Introduction to Superconducting Quantum Circuits

- Review of Superconductivity for Quantum Computing -

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Lecture Overview

Week 1. Introduction to Superconducting Quantum Circuits
Week 2. Review of Mathematics and Microwave Engineering
Week 3. Review of Classical and Quantum Mechanics
Week 4. Review of Superconductivity
Week 5. Quantum Harmonic/Anharmonic Oscillators and Light-Matter Interaction
Week 6. Circuit Quantization Methods
Week 7. Parametrically Pumped Josephson Devices
Week 8. Design and Analysis of Superconducting Resonators
Week 9. Design and Analysis of Superconducting Qubits
Week 10. Design and Analysis of Single-Qubit Device: 3D Cavity
Week 11. Design and Analysis of Single-Qubit Device : 2D Chip
Week 12. Design and Analysis of Two-Qubit Device
Week 13. Design and Analysis of Josephson Parametric Amplifier
Week 14. Term Project
Week 15. Term Project

overall backgrounds, terminologies of quantum computing

mathematical and engineering backgrounds
general superconductivity

Quantum circuit analysis

design and analysis of superconducting RF devices

Keywords in Superconductivity for Quantum Computing

Characteristic Material Properties

Critical Temperature	Gap Energy
London Penetration Depth	Coherence Length
Normal State Conductivity	Electron Mean Free Path
BCS Theory	MB Theory
Quasiparticle	Cooper Pair

Electromagnetic Behaviors

Kinetic Inductance	Surface Impedance

Josephson Junctions

Ambegaokar-Baratoff	Current-Phase Relation
AC Josephson Effect	DC-SQUID
RF-SQUID	

Introduction to Superconductivity

What Is Superconductivity?

- The superconductivity was discovered in 1911 by Dutch physicist Heike Kamerlingh Onnes (won Nobel prize in 1913)
- Superconductors exhibit zero "direct-current" electrical resistance
- Superconductors expel magnetic fields when cooled below a critical temperature (T_c), known as Meissner effect

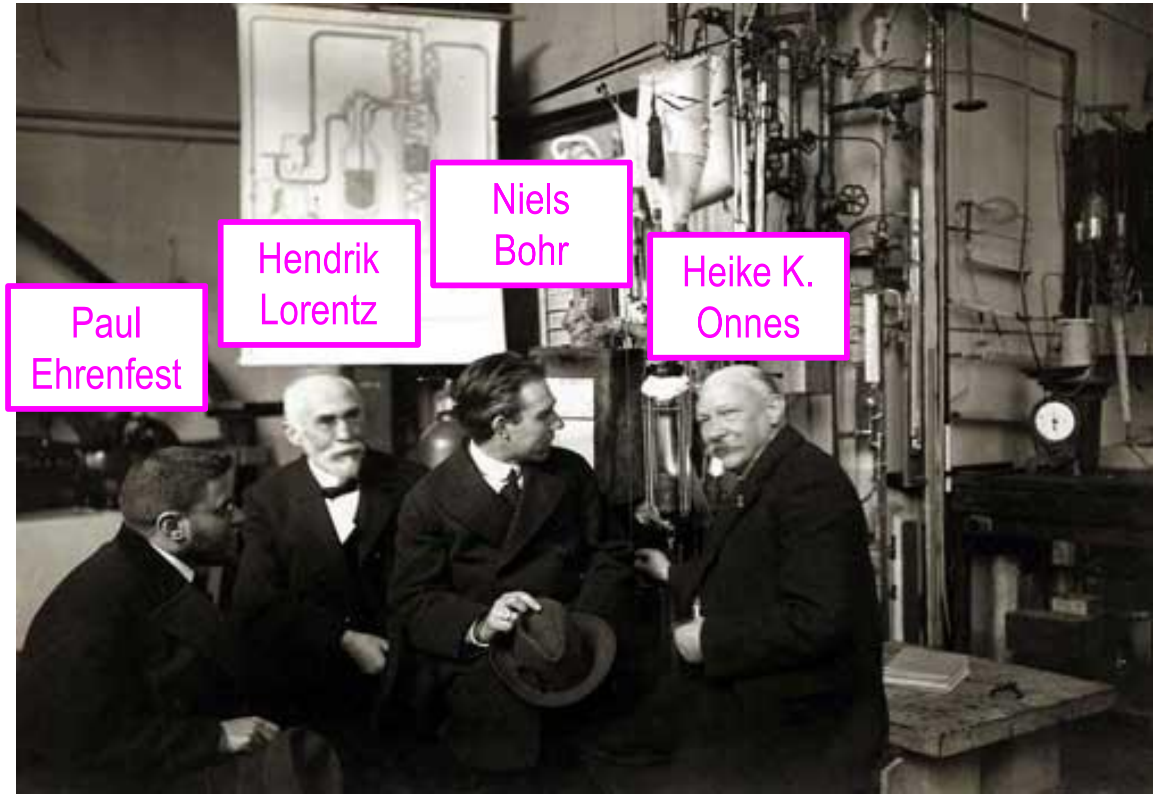

Fig. Heike Kamerlingh Onnes (right), the discoverer of superconductivity.

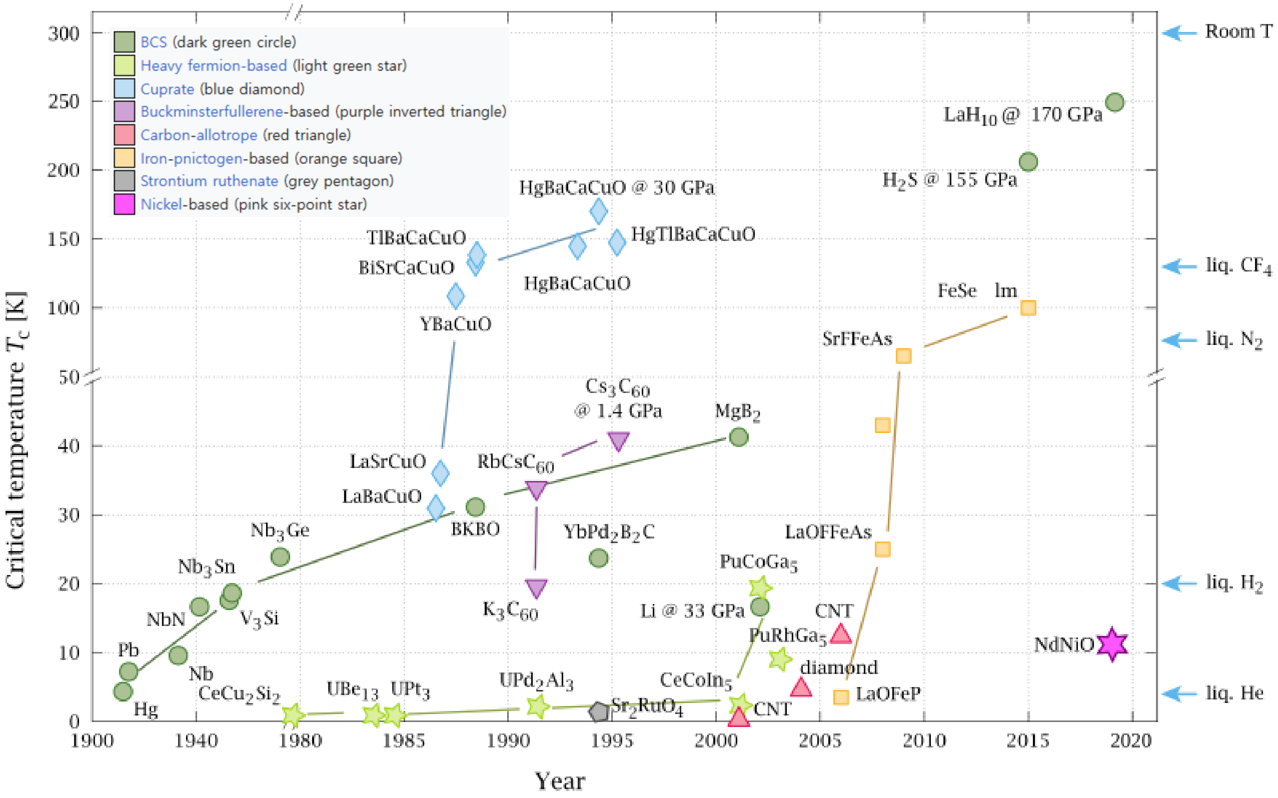

Fig. Timeline of discovering superconducting materials.

Image from https://en.wikipedia.org/wiki/Superconductivity

Characteristic Properties of Superconductors: (1) Critical Temperature

Definition of Critical Temperature

- Characteristic temperature below which a material becomes a superconductor
- When the temperature is above the critical temperature, the material behaves as a normal conductor with resistive losses
- When the temperature is below the critical temperature, the material expel magnetic fields (Meissner effect)
- Common symbol: T_c, unit: [K]

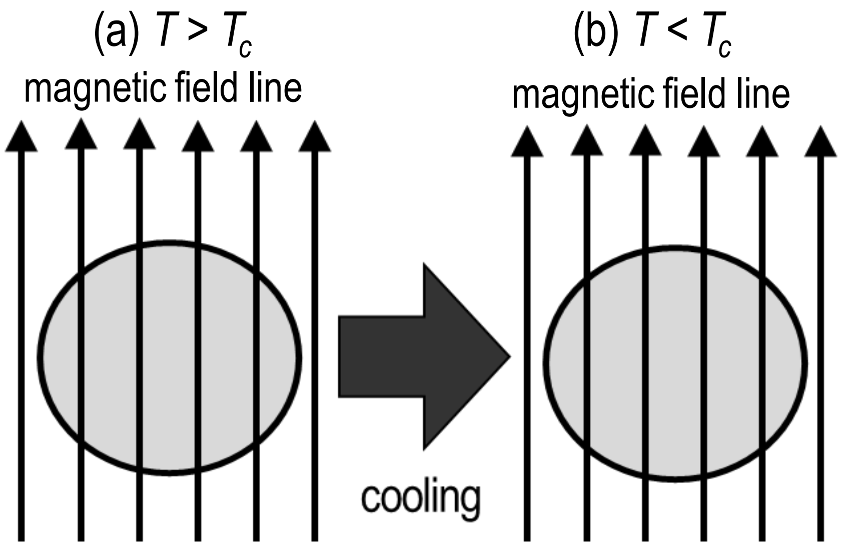

Fig. Magnetic behavior of a perfect conductor with background magnetic field.

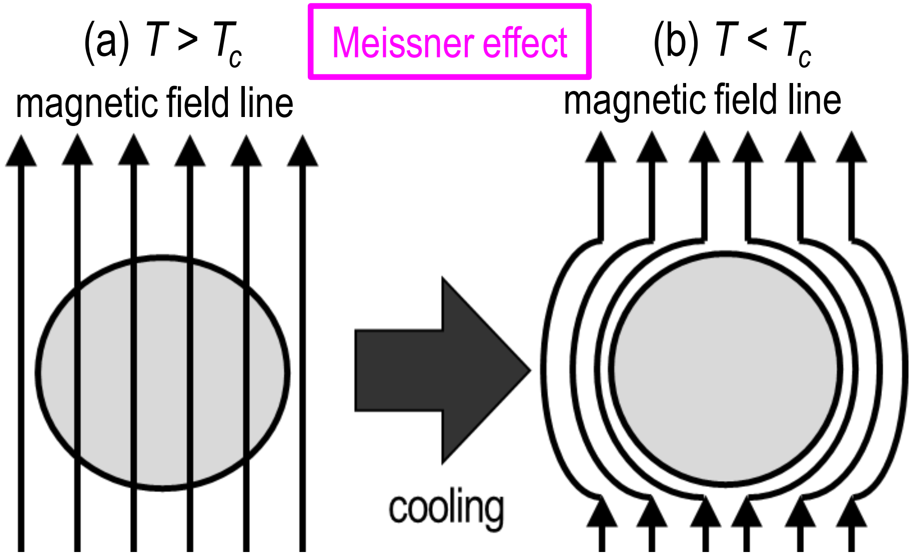

Fig. Magnetic behavior of a superconductor with background magnetic field.

Characteristics of Critical Temperature

- For BCS superconductors, T_c is related to the gap energy of the superconductor.

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Characteristic Properties of Superconductors: (2) Gap Energy

- Definition of Gap Energy
  - The energy required to break a Cooper pair (paired-electrons) in a superconductor
  - This gap represents the energy difference between the superconducting ground state and the first available excited state
  - Common symbol: Δ or E_g unit: [J or eV or Hz] ← depending on the choice for simplicity
  - In this lecture, the energy gap to break a Cooper pair is defined as 2Δ

- Characteristics of Gap Energy
  - According to BCS theory, the microscopic equation at $T = 0$ K is

$$\Delta(0\ \mathrm{K}) \approx 1.75 k_B T_c$$

k_B: Boltzmann's constant

  - The temperature dependent equation is

$$\Delta(T) \approx \Delta(0\ \mathrm{K}) \sqrt{\cos\left(\frac{\pi}{2}\left(\frac{T}{T_c}\right)^2\right)}$$

NOTE:

Gap energy of a superconductor determines the maximum signal frequency to control a superconducting qubit (e.g., aluminum has $2\Delta(0\ \mathrm{K}) \approx 3.6 \times 10^{-4}\mathrm{eV} \approx h \times 90$ GHz)

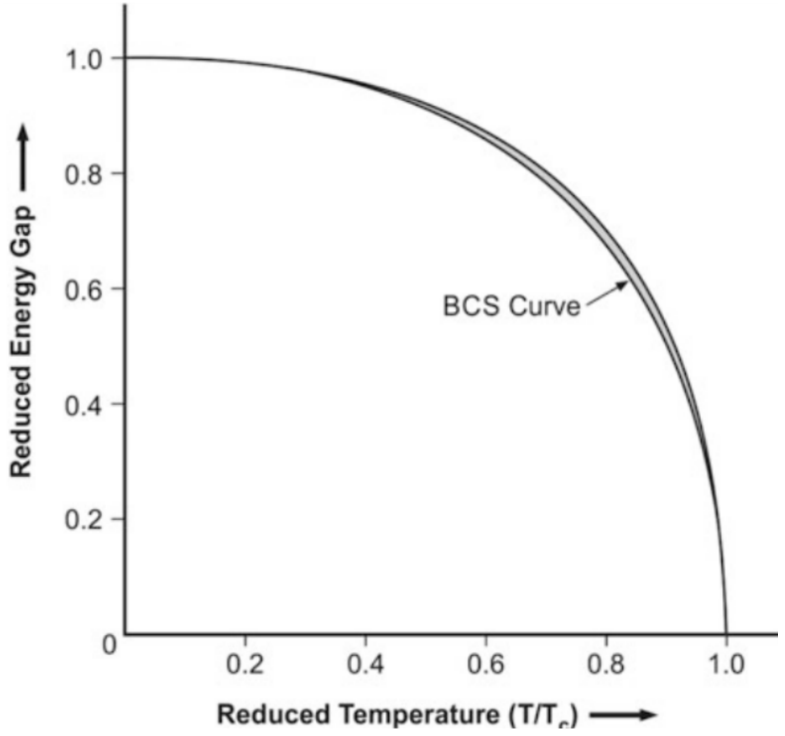


Fig. Reduced energy gap $\Delta(T)/\Delta(0)$ as a function of the reduced temperature. T/T_c

M Tinkham, *Introduction to Superconductivity*, McGraw Hill, Second Edition, (2004).
https://rashid-phy.github.io/me/pdf/notes/Superconductor_Theory.pdf

Characteristic Properties of Superconductors: (3) London Penetration Depth

- Definition of London penetration Depth
  - Distance to which a penetrated magnetic field decays to $1/e$ inside a superconductor
  - Derived from London's theory (developed in 1935) for metallic superconductors
  - Common symbol: λ_L, unit: [m]
- London Equation

$$\boxed{\nabla^2 \dot{\boldsymbol{B}} = \frac{1}{\alpha}\dot{\boldsymbol{B}}} \xrightarrow{\text{London Theory}} \boxed{\nabla^2 \boldsymbol{B} = \frac{1}{\alpha}\boldsymbol{B} = \frac{1}{\lambda_L^2}\boldsymbol{B}} \Rightarrow \text{Solution: } B(x) = B_a e^{-\frac{x}{\lambda_L}}$$

Classical Maxwell's equations

London Equation

- λ_L: Penetration depth

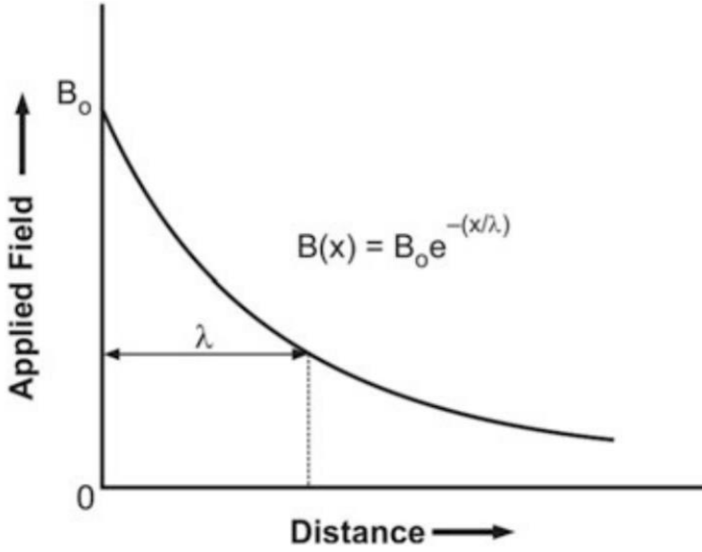


Fig. Field penetration in a superconductor.

- Characteristics of London Penetration Depth
  - The microscopic equation is

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

μ_0: vacuum permeability
n_s: density of superconducting electrons (Cooper pairs)
e: charge of a single electron
m: mass of a single electron

  - The temperature dependent equation is

$$\lambda_L(T) \approx \lambda_L(0\ \mathrm{K}) \frac{1}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}}$$

M Tinkham, *Introduction to Superconductivity*, McGraw Hill, Second Edition, (2004).
https://rashid-phy.github.io/me/pdf/notes/Superconductor_Theory.pdf

Characteristic Properties of Superconductors: (4) Coherence Length

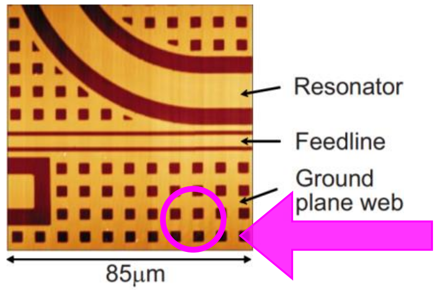
- Definition of Coherence Length
  - The distance over which the superconducting order parameter (or wave function of Cooper pairs) remains correlated
  - The size of a Cooper pair, distance between the paired-electrons
  - Common symbol: ξ, unit: [m]
- Characteristics of Coherence Length
  - The microscopic equation is

$$\xi = \frac{\hbar v_F}{\pi \Delta}$$

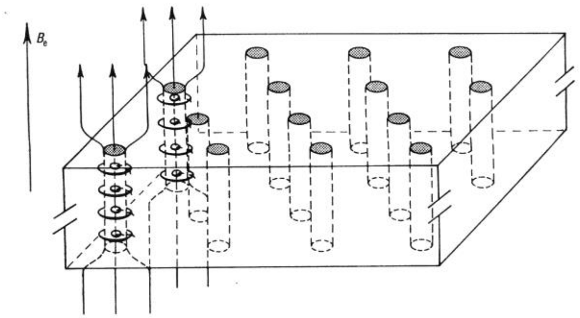
v_F: Fermi velocity
$\hbar$: reduced Planck's constant

  - At absolute zero ($\mathrm{T} = 0$ K), ξ is maximized

NOTE:
vortex can be generated in Type-I superconductors, when the thickness is thin enough.



ex) vortex trap in aluminum (type-I) circuit
vortex trap geometry is often employed in superconducting quantum circuits!
ref: C Song *et al.*, PRB **79**, 174512 (2009)



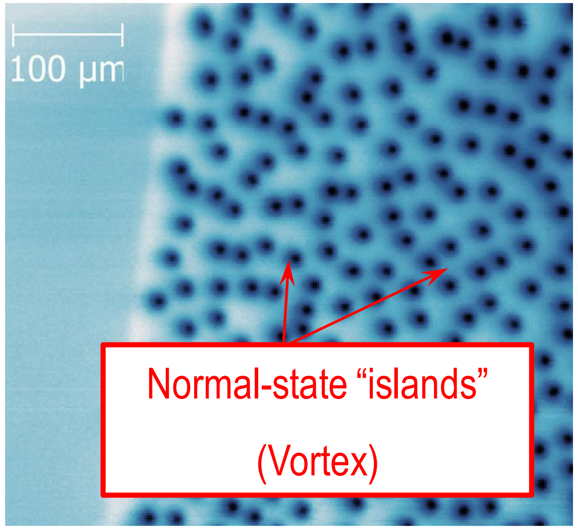


Fig. Micrograph of vortex in superconductor.

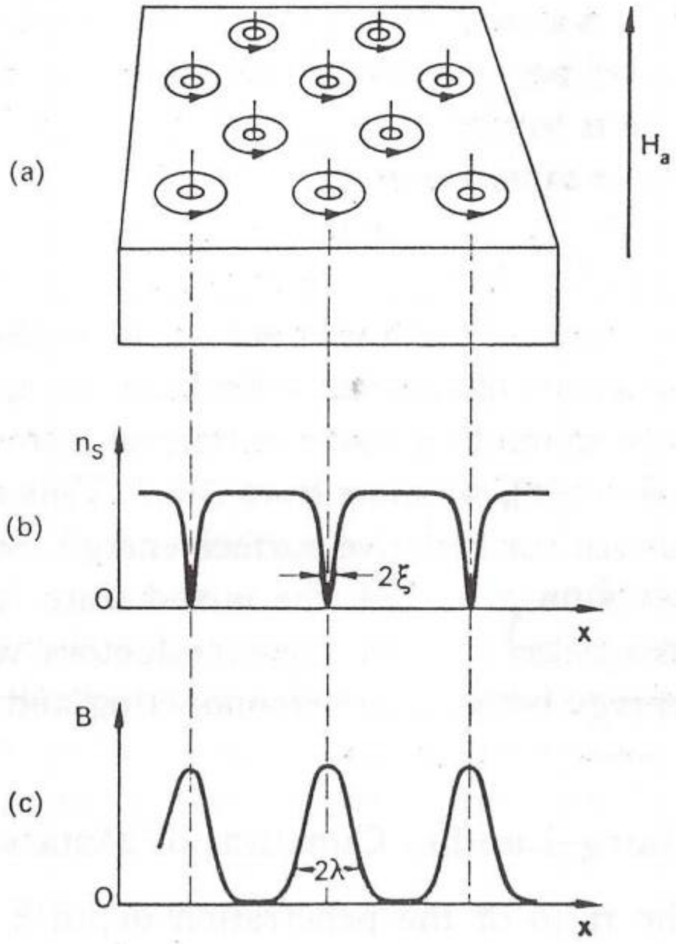


Fig. (a) Fluxon lattice; (b) Cooper pair density; (c) flux density.

https://en.wikipedia.org/wiki/Abrikosov_vortex
M Tinkham, *Introduction to Superconductivity*, McGraw Hill, Second Edition, (2004).
AC Rose-Innes and EH Rhoderick, *Introduction to Superconductivity*, Pergamon Press, Second Edition, (1978)

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Characteristic Properties of Superconductors: (5) Electron Mean Free Path

- Definition of Normal State Resistivity
  - The electrical conductivity of a superconducting material above its T_c
  - Common symbol: ρ_n, unit: $[\Omega \cdot \mathrm{m}]$
- Characteristics of Normal State Resistivity
  - Strongly related to the residual resistance ratio (RRR=$\rho(300\ \mathrm{K})/\rho(T_c)$)
  - For small RRR superconductor, ρ_n is expected to be large
  - For large RRR superconductor, ρ_n is expected to be small
- Definition of Electron Mean Free Path
  - The average distance of a Cooper pair traveling in a superconductor before scattering due to impurities, lattice vibrations (phonons)
  - Common symbol: l or l_e, unit: [m]
- Characteristics of Electron Mean Free Path
  - Strongly related to λ_L and ρ_n of a superconductor
  - Larger l indicates that a superconductor is pure with less defects

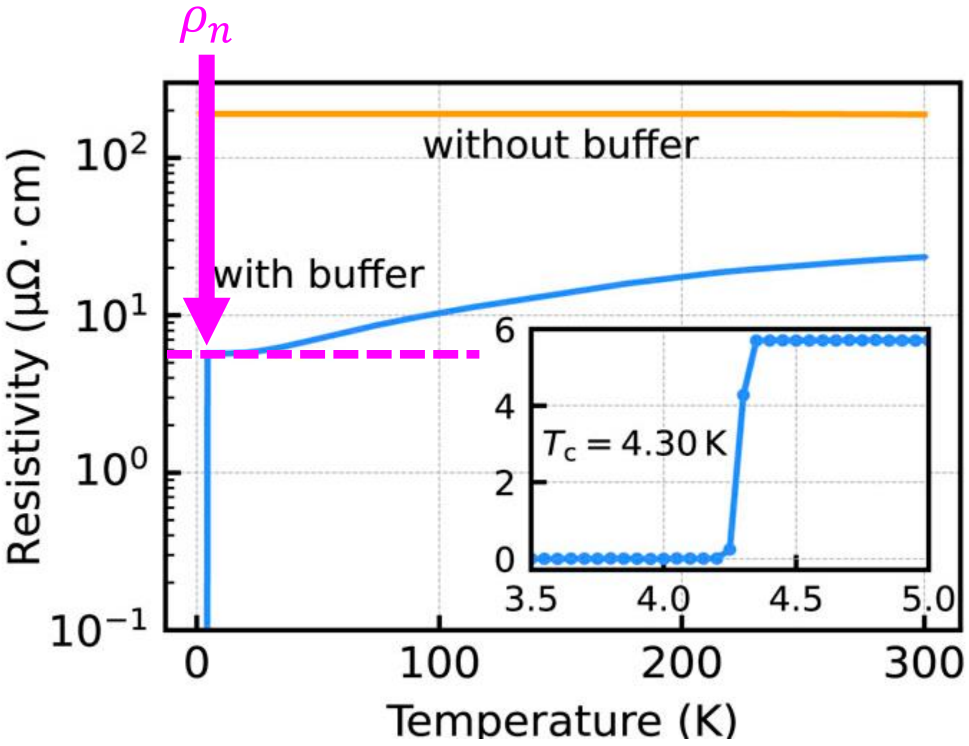


Fig. Temperature dependence of the resistivity of the superconducting tantalum films with and without the Nb buffer layer from 300 to 2 K.

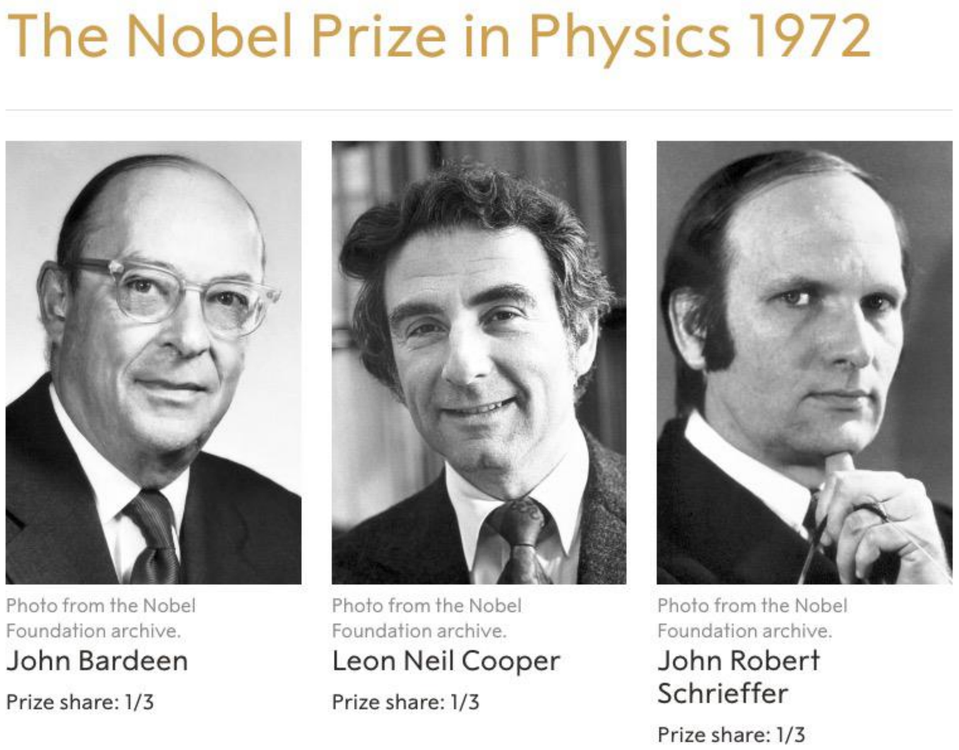
For quantum information processing: superconductors with large RRR and l are usually preferred to improve the film quality

→ ceramic- or alloy-superconductors are not preferred

Y Urade *et al.*, "Microwave characterization of tantalum superconducting resonators on silicon substrate with niobium buffer layer," *APL Mater.*, **12**, 021132 (2024).

Microscopic Theory for Superconductivity: BCS Theory

- Introduction to Bardeen-Cooper-Schrieffer (BCS) Theory
  - The foundational theory of superconductivity that explains how certain materials exhibit zero DC electrical resistance below T_c
  - When $T < T_c$, electrons with opposite spins and momenta form bound pairs known as Cooper pairs
  - Copper pairs form a collective quantum state, known as Bose-Einstein condensation
  - Cooper pairs interact with the crystal lattice via an attractive force, allowing them to move without resistance



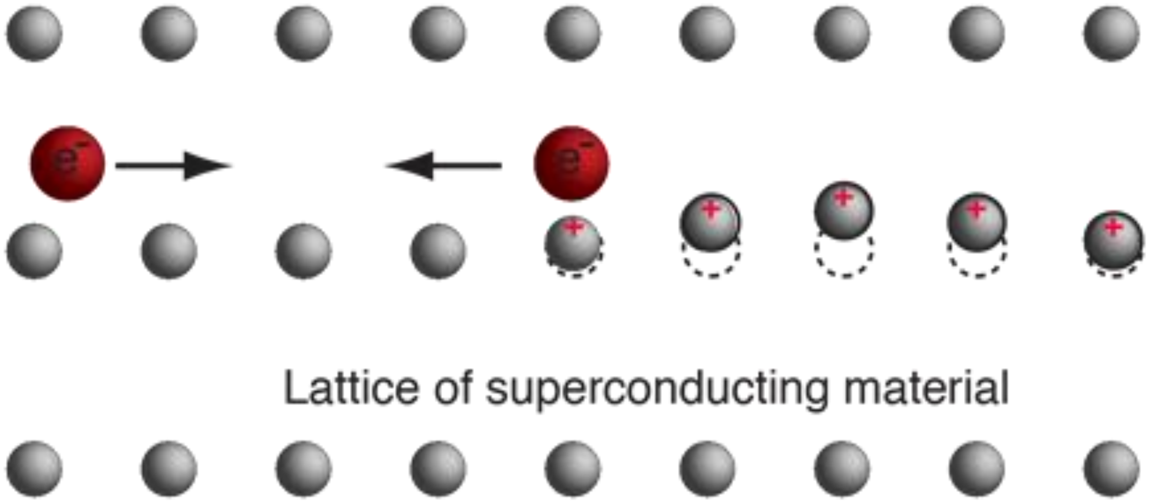


Fig. A conceptual illustration of the BCS theory. The Cooper pair attraction has a passing electron which attracts the lattice.

M Tinkham, *Introduction to Superconductivity*, McGraw Hill, Second Edition, (2004).
http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/coop.html

Types of Superconductors

- Superconductor Classification
  - There are many criteria by which superconductors are classified
  - By properties: if $\frac{\lambda_L}{\xi} > \frac{1}{\sqrt{2}}$ or not

Type-I

$\frac{\lambda_L}{\xi} < \frac{1}{\sqrt{2}}$

Type-II

$\frac{\lambda_L}{\xi} > \frac{1}{\sqrt{2}}$

  - By theory: if explained with BCS theory or not

Conventional (BCS)

Unconventional (non-BCS)

  - By critical temperature: if $T_c > \sim 40$ K or not

Low-temperature

$T_c < \sim 40$ K

High-temperature

$T_c > \sim 40$ K

  - By material structure: if pure element or not

Pure

Alloys / Ceramics

For quantum computing applications: type-I, conventional, low-temperature, pure superconductors are usually preferred (e.g., aluminum, tantalum) due to their low coherence loss and fabrication process

Microscopic Theory for Superconductivity: MB Theory

- Introduction to Mattis-Bardeen (MB) Theory
  - Based on the BCS theory and anomalous skin effect in normal conductors, D.C. Mattis and J. Bardeen developed MB theory
  - When oscillating electromagnetic wave is incident to a superconductor, the current density J and magnetic vector potential A can be analytically described in terms of Fourier components

- Governing Equations of Mattis-Bardeen (MB) Theory
  - $J_x(q)$ and $A_x(q)$ in terms of Fourier components:

$$J_x(q) = -K(q)A_x(q)$$

$K(q)$: Kernel function

  - Complex conductivity $\sigma_s = \sigma_1 - i\sigma_2$ of superconductor, defined at T and ω:

$$\frac{\sigma_1}{\sigma_n} = \frac{2}{\hbar\omega}\int_{\Delta}^{\infty}\frac{[f(E)-f(E+\hbar\omega)](E^2+\Delta^2+\hbar\omega E)}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}dE + \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{-\Delta}\frac{[1-2f(E+\hbar\omega)](E^2+\Delta^2+\hbar\omega E)}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}dE$$

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar\omega}\int_{\max\{\Delta-\hbar\omega,-\Delta\}}^{\Delta}\frac{[1-2f(E+\hbar\omega)](E^2+\Delta^2+\hbar\omega E)}{\sqrt{\Delta^2-E^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}dE$$

$\sigma_n = \frac{1}{\rho_n}$ normal-state conductivity

See the references below

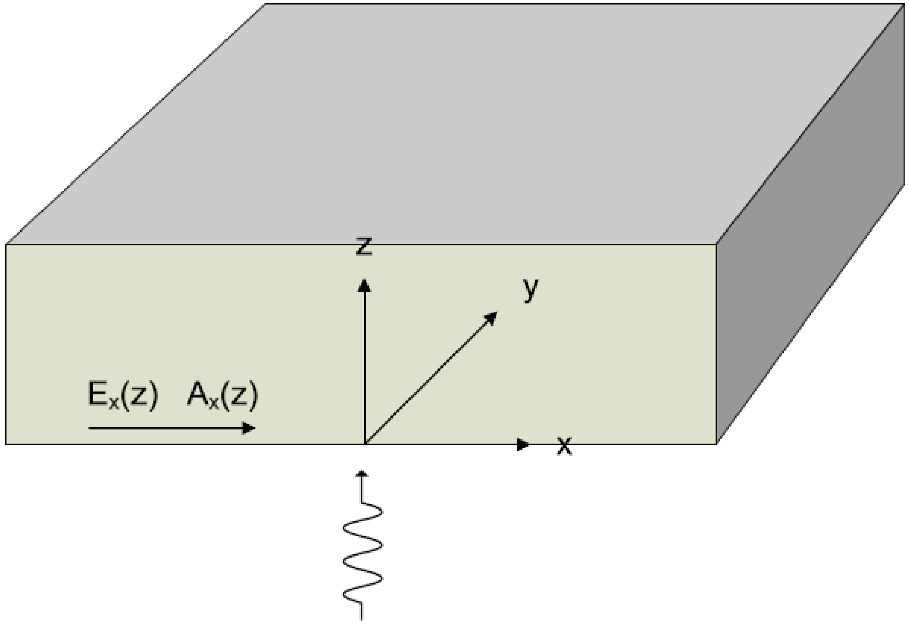


Fig. A plane wave incident onto a bulk superconductor

Input parameters for the MB theory:
$T_c, \Delta(T), \lambda_L(T), \rho_n, \xi$ and T, ω

Output results of the MB theory:
(1) complex conductivity of a superconductor
(2) surface impedance of a superconductor

DC Mattis and J Bardeen, "Theory of the Anomalous Skin Effect in Normal and Superconducting Metals," *Phys. Rev.*, **111**, 412-417 (1958)
M Tinkham, *Introduction to Superconductivity*, McGraw Hill, Second Edition, (2004).
J. Gao, *The physics of superconducting microwave resonators*, Ph.D. thesis, California Institute of Technology (2008)

Electromagnetic Behaviors of Realistic Superconductor

Perfect Electric Conductor (Ideal Conductor)

- When an electromagnetic wave is incident on a surface of a perfect electric conductor, the wave is totally reflected

Realistic Superconductor

- When an electromagnetic wave is incident on a surface of a realistic superconductor, there are reflected and transmitted waves

Boundary Conditions for Perfect Electric Conductor

Mathematical forms:

$$\hat{n} \times \vec{\boldsymbol{E}} = 0$$

$$\hat{n} \cdot \vec{\boldsymbol{D}} = \rho_s$$

$$\hat{n} \cdot \vec{\boldsymbol{B}} = 0$$

$$\hat{n} \times \vec{\boldsymbol{H}} = J_s$$

J_s: surface current density
ρ_s: surface charge density

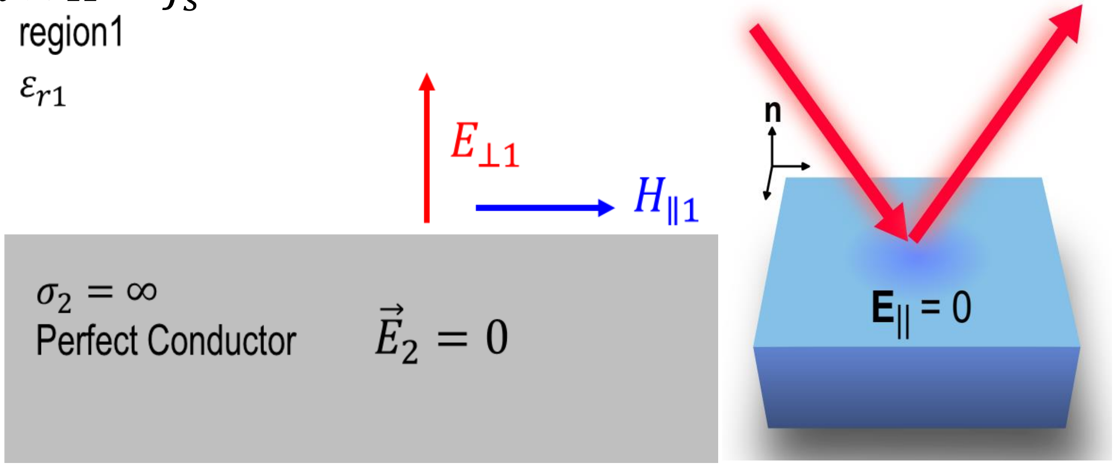


Fig. Conceptual illustrations of an incident wave .on a surface of a perfect conductor.

Boundary Conditions for Realistic Superconductor

Mathematical forms:

$$\boxed{\hat{n} \times \vec{\boldsymbol{E}} = Z_s\left(\hat{n} \times \vec{\boldsymbol{H}}\right)}$$

$$\hat{n} \cdot \vec{\boldsymbol{D}} = \rho_s$$

$$\hat{n} \cdot \vec{\boldsymbol{B}} = 0$$

$$\hat{n} \times \vec{\boldsymbol{H}} = J_s$$

Z_s: surface impedance

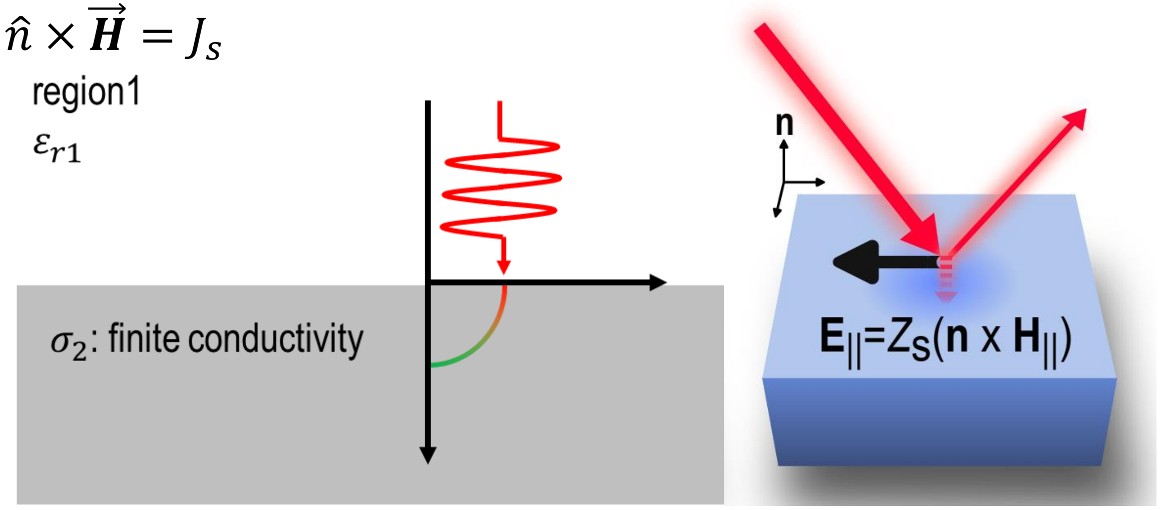


Fig. Conceptual illustrations of an incident wave .on a surface of a superconductor.

Surface Impedance of Superconductor

Surface Impedance Z_s of Superconductor

$$Z_s = R_s + iX_s = R_s + i\omega\mu_0\lambda_{\text{eff}}$$

dissipative part due to quasiparticles

Energy stored in Cooper pairs

Effective kinetic inductance per square

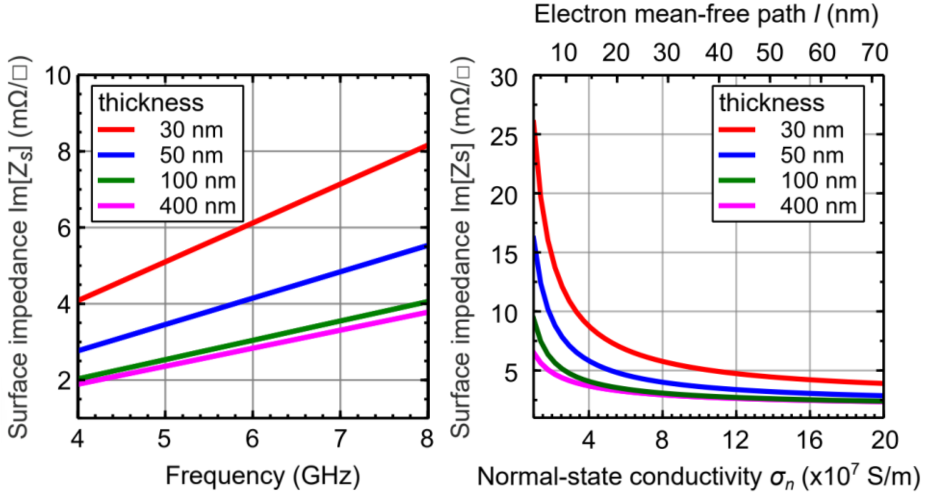
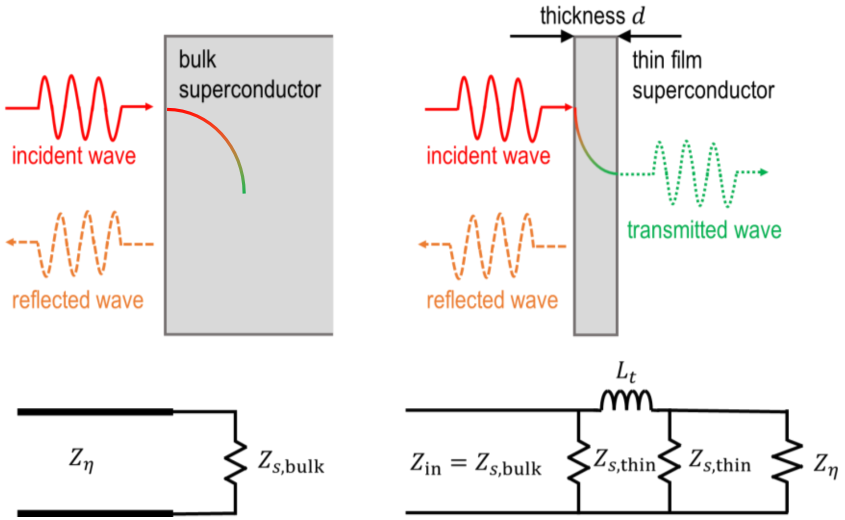


Fig. Example of the calculated imaginary part of the surface impedance of a superconducting niobium film with various thickness.

Equivalent Circuit Diagram of Surface Impedance



- Z_s in bulk-limit

$$Z_{s,\text{bulk}} = \sqrt{\frac{i\mu_0\omega}{\sigma_s}}$$

σ_s: complex conductivity of superconductor

- Z_s in film-limit

$$Z_{s,\text{film}} = \sqrt{\frac{i\mu_0\omega}{\sigma_s}}\coth\left(\sqrt{i\mu_0\omega\sigma_s}\,d\right)$$

d: thickness of a superconducting film

NOTE: calculation of the surface impedance of superconductor is numerically challenging.

Refer to following materials for further information:

[1] J. Gao, *The physics of superconducting microwave resonators*, Ph.D. thesis, California Institute of Technology (2008)

[2] W. Zimmermann *et al.*, "Optical conductivity of BCS superconductors with arbitrary purity," *Phys. C: Supercond. Appl.* 183, 99–104 (1991).

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Introduction to Josephson Junction

- What Is Josephson Junction?
  - A Josephson Junction is a quantum device consisting of two superconductors separated by a thin insulating barrier

- Characteristics of Josephson Junction
  - When no external voltage is applied, a supercurrent flows due to tunneling of Cooper pairs
  - When constant voltage is applied, oscillation of supercurrent occurs
  - Nonlinear inductance: core component for implementing a superconducting qubit

Brian D. Josephson Facts

Brian David Josephson
The Nobel Prize in Physics 1973

Born: 4 January 1940, Cardiff, United Kingdom

Affiliation at the time of the award: University of Cambridge, Cambridge, United Kingdom

Prize motivation: “for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects”

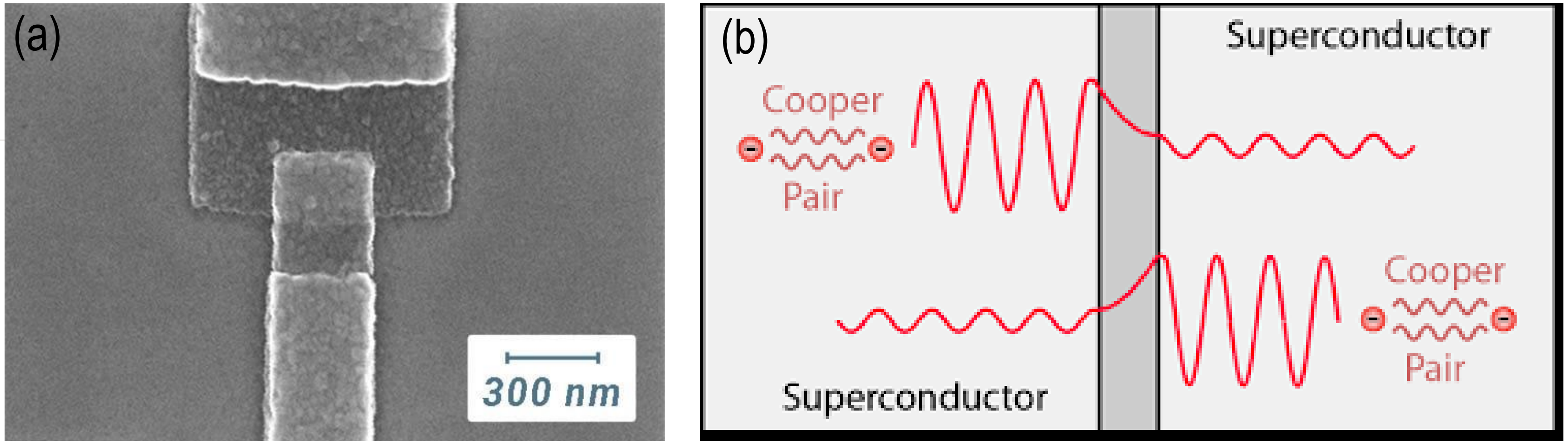


Fig. (a) Example SEM image of a Josephson junction. (b) conceptual illustration of Josephson junction with tunnel barrier.

https://en.wikipedia.org/wiki/Josephson_effect
https://web.mit.edu/6.763/www/FT03/Lectures/Lecture11.pdf

Characteristic Parameters of Josephson Junction

- Characteristic Parameters of Typical Josephson Junction
  - Critical current I_c: maximum current of Josephson junction
  - Voltage gap V_g: maximum voltage of Josephson junction

$$V_g = \frac{2\Delta(T)}{e}$$

  - Normal-state resistance R_N: resistive behavior of Josephson junction when the voltage is larger than its voltage gap
  - Josephson capacitance C_J: intrinsic capacitance of Josephson junction due to thin dielectric barrier
  - Josephson inductance L_J: equivalent junction inductance

$$L_J = \frac{\Phi_0}{2\pi I_c}$$

$\Phi_0 = \frac{h}{2e}$: quantum of flux

  - Josephson Energy E_J: Energy stored in a junction

$$E_J = \frac{\Phi_0 I_c}{2\pi} = \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_J}$$

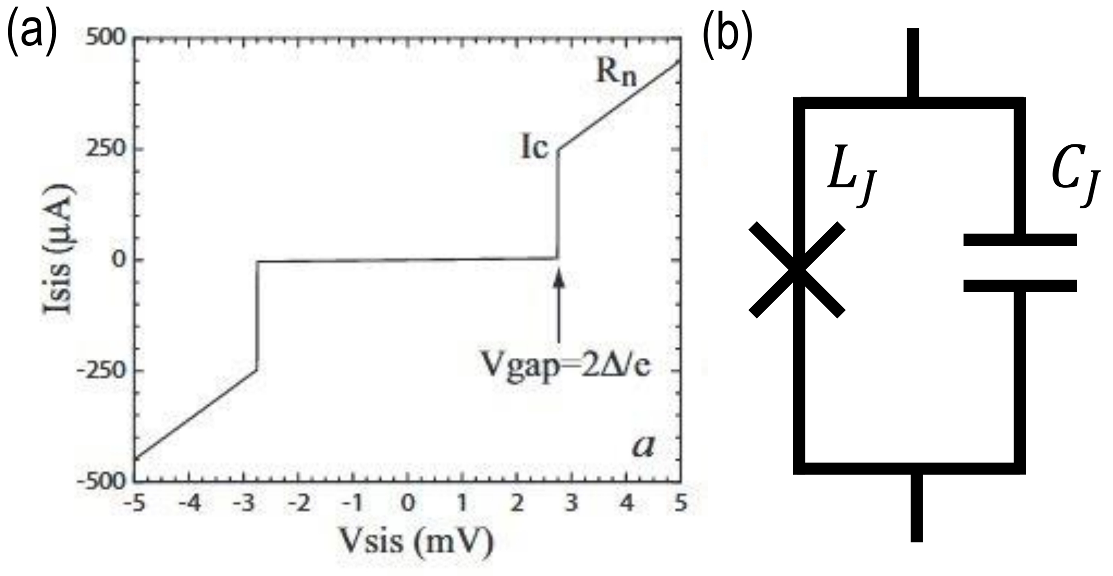


Fig. (a) Current-Voltage characteristic of an ideal SIS junction. (b) Circuit diagram. Here, the resistive part (R_N) is often omitted for quantum applications due to small signal input to the junction.

Example:
for $I_c = 33$ nA Al/AlO$_x$/Al Josephson junction,
$L_J \approx 10$ nH and $E_J/h \approx 16.4$ GHz

https://en.wikipedia.org/wiki/Josephson_effect
https://web.mit.edu/6.763/www/FT03/Lectures/Lecture11.pdf

Josephson Junction: (1) Ambegaokar-Baratoff Relation

- Definition of Ambegaokar-Baratoff Relation
  - Relation between I_c of a Josephson junction to Δ and R_N of a Josephson junction
  - For the same Josephson junction geometry and fabrication process, I_c can be inferred from the junction area

- Equation of Ambegaokar-Baratoff Relation

$$I_c R_N = \frac{\pi}{2e}\Delta(T)\tanh\left(\frac{\Delta(T)}{2k_B T}\right)$$

- Ambegaokar-Baratoff in Quantum Computing
  - To study the fabrication uncertainties and yield-rates of superconducting qubits, Ambegaokar-Baratoff relation is employed
  - To study the temperature dependent behavior of a junction, Ambegaokar-Baratoff relation is employed

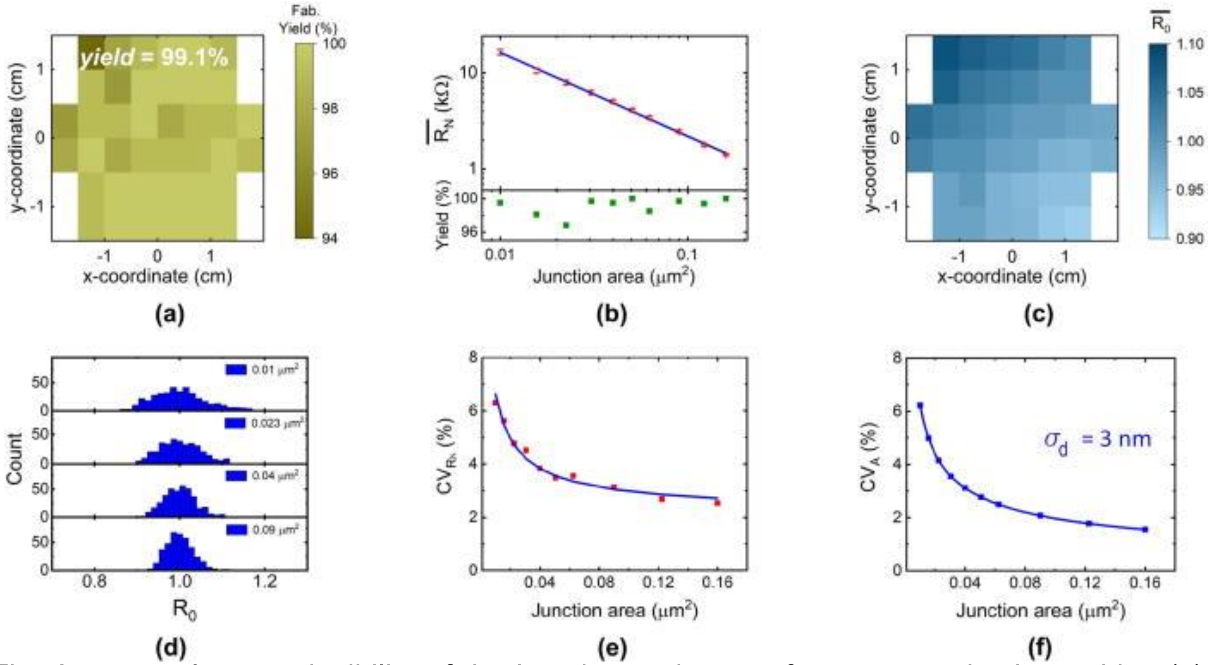


Fig. An example reproducibility of the junction resistance for superconducting qubits. (a) Heat map of the fabrication yield for chips across a 76-mm wafer. (b, top) Mean value of R_N, with error bars (red), vs junction area, and a linear fit (blue). (b, bottom) Fabrication yield vs junction area. (c) Heat map of the average normalized resistance. (d) Histograms of junction resistance for four different junction areas. (e) variation of junction resistance (points) vs junction area. (f) variation of junction area vs junction area.

V Ambegaokar, A Baratoff, "Tunneling Between Superconductors," *Phys. Rev. Lett.*, **11**, 104 (1963)
A Osman *et al*., "Simplified Josephson-junction fabrication process for reproducibly high-performance superconducting qubits," *Appl. Phys. Lett.*, **118,** 064002 (2021)

Josephson Junction: (2) Current-Phase / Voltage-Phase Relation

- **Josephson Equation: (1) Current-Phase Relation**
  - Relation between I_c and phase difference φ of a Josephson junction

$$I(t) = I_c \sin(\varphi(t))$$

$\varphi(t) = \theta_1 - \theta_2$ quantum-mechanical phase difference between two superconducting electrodes

- **Josephson Equation: (2) Voltage-Phase Relation**
  - Relation between V and φ of a Josephson junction

$$\frac{d\varphi(t)}{dt} = \frac{2\pi}{\Phi_0} V(t)$$

Important feature of a Josephson junction to realize a voltage standard device!

- **Josephson Equation: Nonlinear Energy Stored in a Junction**
  - Stored energy in a junction: $E_J(t)$

$$E_J(t) = \int_0^t I(t')V(t')dt' = \frac{\Phi_0 I_c}{2\pi} \int_0^t \sin(\varphi(t'))dt'$$

$$\therefore E_J(t) = -E_{J0} \cos(\varphi(t))$$

$$E_{J0} = \Phi_0 I_c / 2\pi$$

Refer to the following references for the detailed derivations

[1] B.D. Josephson, "Possible new effects in superconductive tunneling," *Phys. Lett.*, **1**, 251-253 (1962).

[2] A.A. Golubov *et al.*, "The current-phase relation in Josephson junctions," *Rev. Mod. Phys.*, **76**, 411 (2004).

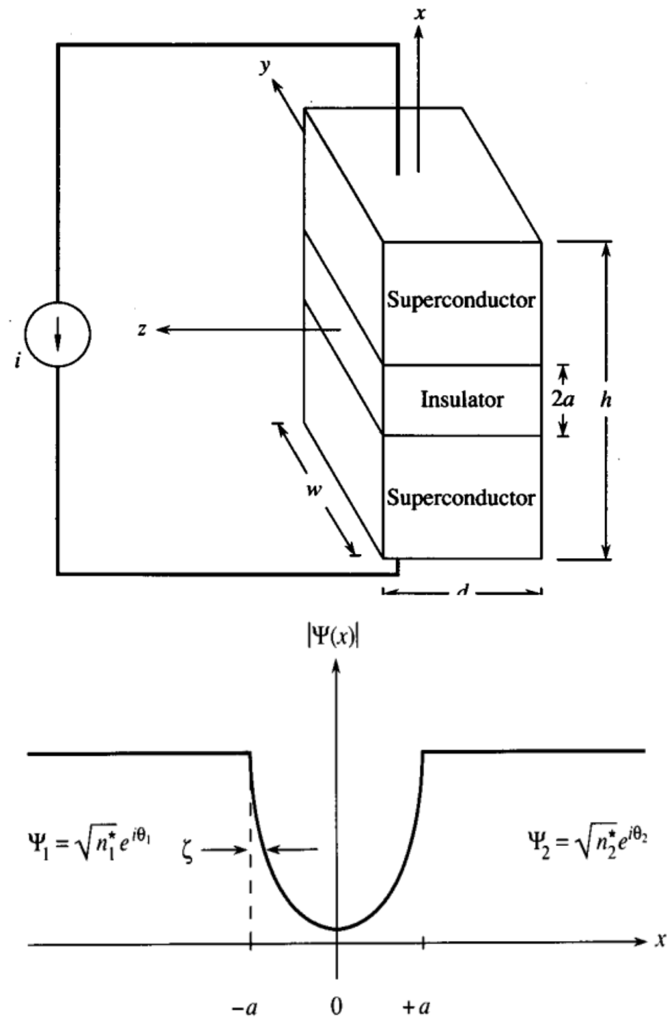


Fig. (a) Schematic drawing of a SIS junction. (b) Wave function of a SIS junction.

https://web.mit.edu/6.763/www/FT03/Lectures/Lecture11.pdf

Josephson Junction: (3) DC+AC Josephson Effect

- Josephson Equation: DC + AC Effect
  - When DC + AC voltage is applied to a Josephson junction, the current can be expressed by Jacobi-Anger expansion as
  - Let the applied voltage to a Josephson junction is

$$V(t) = V_0 + V_S \cos(\omega_s t)$$

  - From the voltage-phase relation, the phase is

$$\varphi(t) = \frac{2\pi}{\Phi_0}\int_0^t V(t')dt' = \varphi_0 + \frac{2\pi}{\Phi_0}V_0 t + \frac{2\pi V_s}{\Phi_0\omega_s}\sin(\omega_s t)$$

  - From the current-phase relation, the current is

$$I(t) = I_c \sin\left(\varphi_0 + \frac{2\pi}{\Phi_0}V_0 t + \frac{2\pi V_s}{\Phi_0\omega_s}\sin(\omega_s t)\right)$$

  - Using the Jacobi-Anger expansion (see Mathematics Lecture Notes), the current can be rewritten as

$$I(t) = I_c \sum_{n=-\infty}^{\infty}(-1)^n\left[J_n\left(\frac{2\pi V_s}{\Phi_0\omega_s}\right)\right]\sin[(\omega_{\text{JJ}} - n\omega_s)t + \varphi_0]$$

DC current will occur when $\omega_{\text{JJ}} = n\omega_s$ is satisfied, that equals to when $V_0 = n\left(\frac{\Phi_0}{2\pi}\right)\omega_s$

https://web.mit.edu/6.763/www/FT03/Lectures/Lecture11.pdf

Types of Josephson Junction Elements: (1) DC-SQUID

- Definition of DC-Superconducting Quantum Interference Device (SQUID)
  - DC-SQUID consists of two Josephson junctions arranged in a superconducting loop
  - DC SQUID has a loop of superconducting material that allows supercurrent to flow without resistance

- Characteristics of DC-SQUID
  - The external magnetic flux is trapped in a loop
  - The external magnetic flux is quantized to Φ_0
  - External magnetic flux Φ_{ext} dependent critical current I_{SQ}

  * Assuming a DC SQUID consist of identical junctions with I_c

$$I_{\text{SQ}}(\Phi_{\text{ext}}) = 2I_c \cos\left(\frac{\pi\Phi_{\text{ext}}}{\Phi_0}\right)$$

DC-SQUID is often utilized to realize "frequency-tunable" superconducting qubits ! (also, Josephson parametric amplifiers)

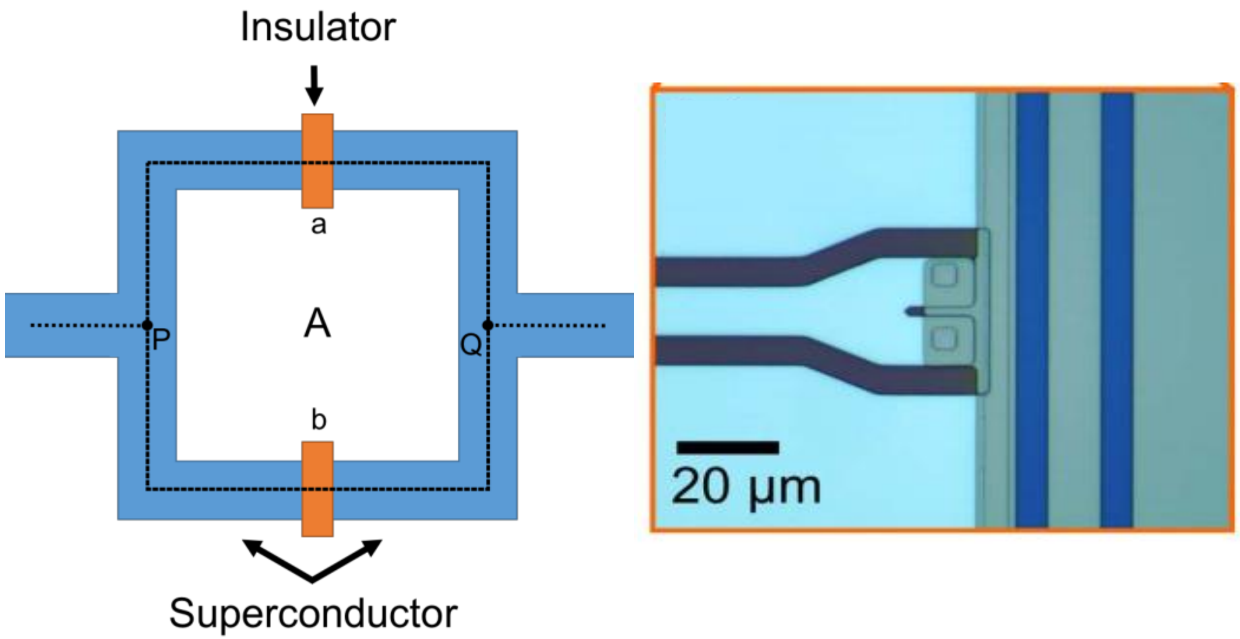
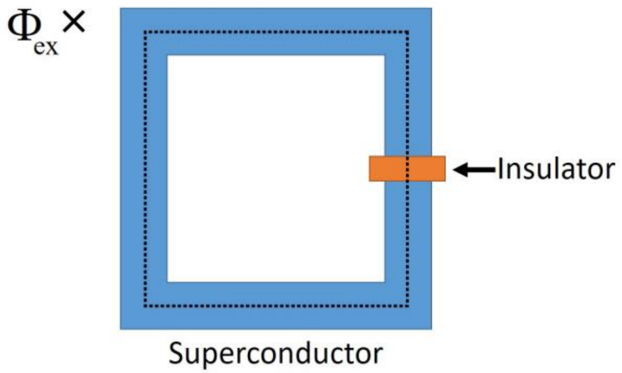


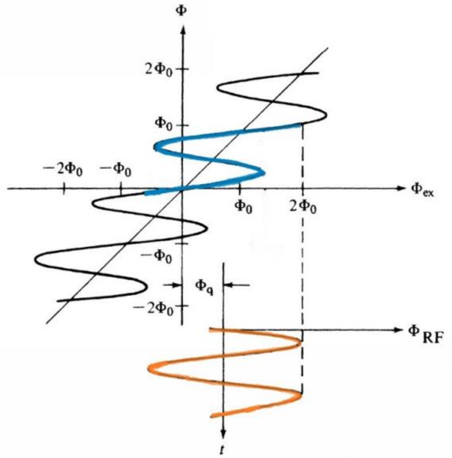
Fig. (a) Schematic drawing of a DC SQUID. (b) Micrograph of a Nb-based DC SQUID.

https://phas.ubc.ca/~berciu/TEACHING/PHYS502/PROJECTS/17SQUID.pdf
https://web.mit.edu/6.763/www/FT03/Lectures/Lecture11.pdf
G Choi *et al*., "Flux-Driven Josephson Parametric Amplifier Fabricated Using the Nb/AlOx/Nb Trilayer Process," *IEEE Trans. Appl. Supercond.*, **33**. 1701504 (2023)

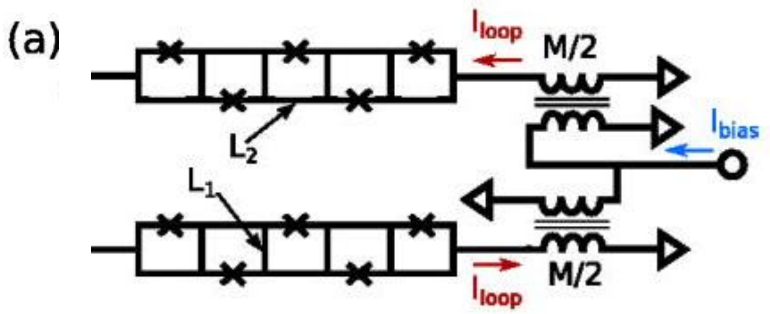
Types of Josephson Junction Elements: (2) RF-SQUID and Others

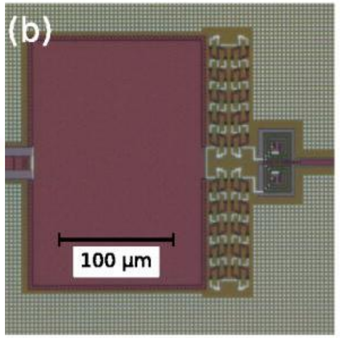
- RF-SQUID
  - A one-junction superconducting loop



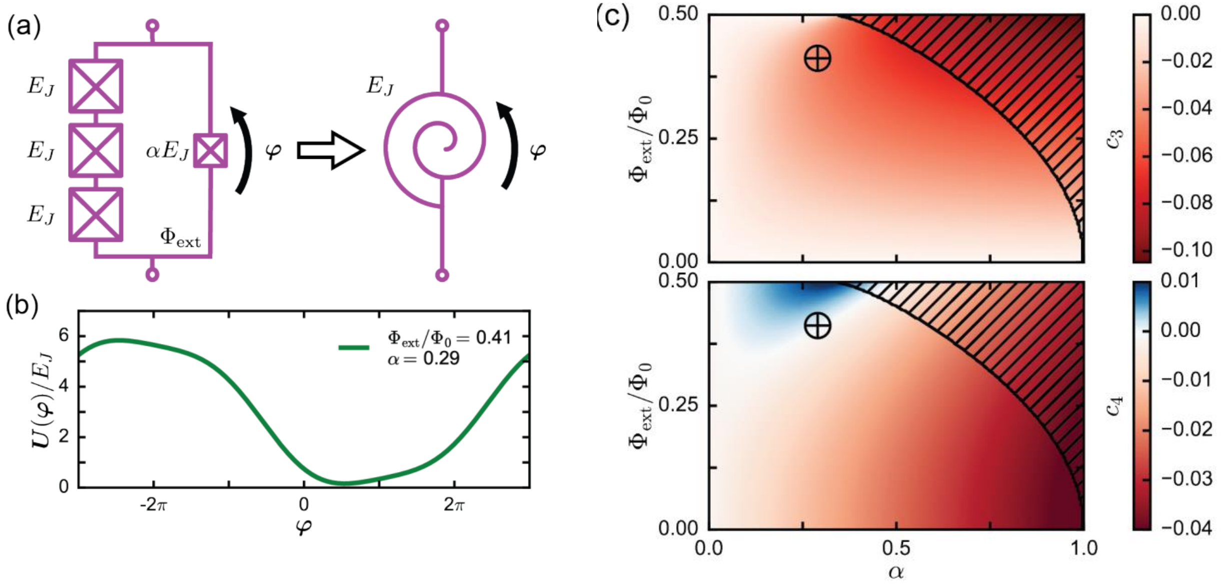


- SNAKE
  - Array of RF-SQUID





- **S**uperconducting **N**onlinear **A**symmetric **I**nductive e**L**ement
  - SNAIL: asymmetric SQUID loop



Details of these SQUID types are beyond the scope…
However, note that every SQUID has its unique features that may be helpful to realize quantum devices!
(e.g., SNAKE is utilized to realize a broadband, high-power Josephson parametric amplifier)

https://phas.ubc.ca/~berciu/TEACHING/PHYS502/PROJECTS/17SQUID.pdf
NE Frattini *et al.*, "3-wave mixing Josephson dipole element," *Appl. Phys. Lett.*, **110**, 222603 (2017)
T White *et al.*, "Readout of a quantum processor with high dynamic range Josephson parametric amplifiers," *Appl. Phys. Lett.* **122**, 014001 (2023)

See Also…

Textbooks:

[1] M Tinkham, *Introduction to Superconductivity*, McGraw Hill, Second Edition, (2004). * recommended

Review Papers:

[1] H Padamsee, *Physics and Accelerator Applications of RF Superconductivity*, Annual Review of Nuclear and Particle Science, (1993)

[2] J. Gao, *The physics of superconducting microwave resonators*, Ph.D. thesis, California Institute of Technology (2008)

[3] A.A. Golubov, M.Yu. Kupriyanov, and E. Il'ichev, "The current-phase relation in Josephson junctions", *Rev. Mod. Phys.*, **76**, 411 (2004).

Open Courses:

[1] T Orlando, *Applied Superconductivity*, MIT OCW, 2005. [Online Available]
https://ocw.mit.edu/courses/6-763-applied-superconductivity-fall-2005/

[2] https://www.wmi.badw.de/fileadmin/WMI/Lecturenotes/SC_and_LT_Physics_1/SC_LT-Physics-Chapter5-2021.pdf