Formalisation of the Dual Calculus in Agda

Computer Science Tripos – Part II – Project Progress Report

Edward While, Fitzwilliam College

October 2020

Project Supervisor: Dmitrij Szamozvancev

Director of Studies: Dr. Robert Harle

Project Overseers: Prof. Jon Crowcroft & Prof. Neil Lawrence

As of the time of writing my project is currently 3 weeks ahead of schedule with all of the core work (minus evaluation) completed and two of the extensions in progress.

I spent the early weeks of the project researching around the problem area and then learning how to program in Agda, a language I had no prior experience in. I have formalised the Dual Calculus in Agda as well as the dual translation of terms in the calculus, and this translation has been proved to be an involution alongside the property that a term is derivable if and only if its dual is derivable. I have developed both a Call-by-Name and Call-by-Value CPS transformation of Dual Calculus terms to Agda programs and completed a proof that demonstrates that these transformations are the duals of each other; thus showing the duality of Call-by-Name and Call-by-Value evaluation strategies.

Of the possible extensions that I laid out in my proposal I have defined an operational semantics for the Dual Calculus, however one property remains to be proved for it to be entirely completed as planned. The proof of the soundness of these operational semantics is in progress and proving to be a good challenge.

The further work I plan to complete is to finish off the above-mentioned extension and then spend any remaining time before 15 February on the final possible extension I listed: demonstrating an equational correspondence between the Dual Calculus and the $\lambda\mu$-calculus. After this I intend to gather evaluation data by comparing the execution times of Agda programs that have been produced by the Call-by-Name and Call-by-Value CPS transformations of some Dual Calculus terms that encode known classical theorems.

The main unexpected difficulties that has arisen was the explosion in complexity of the proofs of various properties of substitution that comes from the Dual Calculus being dual-context and triply mutually recursive resulting in slow and difficult progress on the first two extensions. This was compounded with the issue of trying to stay motivated through the Winter in what's been a testing period for my mental health.