Exercise 1

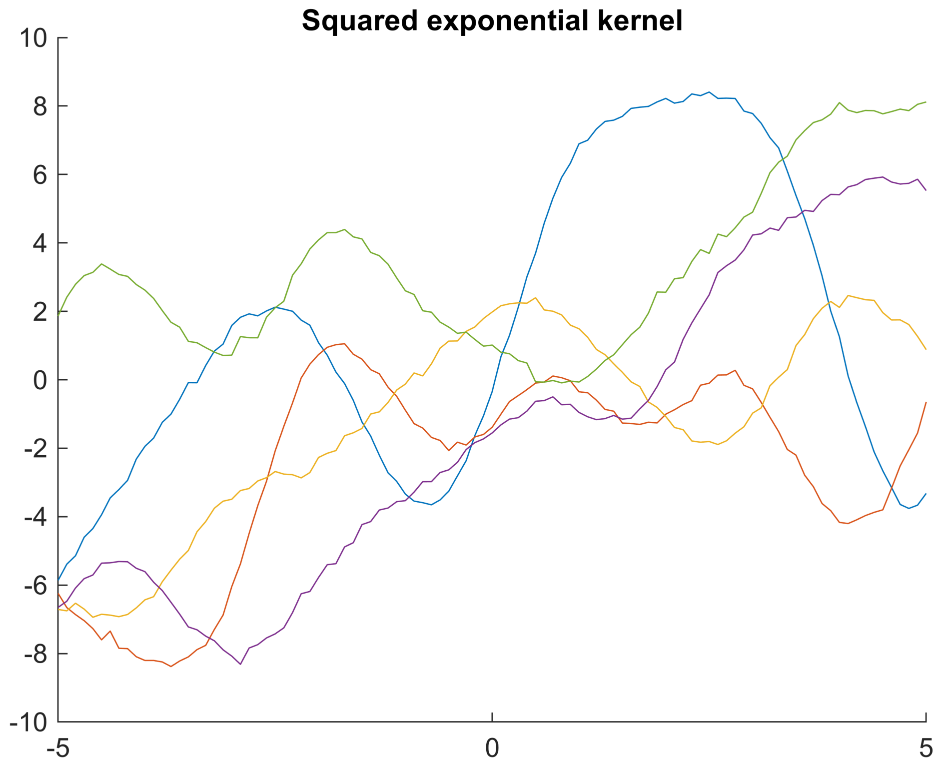
a) Sample from gaussian process.

The code for getting the relevant samples is at the end of the matlab livescript, due to how matlab processes functions I have to order stuff like this.

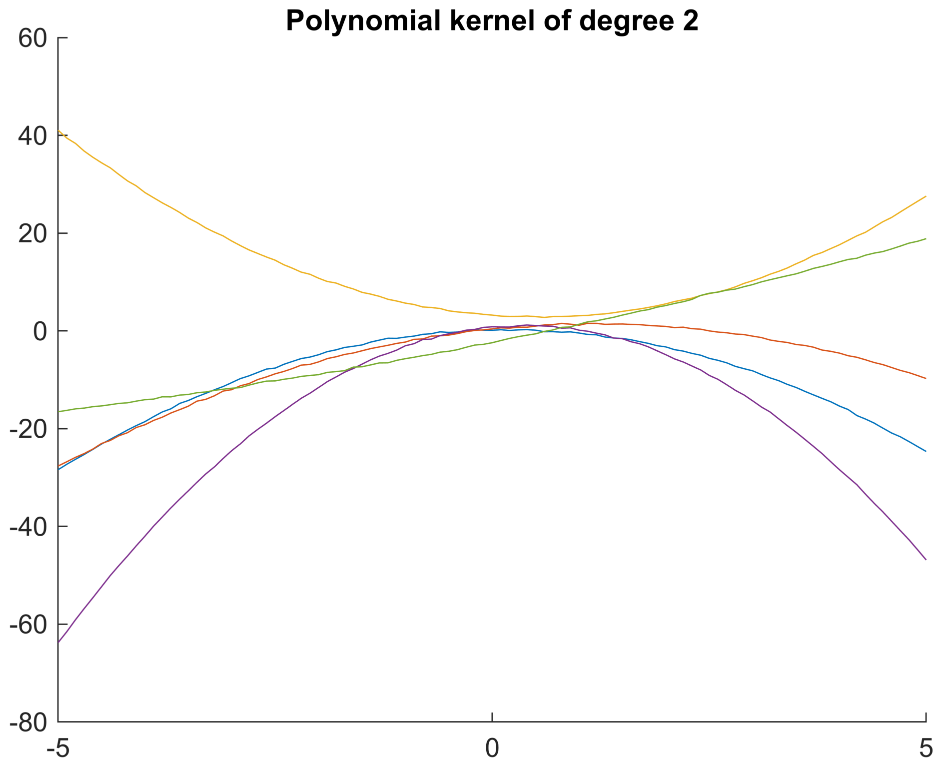
b) Use a set of kernels, draw 5 realizations from them, and then plot them.

There is no strict instructions on the number of features to use, but because we have to plot them I feel that anything more than 2d is hard to visualize. So I will assume that 2 feature vectors is good enough.

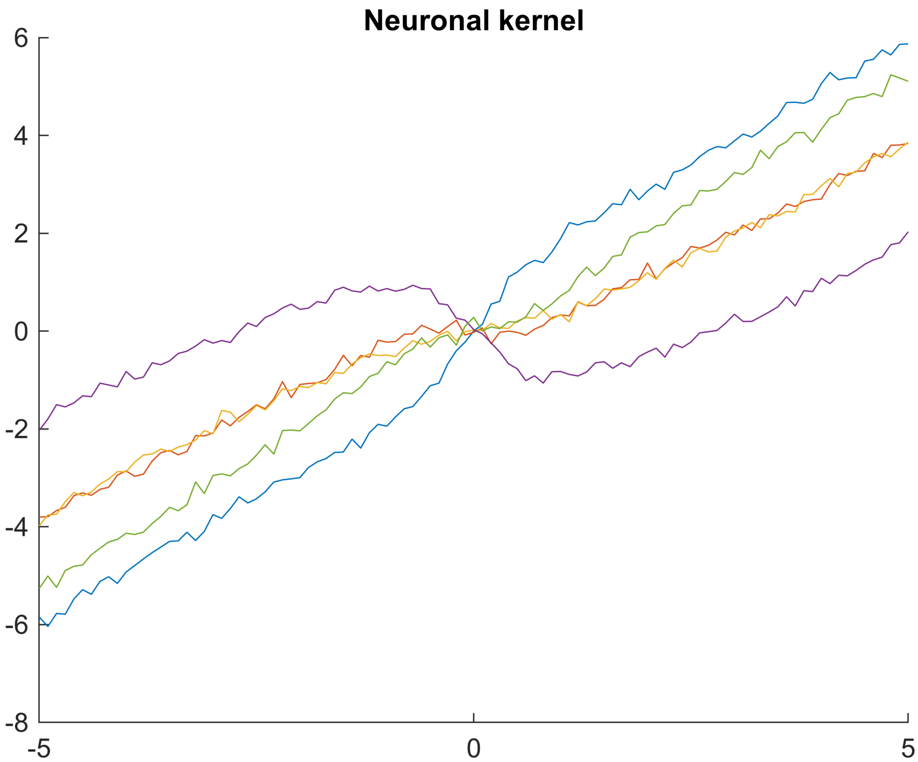
```
% For the square exponential kernel, I need two parameters, the l and the
% variance. For the lengthscale I'll assume 1 is good enough, for the
% variance I'll just the variance formula of the uniform

x_range = [-5 5];
variance = ((x_range(2)-x_range(1)).^2)/12;

% I assumed that the lengthscale is just a 1
% 1 squared is 1, so the denominator is just a division by 2
squared_exp_kernel = @(x, y) variance*exp(-((x-y)^2)/2);
mean_funct = @(x) mean(x, 1);
features = 1;
x_vals = -5:0.1: 5;

clf
hold on
title('Squared exponential kernel')
for i=1:5
    squared_samples = gaussian_process_nice(mean_funct, squared_exp_kernel, x_vals);
    plot(x_vals, squared_samples)
end
hold off
```




```
% For the polynomial kernel of degree 2, I'll assume the alpha=1 is good enough
poly_2_kernel = @(x,y) (x.'*y + 1)^2;

clf
hold on
title('Polynomial kernel of degree 2')
for i=1:5
    poly_samples = gaussian_process_nice(mean_funct, poly_2_kernel, x_vals);
    plot(x_vals, poly_samples)
end
hold off
```



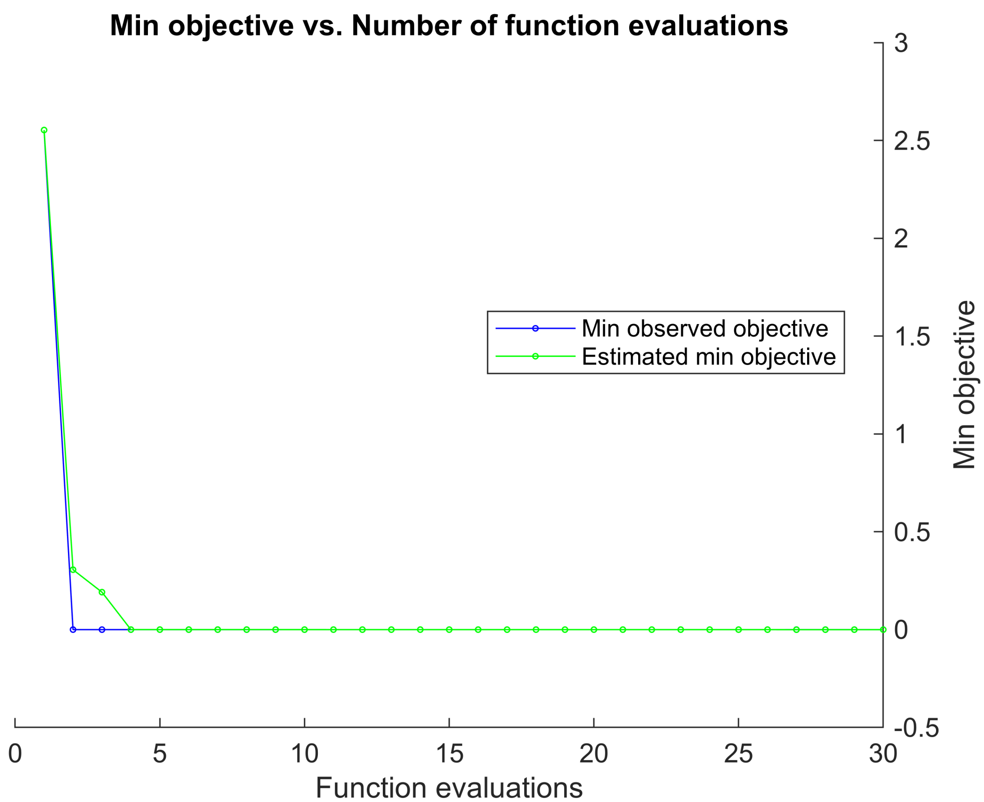

```
% For the neuronal kernel, I need a helper function a
a_func = @(x, y) 2*y.'*sum(x);
neuronal_kern = @(x, y) (2/pi)*asin(a_func(x,y)/(sqrt((1+a_func(x,x))*(1+a_func(y,y)))));

clf
hold on
title('Neuronal kernel')
for i=1:5
    neur_samples = gaussian_process_nice(mean_funct, neuronal_kern, x_vals);
    plot(x_vals, neur_samples)
end
hold off
```



Exercise 2

```
data_x = [-1; -2; 1; 2];
data_y = [1; 4; 1; 4];

% So... there's a way of optimizing the hyper parameters!
% I'm optimizing all of the available values, except for the kernel
% function. Because that was defined in advance
% It already does bayesian optimization, so that is finished.
% Here is the documentation: https://nl.mathworks.com/help/stats/fitrgp.html#butnn96_sep_shared
gprModel = fitrgp(data_x, data_y, 'KernelFunction','squaredexponential', ...
    'OptimizeHyperparameters', {'BasisFunction','KernelScale','Sigma','Standardize'}, ...
    'HyperparameterOptimizationOptions',struct('Verbose',0));
```

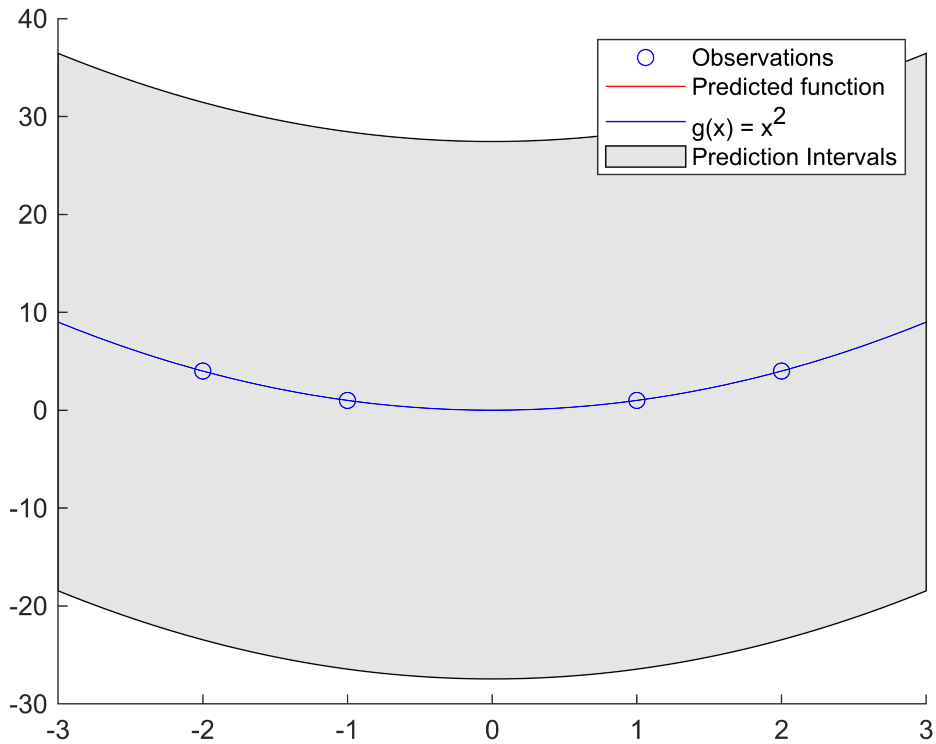
```
Warning: The number of folds K is greater than the number of observations N. K will be set to the value of N.
```




```
% First I'll display the predicted value at 0
pred_at_0 = predict(gprModel, 0)
```

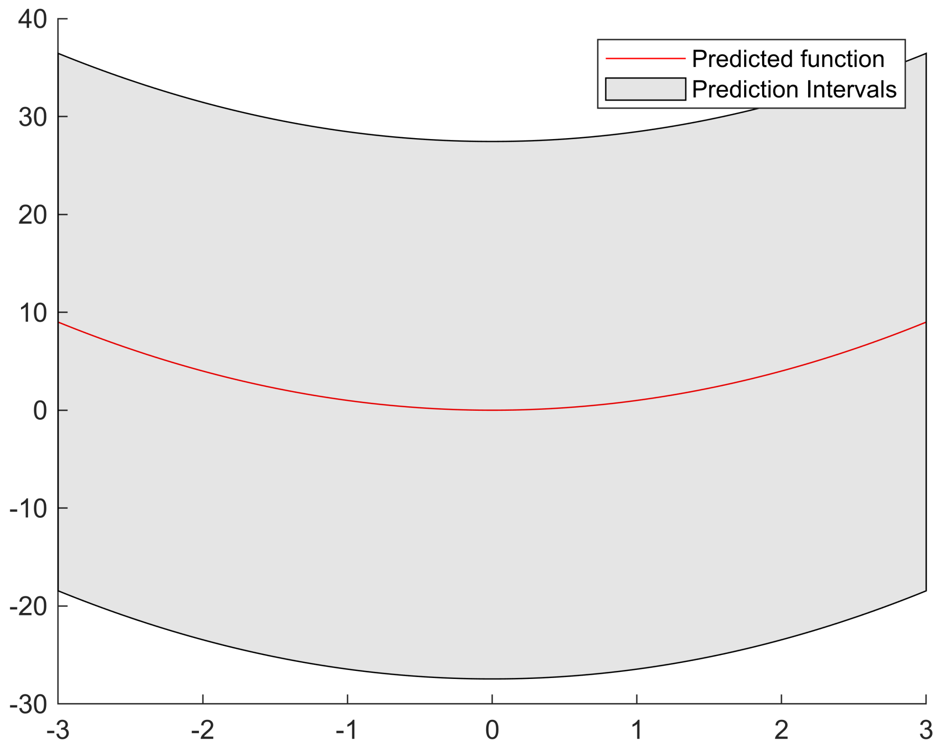
```
pred_at_0 = -6.6570e-16
```

```
x = linspace(-3,3)';
[ypred,~,yint] = predict(gprModel, x);

clf
hold on
% Plotting the observations as well
scatter(data_x,data_y,'b')
plot(x,ypred,'r')
% Plotting reality for comparison, because we know it's x^2
fplot(@(x) x.^2,[-3, 3],'b')
patch([x;flipud(x)],[yint(:,1);flipud(yint(:,2))],'k','FaceAlpha',0.1); % Prediction intervals
legend('Observations', 'Predicted function','g(x) = x^2', 'Prediction Intervals')
hold off
```



So... all of the plots end up overlaying pretty much over each other perfectly. I'll just plot the predicted function and the interval separately again. Even though it's just a straight line... again

```
clf
hold on
% Plotting the observations as well
plot(x,ypred,'r')
% Plotting reality for comparison, because we know it's x^2
patch([x;flipud(x)],[yint(:,1);flipud(yint(:,2))],'k','FaceAlpha',0.1); % Prediction intervals
legend('Predicted function', 'Prediction Intervals')
hold off
```




```
function samples = gaussian_process_nice(mean_funct, kernel_func, x_vals)
    % Method to get multiple samples with a given mean and kernel function
    % x_range is drawn according to a uniform distribution, should contain
    % the minimum and the maximum

    % Assumes X is a vector input

    % Mvnrnd does pretty much most of what I need
    % I just need to generate the relevant values that will be passed to
    % the kernel and mean functions, then pass them to mvnrnd and donezo

    mu = mean_funct(x_vals)';

    % For the kernel_funct... I'll just do 2 for loops for each pairwise
    % element
    sigma = zeros(length(x_vals));

    for i=1:length(x_vals)
        for j=1:length(x_vals)
            sigma(i, j) = kernel_func(x_vals(i), x_vals(j));
        end
    end

    % It's not technically part of the process, but as recommended in b
    % I'll increase sigma (K) with a slight value of the identity matrix,
```

```
    % to make things flow better
    sigma = sigma + 0.01*eye(length(x_vals));

    samples = mvnrnd(mu, sigma);
end
```