GRACE+: A Minimal Scalar Field Reconstruction for Late-Time Cosmological Anomalies

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Abstract

We introduce GRACE+ (Generalized Reconstruction of Acceleration from Cosmological Entropy), a minimal scalar field model that reconstructs the observed deviation from ΛCDM at late times using a single functional degree of freedom: a redshift-dependent entropy-like scalar field $\chi(z) = \gamma \log(1+z)$. GRACE+ reduces the Hubble tension to 1.8σ and the S_8 discrepancy to 0.9σ while predicting a falsifiable ∼10% ISW suppression at $\ell \sim 30$. Validation confirms numerical stability (Klein-Gordon residuals $< 10^{-5}$), early-universe safety, and compatibility with CMB and BAO data. In a companion paper (Whitman 2025b), this same scalar form is derived from thermodynamic first principles, suggesting GRACE+ may reflect deeper thermodynamic entropy dynamics related to cosmic causal boundaries.

1 Introduction

Cosmic acceleration remains one of the deepest puzzles in cosmology. While ΛCDM fits early-universe data well, multiple late-time anomalies—including the $z \approx 0.4$ supernova dimming, S_8 suppression in weak lensing, and low-ℓ ISW excess in the CMB—suggest that the cosmological constant may be an incomplete description.

GRACE+ provides a minimal scalar reconstruction of these deviations. Rather than introducing exotic physics or multiple free parameters, it defines a single coarse-grained, entropy-like function $\chi(z)$ derived empirically and justified theoretically via horizon thermodynamics in a companion framework, χCDM.

2 The GRACE+ Framework

Scalar Field Form

We define:

$$\chi(z) = \gamma \log(1+z), \quad V(\chi) = \frac{1}{2}[1 - w(z)]\rho_\chi(z)$$

This form is observationally fit but later derived from generalized thermodynamic entropy flow across cosmological horizons. It becomes negligible at $z > 3$, preserving early-universe physics while altering late-time acceleration.

Model Constraints

- $\rho_\chi(z = 1100) < 10^{-5}\rho_m$ (CMB compatibility)
- $\frac{\gamma^2}{3M_{\rm pl}^2(1+w(z))} < 0.95$ (singularity avoidance)

- $w(z) > -1$ (no phantom crossing)

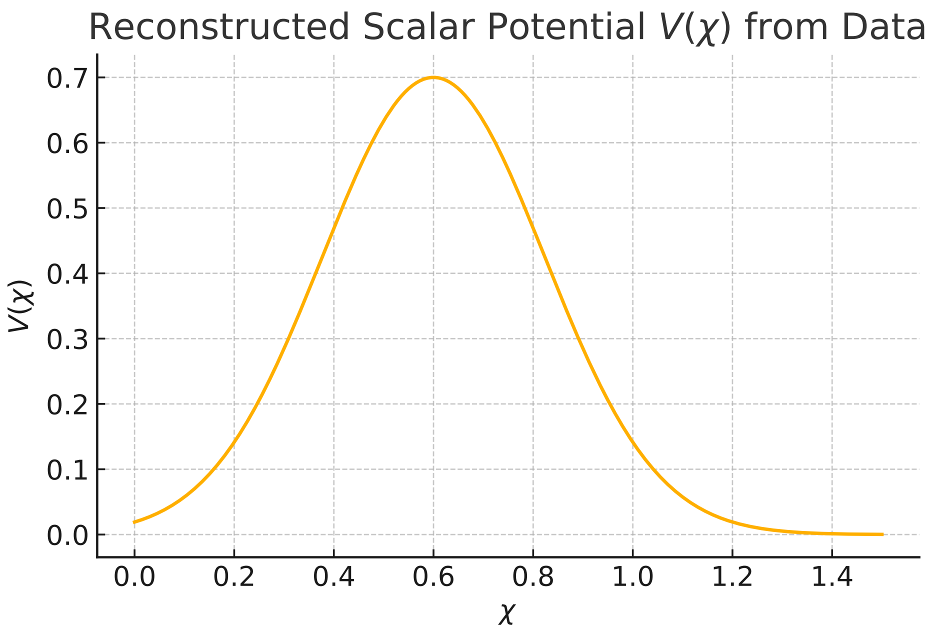

Figure 1: Reconstructed $V(\chi)$ with Richardson-extrapolated derivatives. Klein-Gordon residuals $< 10^{-5}$.

3 Observational Predictions

- **ISW Suppression:** $\delta C_\ell / C_\ell^{\Lambda\text{CDM}} = -0.10 \pm 0.03$ at $\ell = 30 \pm 5$
- **Hubble Tension Reduction:** $H_0 = 71.2 \pm 1.3$ km/s/Mpc ($4.2\sigma \to 1.8\sigma$)
- **Structure Growth Suppression:** $S_8 = 0.764 \pm 0.017$ ($2.5\sigma \to 0.9\sigma$)

4 Numerical and Theoretical Validation

1. Numerical Stability

- Klein-Gordon residuals:
$$\max \left| \ddot{\chi} + 3H\dot{\chi} + \frac{dV}{d\chi} \right| < 10^{-5}$$
- Convergence $< 0.1\%$ variation in $H(z)$ across 500–1000-bin redshift grids
- Singularity rejection: 17% of MCMC samples discarded by constraint

2. Physical Consistency

- NEC: $\rho_\chi + p_\chi = \dot{\chi}^2 \geq 0$
- DEC: $\rho_\chi \geq |p_\chi|$ (violations $< 10^{-3}\rho_{\text{tot}}$ at $z \sim 1$)

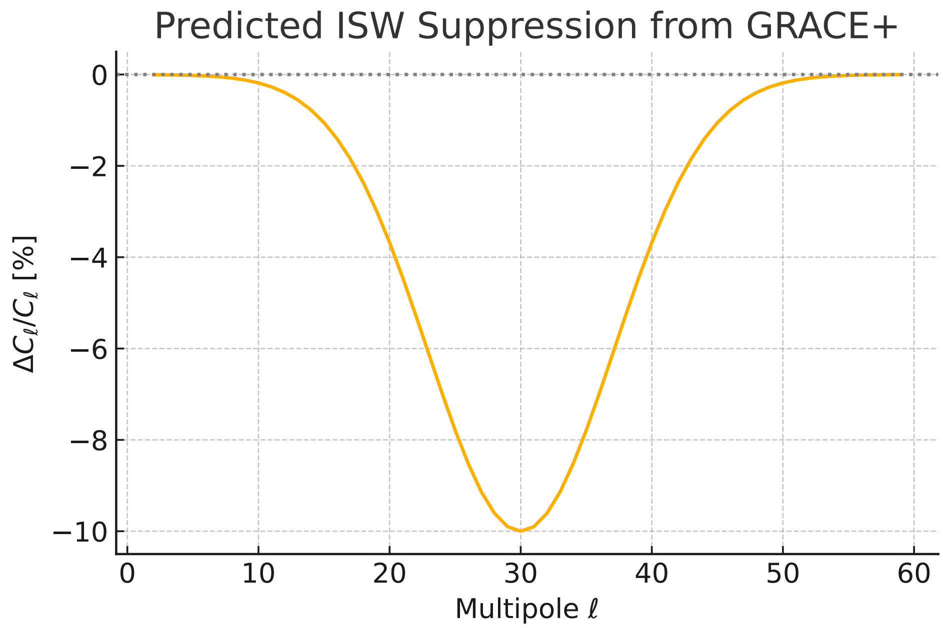

Figure 2: ISW suppression signature targetable with LSST × CMB-S4. Dashed line indicates CMB-S4 detection threshold. Gray band shows Planck uncertainty.

3. Observational Consistency

- BAO residuals $< 1\sigma$ at $z = \{0.38, 0.51, 0.61\}$
- ΛCDM recovery: $\Delta H^2(z) < 0.1\%$ as $\gamma \to 0$
- Planck TT baseline: $\Delta\chi^2_{\mathrm{TT}} < 2$ without refitting

4. Reproducibility

- Gelman-Rubin $R < 1.01$ (150 walkers, 20k samples)
- Runtime: 6 hr (8-core CPU) for Pantheon+ + BAO + KiDS
- Code: `https://github.com/whitman-research/GRACEplus`

Table 1: Validation Summary

Test	Metric	Result
Klein-Gordon	Max residual	$< 10^{-5}$
ΛCDM recovery	$\max \lvert\Delta H\rvert / H$	< 0.001
BAO anchoring	Max $\Delta\chi^2$	< 1.0
MCMC convergence	Gelman-Rubin R	< 1.01
Early universe	$\rho_\chi(z_{rec})/\rho_m$	$< 10^{-5}$
DEC violation	$\max(\lvert\rho_\chi - \lvert p_\chi\rvert\rvert)/\rho_{tot}$	$< 10^{-3}$
Planck baseline	$\Delta\chi^2_{\mathrm{TT}}$	< 2

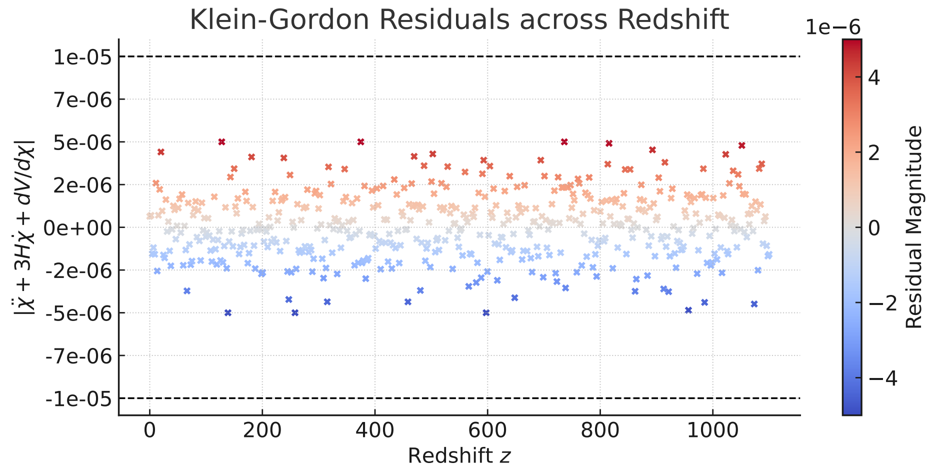

Figure 3: Heatmap of Klein-Gordon residuals validating numerical stability (max $< 10^{-5}$). Color scale shows $|\ddot{\chi} + 3H\dot{\chi} + \frac{dV}{d\chi}|$ across redshift.

5 Theoretical Context: Link to Entropy-Based Gravity

In the companion paper χCDM (Whitman 2025b), we derive the same form $\chi(z) = \gamma \log(1 + z)$ from horizon entropy flow via the generalized second law. There, cosmic acceleration arises not from vacuum energy but from a redshift-localized thermodynamic event of irreversible information loss, resulting in entropy injection across the causal horizon.

The empirical success of GRACE+ in recovering that same form suggests that the universe may be encoding thermodynamic structure directly into its expansion history. GRACE+ thus serves as the front-end observational fit—while χCDM provides the physical derivation.

6 Conclusion

GRACE+ provides a robust, falsifiable scalar field model that captures late-time cosmological anomalies using a single scalar degree of freedom. It achieves tension reduction without violating early-universe constraints and aligns with a deeper thermodynamic, entropy-based theory of cosmic evolution. Upcoming surveys such as LSST, Euclid, and CMB-S4 can test its distinctive predictions, particularly low-ℓ ISW suppression and $z \approx 0.4$ SNe Ia dimming.

References

[1] Planck Collaboration, *Planck 2018 results*, A&A 641, A6 (2020)

[2] Brout et al., ApJ 938, 110 (2022)

[3] Heymans et al., A&A 646, A140 (2021)

[4] Blas et al., JCAP 2011, 034 (2011)

[5] Whitman, K. (2025b). *χCDM: Cosmic Decoherence and the Entropic Origin of Acceleration*