Decision Making of Dynamic Airlines System under Epidemic Environment

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Abstract

Inspired by the fusing command from civil aviation, the decision making of dynamic airline system takes a more essential role in current situation. Firstly, we propose a new decision making model on airline capacity and formulate a constrained optimization problem. After the problem is mathematically simplified into linear constraints, we apply the Differential Evolution Algorithm to obtain the optimal decisions. The empirical experiments shows the necessity to greatly reduce the capacity of airline from the cities with severe outbreaks, which demonstrates excellent potential in broader application[1].

1 Introduction

1.1 Choice of Project

An original network economics research

1.2 Project Scenarios

For the year 2020, people are still suffering from the COVID-19 pandemic. It brings severe challenge to the operation of traditional service industry. The aviation industry is one of those most impacted under the pandemic, since protective measures against infection could set tight restriction to the capacity of the transportation network. For example, governments around the world have prohibited cross-country transportation and the market share of the airline business has shrunk since then. Some methodologies like an event-driven approach have been implemented to quantitatively analyze the economic influence, especially after three major COVID-19 announcements were made by World Health Organization (WHO) [1]. Other studies focus on scenarios variation after post-COVID-19 and how airlines react and recover from the pandemic in the aspect of world airline network (WAN) [2]. Aviation network features like temporal characteristics can also influence the infection rate [3]. Among the countries that waged arduous struggles to the pandemic, China is one of those achieved early recovery. The "Five One" policy adopted by the Civil Aviation Administration of China (CAAC)[2] to prevent imported cases could play a big role. It allows mainland carriers to fly just one flight a week on one route to any other country and foreign airlines to operate just one flight a week to China. From the perspective of airline markets, some foreign airlines also make strategic response to the pandemic crisis and outline key implications for post-COVID-19 competitive landscape, to raise attention and provide recommendations for policy makers [4] [5]. Inspired by such fusing measures and response behavior, we further expand the decision making process into an air traffic control problem. Here we try to provide similar policies for cities in the transportation network, considering both the infection rate and passengers' utility.

1.3 Goals

To achieve our goals, we try to answer the following questions:

- How should we model the aviation network under the pandemic?
- What's the best strategy for each city/for the global network?
- Is there any penalty of anarchy and how do we describe it?

These are novel questions to solve. The solution to the first question is the foundation for our further discussion, and we try to carry out exploratory experiments seeking the solution to the second question. The third question is hard to solve and requires further distribution in such area. The main contributions of this project are the following:

[1]Codes: `github.com/WhiskyChoy/NwEcoPrj`

[2]Official website: `www.caac.gov.cn/en`

- We propose a new decision making model on airline capacity considering both the passengers' demand and the infection severity of different cities.
- We mathematically simplify our model and make it solvable under linear constraint in *IBM CPLEX* solver. We also apply the Differential Evolution Algorithm using the *GeatPy*[6] toolkit to solve our problem and find better result.
- Empirical experiments have been done to show the necessity to ban the airlines from cities with high infection severity and the difficulty to strike a balance between the demand of passengers and public health.

The paper is organized as follows. Section 2 reviews the related work. Section 3 introduces our models, while Section 4 presents our algorithms. Section 5 reports the empirical experiments and results, followed by the discussion and conclusions in Section 6. Lastly, Section 7 briefly introduces the contributions by each group member.

2 Related Work

Recently, researchers have done broad and in-depth studies focusing on how COVID-19 affected our daily life from the perspective of economic, culture and social behaviors. Treating the process of COVID-19 as a susceptible-infected model, Jia Wangping[7] has presented a study in which, COVID-19 data from Jan 22, 2020, to Mar 16, 2020, has been used in time series form for spreading analysis. In the aspects of public transportation, a variety of effctive measures have been successfully implemented to control the COVID-19 [8, 9]. In our paper, decision making of dynamic airline rearrangement is actually a network optimization problem. Shangyao Yan [10] develops an integer multiple commodity network model and a solution algorithm to help carriers simultaneously solve for better fleet routes and appropriate timetables. Some heuristic algorithms such as genetic algorithm [11] and dynamic programming [12] have also beem applied to solve airline network and fleet planning problems. Based on these previous studies, we would like to use a model to solve the network flow problem of airline system for dynamic rearrangement, which might control the epidemic and bring less economic losses.

- **Network flow problem** Network flow is a network that satisfies the following properties. Each edge has a maximum capacity C^{max}, which is the maximum flow that the edge can accommodate. C is the actual traffic flowing through the edge, and there is always C less or equal to C^{max}.
- **Minimum cost maximum flow problem** The minimum cost maximum flow problem is a typical problem in economics and management. Each path in a network is limited by cost and capacity. These kind of research problems are mainly want to find out how to select the path and allocate the traffic passing through the path from a to B can achieve the minimum cost requirement.
- **Osmosis model** Osmosis can be described by a phenonomenon where the spontaneous net of solvent molecules moves through a selectively permeable membrane into a region of higher concentration, in the direction that tends to equalize the two-sided solute concentration. It can also demonstrate a physical process in which any solvent moves across a selectively permeable membrane. In this way, the infection rate between two selective cities can be defined as different levels of solute concentration while flight movements will be regarded as the process of solvent molecules osmosis.

3 Proposed Model

Let us focus on the simplest situation. Considering a group of independent cities, we can draw a simple graph to describe the connectivity between cities, which means the direction and capacity of airlines.

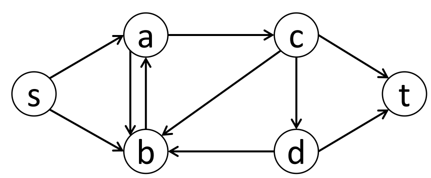


Figure 1: The airline graph of a city group

To further illustrate how these methods applied with empirical data, a general setting of variables and parameters is given as follows, where i, j respectively represents different city or area:

- c_{ij}: the capacity of each airline from city i to j

  $c_{ij} \triangleq f_{ij} \cdot p_{ij}$ (f_{ij}: average daily flights from city i to j; p_{ij}: average passenger capacity per flight. Note that p_{ij} is hard to attained, and in simplified cases it's set as 1)
- d_{ij}: the weighted value for each airline which depends on the related demand and importance

  $d_{ij} \triangleq \frac{t_i}{\sum_k t_k} + \frac{g_j}{\sum_k g_k} \neq d_{ji}$ (t_i: total annual passenger flow of the whole airport city i; g_i: annual GDP of city i)
- u_{ij}: the utility of each airline $u_{ij} \triangleq c_{ij} \cdot d_{ij}$
- S_i: the severity of the epidemic in city i

$S_i \triangleq I_i - E_i$ (I_i: average number of infections in city i per day; E_i: average number of patients cured in city i per day)

- r_{ij}: the rate of infection (a softmax function) $r_{ij} \triangleq e^{S_i \cdot c_{ij}/T}/(e^{S_i \cdot c_{ij}/T} + e^{S_j \cdot c_{ji}/T})$ (T: hyper-parameter)
- ϵ: the threshold of infection rate

Specifically, for those big cities like Beijing and Shanghai, the demand and importance of this airline d_{ij} must be higher than most of cities. For different airlines, if we need to reduce the same amount for capacity c_{ij}, the related utility must decrease more for those airlines with higher r_{ij}. Thus, $u_{ij} = c_{ij} \cdot d_{ij}$ makes sense intuitively.

Besides, the definition of r_{ij} can also be explained intuitively. r_{ij} is regarded as the risk of epidemic transmission, which is related to S_i, S_j (the severity of the epidemic situation in the city i, j) and c_{ij}, c_{ji} (the capacity of airline $i \leftrightarrow j$). Hence, $r_{ij} = f(S_i, S_j, c_{ij}, c_{ji})$, which can be directly transformed into $r_{ij} = S_i \cdot c_{ij}/(S_i \cdot c_{ij} + S_j \cdot c_{ji})$.

To polarize the effect of $S_i \cdot c_{ij}$ and ensure the sign of r_{ij} is positive (considering S_i could be negative if $I_i < E_i$), we apply a softmax function $r_{ij} \triangleq e^{S_i \cdot c_{ij}/T}/(e^{S_i \cdot c_{ij}/T} + e^{S_j \cdot c_{ji}/T})$, which is the definition of r_{ij}. Note that the T here is to adjust the polarization effect. With larger T we will have smaller polarization effect.

To derive the specific form of these variables, we need to selectively choose some datasets $\{f_{ij}, p_{ij}, I_i, E_i, t_i, g_i\}$ in real world as the basis of quantitative variables $\{c_{ij}, d_{ij}, r_{ij}\}$.

Thus, it is natural to formulate a constrained maximization optimization problem:

$$\begin{aligned} \max_{c_{ij}} \; & U = \sum_{i,j} u_{ij} = \sum_{i,j} c_{ij} d_{ij} \\ \text{subject to } \; & c_{ij} \in [0, c_{ij}^{max}], \;\; \forall i \neq j \\ & r_{ij} \leq \epsilon, \;\; \forall i \neq j \end{aligned}$$

In order to avoid nonlinear constraints, we need to simplify the condition $r_{ij} \leq \epsilon$, and then we obtain the linear constraints for $\{c_{ij}\}$:

$$\begin{aligned} & r_{ij} \leq \epsilon, \;\; \forall i \neq j \\ \Leftrightarrow \;\; & e^{S_i \cdot c_{ij}/T}/(e^{S_i \cdot c_{ij}/T} + e^{S_j \cdot c_{ji}/T}) \leq \epsilon, \;\; \forall i \neq j \\ \Leftrightarrow \;\; & (1-\epsilon) e^{S_i \cdot c_{ij}/T} \leq \epsilon e^{S_j \cdot c_{ji}/T}, \;\; \forall i \neq j \\ \Leftrightarrow \;\; & \ln(1-\epsilon) + \cdot S_i \cdot c_{ij}/T \leq \ln \epsilon + S_j \cdot c_{ji}/T, \;\; \forall i \neq j \\ \Leftrightarrow \;\; & S_i \cdot c_{ij} - S_j \cdot c_{ji} + T \cdot \ln \frac{1-\epsilon}{\epsilon} \leq 0, \;\; \forall i \neq j \end{aligned}$$

Like fusing command by civil aviation, if the city s has a serious epidemic situation, then we need to cut some airlines from the figure 1 such that after cutting, there is no path from s to t (eg. capital). The cost of removing airline $i \rightarrow j$ is equal to its capacity c_{ij}. Thus, the problem is changed into a minimum cut problem to find a cut strategy with minimum total cost.

3.1 Algorithm

To solve this integer optimization problem with linear constraints, we firstly try to use CPLEX solver. Unfortunately, the CPLEX solver has some defects to solve the integer optimization problem and always gives the solution full of 0. Then we apply the Differential Evolution Algorithm using the *GeatPy*[6] toolkit to solve it successfully.

Specifically, Differential Evolution (DE) is a powerful yet simple evolutionary algorithm for optimizing real-valued, multimodal functions. Function parameters are encoded as floating-point variables and mutated with a simple arithmetic operation. During mutation, a variable-length, one-way crossover operation splices perturbed best-so-far parameter values into existing population vectors. As for results, DE's performance could be competitive with other fast and converged methods.

4 Performance Experiments

We will conduct several performance experiments based on our proposed models. The quantitative analysis we may do are as follows:

- To make our simulation results more representative, we will introduce several airline routes on behalf of different city scales. By collecting the number of people infected with COVID-19 and the number of susceptible and recovered people in different cities at the same time periods, we will employ the predefined osmosis model to characterize the spread of epidemics in different cities.
- After obtaining the infection rate, we will determine the appropriate number of flights through the established optimization model, so as to maximize the city's utility while reducing the risk of infection as much as possible. The optimization results of different cities may vary, because the weight of routes and infection rates are different. In the most severe case of epidemic, the optimization result is likely to be a route suspension. We will also propose a sensitivity analysis method to figure out how our experimental results change via different parameter choices.
- Under the above model, we will make a decision of whether the target city will cut flights based on the current epidemic situation. By comparing with the "Five One" policy implemented by CAAC, we will analyze whether our proposed network optimization

model is reasonable, and further put forward how to better formulate relevant policies through the model.

4.1 General Settings

We have chosen nine cities with the highest annual flight movements in 2019 and Wuhan, the pandemic center, to construct our airline network with pairwise combinations, as shown in Table 1. In particular, we only consider Beijing Capital International Airport and Shanghai Pudong International Airport since these two airports occupy more market shares in the corresponding city.

Table 1: The city group for proposed model

1	2	3	4	5
Beijing	Shanghai	Guangzhou	Chengdu	Shenzhen
6	7	8	9	10
Kunming	Xi'an	Chongqing	Hangzhou	Wuhan

As mentioned in section 3.1, we need data from annual passenger flow and annual GDP to represent the weighted value of selected two city-pairs and the average number of infections and cured patients to show the rate of infections. Here we list related data for any given date in table 2. Several other statistic datasets, i.e. the maximum capacity, weighted value, severity of the epidemic and the rate of infection of the corresponding 10×10 city-pairs are calculated in table 3 and table 4, shown in the appendix.

Table 2: Datasets of $\{g_i, t_i, I_i, E_i\}$ (Apr. 16^{th})

City i	g_i / trillion	t_i / million	I_i	E_i
1	3.54	100.2	593	50
2	3.82	76.0	628	496
3	2.36	72.7	499	449
4	1.70	55.3	165	155
5	2.69	52.3	459	429
6	0.65	48.0	53	53
7	0.93	46.9	120	117
8	2.36	44.6	579	576
9	1.54	39.8	181	181
10	1.62	26.8	50008	47283

In order to make our experimental results more reasonable, we have collected the target cities' airline capacities in the past six months from Flightera[3], a website dedicating to provide accurate flight dynamic information for customers, and choose the maximum value as c_{ij}^{max}. As illustrated in Figure 2, we can see that there is a certain symmetry in the maximum number of flights between any two city pairs. In general, cities with more significant values have more access to other cities, as row 1 and column 1 represent the capital city Beijing. Besides, there are no flights on the diagonal since we require departure airport and destination airport are different in our model.

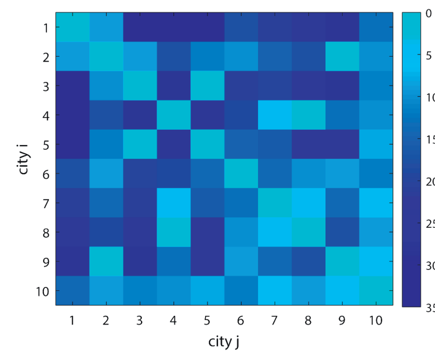
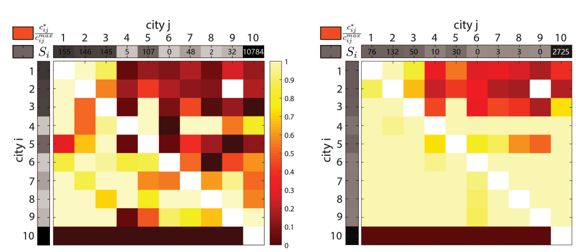


Figure 2: c_{ij}^{max} from targeted city i and city j

4.2 Performance

4.2.1 Relationship between c_{ij}^*/c_{ij}^{max} and S_i

From section 4.1, we have given related required datasets of our model in Apr.16^{th}. As represented in Figure 3, we regard c_{ij}^*/c_{ij}^{max} as our decision variable variation. It's obvious that the more severe the pandemic in any given city becomes, the smallest airline capacity it will have. Note that there exists no flight route between Shanghai and Hangzhou, airline capacity remains zero. Without loss of generality, we consider another date which represents different levels of severity, shown in figure 3 (a). We can easily find out there are obvious difference in some cities less affected by the epidemic.

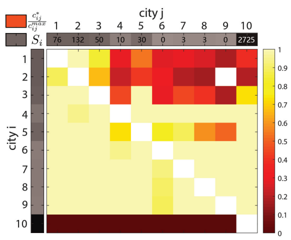


(a) Mar. 27^{th} (b) Apr. 16^{th}

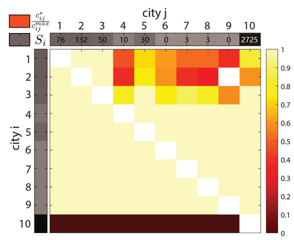
Figure 3: Decision making c_{ij}^*/c_{ij}^{max} of different date with different epidemic severity $\{S_i\}$ ($\epsilon = 0.6, T = 1000$)

[3] https://www.flightera.net/en/

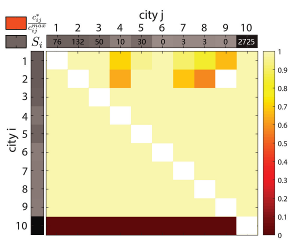
4.2.2 Relationship between c_{ij}^*/c_{ij}^{max} and (ϵ, T)



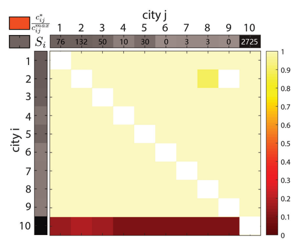
(a) $\epsilon = 0.6, T = 1000$



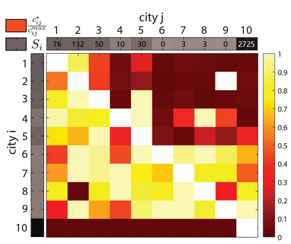
(b) $\epsilon = 0.7, T = 1000$



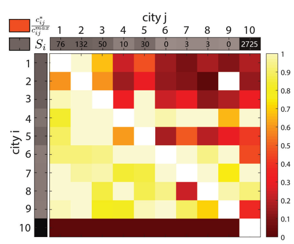
(c) $\epsilon = 0.8, T = 1000$



(d) $\epsilon = 0.9, T = 1000$



(e) $\epsilon = 0.8, T = 100$



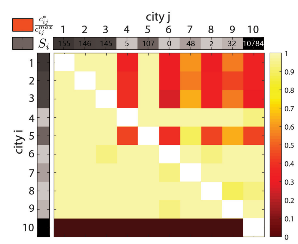
(f) $\epsilon = 0.9, T = 100$

Figure 4: Decision making c_{ij}^*/c_{ij}^{max} using different (ϵ, T) on Apr. 16^{th}

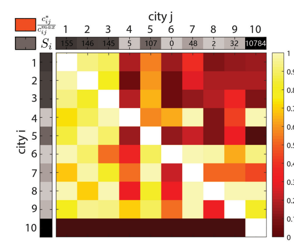
According to our proposed model, one constraint except satisfying the maximum airline capacity needs to be considered more because we aim to reduce the infection rate to an acceptable level. In this way, the threshold of infection rate ϵ and hyper-parameter T should behave well to make our experimental results more practical and reasonable. In figure 4 (a) to (b), we have demonstrated the influence of parameter choices on decision variable variation in depth. In order to control covariates, we still choose Apr.16^{th} as the target experimental date, which represents the same level of severity. It shows that more flights will remain operational for the corresponding city pairs when we set ϵ a higher value. Comparing (a) with (e) in and (b) with (f) in figure 4 , we can conclude similarly that with larger T, smaller number of flights will be suspended because of the smaller effect of polarization in our constraint. Shown as before, flights remains zero in row 2 and column 9 and decision making variable of Wuhan vary small in different parameter choices.

4.2.3 Relationship between c_{ij}^*/c_{ij}^{max} and c_{ij}^{max}

In section 4.2.1, we have discussed thoroughly how the parameters affect our experimental results. But there still remains a question that c_{ij}^{max} in some city pairs has small value, which can impose restrictions on our optimized value to zero. With this regard, we choose two different values of c_{ij}^{max} among our constructed airline network. As presented in figure 5, more proportional number of flights will be cut with higher c_{ij}^{max} under the same severity of pandemic.It may Probably due to the fact that more flights can't be satisfied under this level of severity and result in smaller c_{ij}^*/c_{ij}^{max}.



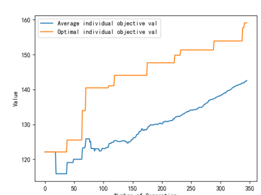
(a) $c_{ij}^{max} = 20$



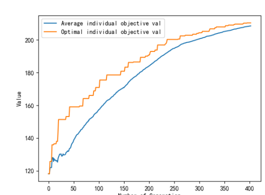
(b) $c_{ij}^{max} = 40$

Figure 5: Decision making c_{ij}^*/c_{ij}^{max} under different max capacity c_{ij}^{max} on Mar. 27^{th} $(\epsilon = 0.75, T = 1000)$

4.2.4 Convergence Results under Different (ϵ, T)



(a) $(\epsilon = 0.6, T = 500)$



(b) $(\epsilon = 0.6, T = 1000)$

Figure 6: Convergence Results of Differential Evolution Algorithm under Different (ϵ, T) on Apr. 16^{th} with $n = 10$

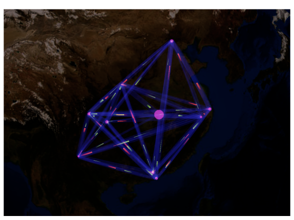
As is shown in Figure 6, we can discover that for the larger T with more relaxtion constraints, the optimal value could be obtained with less convergence steps.

5 Conclusion

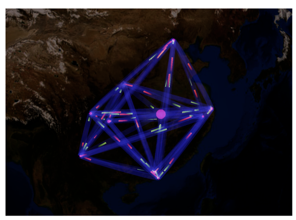
In this paper we focus on the assistant decision task for airline capacity control under pandemic like COVID-19. We try to model the tradeoff between the satisfaction of passengers' transportation demand and the avoidance of any further disease spreading. Our model reflects such tradeoff well and shows the difficulty of striking a balance. For getting opitmal solution we use both the IBM CPLEX solver and the Differential Evolution Algorithm, and we could get satisfactory result in the latter way. However, to answer the quesiton about anarchy penalty, that is, the negative influence of a unregulated aviation industry under global pandemic, we should further study how the capacity of aviation network could influence the infection severity in different cities with airports.

To better illustrate the relationship between decision making with different dates/cities/ϵ/T, we also apply echarts to our Dynamic Airline System in 3D, which can be visited online[4].

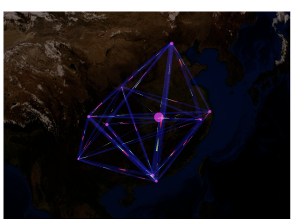
From figure 7(a) and 7(b) we can see with relaxation on parameter T, the width of the airlines apparently increases. From figure 7(b) and figure 7(d), we know only with relaxation on parameter T and lower severity at Wuhan, a small proportion of the airline capacity from Wuhan is allowed to be recovered. Finally, we also visualize the airline capacity in absolute way, that is, use C_{ij} rather than C_{ij}/C_{ij}^{max}, and we can see from figure 7(a) and 7(c) that the actual airline capacity between cities with low d_{ij} is not that large.



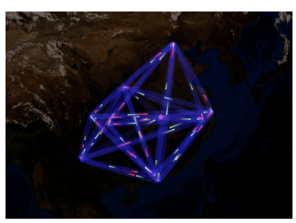
(a) Relative $\epsilon = .7, T = 1000$



(b) Relative $\epsilon = .9, T = 1000$



(c) Absolute $\epsilon = .7, T = 1000$



(d) Relative $\epsilon = .9, T = 1000$

Figure 7: 3D Airline System with Relative Capacity and Absolute Capacity. (a), (b): Feb. 16^{th}; (c), (d): Apr. 16^{th}

6 Future Work

6.1 For the Model: Solving Potential Problems

1. C_{ij} **to/between High Risk Cities** In our current model, we can describe the decision for the airline capicity from citis with high infection risk to those rather safer, but when deciding the airline capacity to or between high risk cities, it might require further consideration. In such conditions, our current model still keep some number of airlines, and it's up to the potential customers to decide whether to utilize such opportunity and go to high risk areas or not. It make sense considering the demand of specific customers, but when considering the public health, the conclusion could be different.
2. **Other Representation of** S_i We use $I_i - E_i$ to represent S_i, and it would help to show the severity within a given time period even if E_i is larger: although S_i would be negative, our model can accept such input about severity and it would hold the meaning that the condition is getting better. However, under traditional viewpoint, we could also define S_i as the number of active cases at a given time point, since such cases could still be infectious and should be taken into consideration.

6.2 For the Data: Adopting Further Promotion

To some extent, our experimental results shares the same trend of whether to maintain current operational level or to cut flights between two targeted city pairs. Beyond "Five One" policy, CAAC also introduces "Circuit Breaker" and "Reward" mechanism for airlines based on passenger nucleic acid test results upon arrival in order to contain the number of imported cases of COVID-19. As an incentive, carriers will be allowed to increase the number of international flights to two per week on one route if the number of passengers who have a positive nucleic acid test on their flights stands at zero for three consecutive weeks. Since Dec.16^{th}, the "circuit breaker" mechanism has been upgraded that the airline must suspend the operation of the route for two weeks if the number of passengers who test positive for the coronavirus reaches five. Note that these dynamic updated policies will also require our network optimization model to show more robust and resonable results in combination with more demanding data. With this regard, we will further analyze how to promote our constructed model in compliance with the newly operational policy.

7 Contribution

In short, all small pieces of this project are constructed by us together.

- Yu Fangchen: mathmetical modeling, plotting figures and writing Section 3 Proposed Model (survey of datasets $\{d_{ij}\}$)

[4] `demo.network-economics.whiskychoy.com`

- Cai Weilin: mathmetical modeling, writing code, writing Section 1 Introduction and Section 6 Future Work (survey of datasets $\{c_{ij}^{max}\}$)
- Li Chi: literature research, results analysis and writing Section 4 Performance Experiments (survey of datasets $\{c_{ij}^{max}\}$)
- Chen Weibin: literature research and writing Section 2 Related Work (survey of datasets $\{S_i\}$)

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Appendix: Datasets of $\{c_{ij}^{max}\}$, $\{d_{ij}\}$ and $\{S_i(t)\}$

$\{c_{ij}^{max}\}$	City $j = 1 \sim 10$									
City $i = 1 \sim 10$	1	2	3	4	5	6	7	8	9	10
1		9	30	32	33	18	21	23	28	13
2	9		9	18	12	10	15	18		10
3	30	10		27	1	21	20	23	27	11
4	33	18	27		26	19	5	1	13	10
5	34	12	2	26		15	16	24	24	8
6	18	9	20	19	14		14	10	9	12
7	21	14	21	5	16	13		5	14	4
8	23	19	23	2	24	10	5		18	9
9	28	1	27	13	25	9	14	18		6
10	14	9	11	10	8	12	4	9	6	

$\{d_{ij}\}$	City $j = 1 \sim 10$									
City $i = 1 \sim 10$	1	2	3	4	5	6	7	8	9	10
1		0.358	0.290	0.258	0.305	0.209	0.222	0.289	0.251	0.255
2	0.302		0.247	0.215	0.262	0.166	0.179	0.246	0.208	0.212
3	0.296	0.309		0.209	0.256	0.160	0.173	0.241	0.202	0.206
4	0.265	0.278	0.210		0.225	0.129	0.142	0.210	0.171	0.175
5	0.260	0.273	0.204	0.173		0.124	0.137	0.204	0.165	0.169
6	0.252	0.265	0.197	0.166	0.212		0.129	0.197	0.158	0.162
7	0.250	0.263	0.195	0.164	0.210	0.114		0.195	0.156	0.160
8	0.246	0.259	0.191	0.159	0.206	0.110	0.123		0.152	0.156
9	0.237	0.251	0.182	0.151	0.198	0.101	0.115	0.182		0.147
10	0.214	0.228	0.159	0.128	0.175	0.078	0.092	0.159	0.120	

$\{S_i(t)\}$	Date t: Feb. $16^{\mathrm{th}} \sim$ Apr. 16^{th}									
City $i = 1 \sim 10$	1	2	3	4	5	6	7	8	9	10
Feb 16	263	190	287	87	80	2	0	339	12	37692
Feb 22	206	105	271	84	196	23	18	239	95	38450
Feb 29	129	47	264	80	278	36	65	132	141	34437
Mar 06	115	33	237	62	298	30	95	44	109	29895
Mar 16	79	30	189	43	232	0	82	0	74	19935
Mar 22	114	72	154	23	158	0	76	1	48	16861
Mar 28	157	153	144	4	100	0	46	3	30	9973
Apr 03	139	175	140	12	70	0	30	3	5	4588
Apr 10	107	113	129	19	49	0	19	3	2	3145
Apr 16	76	132	50	10	30	0	3	3	0	2725