APPLIED SUPERCONDUCTIVITY LABORATORY

Introduction to Superconducting Quantum Circuits

- Parametrically Pumped Josephson Devices -

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Lecture Overview

Week 1. Introduction to Superconducting Quantum Circuits

Week 2. Review of Mathematics and Microwave Engineering

Week 3. Review of Classical and Quantum Mechanics

Week 4. Review of Superconductivity

Week 5. Quantum Harmonic/Anharmonic Oscillators and Light-Matter Interaction

Week 6. Circuit Quantization Methods

Week 7. Parametrically Pumped Josephson Devices

Week 8. Design and Analysis of Superconducting Resonators

Week 9. Design and Analysis of Superconducting Qubits

Week 10. Design and Analysis of Single-Qubit Device: 3D Cavity

Week 11. Design and Analysis of Single-Qubit Device : 2D Chip

Week 12. Design and Analysis of Two-Qubit Device

Week 13. Design and Analysis of Josephson Parametric Amplifier

Week 14. Term Project

Week 15. Term Project

overall backgrounds, terminologies of quantum computing

mathematical and engineering backgrounds
general superconductivity

Quantum circuit analysis

design and analysis of superconducting RF devices

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Keywords in Parametrically Pumped Josephson Devices

Josephson Parametric Amplifiers

Lumped Josephson Parametric Amplifiers
CPW-Shunted Josephson Parametric Amplifiers
Impedance-Matched Josephson Parametric Amplifiers

Pumpistor Method
Coupled-Mode Network Theory
Harmonic Balance Method

Amplifier Gain
Gain Bandwidth
Ripple
Added Noise

Traveling Wave Parametric Amplifiers

Lumped-Element JJ TWPA
Lumped-Element SNAIL TWPA
Kinetic-Inductance TWPA

Lumped RLC Element
Harmonic Balance Method

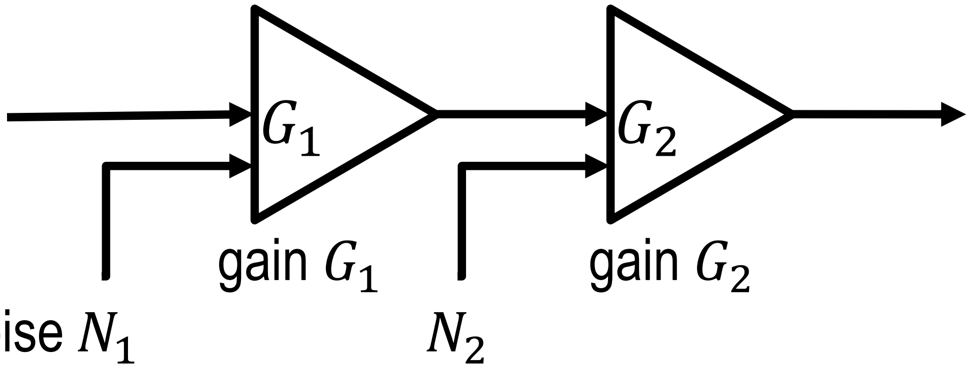
Amplifier Gain
Gain Bandwidth
Ripple
Added Noise

Introduction to Microwave Signal Amplifiers

Microwave Engineering Principles of Noise and Gain in Cascaded Amplifiers

input signal S_{in}

input noise N_{in}



output signal $S_{\text{out}} = G_2 G_1 S_{\text{in}}$

output noise $N_{\text{out}} = G_2[G_1(N_{\text{in}} + N_1) + N_2]$

For *quantum-limited noise* and *high-gain* amplifier *at the 1st* stage,

$$\text{SNR} = \frac{S_{\text{out}}}{N_{\text{out}}} = \frac{G_1 S_{\text{in}}}{G_1(N_{\text{in}}+N_1)+N_2} \approx \frac{S_{\text{in}}}{N_{\text{in}}+\underline{N_1}} \Leftarrow \text{quantum-limited}$$

Commercially Available Classical Amplifiers

- Pros:

(1) Large gain > 30 dB and small ripples < 1 dB

(2) Large input signal saturation power ~ 10 dBm

- Cons:

(1) Large noise ~ 5 K → not-suitable for the 1st amplification

(2) Large operating power supply → large heat intrusion

Quantum-Limited Noise Amplifiers

- Pros:

(1) Quantum-limited noise ~ 0.25 K → best option for the 1st amplification

(2) Small operating power supply → small heat intrusion

- Cons:

(1) Relatively large ripples in the gain bandwidth ~ a few dB

(2) Small input saturation power ~ -110 dBm

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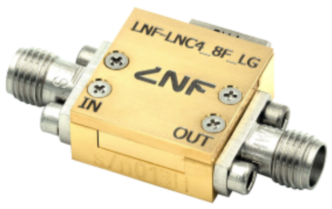
Introduction to Microwave Signal Amplifiers: Classical HEMT Amplifier

- Classical High Electron Mobility Transistor (HEMT) Amplifiers
  - State-of-the-art performance: gain ~ 40 dB… but typical commercial products exhibit gain ~ 25 dB
  - Noise figure: ~ 5 K → not-suitable for the 1st amplification but suitable for the 2nd or 3rd amplification stages

Datasheet

LNF-LNC4_8F_LG

4-8 GHz Cryogenic Low Noise Amplifier



Product Features	
RF Bandwidth	4-8 GHz
Noise Temperature	1.5 K
Noise Figure	0.022 dB
Gain	26 dB
DC power (typical)	V_{ds} = 0.7 V, I_{ds} = 10 mA*
RF Connectors	Female SMA**
DC Connectors	9-pin Female Nano-D
One gate and one drain supply only	

* See test report for actual optimum bias for your unit

** Contact factory for alternative configuration

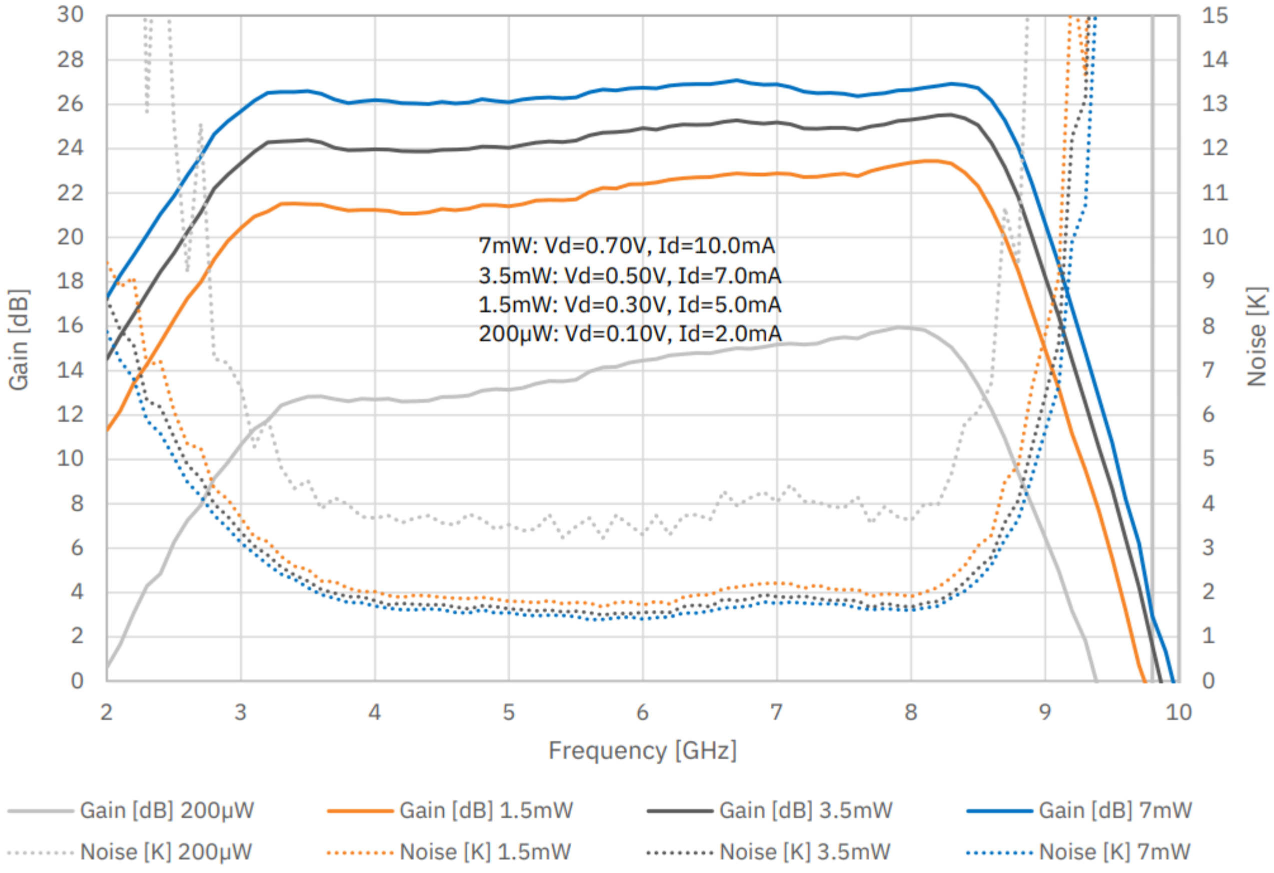


Fig. Features and performance metrics of a commercially available HEMT amplifier from Low Noise Factory (LNF-LNC4_8F_LG model)

HEMT amplifier datasheet from: https://lownoisefactory.com/product/lnf-lnc4_8f_lg/

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Introduction to Quantum-Limited Noise Amplifiers

- How to Improve Readout Fidelity of Superconducting Qubits?
  - Better SNR (signal-to-noise-ratio) ➔ large gain (> 20 dB) with quantum-limited noise (1/2 photon level)
  - Quantum-limited noise amplifiers can be implemented *using parametrically pumped superconducting Josephson elements!*

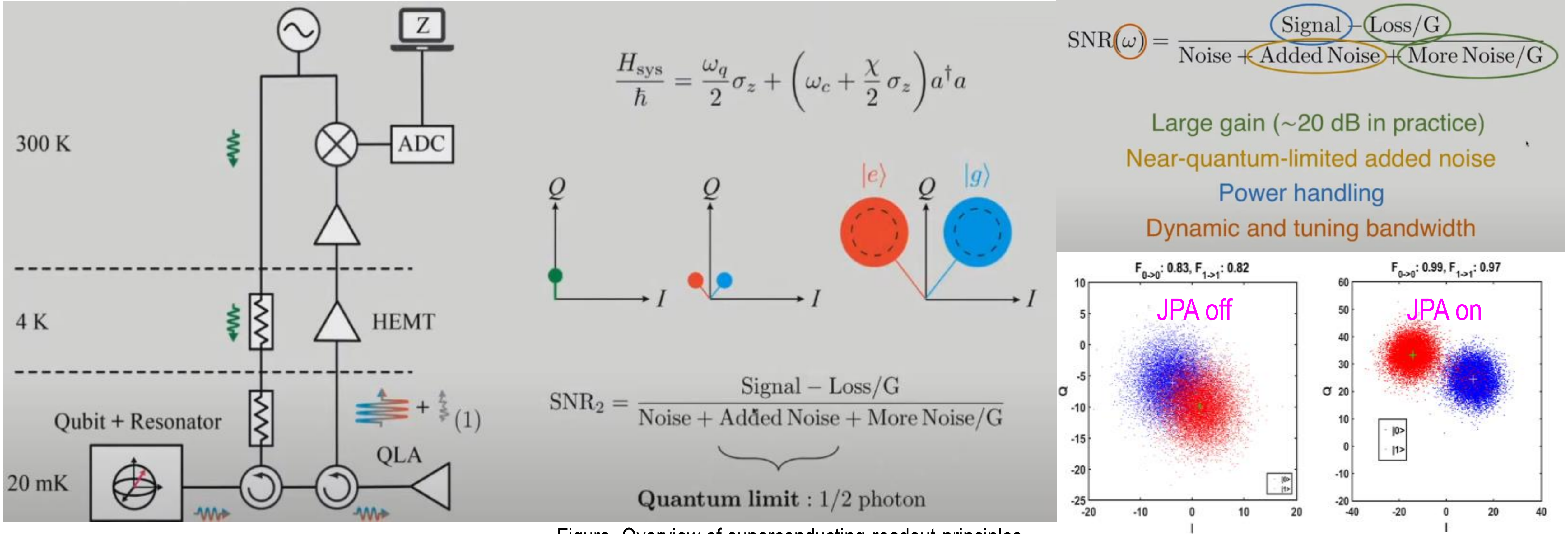


Figure. Overview of superconducting readout principles

V Sivak, What can SNAILs do for Quantum-Limited Amplifiers?, https://www.youtube.com/watch?v=HnF7iGA0H-0
R Yang, *et al.*, "Fabrication of the Impedance-Matched Josephson Parametric Amplifier and the Study of the Gain Profile," ***IEEE Trans. Appl. Supercond***, vol. 30, no. 4, 2020.

Introduction to Quantum-Limited Noise Amplifiers

- Superconducting Josephson Elements as Nonlinear Media in Quantum Optics
  - Superconducting QUantum Interference Device (SQUID) is the most important component
  - A SQUID ➔ a closed loop formed by two Josephson junctions
  - A SQUID's nonlinear inductance➔ nonlinear media in optics

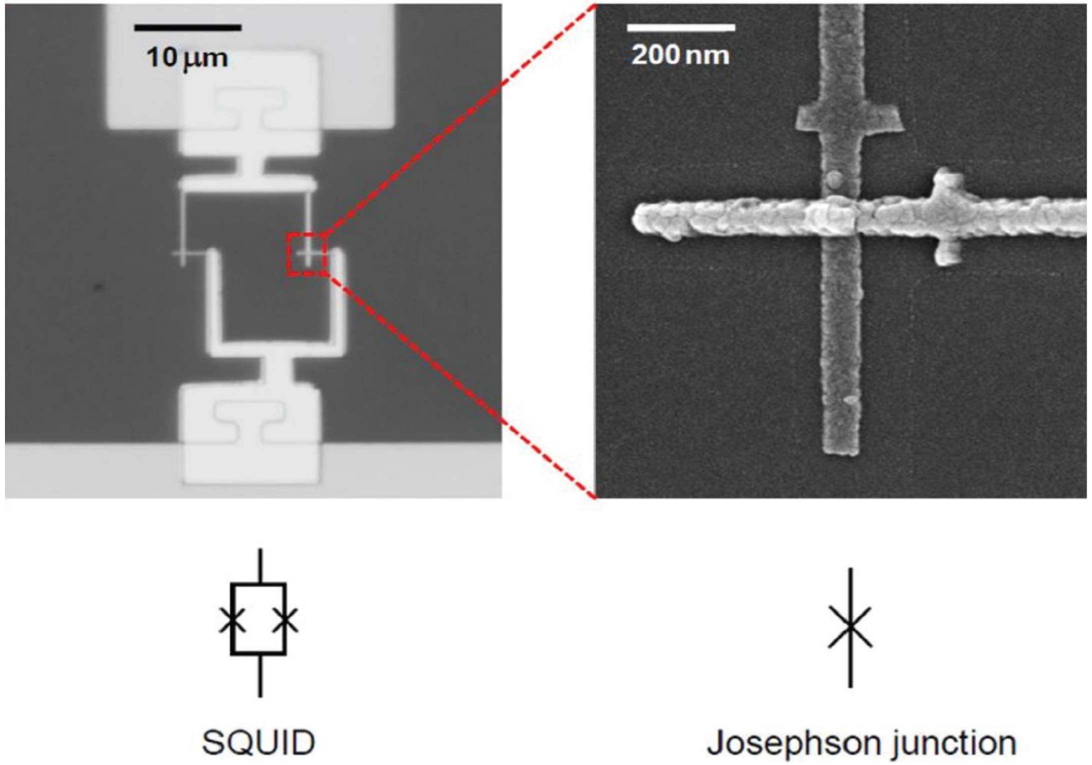


Figure. Example images of SQUID and Josephson junction.

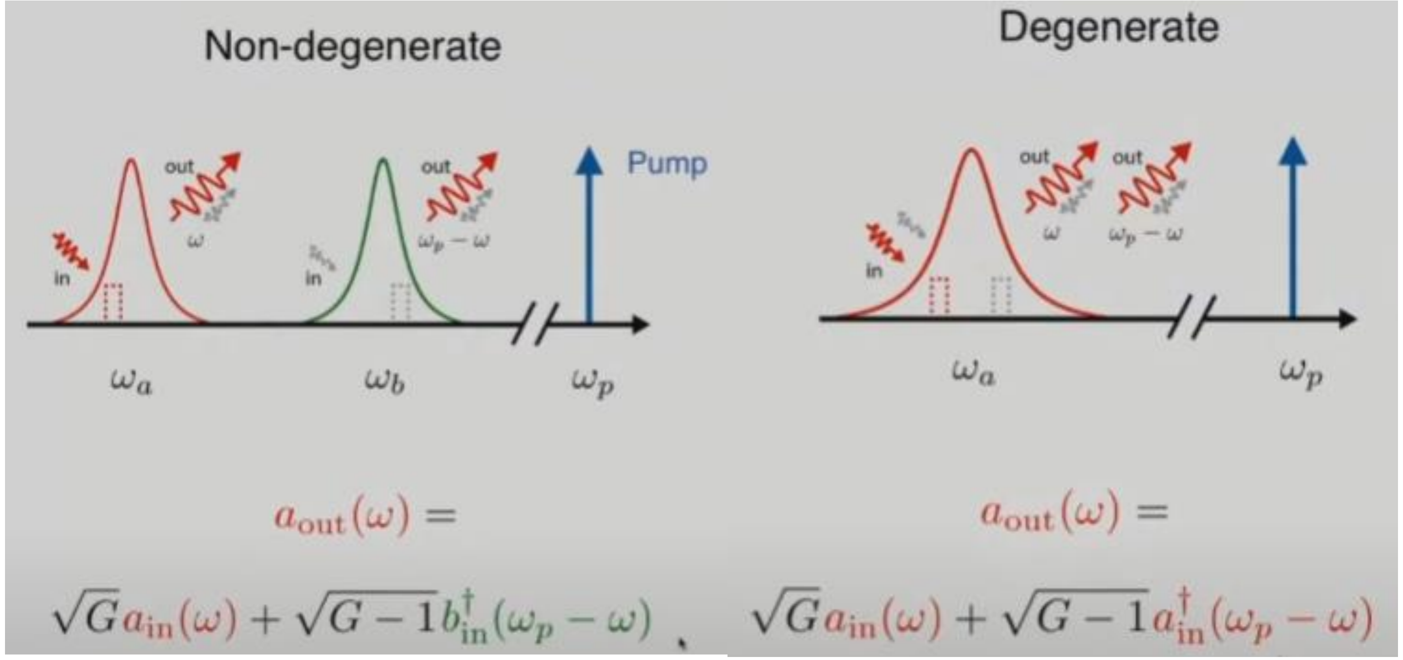
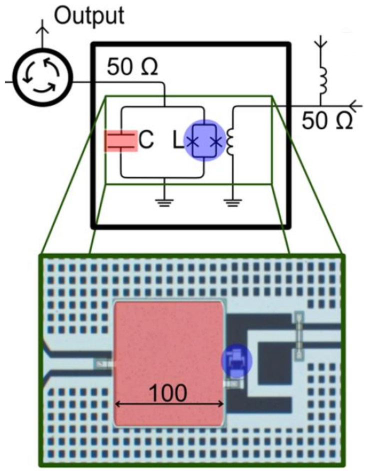


Figure. Comparison of non-degenerate and degenerate parametric amplification modes.

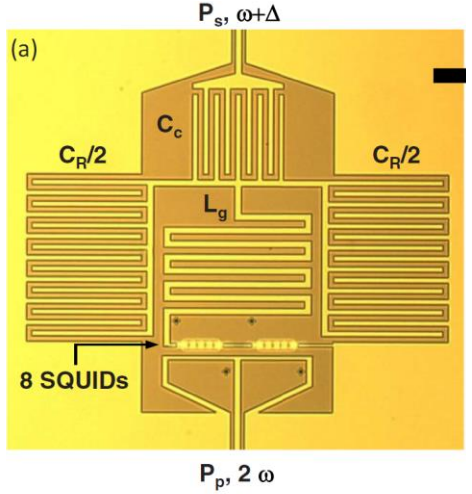
NE Frattini, VV Sivak, *et al*., "Optimizing the Nonlinearity and Dissipation of a SNAIL Parametric Amplifier for Dynamic Range," ***Phys. Rev. Appl***., vol. 10, 2018.

Selected Topologies of Superconducting Quantum-Limited Noise Amplifiers

- Quantum-Limited Noise Amplifiers Based on Josephson Elements
  - Lumped-element Josephson parametric amplifier (LJPA)

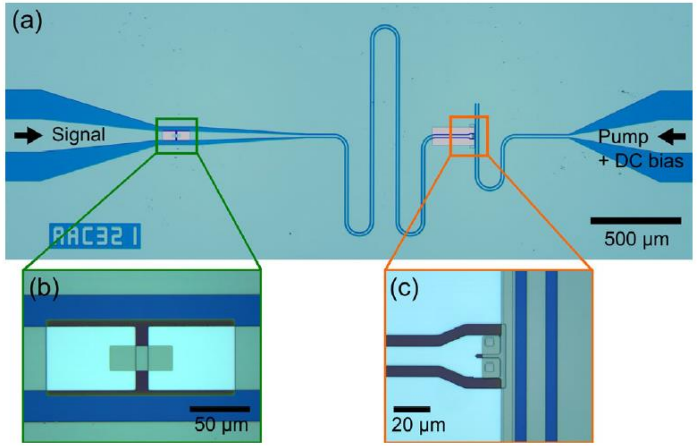


SQUID LJPA

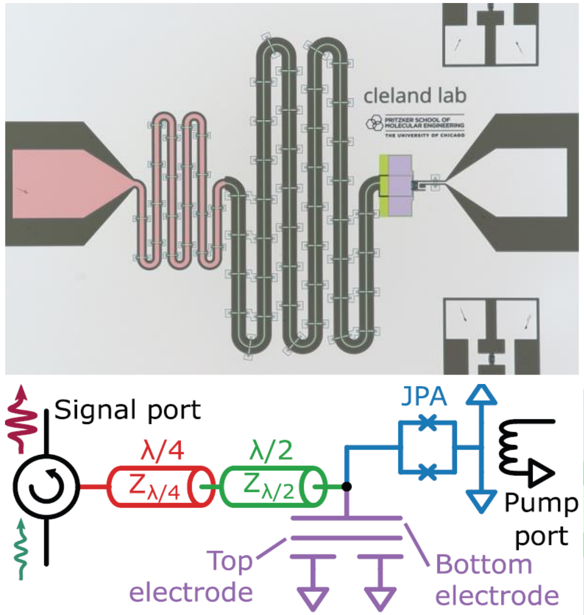


SQUID array LJPA

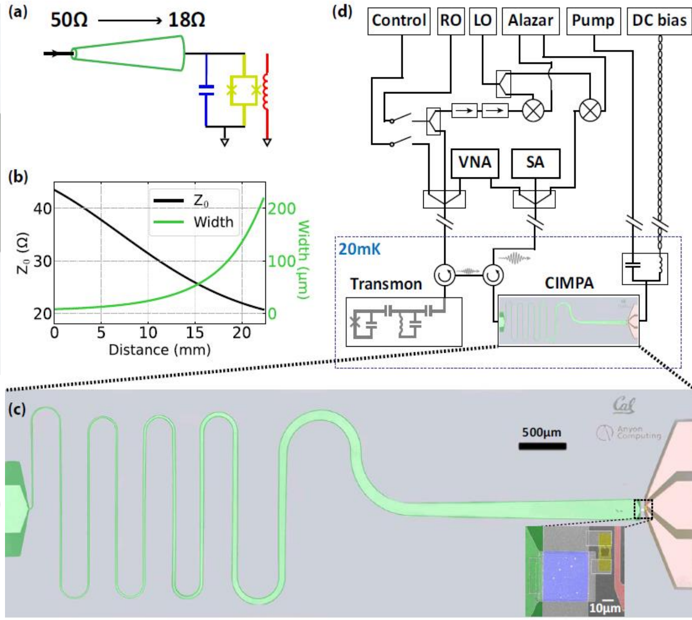
  - CPW resonator-shunted Josephson parametric amplifier



  - Impedance-matched Josephson parametric amplifier (IMPA)



Two-pole transformer IMPA



Klofenstein taper IMPA

See further details at:

[SQUID LJPA] JY Mutus *et al.*, *Appl. Phys. Lett.* **103**, 122602 (2013)

[SQUID Array LJPA] X Zhou *et al.*, *Phys. Rev. B,* **89**, 214517 (2014)

[CPW shunted JPA] G Choi *et al.*, *IEEE Trans. Appl. Supercond.*, **33**, 1701504 (2023)

[Two-Pole Transformer IMPA] J Grebel *et al.*, *Appl. Phys. Lett.* **118**, 142601 (2021)

[Klopfenstein Taper IMPA] B Qing *et al.*, *Phys. Rev. Research*, **6**, L012035 (2024)

How to design and analyze Josephson parametric amplifiers?

Analysis of Parametrically-Pumped Josephson Junction: (1) Pumpistor Model

- Equivalent Lumped-Element Modeling Method for Parametrically-Pumped Josephson Devices
  - SQUID can be modelled into lumped nonlinear inductor circuit model, known as pumpistor
  - A pumped SQUID can be represented by an inductor and negative resistor

(1) For a SQUID loop with an external DC flux Φ_{DC} and AC pump $\Phi_{\mathrm{AC}}\cos(\omega_P t+\theta_P)$,

$$\Phi(t)=\Phi_{\mathrm{DC}}+\Phi_{\mathrm{AC}}\cos(\omega_P t+\theta_P),$$

(2) The net supercurrent I through the SQUID loop is:

$$I=I_c\left|\cos\left(\frac{\pi\Phi(t)}{\Phi_0}\right)\right|\sin(\phi(t)),$$

where Φ_0 is quantum of flux, 2.067×10^{-15} [Wb] and I_c is the critical current of SQUID

(3) By normalizing the flux as $F=\pi\Phi_{\mathrm{DC}}/\Phi_0$ and $\delta f=\pi\Phi_{\mathrm{AC}}/\Phi_0$:

$$I=I_c\left|\cos\left(\frac{\pi\Phi(t)}{\Phi_0}\right)\right|\approx I_c\cos(F)-I_c\sin(F)\delta f\cos(\omega_P t+\theta_P),$$

(4) Assuming the SQUID phase as $\phi(t)=\phi_s\cos(\omega_s t+\theta_s)$:

$$\sin(\phi(t))\approx\sum_{n=-\infty}^{\infty}J_n(\phi_s)\sin\left(n\left(\omega_s t+\theta_s+\frac{\pi}{2}\right)\right),$$

where J_n is n^{th} order Bessel function of the first kind

(5) The equivalent SQUID inductance L_{SQ} can be expressed as $L_{\mathrm{SQ}}^{-1}=L_{\mathrm{J}}^{-1}+L_{\mathrm{P}}^{-1}$

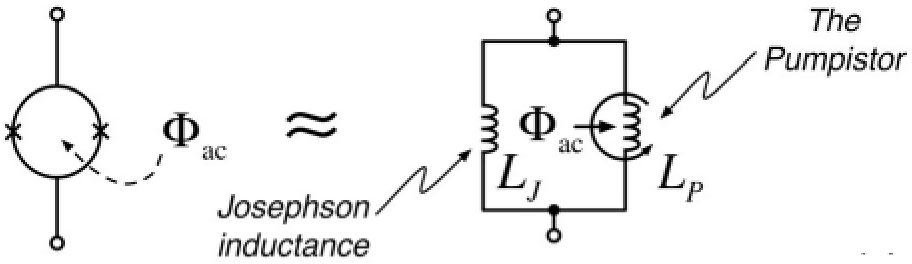


Figure. Equivalent circuit model for SQUID loop as a pumpistor.

$$L_J=\frac{L_{J0}}{\cos(F)}\left[\frac{\phi_s}{2J_1(\phi_s)}\right],$$

$$L_P=\frac{-2\,e^{j\Delta\theta}}{\delta f}\frac{L_{J0}}{\sin(F)}\left[\frac{\phi_s}{2J_1(\phi_s)-2e^{j2\Delta\theta}J_3(\phi_s)}\right],$$

where

L_{J0}: SQUID's bare inductance,

$\Delta\theta$: $2\theta_s-\theta_P$ with "3-wave mixing assumption"

KM Sundqvist, *et al.*, "The pumpistor: A linearized model of a flux-pumped superconducting quantum interference device for use as a negative-resistance parametric amplifier," *Appl. Phys. Lett.*, **103**, 102603 (2013).

- **Example Study Using the Pumpistor Model: CPW Resonator-Shunted Parametric Amplifier**
  - Geometric features of the example:

(1) A SQUID is shunted by a quarter-wavelength CPW resonator

(2) A CPW resonator is capacitively coupled (C_c) to a signal in/out port

(3) I_c of SQUID: 4.3 μA, $C_c = 5.4$ fF, $\omega_{\mathrm{CPW}}/2\pi = 5$ GHz

  - Step-by-step analysis:

(1) Calculate the equivalent inductance and resistance of a SQUID for a given DC bias and pump bias

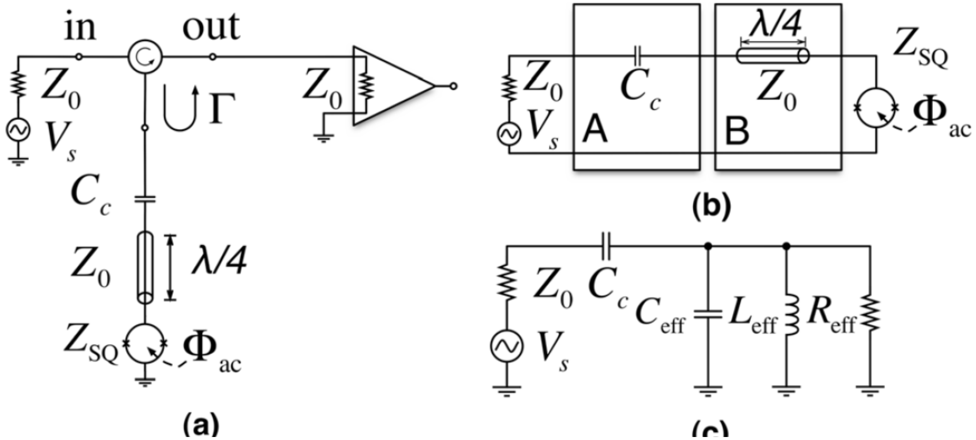
(2) Calculate the equivalent impedance of the SQUID

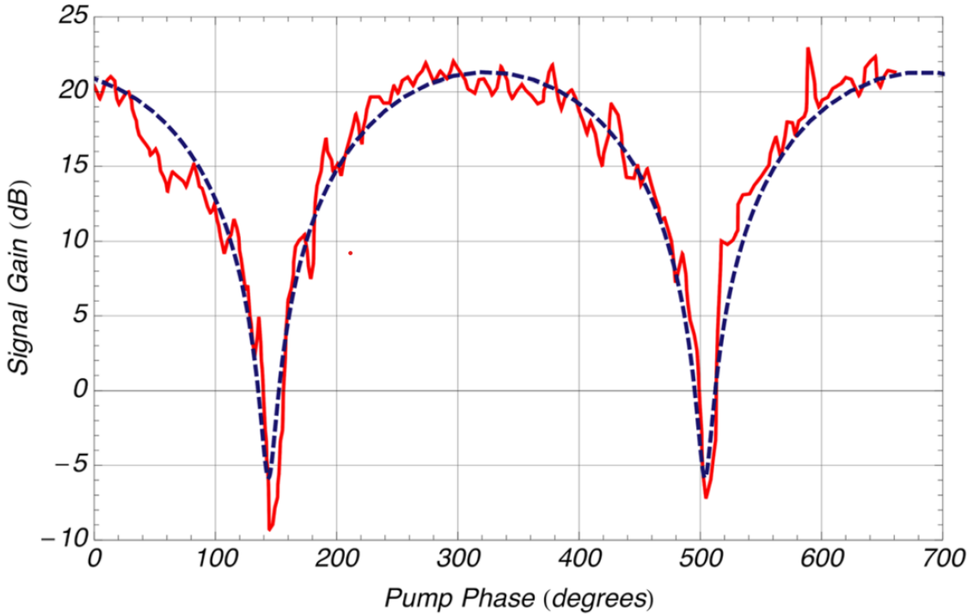
(3) Calculate the S-parameters of the device except for the SQUID

(4) Calculate the total reflection coefficient of the amplifier

(5) Calculate the gain of the amplifier as a function of the signal frequency

See KM Sundqvist *et al.*, *Appl. Phys. Lett.*, **103**, 102603 (2013) and also KM Sundqvist *et al.*, *EPJ Quantum Technol.*, **1**, 6 (2014) for further details on the analysis method





KM Sundqvist, *et al.*, "The pumpistor: A linearized model of a flux-pumped superconducting quantum interference device for use as a negative-resistance parametric amplifier," *Appl. Phys. Lett.*, **103**, 102603 (2013).

Analysis of Parametrically-Pumped Josephson Junction: (2) Coupled-Mode Network

- Synthesis Method of Parametrically Coupled Networks: (1) Equation of Motion
  - Quantum mechanical picture for the electrical circuits → introducing the equation of motion to the electrical circuits

* the equation of motion for voltage vector $\vec{v} = \{a_j(\omega_j^s)\}^{\mathrm{T}}$ and input voltage vector $\vec{v}_{\mathrm{in}} = \{a_j^{\mathrm{in}}(\omega_j^s)\}$:

$$-i\gamma_0 \mathbf{M}\vec{v} = \mathbf{K}\vec{v}_{\mathrm{in}}$$

where,

$$\mathbf{K} \equiv \mathrm{diag}\left(\sqrt{\gamma_j^{\mathrm{ext}}}\right) \qquad \beta = \frac{C_{jk}}{2\gamma_0} \text{ (off-diagonal coupling)}$$

$$\gamma_0 = \sqrt[N_P]{\textstyle\prod_k \gamma_k} \text{ (overall dissipation)} \qquad C_{jk} = \omega_0\sqrt{\frac{Z_j}{Z_k}} \text{ coupling strength}$$

$$\Delta_k \equiv \frac{1}{\gamma_0}\left(\omega_k^s - \omega_k + i\frac{\gamma_k}{2}\right)$$

$$\mathbf{M} = \begin{bmatrix} \Delta_1 & \beta_{12} & \cdots & \beta_{1N} & \beta_{11^*} & \cdots & \beta_{1N^*} \\ \beta_{21} & \Delta_2 & \cdots & \beta_{2N} & \beta_{21^*} & \cdots & \beta_{2N^*} \\ \vdots & & \ddots & \vdots & \vdots & \ddots & \vdots \\ \beta_{N1} & \cdots & & \Delta_N & \beta_{N1^*} & \cdots & \beta_{NN^*} \\ \beta_{1^*1} & \cdots & & \beta_{1^*N} & -\Delta_1^* & \cdots & \beta_{1^*N^*} \\ \vdots & & & \vdots & \vdots & \ddots & \vdots \\ \beta_{N^*1} & \cdots & & \beta_{N^*N} & \beta_{N^*1^*} & \cdots & -\Delta_N^* \end{bmatrix}$$

* from the input voltage $\vec{v}_{\mathrm{in}}$ & output voltage $\vec{v}_{\mathrm{out}}$ amplitudes,

$$\vec{v}_{\mathrm{in}} + \vec{v}_{\mathrm{out}} = \mathbf{K}\vec{v}$$

* then the scattering matrix $\mathbf{S}$ and $\{j,k\}$ component S_{jk} are:

$$\mathbf{S} = i\frac{1}{\gamma_0}\mathbf{K}\mathbf{M}^{-1}\mathbf{K} - \mathrm{I} \text{ and } S_{jk} = i\frac{\sqrt{\gamma_j^{\mathrm{ext}}\gamma_k^{\mathrm{ext}}}}{\gamma_0}[\mathbf{M}^{-1}]_{jk} - \delta_{jk}$$

L Ranzani, J Aumentado, "Graph-based analysis of nonreciprocity in coupled mode system," ***New J. Phys***., vol. 17, p. 023024, 2015.

Analysis of Parametrically-Pumped Josephson Junction: (2) Coupled-Mode Network

Synthesis Method of Parametrically Coupled Networks: (2) Graph-Based Representations

- Graph notation to describe the parametrically coupled networks

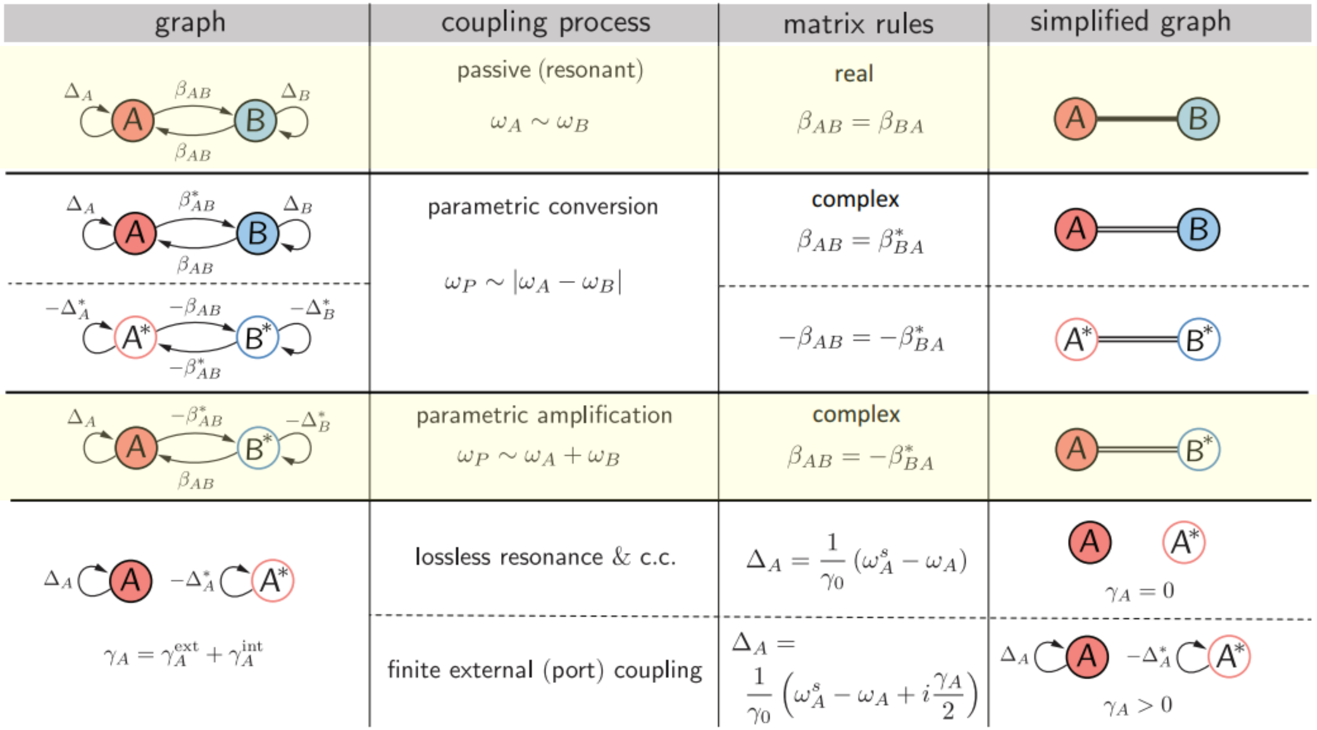


Fig. Rules for describing the equation-of-motion of parametrically coupled network

* for bandpass filter design,

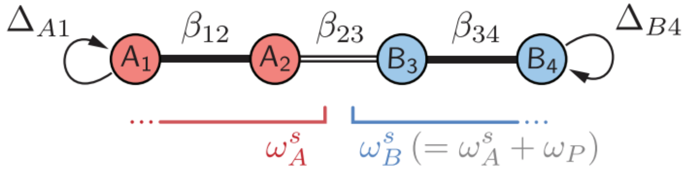
$$\therefore S_{\text{in,in}}(\omega^s = \omega_{\text{target}}) = 0$$

$$\boxed{S_{jk} = i\frac{\sqrt{\gamma_j^{\text{ext}}\gamma_k^{\text{ext}}}}{\gamma_0}[\mathbf{M}^{-1}]_{jk} - \delta_{jk}}$$

* for parametric amplifier design,

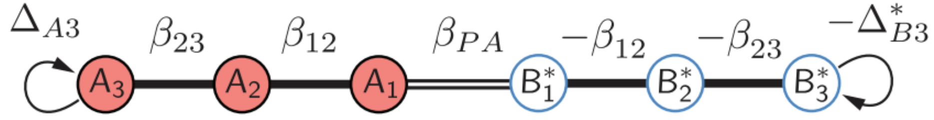
$$\therefore S_{\text{in,in}}(\omega^s = \omega_{\text{target}}) = \sqrt{G}\ (\text{where } G\text{: target gain})$$

- Example (1): 4-mode coupled network and its $\mathbf{M}$



$$\mathbf{M} = \begin{bmatrix} \Delta_{A1} & \beta_{12} & 0 & 0 \\ \beta_{12} & \Delta_{A2} & \beta_{23} & 0 \\ 0 & \beta_{23}^* & \Delta_{B3} & \beta_{34} \\ 0 & 0 & \beta_{34} & \Delta_{B4} \end{bmatrix}$$

- Example (2): 6-mode coupled network and its $\mathbf{M}$



$$\mathbf{M}_{\text{PA}} = \begin{bmatrix} \Delta_{A3} & \beta_{23} & 0 & 0 & 0 & 0 \\ \beta_{23} & \Delta_{A2} & \beta_{12} & 0 & 0 & 0 \\ 0 & \beta_{12} & \Delta_{A1} & \beta_{\text{PA}} & 0 & 0 \\ 0 & 0 & -\beta_{\text{PA}}^* & -\Delta_{B1}^* & -\beta_{12} & 0 \\ 0 & 0 & 0 & -\beta_{12} & -\Delta_{B2}^* & -\beta_{23} \\ 0 & 0 & 0 & 0 & -\beta_{23} & -\Delta_{B3}^* \end{bmatrix}$$

O Naaman, J Aumentado, "Synthesis of Parametrically Coupled Networks," ***PRX Quantum***, vol. 3, p. 020201, 2022.

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Analysis of Parametrically-Pumped Josephson Junction: (2) Coupled-Mode Network

Synthesis Method of Parametrically Coupled Networks: (3) Electrical Coupling Elements

- Electrical circuit components (J-inverter, K-inverter) can be expressed as follows:

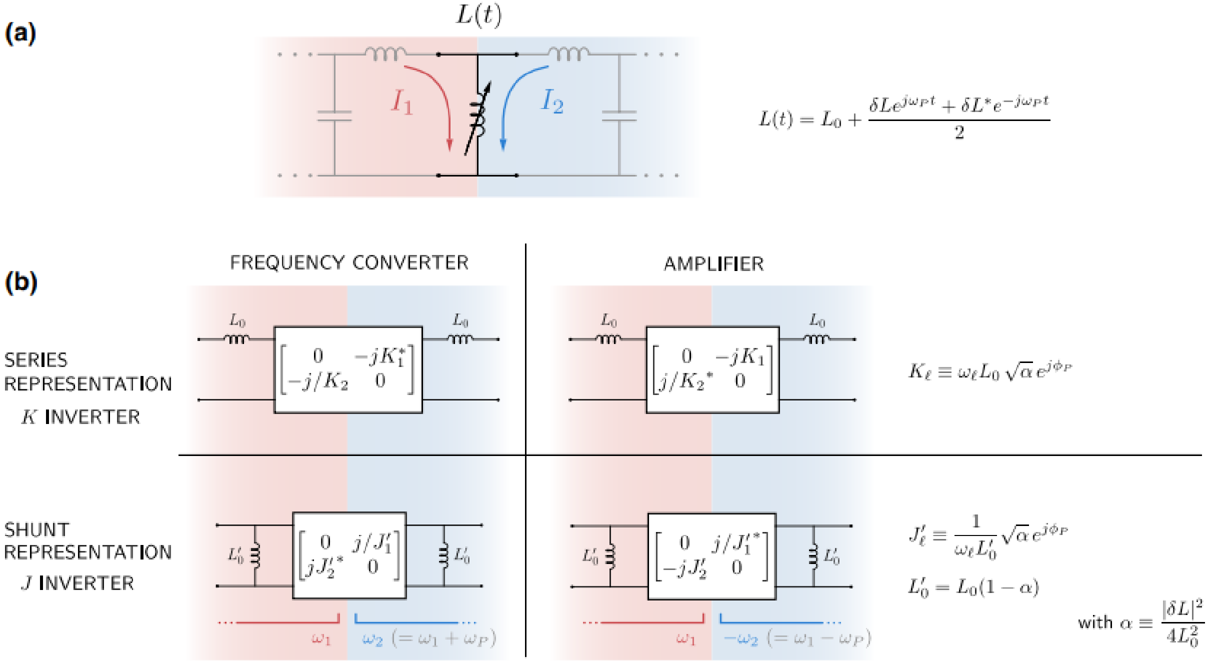


Fig. Modulated inductance with idler current I_1 and I_2 and equivalent J/K inverter.

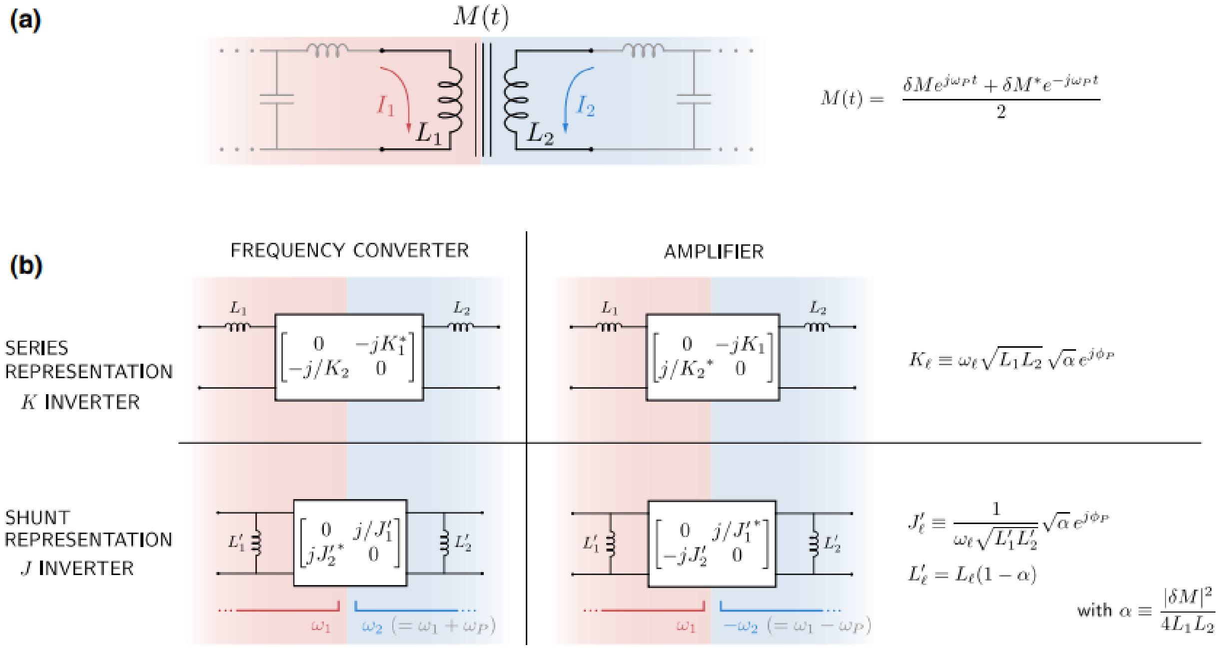
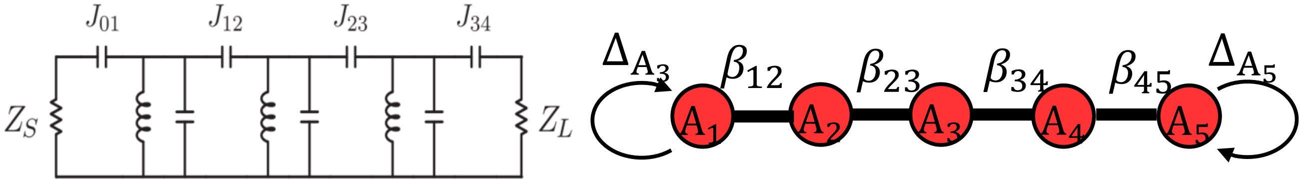
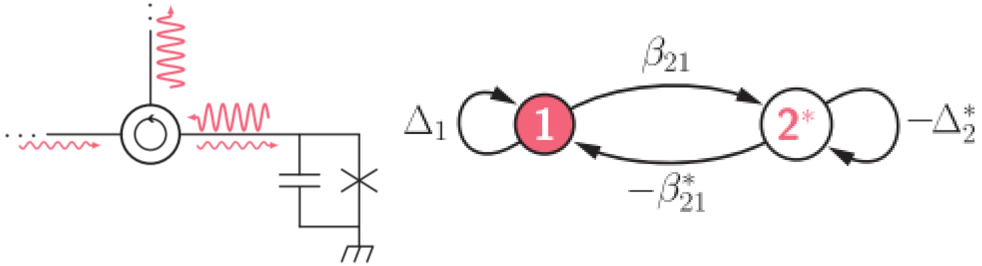


Fig. Mutual inductance with idler current I_1 and I_2 and equivalent J/K inverter.

- Example (1): bandpass filter



- Example (2): Josephson parametric amplifier



L Ranzani, J Aumentado, "Graph-based analysis of nonreciprocity in coupled mode system," ***New J. Phys***., vol. 17, p. 023024, 2015.
O Naaman, J Aumentado, "Synthesis of Parametrically Coupled Networks," ***PRX Quantum***, vol. 3, p. 020201, 2022.

Analysis of Parametrically-Pumped Josephson Junction: (2) Coupled-Mode Network

- Example Study Using the Coupled-Mode Network Method: Impedance-Matched Parametric Amplifier
  - Impedance matched Josephson parametric amplifier (center frequency: 6 GHz, bandwidth: 600 MHz, gain: 20 dB)

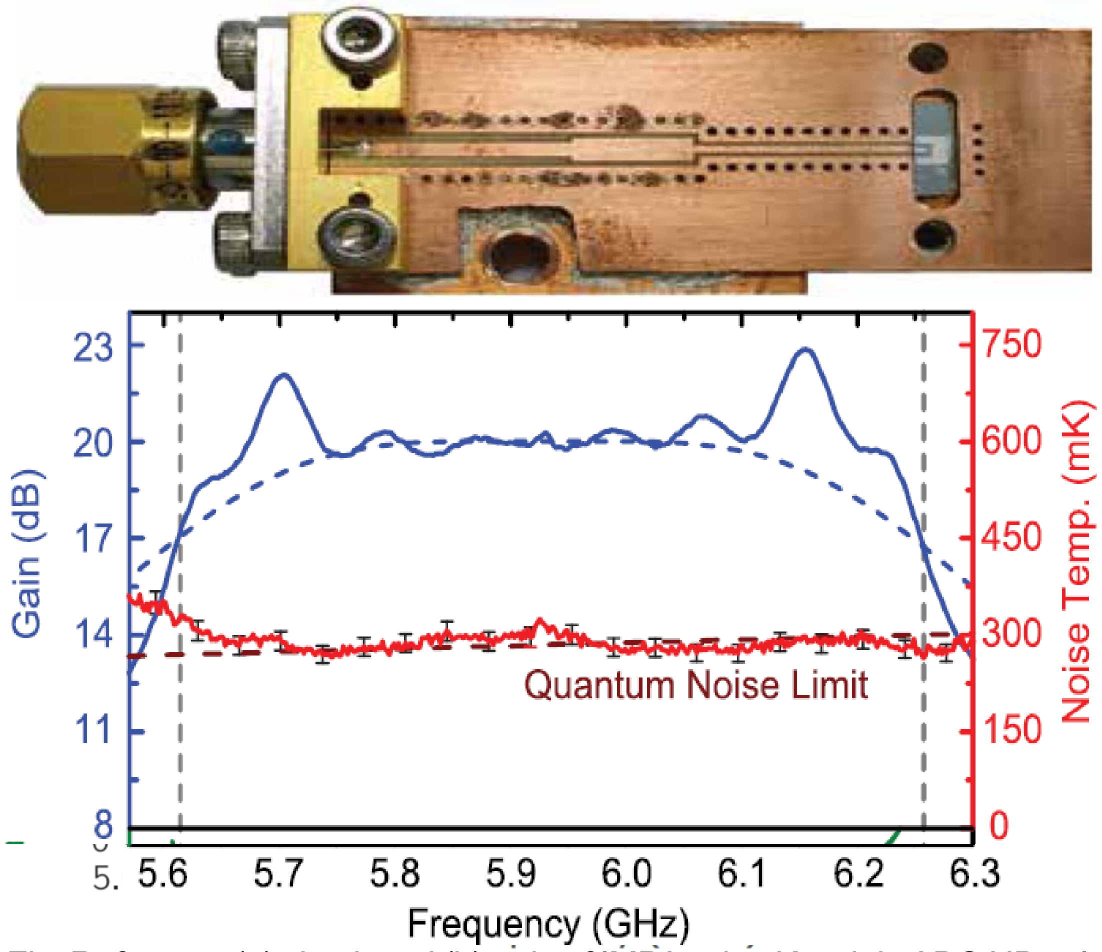


Fig. Reference (a) circuit and (b) gain of IMPA using Keysight ADS HB solver

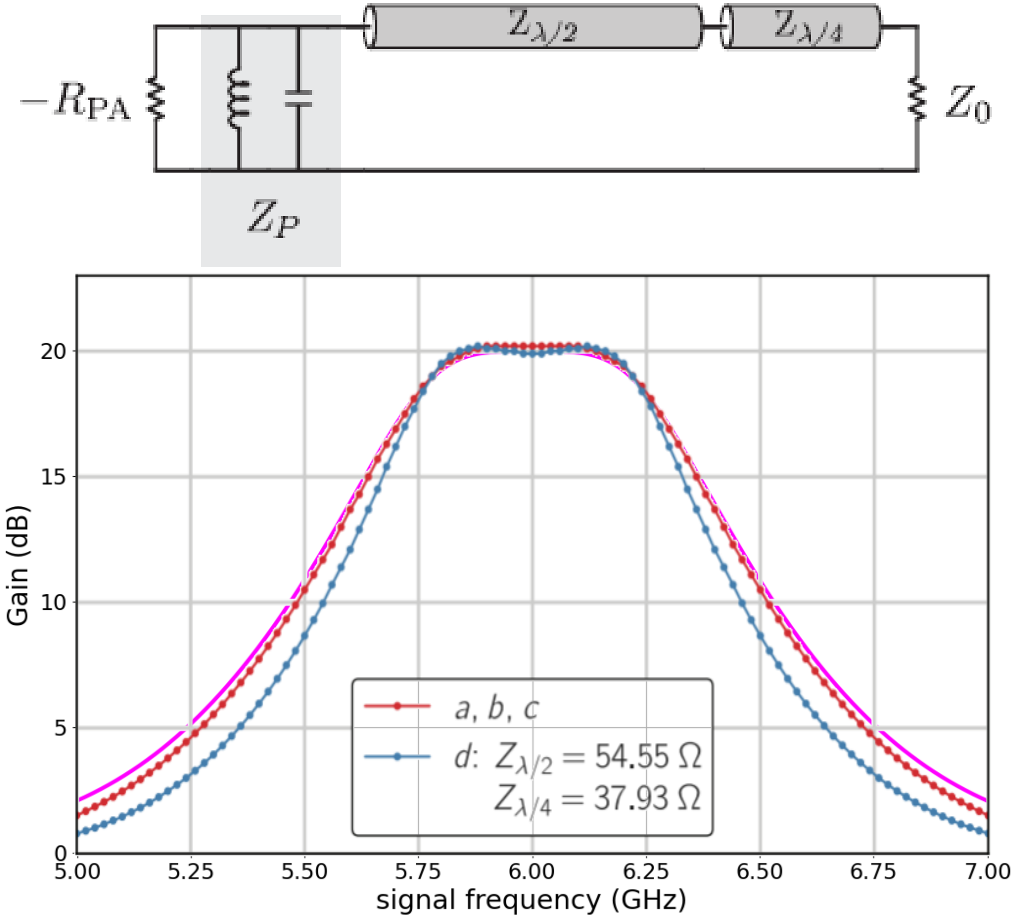


Fig. Reproduce results of IMPA using network synthesis method

O Naaman, J Aumentado, "Synthesis of Parametrically Coupled Networks," ***PRX Quantum***, vol. 3, p. 020201, 2022.
T Roy, *et al*., "Broadband parametric amplification with impedance engineering: Beyond the gain-bandwidth product," ***Appl. Phys. Lett***., vol. 107, p. 262601, 2014.

Analysis of Parametrically-Pumped Josephson Junction: (2) Coupled-Mode Network

- Example Study Using the Coupled-Mode Network Method: Impedance-Matched Parametric Amplifier
  - Step-by-step analysis:

(1) Draw the network graph

(2) Calculate network prototype coefficients

(3) Assign coupling rates

(4) Write down the equation-of-motion matrix

(5) Calculate the S-parameters

(6) Determine parametric coupler implementation

→ SQUID, SNAIL, RF-SQUID array,…etc.

(7) Determine resonator implementation

→ CPW, lumped-element, microstrip,…etc.

(8) Calculate the immittance inverters

(9) Calculate component values

(10) Adjust the final circuit layout

See O Naaman, *PRX Quantum*, **3**, 020201 (2022) for further details on the analysis method

Table. Chebyshev coefficients calculated for an amplifier prototype.

Signal gain G (dB)	Ripple R (dB)	Order	g_0	g_1	g_2	g_3	g_4	g_5
17	0.1	2	1.0	0.2769	0.1451	1.1528		
		3	1.0	0.5595	0.5410	0.3098	0.8674	
		4	1.0	0.7489	0.8450	0.9181	0.2767	1.1528
	0.5	2	1.0	0.3981	0.2206	1.1528		
		3	1.0	0.7062	0.7029	0.4326	0.8674	
		4	1.0	0.8533	0.9943	1.1289	0.3667	1.1528
	1.0	2	1.0	0.4567	0.2642	1.1527		
		3	1.0	0.7822	0.7854	0.5095	0.8674	
		4	1.0	0.8892	1.0592	1.2252	0.4182	1.1527
20	0.1	2	1.0	0.2204	0.1310	1.1055		
		3	1.0	0.4656	0.5126	0.2707	0.9045	
		4	1.0	0.6370	0.8200	0.8243	0.2683	1.1055
	0.5	2	1.0	0.3184	0.1982	1.1055		
		3	1.0	0.5899	0.6681	0.3753	0.9045	
		4	1.0	0.7296	0.9671	1.0147	0.3525	1.1055
	1.0	2	1.0	0.3666	0.2366	1.1055		
		3	1.0	0.6545	0.7488	0.4397	0.9045	
		4	1.0	0.7629	1.0310	1.1032	0.3999	1.1055
25	0.1	2	1.0	0.1546	0.1069	1.0579		
		3	1.0	0.3520	0.4541	0.2214	0.9453	
		4	1.0	0.4997	0.7559	0.7044	0.2485	1.0579
	0.5	2	1.0	0.2246	0.1608	1.0579		
		3	1.0	0.4487	0.5939	0.3039	0.9453	
		4	1.0	0.5768	0.8950	0.8679	0.3226	1.0579
	1.0	2	1.0	0.2599	0.1912	1.0579		
		3	1.0	0.4992	0.6682	0.3537	0.9453	
		4	1.0	0.6060	0.9554	0.9450	0.3632	1.0579
30	0.1	2	1.0	0.1107	0.0849	1.0321		
		3	1.0	0.2722	0.3920	0.1839	0.9689	
		4	1.0	0.4013	0.6794	0.6112	0.2256	1.0321
	0.5	2	1.0	0.1615	0.1272	1.0321		
		3	1.0	0.3488	0.5139	0.2506	0.9689	
		4	1.0	0.4663	0.8071	0.7530	0.2899	1.0321
	1.0	2	1.0	0.1875	0.1507	1.0321		
		3	1.0	0.3892	0.5796	0.2901	0.9689	
		4	1.0	0.4918	0.8630	0.8204	0.3245	1.0321

Analysis of Parametrically-Pumped Josephson Junction: (3) Harmonic Balance

- Harmonic Balance Method for Nonlinear Josephson Junction Modeling
  - Frequency domain method for calculating the steady state of the nonlinear Josephson elements

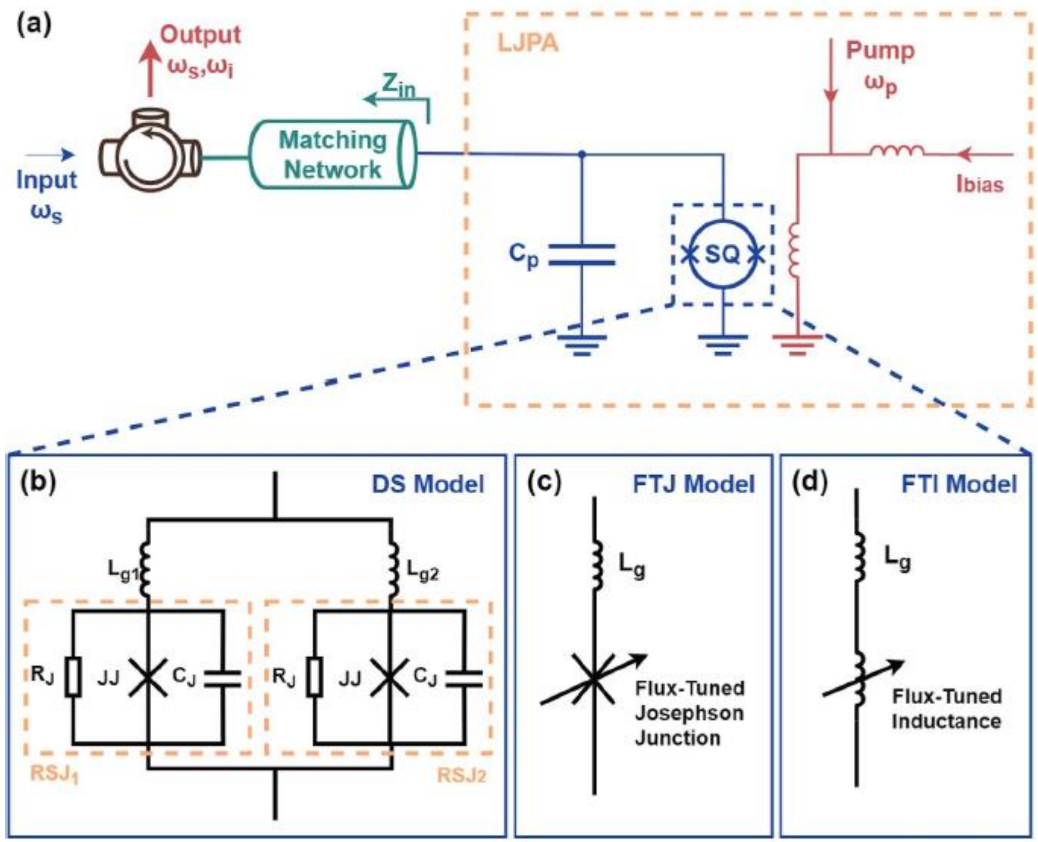


Fig. Circuit diagram for Josephson circuits.

- Numerical modeling of Josephson elements in harmonic balance

$$I = I_c \sin\varphi$$

$$R_J = \begin{cases} R_L & \text{for } |V| < V_g \\ R_N & \text{for } |V| > V_g \end{cases}$$

- Equation of motion for Josephson elements

$$\frac{d^2\varphi_s(t)}{dt^2} + \kappa_0 \frac{d\varphi_s(t)}{dt} + \frac{2I_c}{C_p\varphi_0}\cos\left[\frac{\pi\Phi_{\text{ext}}(t)}{\Phi_0}\right]\sin[\varphi_s(t)] = \frac{I_s(t)}{C_p\varphi_0}$$

where $\varphi_s = (\varphi_1 - \varphi_2)/2$, $\varphi_0 = \Phi_0/(2\pi)$, $\kappa_0 = 1/C_p Z_{\text{in}}$

- Available software tools: Keysight ADS, WRSPICE,…
- Available numerical tools: JosephsonCircuits (Julia),…

K Peng, *et al.*, “X-parameter based design and simulation of Josephson traveling-wave parametric amplifiers for quantum computing applications,” *IEEE QCE*, 2022.
K He, *et al.*, “Simulation of a flux-pumped Josephson parametric amplifier with a detailed SQUID model using the harmonic balance method,” *Supercond. Sci. Technol.*, **36**, 045010 (2023).

Analysis of Parametrically-Pumped Josephson Junction: (3) Harmonic Balance

- Example Study Using the Harmonic Balance Method: Impedance-Matched Parametric Amplifier
  - Step-by-step analysis:

(1) Draw the electrical circuit diagram

(2) Define the simulation setup for harmonic balance

- Higher order
- Time step
- Convergence criteria

(3) Simulate the circuit as a function of DC bias

- Find the resonance of the circuit

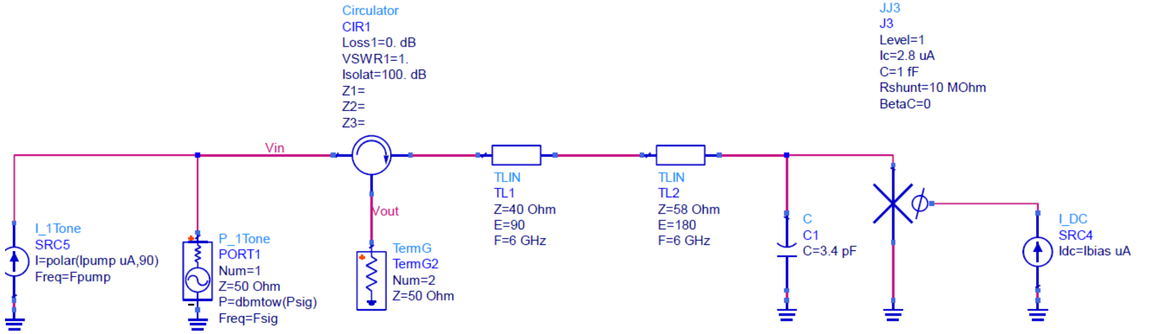
(4) Simulate the circuit as a function of pump bias

- Find the maximum gain profile of the circuit

(5) Simulate the circuit as a function of input signal power

- Find the input signal saturation power (gain 1 dB compression)

See JosephsonCircuits (Julia package)
for further details on the open-source project



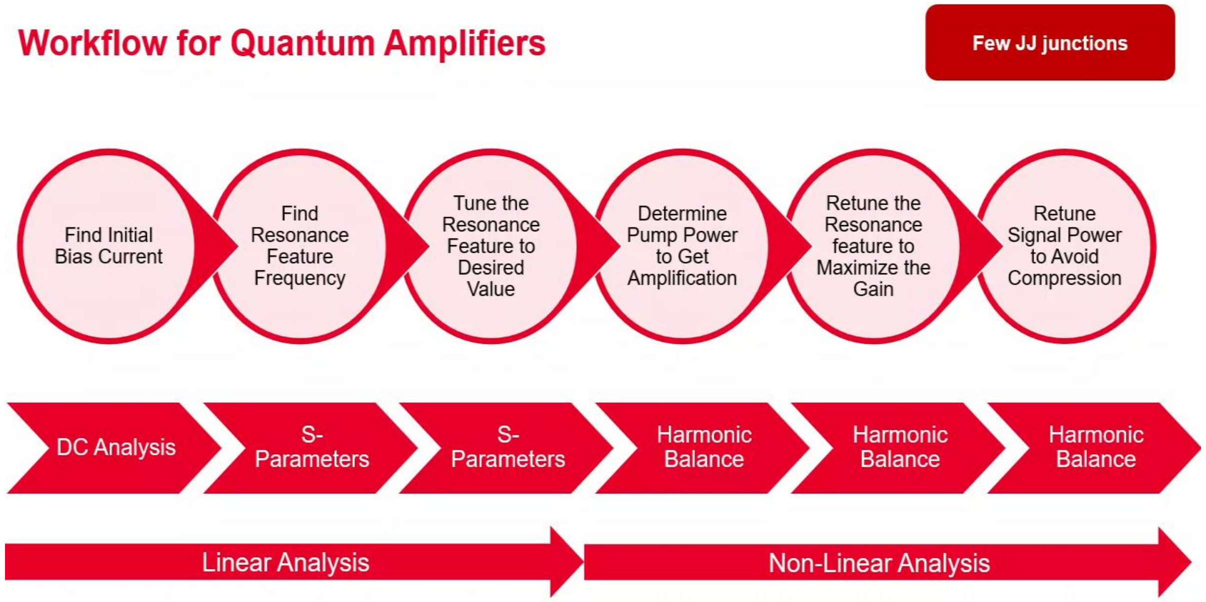


Fig. Example simulation setup in Keysight ADS 2024 program to simulate an IMPA

T Roy, *et al*., "Broadband parametric amplification with impedance engineering: Beyond the gain-bandwidth product," ***Appl. Phys. Lett***., vol. 107, p. 262601, 2014.

Introduction to Traveling Wave Parametric Amplifiers

- Superconducting Traveling Wave Parametric Amplifiers Using Josephson Junctions
  - Josephson Junction-based Travelling Wave Parametric Amplifier (JJ-TWPA)
  - JJ-TWPA ➔ thousands of Josephson junction included unit cell to achieve nonlinear transmission line
  - One unit cell usually consists of: (1) Josephson junction, (2) capacitor, (3) lumped element resonator

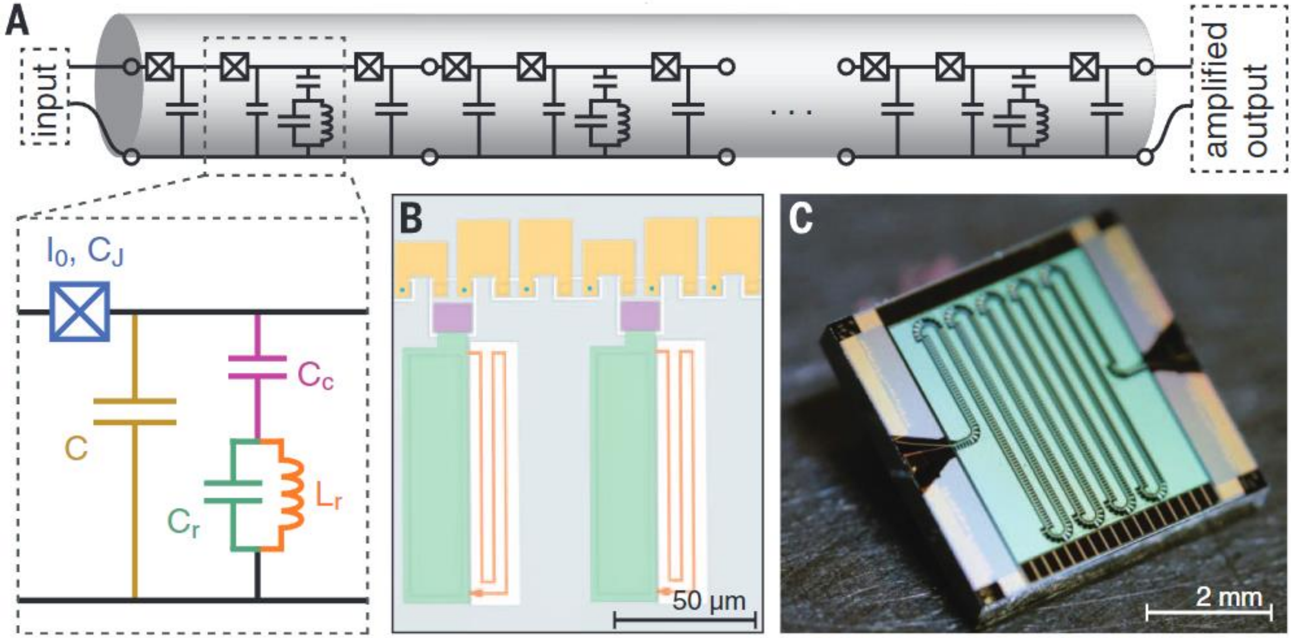


Figure. (a) Circuit diagram of TWPA. One unit cell consists of a Josephson junction, shunted capacitance, and capacitively coupled resonator. Optical images of (b) one unit cell and (c) TWPA.

- JJ-TWPA has broad gain bandwidth over 1 GHz
- JJ-TWPA shows huge dips where phase mismatches

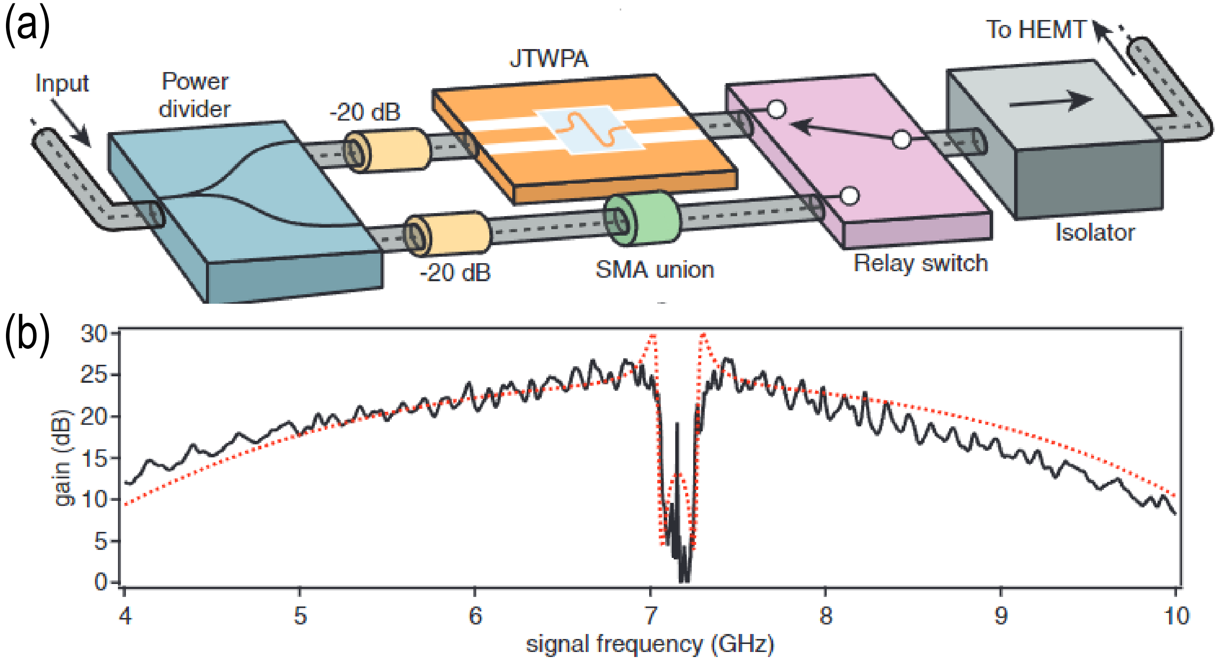


Figure. (a) Cryogenic TWPA measurement setup to extract TWPA's characteristic values. (b) Measured gain profile vs signal frequency.

C Macklin, *et al*., "A near–quantum-limited Josephson traveling-wave parametric amplifier," ***Science***., vol. 350, no. 6258, 2015.

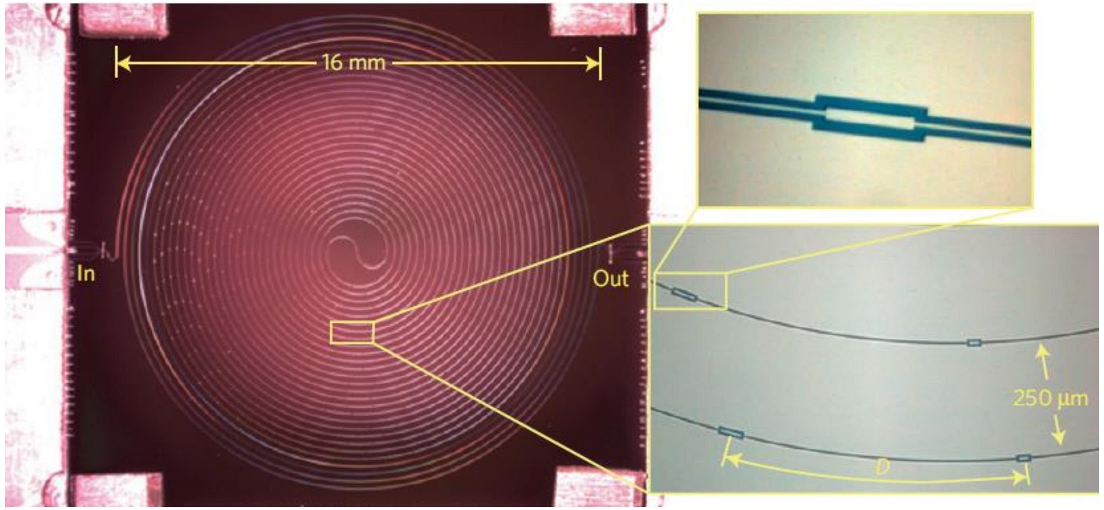
Introduction to Traveling Wave Parametric Amplifiers

- Superconducting Traveling Wave Parametric Amplifiers Using Kinetic Inductance
  - KI-TWPA ➔ large kinetic inductance exhibiting superconducting film to achieve nonlinear transmission line
  - Unlike JPAs, KI-TWPAs do not use Josephson junctions!
  - KI-TWPA is usually implemented with ultra-thin NbTiN or TiN to have large kinetic inductance
  - KI-TWPA has large dynamic ranges up to ~ 1 THz regime
  - KI-TWPA usually has larger noise figure (~ 3 or 4 photons)

NOTE: JJ-TWPA usually exhibits quantum-limited noise (0.5 photon)

  - KI-TWPA usually requires phase-matching technique to design periodically-loaded circuit layout
  - KI-TWPA usually exhibits larger ripples due to the complexity in engineering the kinetic inductance of thin superconducting films

See BH Eom *et al.*, *Nature Phys.*, **8**, 623-627 (2012) for further details on KI-TWPA design and results



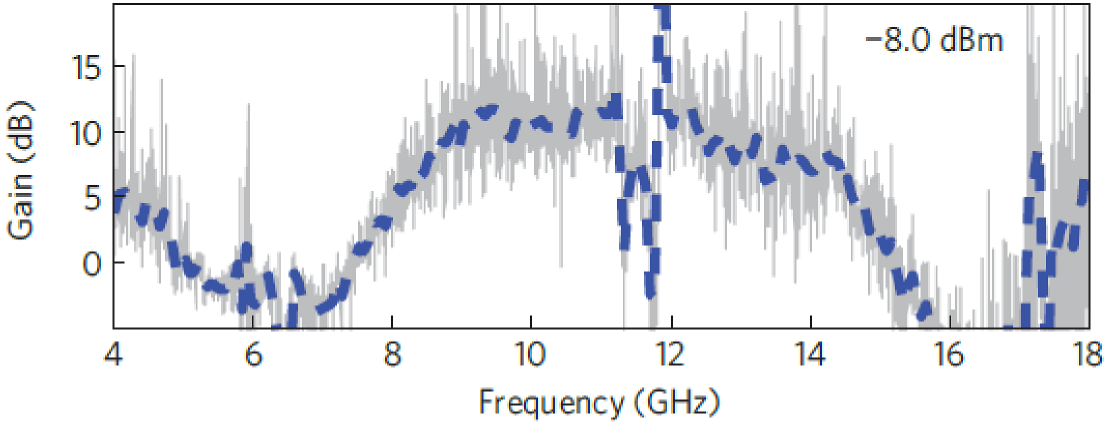


Figure. A picture of the amplifier which consists of a 0.8m length of NbTiN CPW line. Measured gain of the amplifier. The large peak at 11.9 GHz is arising from the shift to lower frequency of the transmission dip produced by the periodic loading.

BH Eom *et al.*," A wideband, low-noise superconducting amplifier with high dynamic range," *Nat. Phys.*, **8**, 623-627 (2012)

See Also…

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[CPW-JPA] G Choi *et al., IEEE Trans. Appl. Supercond.,* **33**, 1701504 (2023)

[IMPA] J Grebel *et al*., *Appl. Phys. Lett.* **118**, 142601 (2021)

[IMPA] R kaufman *et al., Phys. Rev. Appl.,* **20**, 054058 (2023)

Traveling Wave Parametric Amplifiers:

[JJ-TWPA] C Macklin *et al*., *Science*, **350**, 307-310 (2015)

[KI-TWPA] BH Eom *et al., Nature. Phys.,* **8**, 623-627 (2012)

[KI-TWPA] C Bockstiegel *et al., J. Low Temp. Phys.,* **176**, 476-482 (2014)

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[1] KM Sundqvist *et al*., "Negative-resistance models for parametrically flux-pumped superconducting quantum interference devices", *EPJ Quantum Technol*., **1**, 6 (2014).

[2] JY Mutus *et al*., "Strong environmental coupling in a Josephson parametric amplifier", *Appl. Phys. Lett*., **104**, 263513 (2014)

Coupled-Mode Network Method:

[1] L Ranzani, J Aumentado, "Graph-based analysis of nonreciprocity in coupled mode system," *New J. Phys*., **17**, 023024 (2015).

[2] O Naaman, J Aumentado, "Synthesis of Parametrically Coupled Networks," *PRX Quantum*, **3**, 020201 (2022).

Harmonic Balance Method:

[1] K Peng *et al*., "X-parameter based design and simulation of Josephson traveling-wave parametric amplifiers for quantum computing applications," *IEEE QCE 2022*

[2] D Shiri *et al*., "Modeling and Harmonic Balance Analysis of Superconducting Parametric Amplifiers for Qubit Readout: A Tutorial," *IEEE Micorw. Mag*., **25**, 54-73 (2024).