestones in Transmon-Based Research (4/4)

- Demonstration of Quantum Error Correction with Superconducting Qubits (2022~2024)
 - The first demonstration of quantum error correction by increasing the size of the error correction protocol



ETHZ: distance-3 code
Google: distance-3 code
Google: distance-5 code
Hefei: distance-3 code

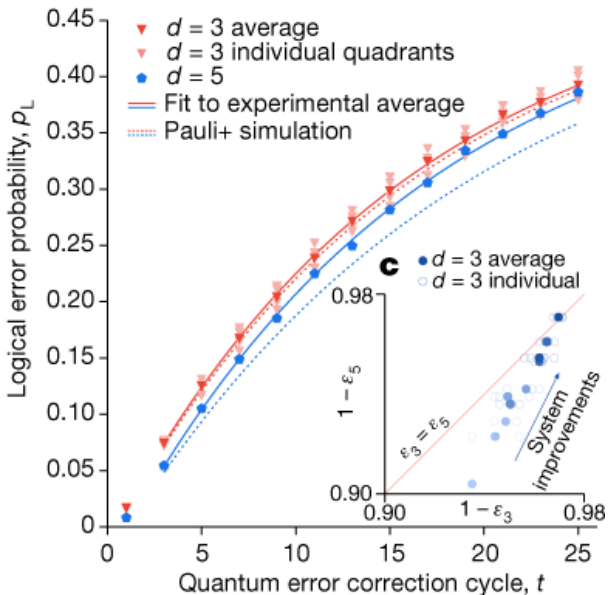
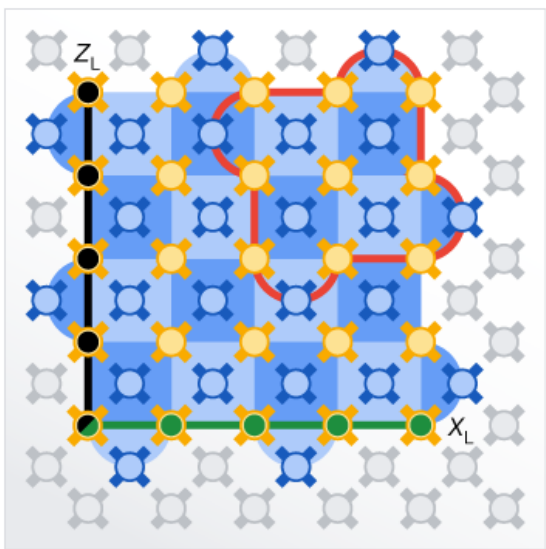


Fig. A schematic of the Google's quantum computer and the logical error rate for different surface code protocol size.

Currently available quantum computers are noisy, necessitating the development of quantum error correction!

S. Krinner, *et al.*, "Realizing repeated quantum error correction in a distance-three surface code," *Nature*, **605**, 669-674 (2022).
Y Zhao, *et al.*, "Realization of an error-correcting surface code with superconducting qubits," *Phys. Rev. Lett.*, **129**, 030501 (2022).
Google Quantum AI, "Suppressing quantum errors by scaling a surface code logical qubit," *Nature*, **614**, 676-681, (2023).

Progress of Superconducting Qubits and Quantum Computers

■ Performance Metrics of Currently Available Quantum Computers

- State-of-the-art superconducting quantum computers: Noisy Intermediate Scale Quantum (NISQ)

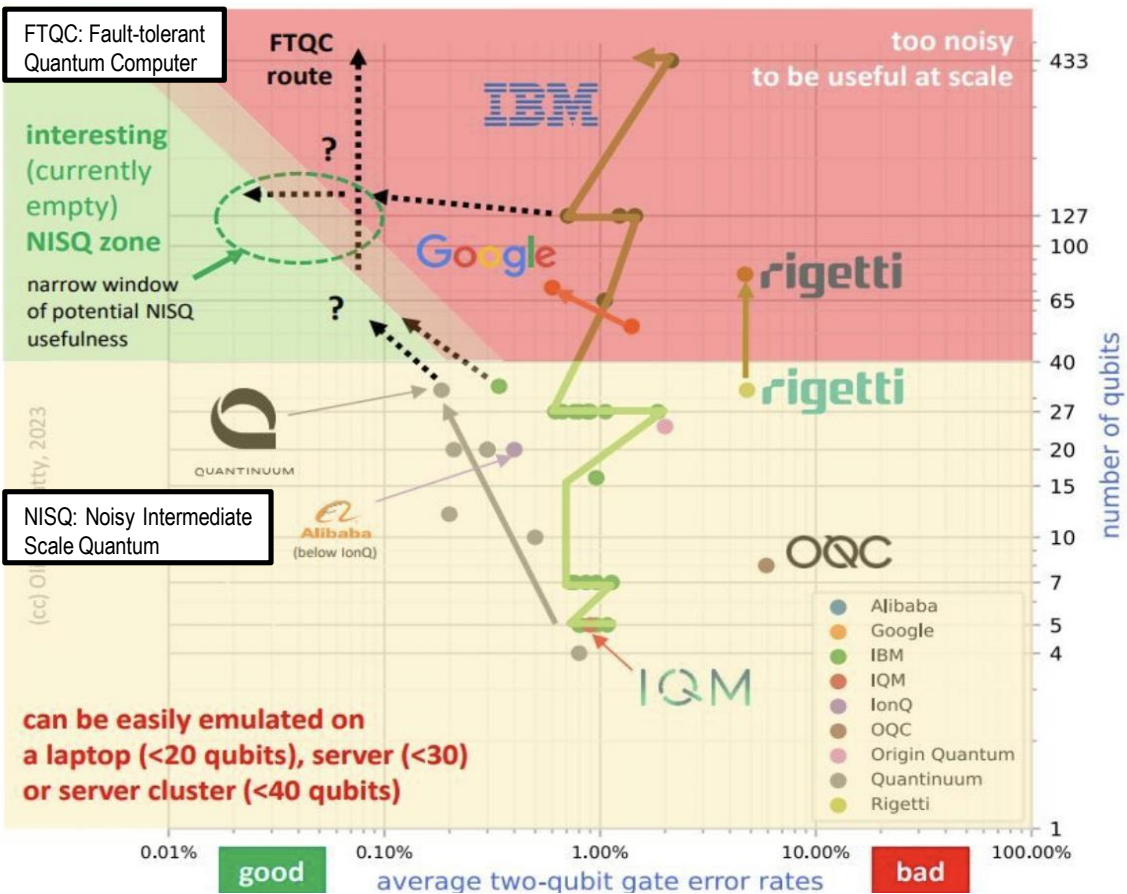


Fig. Progress of quantum computers with two-qubit gate error and the number of qubits

O. Ezratty, "Where are we heading with NISQ?," 2023. [online]. Available: arXiv:2305.09518, 2023.
D. Gil, W.M.J. Green, "The Future of Computing: Bits + Neurons + Qubits," in *IEEE Int. Solid-State Circuits Conf.*, (2020)

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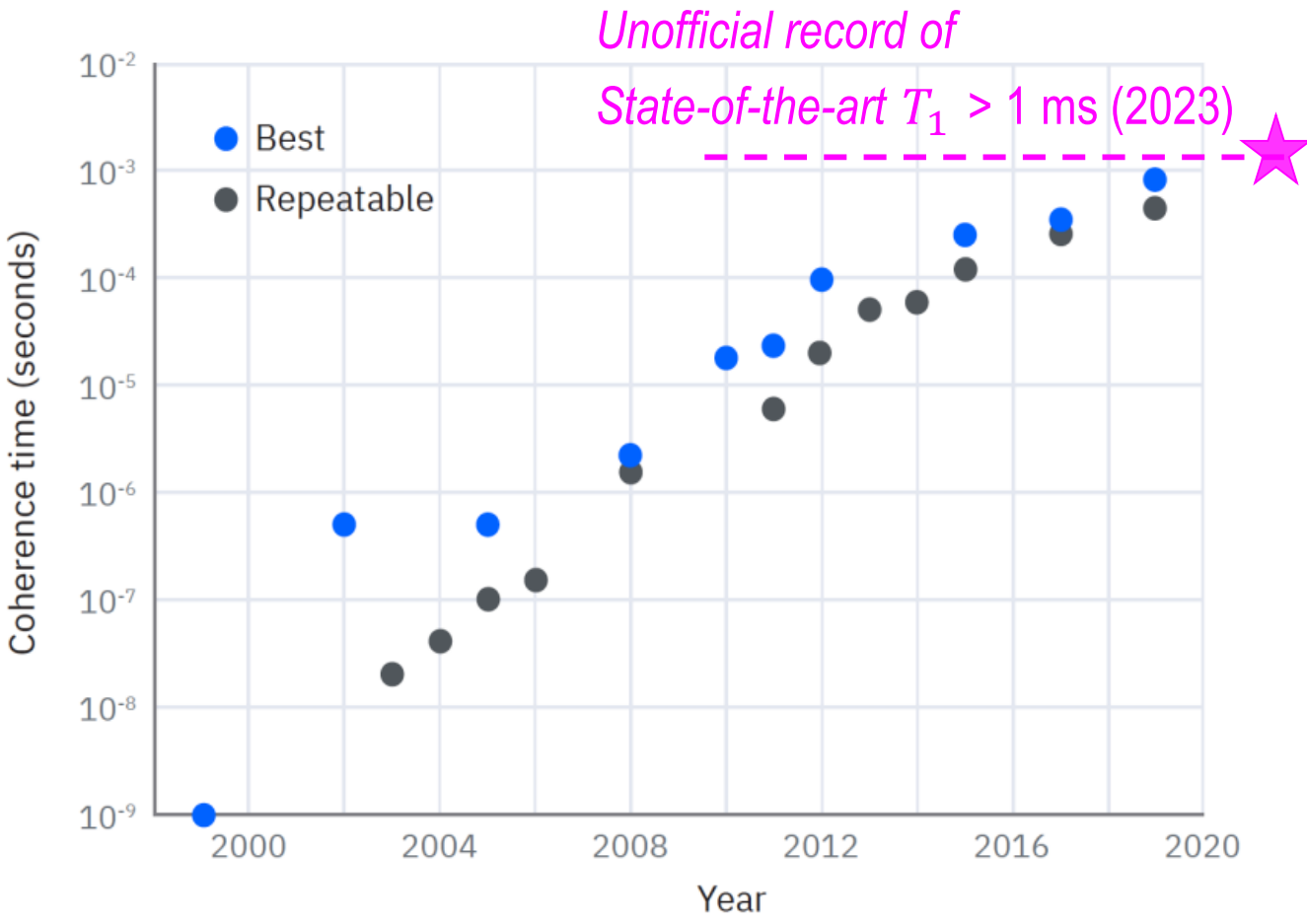


Fig. Historical improvement in coherence time of superconducting qubits

Introduction to Superconducting Transmon Qubit

■ What Is Superconducting Transmon Qubit?

- A transmon qubit **minimizes sensitivity to charge noise by a large shunt capacitance** connected to a Cooper pair box
- Due to the nonlinearity of a Josephson junction, a transmon qubit is anharmonic oscillator

■ Strengths of Transmon Qubits

- Reduced noise sensitivity and long coherence times
- Scalability and reproducibility
- Well-established control and readout schemes

■ Weaknesses of Transmon Qubits

- Dielectric two-level system loss
- Large footprint size
- Weakly anharmonic energy level diagram

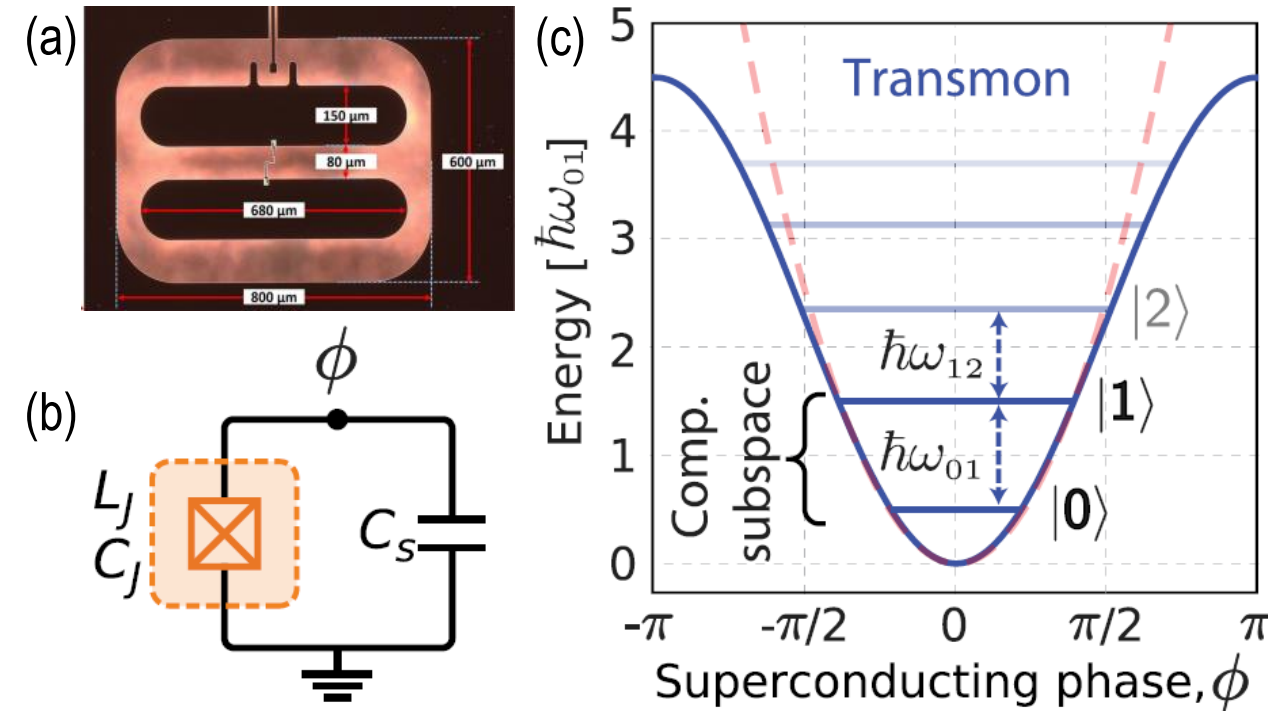


Fig. (a) Example of a transmon qubit. (b) Equivalent circuit diagram. (c) Energy level of a transmon qubit as a function of superconducting phase. The two lowest energy levels can be effectively isolated from others, forming a computational subspace of $|0\rangle$ and $|1\rangle$.

There are many other types of qubit as shown in previous slide!
(in this lecture, we focus on the transmon qubit only)

Transmon Qubit as a Quantum Anharmonic Oscillator (1/2)

■ Nonlinear Josephson Junction Equations

- Recall that from the current-phase relations,

$$I = I_c \sin(\phi) \quad V = \frac{\hbar}{2e} \frac{d\phi}{dt}$$

I_c : critical current of a Josephson junction
 ϕ : phase difference in a Josephson junction

- The potential Josephson energy E_J of a Josephson junction can be expressed as

$$E_J = \frac{1}{L_J} \left(\frac{\Phi_0}{2\pi} \right)^2 = I_c \left(\frac{\Phi_0}{2\pi} \right)$$

L_J : Josephson junction inductance
 $\Phi_0 = \frac{h}{2e}$: quantum of flux

■ Hamiltonian of a Transmon Qubit

- Replace the total capacitance with charging energy
($E_c = e^2/2C_\Sigma$, where $C_\Sigma = C_J + C_s$: total capacitance)

- The Hamiltonian can be expressed as

$$\therefore \mathcal{H} = 4E_c (\hat{n} - n_g)^2 - E_J \cos \hat{\phi}$$

n_g : offset charge in a Josephson junction

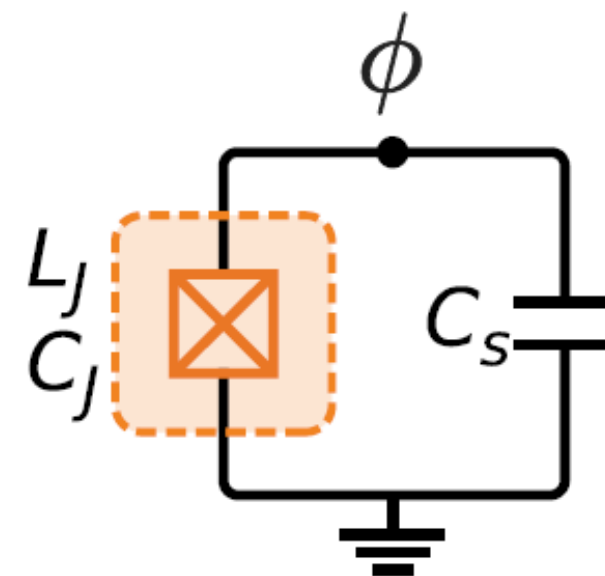


Fig. Equivalent circuit diagram. Here, Josephson inductance and capacitance are represented by L_J and C_J . The shunt capacitance is denoted as C_s

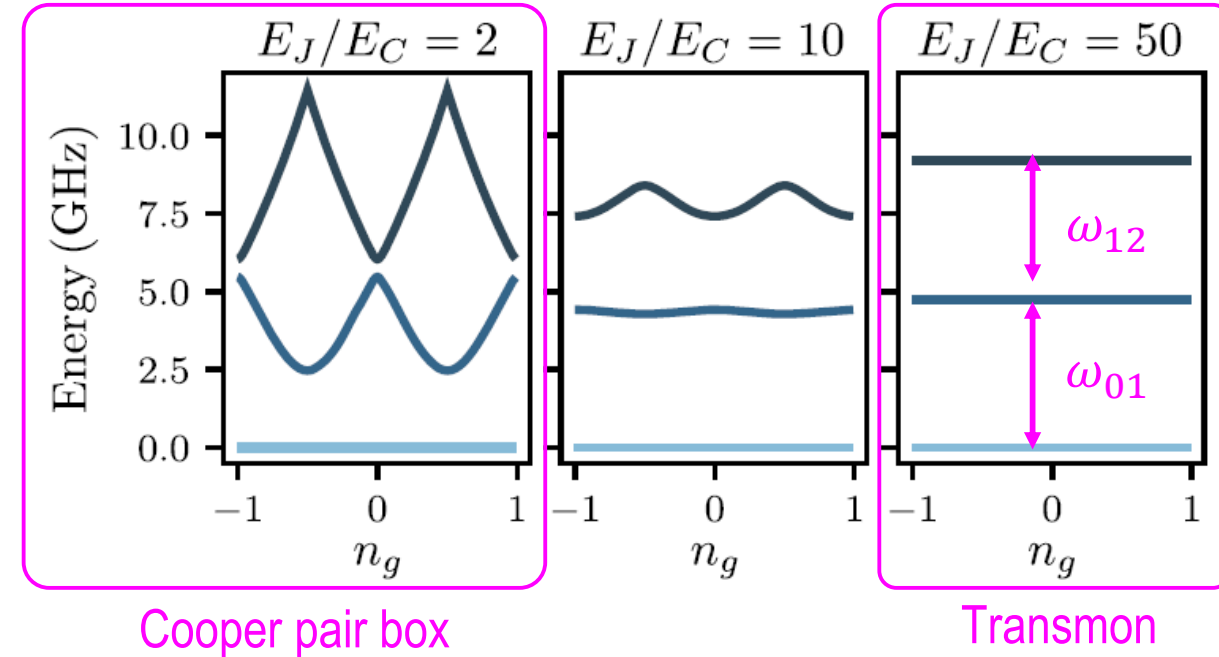
The Hamiltonian of a transmon is same as that of a CPB...
What's the difference?

Transmon Qubit as a Quantum Anharmonic Oscillator (2/2)

■ From Cooper Pair Box to Transmon Qubit

- A typical Cooper pair box exhibits $E_J/E_C \sim 1$ ratio
- In contrast, a transmon qubit exhibits $E_J/E_C \gg 1$ ratio
- Physical meaning: a transmon has large shunt capacitance
- As a result, a transmon qubit suppresses charge noise!

Transmon qubits exhibit almost constant energy levels with respect to charge offset !



■ Characteristic Parameters of Transmon Qubits

- Qubit frequency ω_{01} : energy difference when $|1\rangle$ and $|0\rangle$ (* by multiplying Planck's constant h , frequency \rightarrow energy unit)
- For simplicity, $\omega_{01} \approx \sqrt{8E_J E_C} - E_C$
- Qubit anharmonicity $\alpha = \omega_{12} - \omega_{01}$: energy difference between $|2\rangle \rightarrow |1\rangle$ transition and $|1\rangle \rightarrow |0\rangle$ transition
- For simplicity, $\alpha \approx -E_C$

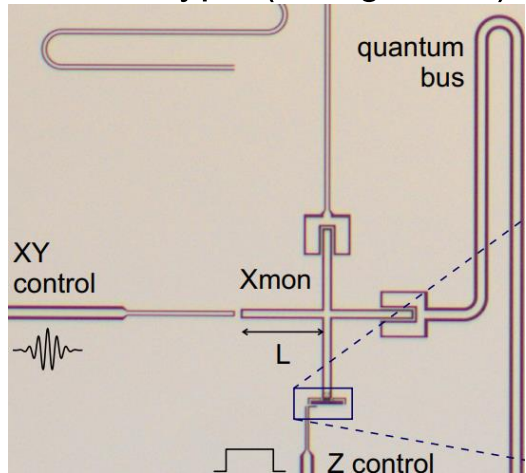
Since the anharmonicity ($\alpha = \omega_{12} - \omega_{01}$) is not large, transmon qubits are weakly anharmonic qubit!

Practical Guidelines to Design Transmon Qubits

- Typical Choice for Josephson Junction Parameters
 - Critical current ~ 30 nA \leftrightarrow junction inductance ~ 10 nH \leftrightarrow junction energy ~ 16 GHz (assuming $\hbar = 1$ for simplicity)
- Typical Choice for Shunt Capacitance
 - capacitance ~ 60 fF \leftrightarrow charging energy ~ 300 MHz (assuming $\hbar = 1$ for simplicity)
- Resulting Qubit Parameters
 - Qubit frequency $\omega_{01} \sim 6$ GHz and qubit anharmonicity $\alpha \sim -330$ MHz
- Various Design Examples of Transmon Qubits

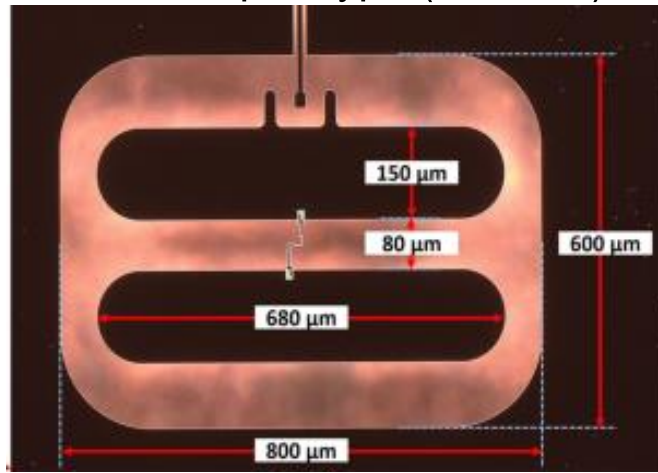
Popular choices

Xmon type (Google-like)



R Barends et al., *Phys. Rev. Lett.*, **111**, 080502 (2013)

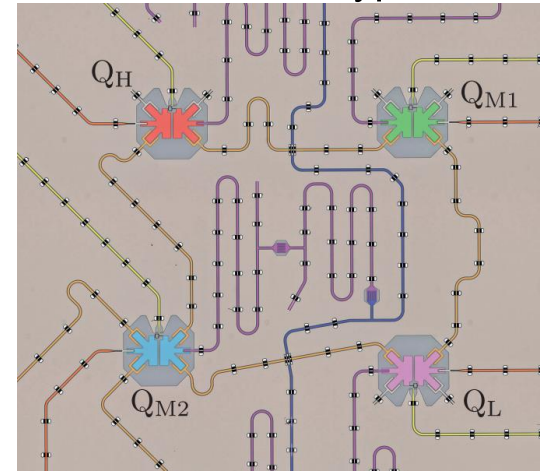
Double pad type (IBM-like)



C Wang et al., *npj Quantum Inf.*, **8**, 3 (2022)

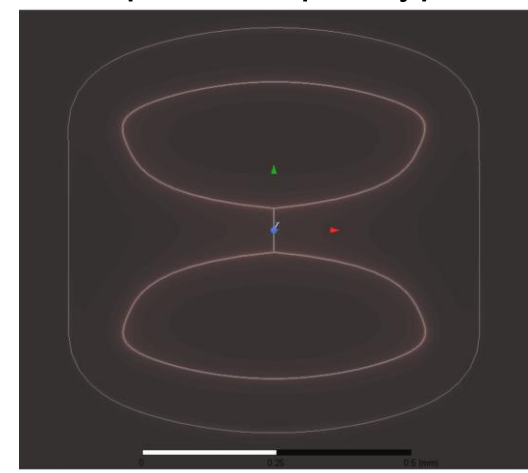
You will design your own transmon qubits in lab sessions

Starmon type



V. Negîrneac et al., *Phys. Rev. Lett.*, **126**, 220502 (2021)

Optimized pad type



S Eun et al., *J. Phys. D.*, **56**, 505306 (2023).

See Also...

■ Review Papers:

- [1] A. Blais, AL Grimsmo, SM Girvin, A Wallraff, "Circuit quantum electrodynamics", *Rev. Mod. Phys.*, **93**, 025005 (2021).
- [2] P Krantz *et al.*, "A quantum engineer's guide to superconducting qubits," *Appl. Phys. Rev.*, **6**, 021318 (2019).
- [3] U Vool, M Devoret, "Introduction to quantum electromagnetic circuits," *Int. J. Circ. Theor. Appl.*, **45**, 897-934 (2017).
- [4] J Koch *et al.*, "Charge-insensitive qubit design derived from the Cooper pair box," *Phys. Rev. A*, **76**, 042319 (2007). ← transmon qubit proposal
- [5] SM Girvin, Circuit QED: Superconducting Qubits Coupled to Microwave Photons, Les Houches, Oxford University Press

■ Open Courses:

- [1] Qiskit Global Summer School (2020): *for circuit quantization, see videos #16~ #21 (but every video is very useful)
<https://youtube.com/playlist?list=PLOFEBzvs-VvrXTMy5Y2lqmSaUjfnhvBHR&si=xE-4OjWXYsxf5gY6>
- [2] Rob Schoelkopf: Experimental Platforms: Superconducting Circuit
<https://youtu.be/nYcNQL6pS0o?si=K4JhxXM-rQGeC3aj>
- [3] Lecture Notes by Prof. Steven M Girvin (Yale University)
<https://girvin.sites.yale.edu/lectures>